

Article

Marine Predators Algorithm for Sizing Optimization of Truss Structures with Continuous Variables

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Abstract: In this study, the newly developed Marine Predators Algorithm (MPA) is formulated to minimize the weight of truss structures. MPA is a swarm-based metaheuristic algorithm inspired by the efficient foraging strategies of marine predators in oceanic environments. In order to assess the robustness of the proposed method, three normal-sized structural benchmarks (10-bar, 60-bar, and 120-bar spatial dome) and three large-scale structures (272-bar, 942-bar, and 4666-bar truss tower) were selected from the literature. Results point to the inherent strength of MPA against all state-of-the-art metaheuristic optimizers implemented so far. Moreover, for the first time in the field, a quantitative evaluation and an answer to the age-old question of the proper convergence behavior (exploration vs. exploitation balance) in the context of structural optimization is conducted. Therefore, a novel *dimension-wise diversity index* is adopted as a methodology to investigate each of the two schemes. It was concluded that the balance that produced the best results was about 90% exploitation and 10% exploration (on average for the entire computational process).

Keywords: marine predators algorithm; truss optimization; dimension-wise diversity index; large-scale structures



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1. Introduction

The weight minimization of truss structures, as an all-encompassing optimization task which indirectly incorporates manufacturing, transportation, and lifecycle costs, has been an active field of research in the last two decades. Owing to the economic implications that this category of engineering optimization has on society, several so-called gradient-based optimization methodologies, which function by using the derivative information of the search space, were employed early on in the field. With the recent introduction of metaheuristic search methodologies, however, which function as global stochastic optimization methods, engineers can now more fully explore the truss design space more efficiently and propose a dichotomy of optimized designs to be scrutinized for feasibility later. Metaheuristics generally divide the optimization process into two phases: exploration and exploitation. This typically involves the population of solution candidates to heuristically navigate themselves, either searching in their local vicinity or with larger strides, efficiently. Research trend in the field is currently focused on developing these heuristic paradigms to produce lighter-weight trusses with fewer structural evaluations (computational effort). The objective, therefore, is to propose algorithms which respond to these very pressing needs.

1.1. Related Work

There exists a rich body of literature involving the use of metaheuristic algorithms for truss optimization problems. The No Free Lunch (NFL) Theorem proposed by Wolpert et al. [1] explicitly states that no single metaheuristic is capable of solving all optimization problems with an equal level of effectiveness. This implies that every optimization problem needs

to be recognized as belonging to a distinct problem category with a set of well-suited optimization methodologies associated with it [2].

A number of MH algorithms have been used for truss optimization problems by various authors over the years. For instance, Lee and Geem implemented the Harmony Search (HS) algorithm for the weight minimization of truss structures with continuous variables [3]. Sonmez adopted the Artificial Bee Colony (ABC) optimizer for structural optimization with continuous and discrete variables [4,5]. Degertekin and Hayalioglu investigated the applicability of the Teaching Learning Based Optimizer (TLBO) [6]. Bekdas et al. recognized the feasibility of the Flower Pollination Algorithm (FPA) for its robust global and local search aspects in structural design [7]. Kaveh and Bakshoopri presented the Cuckoo Search (CS) algorithm in the context of truss sizing optimization [8]. Kaveh and Khayatazad employed the so-called Ray Optimizer (RO) algorithm for the weight minimization of truss structures [9]. A Cultural Algorithm (CA) was successfully implemented for the optimal design of truss structures with stress and deflection constraints by Jalili and Hosseinzadeh [10]. Similarly, Kaveh and Mahdavi showcased the robustness of the Colliding Bodies Optimizer (CBO) for truss design problems [11]. Recently published studies in the field include Kaveh and Bakshoopri [12] and Kooshkbaghi and Kaveh [13] with their implementation of the Water Evaporation Optimizer (WEO) and the Artificial Coronary Circulation Systems (ACCS) algorithms, respectively, for sizing optimization of truss structures. Ozbasaran and Yildirim evaluated the performance of the Crow Search Algorithm (CSA) for the weight minimization of truss structures [14]. Degertekin et al. [15] adopted the recently proposed Heat Transfer Search (HTS) algorithm for the sizing optimization of truss structures. Azizi et al. considered the utilization of the Material Generation Algorithm (MGA) for the optimum design of truss structures [16]. Furthermore, Bodalal (present author) [17] presented a critical analysis of the recently developed Political Optimizer (PO) algorithm for the weight minimization of both medium- and large-sized structural systems.

Hybridized/modified algorithms that attempted to compensate for the shortcomings of direct MH implementations naturally followed over the years. Recognizing this fact, a hybridized PSO, ACO, and HS algorithm (HPSACO) was presented by Kaveh et al. in the hopes of improving structural design results further [18]. Khatibinia and Yazdani proposed the Accelerated Multi-Gravitational Search Algorithm (AMGSA) to overcome the performance issues of the conventional Gravitational Search Algorithm (GSA) [19]. Kaveh et al. developed an Improved Ray Optimizer (IRO) algorithm to further enhance designs yielded from the previous direct RO implementation study [20]. A hybrid Imperialist Competitive Algorithm (ICA) and Harris Hawk Optimizer (HHO) was developed by Kaveh et al. (ICHHO) to improve the structural results reported in an earlier ICA truss optimization paper [21]. Jafari et al. hybridized the standard PSO with CA to propose the Particle Swarm Optimizer Cultural (PSOC) algorithm in the hopes of yielding a more harmonized balance between exploration and exploitation [22].

It is therefore apparent from the aforementioned studies that the implementation of newer metaheuristics is directly associated with improved optimization methodologies developed afterwards since the strengths and weaknesses become clear as a consequence.

1.2. Motivation and Contribution

Recently, a newly developed swarm-based metaheuristic optimizer, called the Marine Predators Algorithm (MPA), was proposed by Faramarzi et al. in 2020 [23]. MPA is inspired by the efficient foraging strategies of marine predators in oceanic environments. Search agents are modeled as marine prey (food), while the best-performing solution in the population acts as the marine predator (hunter). Faramarzi et al. [23] evaluated the efficiency of MPA with 29 function benchmarks, three classical engineering problems, and two real-world problems. Results revealed the superiority of MPA over all contemporary algorithms in comparison. As a result, MPA was seen by many researchers as a promising solution technique. This led to MPA's utilization for many challenging engineering

optimization problems. Notable examples from literature include the parameter extraction of single/double diode PV modules [24], image feature selection [25], fan control scheduling for ventilation purposes [23], CNC lathe machine optimization [26], COVID-19 detection/image classification [27], and antenna design [28].

The primary purpose of this study (therefore) is to extend the application of the recently developed Marine Predators Algorithm to the weight minimization of truss structures. Six challenging truss benchmarks (three normal-sized and three large-scale) are consequently optimized to test the efficiency of MPA. The proposed algorithm was selected because it efficiently incorporates both Lévy and Brownian motion models into its solution updating methodology. Several studies related to structural optimization (and other similarly challenging problems) have repeatedly underscored the effectiveness of the Lévy flight/walk motion model to escape local optima (this is especially the case for constrained, multi-model optimization problems). Notable articles in the field include the implementation of the Cuckoo Search (CS) algorithm by Kaveh and Bakshoopri [8] for truss optimization. Inherent to the algorithm was the Lévy motion model, and conclusions were made favoring the motion pattern for truss optimization. A similar conclusion was reached with implementing the Dragonfly Algorithm (DA) by Jawad et al. for truss weight minimization [29]. Lévy flight techniques (intrinsic to DA operations) proved crucial to the algorithm's excellent performance, allowing it to reach global truss solutions with the least computational effort. The results were furthermore corroborated with a similar metaheuristic truss optimization study conducted by Etaati et al. [30]. Along those lines, other studies related to the field have also reported the advantage that Lévy-based MH algorithms possess over other techniques for truss optimization problems (see Refs. [31–35]). Based on the conclusions made by a sample of studies related to the field, MPA (with its basic Lévy motion position updating scheme) is expected to perform exceptionally well when applied to the challenging truss optimization problem. The testing of MPA in this study is therefore justified from a literature review perspective.

Furthermore, in recognition of the need for researchers to have better A.I. optimization methodologies at their disposal, the author noticed a research gap where no study, as of the time of this writing, has ever attempted to quantitatively determine the correct value of algorithmic “exploration” and “exploitation” for truss optimization problems. Given the ever-increasing involvement of AI-based methods in structural engineering and the weaknesses of conventional analytical techniques currently at our disposal, the importance of having a basic numerical estimation for the combination of these two schemes will immensely aid future researchers in the field whenever they develop new algorithms. Truss optimization problems are well-known for being highly non-linear, non-convex, and multimodal design problems with respect to their design variables [36]. Problem complexity is further compounded by the hundreds, if not thousands, of design constraints imposed by dint of the various construction/building codes typically required by government organizations. The sizing optimization of truss structures (by definition) involves selecting the appropriate cross-sectional areas of truss members to minimize structural weight, all the while conforming to the predefined design constraints in the form of stress and deflection limitations. Optimized cross-sectional areas are consequently required to be selected from a practical range usually preordained by commercial section availability. Fabrication constraints, as they are commonly called, are thus an added computational challenge to the algorithm and take the form of decision variable limitations. Furthermore, the truss optimization problem is notorious for its high computational cost. Objective function evaluations necessarily involve the simultaneous solution of thousands of force-displacement equations using matrix operations. This differentiates structural optimization problems from other simpler engineering design tasks where the objective function estimation process does not require lengthy computation (see Refs. [37–39]). In order to reduce the amount of time required to obtain optimized solutions, MH algorithms that efficiently navigate the challenging design search space with the least amount of structural analysis are therefore highly sought after. The truss sizing problem, as a result, continues to be an

active research problem solved by many researchers in the field [40–42]. From the above discussion, it becomes clear that the sizing optimization of truss structures provides a particularly challenging optimization problem in which the MH exploration–exploitation tradeoff trend can be thoroughly investigated. That is in addition to the fact that most studies in the field usually only consider final results such as optimized weights, standard deviations, and convergence speeds. However, no discussions or calculations are made to justify the reported results in light of the two fundamental metaheuristic search concepts mentioned earlier: exploration and exploitation. To fill this gap, the computational behavior of MPA, used as a base algorithm in this work (and justified in the previous paragraph), is analyzed using a novel dimension-wise diversity index (previously developed by Morales-Castañeda et al. [43]). Metaheuristic search aspects can subsequently be easily adjusted to these newly found values and theoretically achieve comparable optimization performance.

The contributions of this study can be summarized as follows:

- (1) A novel application of MPA for structural optimization problems is presented.
- (2) Six challenging truss benchmarks (including three real-sized structures) are optimized using the proposed technique.
- (3) For the first time in the field, a numerical quantification of the proper balance between metaheuristic exploration vs. exploitation for truss optimization problems is presented.
- (4) Finally, a critical assessment of MPA is conducted in light of the detailed findings reported in the study (exploration-exploitation balance, optimization trend, and statistical results).

The rest of this paper is organized as follows: Section 2 formulates the truss sizing problem. Section 3 explains the working principle of the MPA algorithm. Section 4 describes the software implementation of MPA for truss sizing optimization. Section 5 presents the methodology for quantifying the exploration vs. exploitation balance. Section 6 compares MPA results with those of other optimizers in literature. Section 7 summarizes MPA performance in relation to the truss optimization results. Finally, some concluding remarks/recommendations are made in Section 8.

2. Problem Formulation

The sizing optimization of truss structures can be regarded as the problem of selecting the appropriate cross-sectional areas of structural members (*with the goal of minimizing structural weight*) while conforming to the pre-imposed design constraints in the form of stress and deflection limitations. Mathematically, this can be expressed as:

Find optimal values:

$$\vec{A} = \{A_1, A_2, A_3, \dots, A_d\} \quad (1)$$

To minimize:

$$W(\vec{A}) = \sum_{i=1}^{NE} \rho_i A_i L_i, \quad i = 1, \dots, NE \quad (2)$$

Subject to constraints:

$$\begin{cases} \sigma_L \leq \sigma_i \leq \sigma_U \\ \delta_L \leq \delta_j \leq \delta_U \\ 0 \leq \sigma_k^b \leq \sigma_U^b \end{cases} \quad (3)$$

where $W(\vec{A})$ denotes the overall truss weight, NE denotes the number of structural elements, d is the number of truss sizing variables, ρ_i represents the mass density of the structural material, L_i denotes the truss member length, A_i is the cross-sectional area of the i th member, and σ_i , σ_k^b , and δ_j represent the normal stress, buckling stress, and nodal displacement for the i th member, k th compressive element, and j th node, respectively.

Multiple constraint-handling methodologies have recently been proposed to convert constrained problems into unconstrained ones. Readers are encouraged to read Coello Coello [44] for a more in-depth discussion on this matter. Nevertheless, in structural design, the objective function formulation for the truss optimization problem saw few changes over the years. Studying the research trend shows the adoption of three main types of objective function formulations: (1) Fitness functions with static penalty factors [45]. (2) Fitness function with dynamic penalty factors [46]. (3) Fitness function with relaxed penalty factors [47]. Of the three, formulations 2 and 3 are the most adopted, and their effects have been shown to differ only slightly (among themselves) on overall algorithmic performance [48]. Consequently, a quantitative comparison between different MH algorithms using the aforementioned formulations is justified. In this work, in order to efficiently handle the design constraints outlined in Equation (3), formulation 3 is adopted, and the total number of evaluated constraint violations m for a given truss design are first normalized and then subsequently merged into a single variable V , which represents the total penalized value of the structure. This is mathematically shown as follows [17]:

$$V = \sum_{i=1}^m v_i = \sum_{i=1}^{NE} \left(\left| \frac{\sigma_i(x)}{\sigma_{L,U}} \right| - 1 \right) + \sum_{j=1}^{NN} \left(\left| \frac{\delta_j(x)}{\delta_{L,U}} \right| - 1 \right) + \sum_{k=1}^{NC} \left(\left| \frac{\sigma_k^b}{\sigma_U^b} \right| - 1 \right) \quad (4)$$

where NN and NC are the *number of nodes* and the *number of compressive members*, respectively. Constraints violations with values greater than zero are accounted for, while those less than that are neglected.

It therefore follows that the ultimate form of the penalized fitness function F_p minimized in this study is as follows [17]:

$$F_p(\vec{A}) = W(\vec{A}) * \alpha * (1 + V)^\beta \quad (5)$$

where the variables α and β in Equation (5) are the penalty scaling factor and penalty exponent, respectively, the value of which is kept as one (part of the relaxed objective function scheme) [17]. This allows the algorithm to freely search the computational domain, unimpeded. Global optimum truss weights, when obtained, would either have zero constraint violations or be so small that they could be regarded as negligible.

3. Marine Predators Algorithm (MPA)

3.1. Inspiration

The Marine Predators Algorithm (MPA) is a recently developed swarm-based meta-heuristic optimizer that mathematically models the efficient foraging strategies of Marine Predators (MP) in oceanic environments. The term “marine predator” typically describes aquatic creatures, such as sharks, whales, dolphins, and swordfish, i.e., fish that eat other fish. While searching for prey (food), MPs generally adopt one of two motion models: (1) Lévy motion or (2) Brownian motion. MPs constantly switch between the two motion patterns based on “encounter policies.” Simply put, if an MP finds itself in an area of sea with a low/patchy concentration of prey, it starts moving around in a Lévy fashion. Conversely, if abundant prey is present (high concentration), Brownian motion is preferred. Therefore, MPs successfully forage by constantly alternating between Lévy and Brownian motion based on prey concentration in their local area.

Numerous mathematical models have been developed to predict the exact moment at which MPs change motion patterns. The best technique developed so far is by relating it to something called the “velocity ratio” (represented as $\gamma = \frac{\vec{v}_{prey}}{\vec{v}_{predator}}$). It is reasoned that depending on the speed of the prey versus that of an MP, a switch in motion type will occur to minimize both the time and effort involved in encountering it. From the above discussion, three different MP hunting tactics (in relation to the “velocity ratio”) can be summarized as follows:

- **High Velocity Ratio Scenario ($\gamma \geq 10$):**

This scenario occurs when the intended prey is much faster than the predator ($\overrightarrow{v_{prey}} \gg \overrightarrow{v_{predator}}$). In this case, MPs hunt by not moving at all. This is logical since the probability that the prey will pass your field of view is high.

- **Unit Velocity Ratio Scenario ($\gamma = 1$):**

This scenario occurs when the velocities of prey and predator are equal ($\overrightarrow{v_{prey}} = \overrightarrow{v_{predator}}$). In this case, marine predators adopt the opposite foraging tactic of their prey, i.e., if the intended prey is moving in a Brownian fashion (randomly), the predators move with Levy motion.

- **Low Velocity Ratio Scenario ($\gamma \leq 0.1$):**

This scenario occurs when prey move slower than predators ($\overrightarrow{v_{prey}} \ll \overrightarrow{v_{predator}}$). In this scenario, marine predators must actively search for their food, and the much slower prey are often concentrated in swarms. This dictates the MP adopt the Levy flight/walk foraging mechanism, i.e., deeply search nearby areas with small steps and explore other areas with “relocations” until the prey swarm is found. Figure 1 graphically summarizes the three foraging scenarios described earlier for greater clarity.

One last observation related to MP behavior is their sudden switch of motion patterns (Lévy flight/walk or Brownian motion) when in the immediate proximity of “natural environmental barriers” such as eddy currents or Fish Aggregating Devices (FADs). Predators were found to change tactics depending on the type of obstruction.

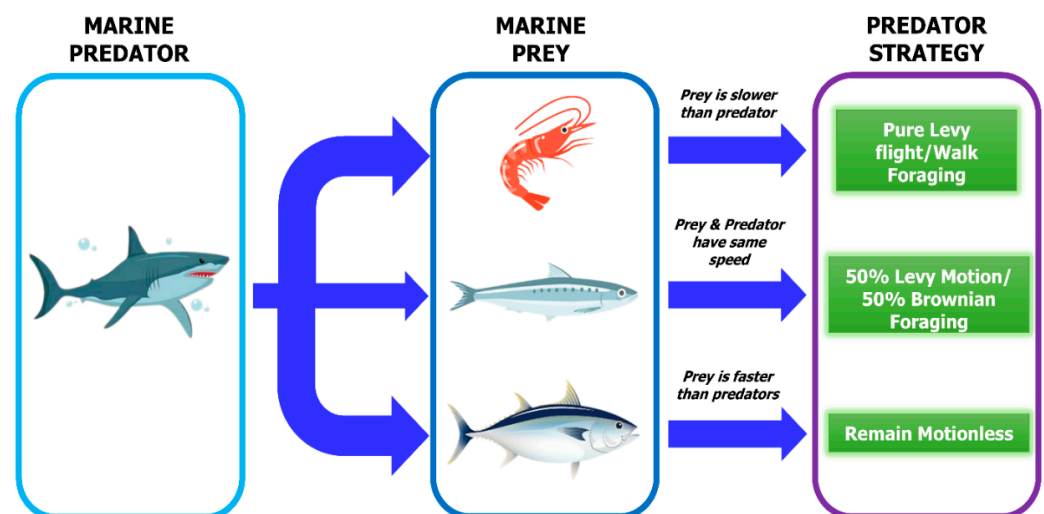


Figure 1. Typical foraging tactics of marine predators in oceanic environments.

3.2. Mathematical Formulation

As mentioned earlier, one of the primary purposes of this study is to extend the application of the recently developed Marine Predators Algorithm (MPA) to the weight minimization of truss structures. As such, MPA operation in relation to the inspiration section described earlier is presented here. Similar to other MH algorithms, MPA divides the optimization process into exploration and exploitation. However, unlike other algorithms, it transitions from pure exploration to pure exploitation with an intermediate stage comprised of 50% exploration and 50% exploitation. The following is an overview of how MPA operates:

The algorithm’s optimization sequence begins by randomly generating the initial population of search agents. Fitness functions are subsequently estimated, and the best-performing solution serves as the marine predator, while the remaining solutions act as prey. Now that the search space has multiple prey and one MP (the best solution), the

hunt for the global optimum begins. The three encounter policies (based on velocity ratios) described earlier are modelled in MPA by dividing the optimization process into three equal phases. In the first third, solution candidates (prey) update their positions with respect to the MP (best solution) in a Brownian fashion. In the second phase, solution candidates are divided in half, and each half has its position vectors updated either in a Lévy or Brownian manner in relation to the top solution. Finally, in the last third of iterations, the MP has its position vector multiplied by the Levy operator, and the prey update their solutions based on it. While this all happens, the best-performing solution is constantly updated for every iterative cycle. Overall, MPA is a relatively simple metaheuristic, with only a few performance parameters to be considered.

The following subsections represent a clear and concise mathematical derivation of all MPA mathematical procedures discussed earlier.

3.2.1. Initialization

The MPA optimization cycle begins by uniformly distributing the population of search agents over the computational domain using Equation (6).

$$\vec{X}_i = \vec{X}_L + \overrightarrow{rand} \otimes (\vec{X}_U - \vec{X}_L), \quad i = 1, \dots, PopSize \quad (6)$$

where \vec{X}_L and \vec{X}_U are vectors representing the lower and upper design variable bounds, respectively, \overrightarrow{rand} is a vector of random numbers uniformly generated between [0, 1], and \otimes is an operator indicative of entry-wise multiplication. The best-performing solution in the population is denoted by \overrightarrow{Elite} , and all solution candidates update their positions with respect to it.

3.2.2. Optimization Scenarios

The MPA algorithm models the three “encounter policies” discussed earlier by dividing the entire optimization process into three equal parts. In all phases, candidate solutions update their position vectors with reference to the “best solution.” Therefore, the \overrightarrow{Elite} vector is key to MPA. The three phases, along with their details, are discussed below:

- **Phase I** ($iter < \frac{1}{3}MaxIt$)

The first third of iterations models the high velocity ratio scenario ($\gamma \geq 10$) where prey move faster than predators. In the algorithm, this is mathematically modelled as follows:

$$\begin{aligned} \overrightarrow{stepsize} &= \vec{R}_B \otimes \left(\overrightarrow{Elite}(t) - \vec{R}_B \otimes \vec{X}_i(t) \right), \quad i = 1, \dots, PopSize \\ \vec{X}_i(t+1) &= \vec{X}_i(t) + P \cdot \overrightarrow{rand} \otimes \left(\overrightarrow{stepsize} \right) \end{aligned} \quad (7)$$

where $\vec{X}_i(t+1)$ is the updated solution vector, P is a constant scaling factor, which is considered here an algorithmic parameter, and \vec{R}_B is a vector of random numbers generated according to Brownian motion. The vector \vec{R}_B is multiplied by $\vec{X}_i(t)$ (marine prey), since fast prey usually display random motion. Readers interested in the probabilistic governing equation for \vec{R}_B are referred to reference [23] for more information.

- **Phase II** ($\frac{1}{3}MaxIt < it < \frac{2}{3}MaxIt$)

Serving as an intermediate stage from mere exploration to mere exploitation, phase II of MPA maps the behavior of marine predators according to the unit velocity ratio scenario ($\gamma = 1$). Observations have shown that both predators and prey are searching for their food,

and therefore, they adopt the opposite strategies of each other. This is modelled with half the prey population updating their positions according to Lévy strategy and the other half according to Brownian motion, i.e., 50% exploration and 50% exploitation. Mathematically, this is written as follows:

For the First Half of the Population (Exploitation):

$$\begin{aligned}\overrightarrow{stepsize} &= \overrightarrow{R_L} \otimes \left(\overrightarrow{Elite(t)} - \overrightarrow{R_L} \otimes \overrightarrow{X_i(t)} \right), \quad i = 1, \dots, PopSize/2 \\ \overrightarrow{X_i(t+1)} &= \overrightarrow{X_i(t)} + P.rand \otimes \left(\overrightarrow{stepsize} \right)\end{aligned}\quad (8)$$

where $\overrightarrow{R_L}$ is a vector of random numbers based on the Lévy distribution [23].

For the Second Half of the Population (Exploration):

$$\begin{aligned}\overrightarrow{stepsize} &= \overrightarrow{R_B} \otimes \left(\overrightarrow{R_B} \otimes \overrightarrow{Elite(t)} - \overrightarrow{X_i(t)} \right), \quad i = PopSize/2, \dots, PopSize \\ \overrightarrow{X_i(t+1)} &= \overrightarrow{X_i(t)} + P.CF \otimes \left(\overrightarrow{stepsize} \right)\end{aligned}\quad (9)$$

$$CF = \left(1 - \frac{t}{MaxIt} \right)^{\left(\frac{2t}{MaxIt} \right)} \quad (10)$$

where CF is a control parameter that dictates the step size length of predator motion.

- **Phase III ($iter > \frac{2}{3}MaxIt$)**

The final third of MPA iterations simulates the low velocity ratio scenario ($\gamma \leq 0.1$) and serves as a pure exploitation phase. This can be mathematically formulated as follows:

$$\begin{aligned}\overrightarrow{stepsize} &= \overrightarrow{R_L} \otimes \left(\overrightarrow{R_L} \otimes \overrightarrow{Elite(t)} - \overrightarrow{X_i(t)} \right), \quad i = 1, \dots, PopSize \\ \overrightarrow{X_i(t+1)} &= \overrightarrow{Elite(t)} + P.CF \otimes \left(\overrightarrow{stepsize} \right)\end{aligned}\quad (11)$$

Search agents update their positions around “top predators” in this phase, and therefore, the refinement of solutions is conducted after exploring the search space for 2/3 of the iterations.

Finally, to account for the unusual activity of MPs when confronted with FADs, the population of solution candidates is allowed to update their position vectors according to a predefined probability factor assigned by the user. One of two equations is executed, and they are mathematically presented as follows:

$$\overrightarrow{X_i(t+1)} = \begin{cases} \overrightarrow{X_i(t)} + CF \left[\overrightarrow{X_L} + rand \otimes (\overrightarrow{X_U} - \overrightarrow{X_L}) \right] \otimes \overrightarrow{U}, & r \leq FADs \\ \overrightarrow{X_i(t)} + CF[FADs(1-r) + r] \left(\overrightarrow{X_{rand1}(t)} - \overrightarrow{X_{rand2}(t)} \right), & r > FADs \end{cases} \quad (12)$$

where FADs is the user-defined probability factor, \overrightarrow{U} is a binary vector, r is a random number between [0, 1], and rand1 and rand2 are subscripts indicating random prey 1 and randomly selected prey 2. This additional step occurs for every computational cycle and can be regarded as a measure to prevent the population from being snarled into local optima. The pseudocode of the proposed MPA algorithm is depicted in Figure 2.

```

Initialize performance parameters: PopSize, P, and FADs;
Initialize search agents (Prey)  $\vec{X}_i$  ( $i = 1, 2, 3, \dots, \text{PopSize}$ );
while ( $it \leq \text{MaxIt}$ ) do
    Estimate fitness values and update top predator (Elite Prey)
    % Update Position Vectors (Optimization Scenarios) %%%%%%%%%%
    for every search agent ( $i \rightarrow \text{PopSize}$ ) do
        if ( $it < \text{MaxIt}/3$ ) do
            Update prey  $\vec{X}_i$  using Eq. (7);
        else if ( $\text{MaxIt}/3 < it < 2\text{MaxIt}/3$ ) do
            % For the First Half of the Population %
            Update prey  $\vec{X}_i$  using Eq. (8);
            % For the Second Half of the Population %
            Update prey  $\vec{X}_i$  using Eq. (9);
        else if ( $it > 2\text{MaxIt}/3$ ) do
            Update prey  $\vec{X}_i$  using Eq. (11);
        end if
    end for
    Calculate new fitness values and update top predator (with marine memory saving)
    % Eddy Formation and FADs effect %%%%%%%%%%
    for every search agent ( $i \rightarrow \text{PopSize}$ ) do
        Update prey  $\vec{X}_i$  with either of the two formulae in Eq. (12) based on random number  $r$ ;
    end for
end while
Return bestFitness,  $\vec{X}_{\text{Elite}}$ ;

```

Figure 2. Pseudocode of the MPA algorithm.

4. Software Implementation for the MPA Truss Optimization Problem

This section presents the step-by-step software implementation sequence for solving the truss sizing optimization problem. The steps carried out by the algorithm are outlined below:

- Step (1) Initialize MPA control parameters: Set values for Population Size (PopSize), Scaling Factor (P), Fish Aggregating Devices (FADs), and Maximum Number of Iterations (MaxIt). Iteration counter value initialized to zero ($it = 0$).
- Step (2) Load truss benchmark data: Information such as nodal coordinates, member connectivity, material properties, loading conditions, and design variable groupings for a particular structural problem are initialized in this step.
- Step (3) Generate randomized truss designs: Using Equation (6), a pool of random truss solutions is generated. Solution vectors represent truss cross-sectional area values. Each design variable value corresponds to the area of a particular set of truss members.
- Step (4) Evaluate the objective function values: The objective function values $W(\vec{A})$ for all truss designs are evaluated in this step. Initialized designs are analyzed using well-known Finite Element Method process (namely, *direct stiffness method*). Member stresses and nodal deflections are subsequently estimated and compared to their pre-defined limits (see Equation (4)).
- Step (5) Determine “Top Predator”: The truss with the lowest weight (best solution) is assigned as the “Elite Predator.” All other solutions update their position values based on it.
- Step (6) Main MPA Optimization Sequence Begins: Based on the three MP hunting scenarios discussed earlier, new truss designs are generated from Equation (7) to (9) or (11). This step is applied to all members of the population.
- Step (7) Exploration through FADs: Newer truss designs are generated through Equation (12)

- Step (8) **Objective Function Evaluation:** Newly generated truss designs have their objective functions evaluated in this step. A greedy selection is performed, whereby old and new solutions are compared. Those solutions exhibiting lower truss weights are adopted, and heavier ones are discarded.
- Step (9) **Check Stopping Criteria:** If the number of iterations exceeds the maximum pre-defined limit (MaxIt), stop the program and display the best truss design. Otherwise, return to step 5.

5. Quantification of the Exploration–Exploitation Balance

In the field of metaheuristic optimization, there is a general consensus that for an algorithm to achieve satisfactory computational performance for a given optimization task, knowledge of the correct balance between exploration and exploitation is a necessary pre-requisite. The question that naturally arises when tackling the problem of truss optimization is:

“What is the correct balance between an algorithm’s global and local search aspects that will yield appropriate results for truss optimization problems?”

For many years, this has been an open question in the field, and no attempt has yet been made to provide a quantitative answer to this straightforward yet crucial question. As mentioned earlier, one of the main contributions of this study, in addition to implementing MPA for truss sizing optimization, is to give a percentile quantity for the proper amount of exploration and exploitation required for structural sizing problems.

To that end, a very recent study conducted by Morales-Castañeda et al. [43] has extended the concept of a dimension-wise diversity index in an attempt to numerically quantify algorithmic “exploration” and “exploitation.” In their methodology, the distance between search agents from their median (*on a dimension-wise basis*) is utilized to estimate the diversity among solution candidates for any single iteration. As the population of search agents come together, the diversity decreases, and vice versa. Population diversity, therefore, is defined as follows:

$$Div_j = \frac{1}{PopSize} \sum_{i=1}^{PopSize} |median(x^j) - x_i^j| \quad (13)$$

$$Div = \frac{1}{d} \sum_{j=1}^d Div_j \quad (14)$$

where $median(x^j)$ denotes the median of j th dimension for the entire population and x_i^j is the j th design variable of search agent “ i .”

To simplify matters for readers, Equation (13) estimates the average diversity for all members of the population in a single dimension, while Equation (14) provides the average diversity for the entire population across all dimensions. It is worth mentioning here that the values of both equations are computed for every iteration.

After completing the entire optimization process, the percentage of exploration and exploitation for every iteration is computed as follows,

$$\%EXPLORE = \left(\frac{Div_t}{Div_{max}} \right) \times 100 \quad (15)$$

$$\%EXPLOIT = \left(\frac{|Div_t - Div_{max}|}{Div_{max}} \right) \times 100 \quad (16)$$

where Div_{max} represents the maximum diversity obtained in the optimization process and Div_t is the dimension-wise population diversity at iteration t . Averaging the entire results of exploration and exploitation for every iteration gives us the required balance of intensification and diversification for the problem.

The provided quantities of both “exploration” and “exploitation,” estimated earlier, are expected to serve as a preliminary rule-of-thumb for any future algorithms specifically developed to tackle structural optimization problems. By adopting the diversity methodology presented in this study, researchers can easily adjust their algorithms to these average exploration/exploitation values and (in theory) achieve comparable structural optimization performance. The findings in that regard are expected to dramatically reduce both the time and effort involved in producing better structural optimization techniques.

For more information, readers are referred to reference [43] for an in-depth explanation of the diversity index and its application to estimate metaheuristic exploration vs. exploitation.

6. Numerical Examples

Six classical truss benchmarks (three medium-sized and three large-scale) were selected from literature to test MPA efficiency. The adopted benchmarks along with their loading conditions are listed below:

- A 10-bar planar truss subjected to two independent loading scenarios,
- A 60-bar planar truss ring subjected to three loading conditions,
- A 120-bar spatial truss dome with a single loading condition,
- A 272-bar electric transmission tower with twelve loading conditions,
- A 26-storey, 942-bar truss tower subjected to a single loading condition,
- A 62-storey, 4666-bar truss tower with three loading conditions.

The example problems listed above were selected for testing MPA performance based on three criteria: (1) truss type (small or large), (2) solution instances in the literature, and (3) number of design constraints. For criterion 1, both medium- and large-scale trusses were chosen from the literature to account for the various sizes of structures in daily practice. The underlying logic is the crucial necessity of testing MPA robustness for all structural problem types. Criterion 2 was considered in order to ensure the existence of sufficient comparative studies to validate MPA results (i.e., an ample amount of studies in previous works dealing with the benchmarks provides MPA with the necessary number of comparative solutions to draw clear conclusions regarding its performance). Finally, for Criterion 3, the benchmarks containing the greatest number of design constraints were selected to fully challenge MPA's operation principle. The number of design constraints automatically translates to the number of nodes and structural members a truss possesses. Basically, the greater the number of design constraints a problem has, the more difficult it is to optimize. The benchmarks employed and chosen in this study, therefore, represent a compromise between all three aforementioned selection criteria.

MPA has three control parameters that require tuning before optimization. The parameters are PopSize (Population Size), P (scaling factor), and FADs (Fish Aggregating Devices). Following a detailed sensitivity analysis in which the “one-parameter-at-a-time” methodology was employed (further details are available in ref. [17]), a combination of 15 for PopSize, 0.1 for FADs, and 0.3 for P was found appropriate for optimal MPA performance. Unless stated otherwise, this combination of parameter values will be used for all benchmarks investigated. The Maximum Number of Iterations (MaxIt), which governs the upper limit of objective function evaluations, was kept as a user-defined variable best adjusted according to the optimization problem. This has been done in full conformity with literary practice in the field. The methodology of arriving at suitable stopping criteria for the different truss benchmarks solved in this study involves testing various MaxIt values (typically adopted from a range) and seeing what value provides the best algorithmic performance with the least amount of computational effort (further details are available in Refs. [4,5,7,17]). This way, algorithms having different computational budgets (fitness calls vs. iterations) are accounted for, and MHs requiring more iterations to achieve good results naturally perform poorly in terms of convergence speeds. The maximum number of MPA function evaluations (MaxFE) and computational budget, therefore, can be estimated from MaxIt using the following relation: $MaxIt * (2 * PopSize)$. It should be noted that in

some instances, MPA converges well before reaching the MaxFE limit. Accurate function evaluation values are therefore reported with the help of incremental counters added to the original MATLAB code (accurate computational convergence speeds are thus reported). Using the above-described methodology, a fair algorithmic comparison is achieved in terms of computational speed.

Finally, in implementing MPA, 20 independent runs (as justified by ref. [49]) were conducted for each benchmark. The best design, the average, the worst, the standard deviation (SD), and the number of function evaluations (NFE) are reported and compared with other optimizers published in literature. To avoid additional computational complexity, the baseline reference results used for comparative purposes were not produced in a system with the same computational configuration (i.e., comparative results were quoted from the literature). Every effort was made, nonetheless, to simulate (as much as possible) the computing conditions and practices followed by those in the field. The Friedman ranking test was also conducted for every truss benchmark solved. Winners and losers are drawn based on three important algorithmic categories: (1) Best Weight, (2) Average Weight, and (3) Number of Function Evaluations. This way, the proposed method was tested against other algorithms based on solution quality, robustness, and speed. MPA was executed using MATLAB, and all runs were carried out on a personal laptop with a 1.8 GHz Intel Corei5 processor and 4 GB of RAM.

6.1. A 10-Bar Planar Truss Structure

The first benchmark considered in this study is the 10-bar planar truss structure (shown in Figure 3). Two independent loading scenarios were investigated: Case I uses values of $P_1 = 100$ kips (445 kN) and $P_2 = 0$, whereas Case II uses values of $P_1 = 150$ kips (667 kN) and $P_2 = 50$ kips (222.5 kN). Further details regarding the material properties, design constraints, and cross-sectional area bounds for the truss are conveniently summarized in Table 1. In total, the problem has 32 non-linear design constraints (10 tension, 10 compression, and 12 displacement constraints).

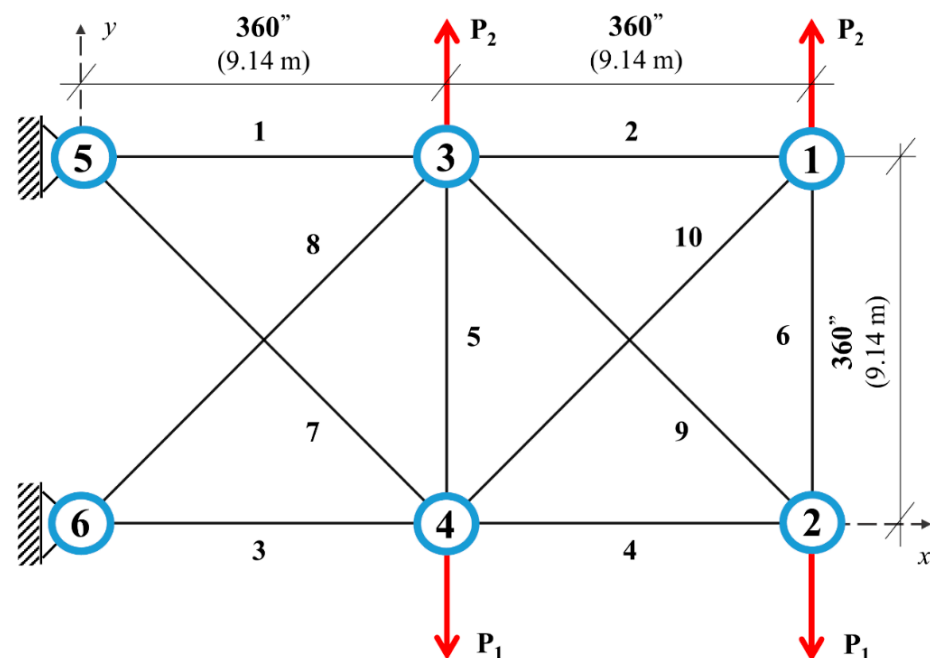


Figure 3. Schematic of the 10-bar planar truss structure.

Table 1. Structural parameters for the 10-bar planar truss [4,17].

Property	Value
Material Density	0.1 lb/in ³
Young's Modulus	10,000 ksi
Number of Area Groups	10
Maximum Normal Stresses	±25 ksi
Maximum Nodal Displacements (x, y)	±2 in.
Range of Cross-sectional Areas	0.1~35.0 in ²

Tables 2 and 4 compare the results found by MPA against other studies for Cases I and II of the benchmark, respectively. Following a thorough test with various MaxIt values, a stopping criterion of 350 iterations (10,500 function evaluations) was set for MPA to solve both cases of the benchmark. Tabulated results reveal MPA produces the lightest structural designs (5060.859 lb and 4677.00 lb) with the least computational effort (9570 and 9720 function evaluations) for both scenarios investigated. Moreover, the two corresponding optimization runs recorded a CPU time of 3.45 s. In total, MPA results were compared with eight other metaheuristic algorithms for Case I and nine other algorithms for Case II. On average, MPA showed a 50.64% reduction in computational effort over all other algorithms for Case I and a 51.41% reduction for Case II of the benchmark (about half). In terms of solution robustness, MPA was superior to all previously published algorithms, with the exception of the ACCS optimizer implemented by Kooshkbaghi and Kaveh [13] for Case II of the benchmark, where a slightly lower standard deviation was reported over MPA. Nonetheless, the difference is so insignificant as to make the algorithms similar in that respect. Tables 3 and 5 show the Friedman rank test results for Cases I and II of the 10-bar truss benchmark. Results show MPA ranking first in all three algorithmic categories tested in this study.

Studying MPA exploration vs. exploitation tradeoff trends for both Cases I and II of the benchmark, an approximate value of 10% exploration and 90% exploitation was reported. Figure 4 visually depicts the symmetric tradeoff between the two aforementioned search schemes. A closer look at the figure shows that as exploration decreases throughout the optimization process, there is an equal and corresponding increase in exploitation (perfectly symmetrical). Moreover, the rough tradeoff spikes in the figure indicate MPA's constant reliance on Lévy operators and FADs to produce good results beyond its current neighborhood.

Finally, the estimated values of exploration vs. exploitation show that MPA is, in essence, an "exploitation-dominated" metaheuristic. This completely contradicts the widely held belief that a robust exploration-dominated algorithm is preferred for truss optimization problems. The significance of having an initial estimate of exploration and exploitation for truss optimization problems is that future researchers can use the values to adjust their algorithms to produce similar results.

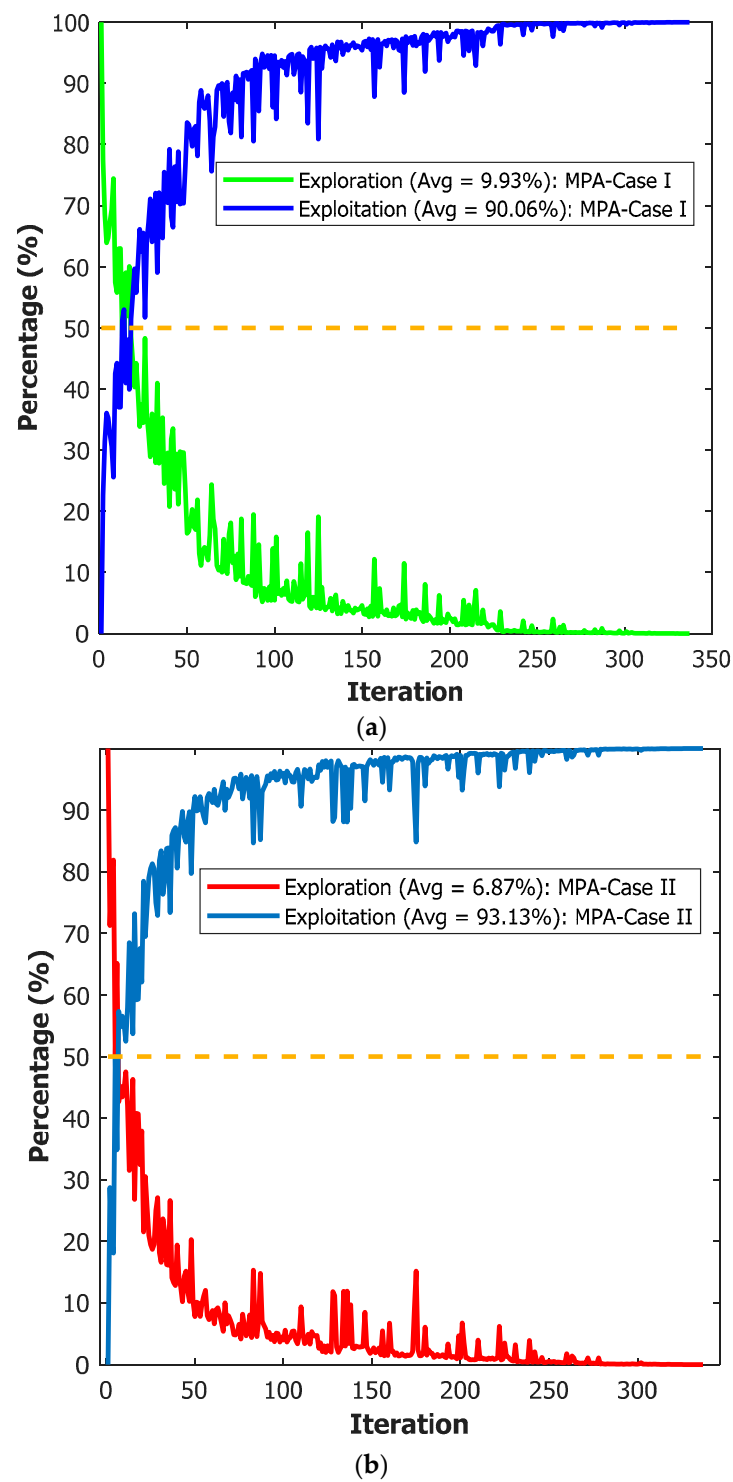


Figure 4. Exploration to exploitation tradeoff trend for the 10-bar planar truss structure. (a) Case I and (b) Case II.

Table 2. Optimized result comparison for the 10-bar planar truss (Case I).

Design Variable (in ²)	(ABC-AP) [4]	(TLBO) [6]	(mTLBO) [50]	(HPSSO) [51]	(WEO) [12]	(CPA) [52]	(CSA) [53]	(ECSA) [53]	This Study	
									(MPA) _{Best}	(MPA) _{Worst}
A ₁	30.548	30.4286	30.6684	30.53838	30.5755	30.5022	33.6116	30.5096	30.5066	30.629
A ₂	0.100	0.1000	0.1000	0.1	0.1000	0.1000	0.1478	0.1000	0.1	0.10997
A ₃	23.180	23.2436	23.1584	23.15103	23.3368	23.2170	22.9345	23.2253	23.1736	23.023
A ₄	15.218	15.3677	15.2226	15.20566	15.1497	15.2204	13.9637	15.2315	15.2499	15.3
A ₅	0.100	0.1000	0.1000	0.1	0.1000	0.1001	0.1050	0.1000	0.1	0.1
A ₆	0.551	0.5751	0.5421	0.548897	0.5276	0.5587	0.3611	0.5517	0.5558	0.54452
A ₇	7.463	7.4404	7.4654	7.465322	7.4458	7.4548	7.9202	7.4561	7.4566	7.4722
A ₈	21.058	20.9665	21.0255	21.06437	20.9892	21.0371	22.0883	21.0276	21.0229	21.018
A ₉	21.501	21.5330	21.4660	21.52935	21.5236	21.5295	19.6785	21.5239	21.5494	21.544
A ₁₀	0.100	0.1000	0.1000	0.1	0.1000	0.1002	0.1041	0.1000	0.1	0.10001
Weight (lb)	5060.88	5060.96	5060.97	5060.86	5060.99	5060.92	5095.40	5060.91	5060.859	5061.86
Avg. Weight (lb)	N/A	5062.08	5064.808	5062.28	5062.09	5062.45	5290.79	5063.41	5061.12	-
St. Deviation (lb)	N/A	0.79	6.3707	4.325	2.05	3.77	125.89	5.43	0.304	-
Con. Violation	None	None	None	None	None	None	None	None	None	None
No. of Evaluations	500,000	16,872	13,767	14,118	19,540	23,700	18,706	16,401	9570	9810
Avg. Exploration	–	–	–	–	–	–	–	–	9.93%	–
Avg. Exploitation	–	–	–	–	–	–	–	–	90.06%	–

Note: 1 in² = 6.452 cm²; 1 lb. = 0.4536 kg.**Table 3.** Friedman rank test results for the 10-bar planar truss (Case I).

Category	(ABC-AP) [4]	(TLBO) [6]	(mTLBO) [50]	(HPSSO) [51]	(WEO) [12]	(CPA) [52]	(CSA) [53]	(ECSA) [53]	This Study	
									(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Weight)	3	6	7	2	8	5	9	4	1	–
Friedman Rank (Avg.)	9	2	7	4	3	5	8	6	1	–
Friedman Rank (NFE)	9	5	2	3	7	8	6	4	1	–
Averaged Rank	7	4.333	5.333	3	6	6	7.6667	4.6667	1	–

Table 4. Optimized result comparison for the 10-bar planar truss (Case II).

Design Variable (in ²)	(ABC-AP) [4]	(MSPSO) [54]	(TLBO) [6]	(WEO) [12]	(PGO) [49]	(ACCS) [13]	(ICA) [21]	(HHO) [21]	(ICHHO) [21]	This Study	
										(MPA) _{Best}	(MPA) _{Worst}
A ₁	23.4692	23.4432	23.524	23.5804	23.5326	23.522	23.9224	24.335	23.4119	23.4469	23.580
A ₂	0.1005	0.1000	0.1000	0.1003	0.1000	0.1	0.1	0.1	0.1	0.10002	0.1243
A ₃	25.2393	25.3718	25.441	25.1582	25.0068	25.364	25.6412	22.188	25.0207	25.3183	25.199
A ₄	14.354	14.1360	14.479	14.1801	14.4241	14.503	15.0520	17.095	14.4040	14.3490	14.295
A ₅	0.1001	0.1000	0.1000	0.1002	0.1000	0.1	0.1	0.1	0.1	0.10003	0.1001
A ₆	1.9701	1.9699	1.995	1.9708	1.9721	1.97	1.9690	2.0380	1.9727	1.96979	1.9679
A ₇	12.4128	12.4335	12.334	12.4511	12.4286	12.417	12.2295	13.095	12.4683	12.40842	12.429
A ₈	12.8925	13.0173	12.689	12.9349	12.8215	12.938	12.4021	13.06	12.9778	12.92012	12.919
A ₉	20.3343	20.2717	20.354	20.3595	20.4603	20.058	19.9683	19.778	20.3544	20.27331	20.309
A ₁₀	0.1000	0.1000	0.1000	0.1001	0.1000	0.1	0.1029	0.1018	0.1	0.10001	0.1015
Weight (lb)	4677.077	4677.26	4678.31	4677.31	4677.17	4677.267	4680.14	4714.56	4677.50	4677.00	4679.22
Avg. Weight (lb)	N/A	4681.45	4680.12	4679.06	4677.88	4677.909	4756.29	5024.08	4682.84	4677.81	-
St. Deviation (lb)	N/A	2.19	1.016	2.07	0.72	0.455	97.273	649.89	3.816	0.58	-
Con. Violation	None	None	None	None	None	None	None	None	None	None	None
No. of Evaluations	500,000	25,000	14,857	19,890	17,580	12,000	20,000	20,000	20,000	9720	9840
Avg. Exploration	–	–	–	–	–	–	–	–	–	6.87%	–
Avg. Exploitation	–	–	–	–	–	–	–	–	–	93.12%	–

Note: 1 in² = 6.452 cm²; 1 lb. = 0.4536 kg.

Table 5. Friedman rank test results for the 10-bar planar truss (Case II).

Category	(ABC-AP) [4]	(MSPSO) [54]	(TLBO) [6]	(WEO) [12]	(PGO) [49]	(ACCS) [13]	(ICA) [21]	(HHO) [21]	(ICHHO) [21]	This Study	
										(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Weight)	2	4	8	6	3	5	9	10	7	1	–
Friedman Rank (Avg.)	10	6	5	4	2	3	8	9	7	1	–
Friedman Rank (NFE)	10	9	3	5	4	2	7	7	7	1	–
Averaged Rank	7.333	6.333	5.333	5	3	3.333	8	8.6667	7	1	–

6.2. A 60-Bar Planar Truss Structure

The 60-bar planar truss structure is schematically shown in Figure 5. The inner and outer radii of the ring are $R_i = 90$ in (2.28 m) and $R_o = 100$ in (2.54 m), respectively. Three loading conditions are considered to act simultaneously on the structure (further details are available in ref. [13]), and Table 6 conveniently lists the adopted design parameters for the truss. A total of 168 non-linear design constraints are imposed on the structure (120 tension/compression and 48 node displacements) for every independent loading condition.

Table 6. Structural parameters for the 60-bar planar truss ring [13].

Property	Value
Material Density	0.1 lb/in ³
Young's Modulus	10,000 ksi
Number of Area Groups	25
Maximum Normal Stresses	± 10 ksi
Maximum Nodal Displacements (x, y)	± 1.75 in. (Node 4), ± 2.75 in. (Node 19), ± 2.25 in. (Node 13)
Range of Cross-sectional Areas	0.5 ~ 4 in ²

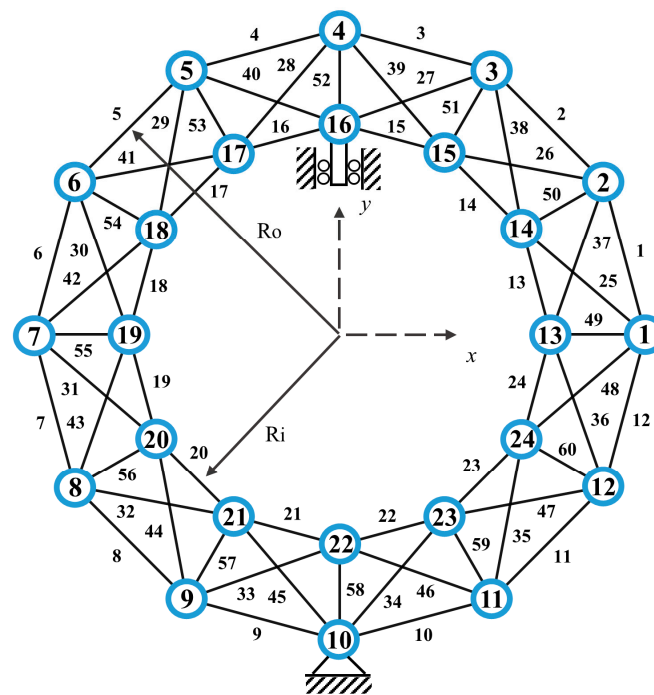


Figure 5. Schematic of the 60-bar planar truss structure (not drawn to scale).

Table 7 lists and compares the results obtained by MPA with other methods quoted from literature. A stopping criterion of 800 iterations (24,000 function evaluations) was deemed suitable for the benchmark. Among feasible solutions, MPA achieves the lightest structural design (309.00 lb) with the least computational effort (23,970 function evaluations). Other algorithms showcasing more lightweight designs, such as the Method of Centers and Force Formulation (MC-FF) algorithm proposed by Farshi and Alinia-Ziazi [55], were found to have constraint violations and are therefore infeasible. Moreover, the final weight reported by MFUD [56], although lighter than MPA, lacked all important statistical data to make a fair comparison. Figure 6 shows the comparison between existing and allowable stresses estimated for the optimal 60-bar truss design produced by MPA. All stress values are clearly shown to be within their predefined limits. Table 8 presents the Friedman rank test results for the 60-bar planar truss structure. Results show that MPA ranks first in the “average weight” and “convergence speed” categories. Overall, a rank of 1.6667 was achieved by the proposed method for all three categories (therefore

outperforming all the rest). MPA is thus the clear winner for this normal-sized benchmark. Finally, it is worth noting that in all statistical aspects, MPA's worst run outperformed the best results achieved by both CPM-GA [57] and APM-GA [57]. Meanwhile, the best combination of average exploration and exploitation for MPA's best run was found to be 10.56% and 89.43%, respectively, as shown in Figure 7.

Table 7. Optimized result comparison for the 60-bar planar truss.

Design Variable (in ²)	(MFUD) [56]	(SECr) [58]	(CPM-GA) [57]	(APM-GA) [57]	(MC-FF) [55]	(ACCS) [13]	This Study	
							(MPA) _{Best}	(MPA) _{Worst}
A ₁ (M ₁₁ , M ₁₃)	N/A	2.027	1.1906	1.1202	2.0273	2.1405	2.02824	1.98861
A ₂ (M ₂ , M ₁₄)	N/A	0.5	2.2771	2.0219	0.5000	0.5068	1.77589	0.54147
A ₃ (M ₃ , M ₁₅)	N/A	1.907	0.6055	0.5088	1.7781	1.6508	1.77589	1.90366
A ₄ (M ₄ , M ₁₆)	N/A	1.826	1.5733	1.7273	1.7775	1.7615	1.66753	1.78047
A ₅ (M ₅ , M ₁₇)	0.59	0.633	1.3753	1.5205	0.5793	0.5949	0.57721	0.52531
A ₆ (M ₆ , M ₁₈)	N/A	1.846	0.5087	0.5264	1.8305	1.8235	1.96150	2.09292
A ₇ (M ₇ , M ₁₉)	N/A	1.841	1.9340	1.9032	1.7947	1.7580	1.87626	2.18541
A ₈ (M ₈ , M ₂₀)	N/A	1.005	2.0528	2.1275	0.9830	0.9732	1.00689	1.07834
A ₉ (M ₉ , M ₂₁)	N/A	1.847	1.2390	0.9882	1.9031	2.0860	1.78673	1.56056
A ₁₀ (M ₁₀ , M ₂₂)	1.84	1.885	1.8196	2.0528	1.9497	1.9542	1.85856	1.61833
A ₁₁ (M ₁₁ , M ₂₃)	N/A	0.5	1.6393	2.0528	0.5000	0.5154	0.50000	0.50001
A ₁₂ (M ₁₂ , M ₂₄)	N/A	2.025	0.5264	0.7243	2.0135	2.0438	2.02047	2.01238
A ₁₃ (M ₂₅ , M ₃₇)	N/A	1.25	2.1979	1.9604	1.2441	1.2434	1.24579	1.26097
A ₁₄ (M ₂₆ , M ₃₈)	N/A	1.022	1.2346	1.2302	1.0156	1.0397	1.01628	1.08643
A ₁₅ (M ₂₇ , M ₃₉)	0.77	0.752	1.0498	0.9970	0.6896	0.7461	0.70831	0.59603
A ₁₆ (M ₂₈ , M ₄₀)	N/A	0.771	0.7595	0.6056	0.7233	0.6563	0.81626	0.82665
A ₁₇ (M ₂₉ , M ₄₁)	N/A	0.928	0.6143	0.7287	1.0578	0.9692	1.08483	1.15086
A ₁₈ (M ₃₀ , M ₄₂)	N/A	1.128	1.1202	1.0938	1.1226	1.1296	1.12470	1.14790
A ₁₉ (M ₃₁ , M ₄₃)	N/A	1.144	1.1158	1.1158	1.1512	1.1599	1.14808	1.13739
A ₂₀ (M ₃₂ , M ₄₄)	N/A	0.922	1.1554	1.1686	1.0664	0.9638	0.98191	1.03903
A ₂₁ (M ₃₃ , M ₄₅)	N/A	1.046	1.1862	1.0674	1.0467	1.0646	1.04983	1.06522
A ₂₂ (M ₃₄ , M ₄₆)	N/A	0.75	1.0718	1.0630	0.7039	0.7414	0.75463	0.62340
A ₂₃ (M ₃₅ , M ₄₇)	N/A	1.024	0.7903	0.5879	1.0280	1.0623	1.03862	1.00742
A ₂₄ (M ₃₆ , M ₄₈)	N/A	1.252	1.2654	1.0674	1.2588	1.2694	1.25500	1.23458
A ₂₅ (M ₄₉ ~M ₆₀)	1.16	1.151	1.2698	1.2698	1.1475	1.1673	1.14838	1.13372
Weight (lb)	308.07	309.58	315.480	311.876	308.59	309.894	309.00	310.587
Avg. Weight (lb)	N/A	N/A	337.339	333.0190	N/A	310.714	309.74	-
St. Deviation (lb)	N/A	N/A	N/A	N/A	N/A	0.772	0.411	-
Con. Violation	Unknown	2.6×10^{-3}	None	None	0.10×10^{-3}	None	None	None
No. of Evaluations	N/A	N/A	800,000	800,000	N/A	72,900	23,970	24,390
Avg. Exploration	-	-	-	-	-	-	10.56%	-
Avg. Exploitation	-	-	-	-	-	-	89.43%	-

Note: 1 in² = 6.452 cm²; 1 lb. = 0.4536 kg.

Table 8. Friedman rank test results for the 60-bar planar truss.

Category	(MFUD) [56]	(SECr) [58]	(CPM-GA) [57]	(APM-GA) [57]	(MC-FF) [55]	(ACCS) [13]	This Study	
							(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Weight)	1	4	7	6	2	5	3	-
Friedman Rank (Avg.)	6	6	4	3	6	2	1	-
Friedman Rank (NFE)	6	6	3.5	3.5	6	2	1	-
Averaged Rank	4.333	5.333	4.8333	4.16667	4.6667	3	1.6667	-

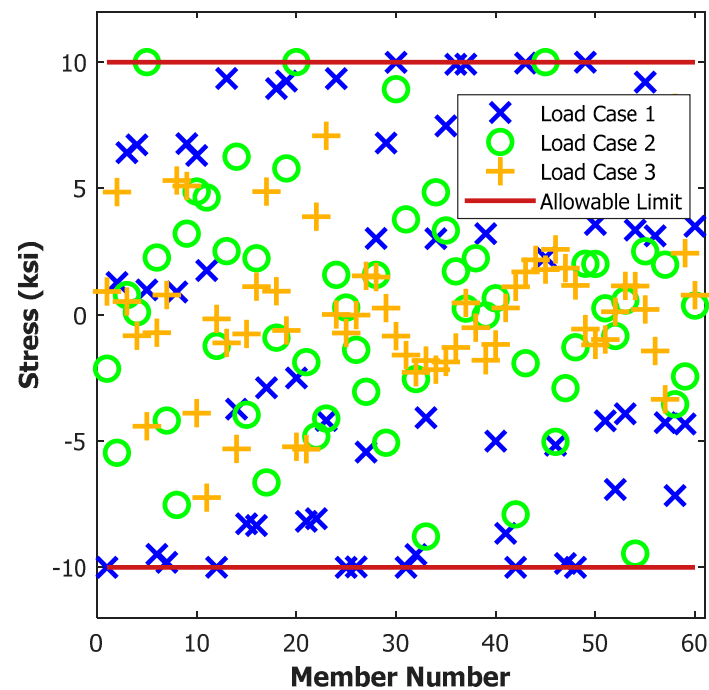


Figure 6. Stress constraint values estimated for the optimized 60-bar planar truss structure.

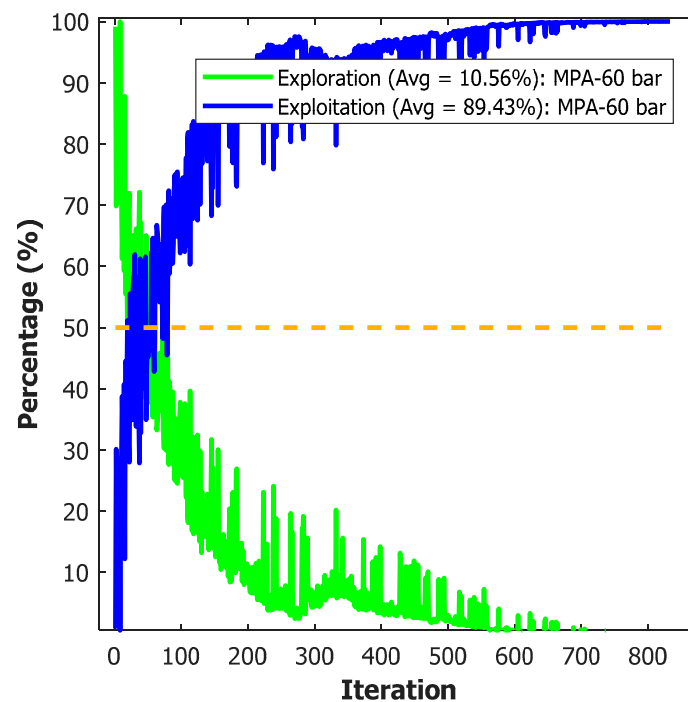


Figure 7. Exploration to exploitation tradeoff trend for the 60-bar planar truss.

6.3. A 120-Bar Spatial Dome Structure

The third benchmark considered in this study is the 120-bar truss dome (shown in Figure 8). A single loading scenario comprised of asymmetrical vertical forces is assumed to act on the structure as follows: -13.49 kips for node 1, -6.744 kips for nodes 2 through 14, and -2.248 kips for the rest of the nodes. Table 9 conveniently summarizes the structural parameters related to the truss, and the allowable stresses for every member

$(\sigma_{allow})_i$ are estimated according to the provisions laid down by AISC-ASD [22], briefly described below:

$$\begin{cases} (\sigma_{allow})_i^+ = 0.6F_y, & \text{for } \sigma_i \geq 0 \\ (\sigma_{allow})_i^-, & \text{for } \sigma_i < 0 \end{cases} \quad (17)$$

Here, $(\sigma_{allow})_i^-$ is calculated according to the slenderness ratio λ_i and is estimated as follows [22]:

$$(\sigma_{allow})_i^- = \begin{cases} \left(1 - \frac{\lambda_i^2}{2C_c^2}\right)F_y, & \text{for } \lambda_i < C_c \\ \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3}\right), & \text{for } \lambda_i \geq C_c \end{cases} \quad (18)$$

where E represents Young's Modulus, F_y the material Yield Stress, C_c is the slenderness ratio dividing the elastic and inelastic regions ($C_c = \sqrt{2\pi^2 E / F_y}$), and λ_i is the truss member slenderness ratio ($\lambda_i = kL_i / r_i$). The effective length factor (k) is assumed to be 1.0 for the truss member pin-pin scenario, while r_i denotes the radius of gyration. The benchmark has 387 non-linear design constraints (240 tension/compression and 147 displacement constraints).

Table 9. Structural parameters for the 120-bar spatial truss [22,53].

Property	Value
Material Density	0.288 lb/in ³
Young's Modulus	30,450 ksi
Yield Stress	58.0 ksi
Number of Area Groups	7
Maximum Nodal Displacements (x, y, z)	± 0.1969 in.
Range of Cross-sectional Areas	0.775~20.0 in ²

The results obtained by MPA and several other optimization techniques are shown in Table 10. A Maximum Iteration value of 425 (12,750 function evaluations) was used as a stopping criterion. The proposed method obtained a final weight of 33,249.46 lbs. in exactly 12,630 function evaluations. No constraint violations were reported, as depicted in Figure 11. A look at the tabulated results shows MPA outperforming all ten algorithms used to solve the benchmark. The only method yielding similar truss weight was the Hybrid Imperialist-Competitive Harris Hawk Optimizer (ICHHO) developed by Kaveh et al. [21]. However, upon closer inspection, the algorithm proved insufficient in other essential aspects such as computational effort and solution robustness. Quantitatively, MPA was observed to require 2370 fewer function evaluations (15.80% reduction) over ICHHO. Moreover, inspecting the weight yielded by MPA's worst run, it was found to be lighter than the best designs proposed by CBO [59], CSA [53], ICA [21], and HHO [21] with, once again, fewer computational efforts. The convergence history for the 120-bar truss benchmark is shown in Figure 9. The algorithm shows a sharp reduction in optimized weight prior reaching the 100 iteration mark. Beyond that, refinements to optimized solutions occur at a slower pace. No significant difference is apparent between MPA's best and worst runs. Overall, a nearly similar convergence profile is apparent for the two. This indicates excellent solution robustness by MPA. The Friedman rank test results for the 120-bar truss structure are shown in Table 11. The results indicate that MPA achieved first place in all important algorithmic metrics.

The tradeoff convergence trend for the 120-bar truss dome shows no new aspects to it. A symmetrical tradeoff between exploitation and exploration occurs throughout the optimization process, with a sharp reduction in exploration ability after 100 iterations as shown in Figure 10. Consequently, the average exploration drops from its usual value of 9% to 7.74%. This may be attributable to MPA concentrating on one region of the search space after initial exploration. The rough trend, with occasional spikes in diversity, seems to be a common feature innate to the proposed algorithm. It is clear that this can only be found during the second and third divisions (Phase II and III) of MPA. The Lévy strategy, in addition to the added explorative search achieved through FADs, is absolutely crucial to MPA's excellent performance as a structural optimizer.

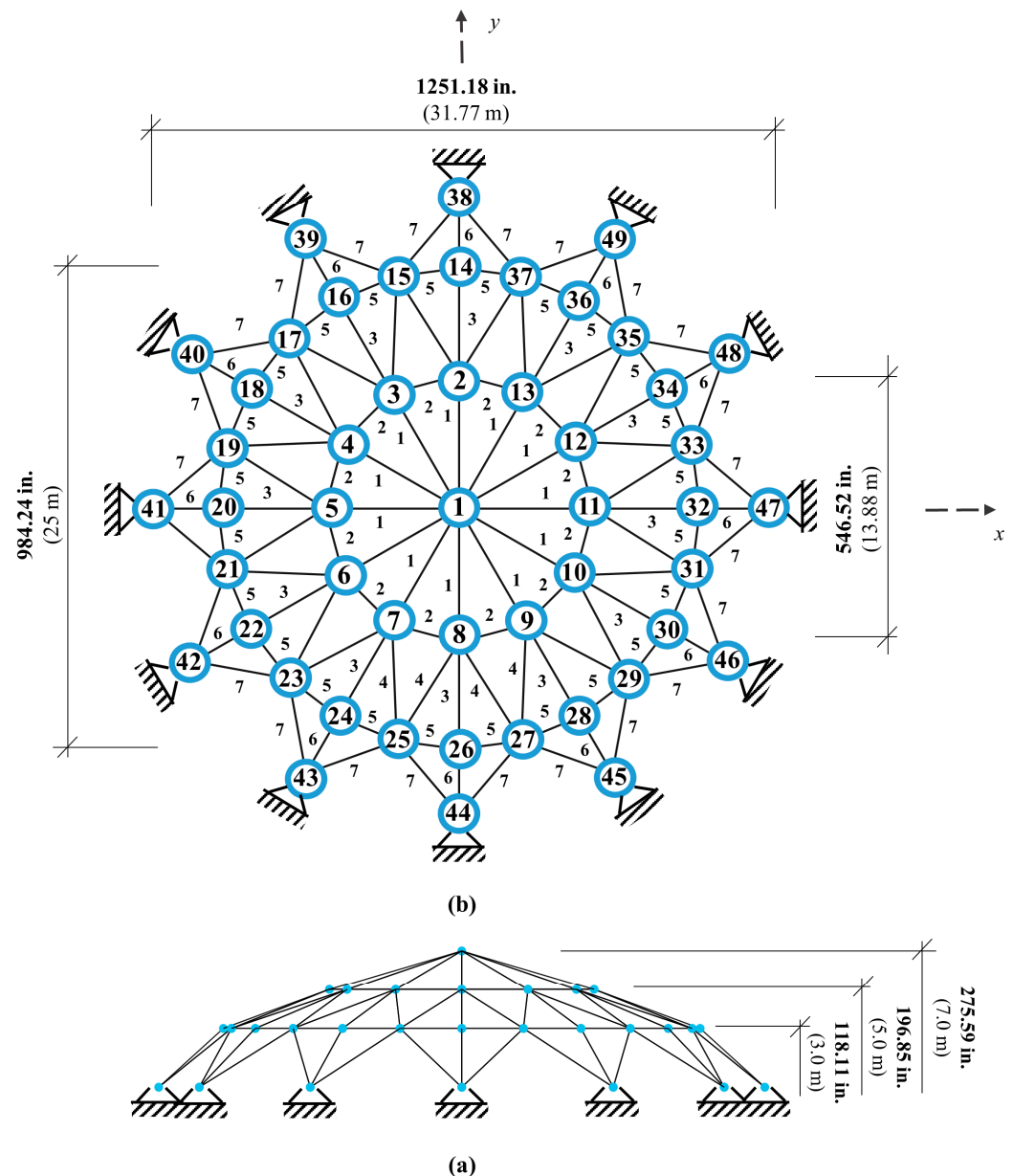


Figure 8. Schematic of the 120-bar spatial truss dome. (a) Front View; (b) Top View.

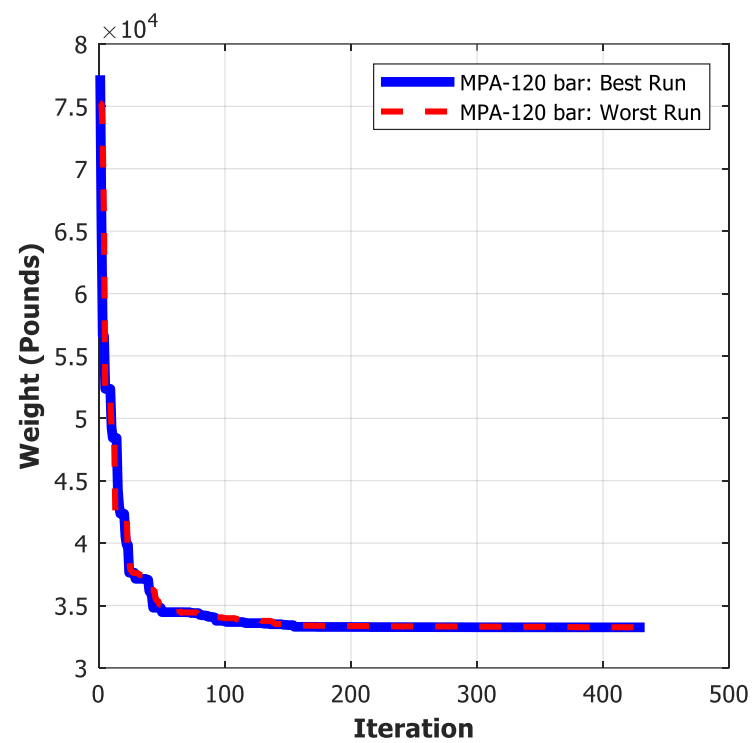


Figure 9. Convergence history for the 120-bar spatial truss dome.

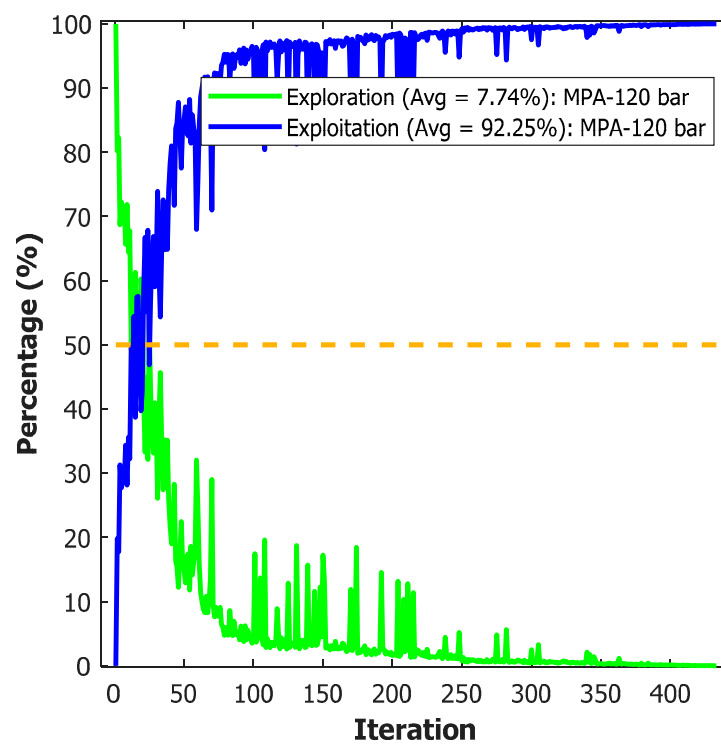


Figure 10. Exploration to exploitation tradeoff trend for the 120-bar truss dome.

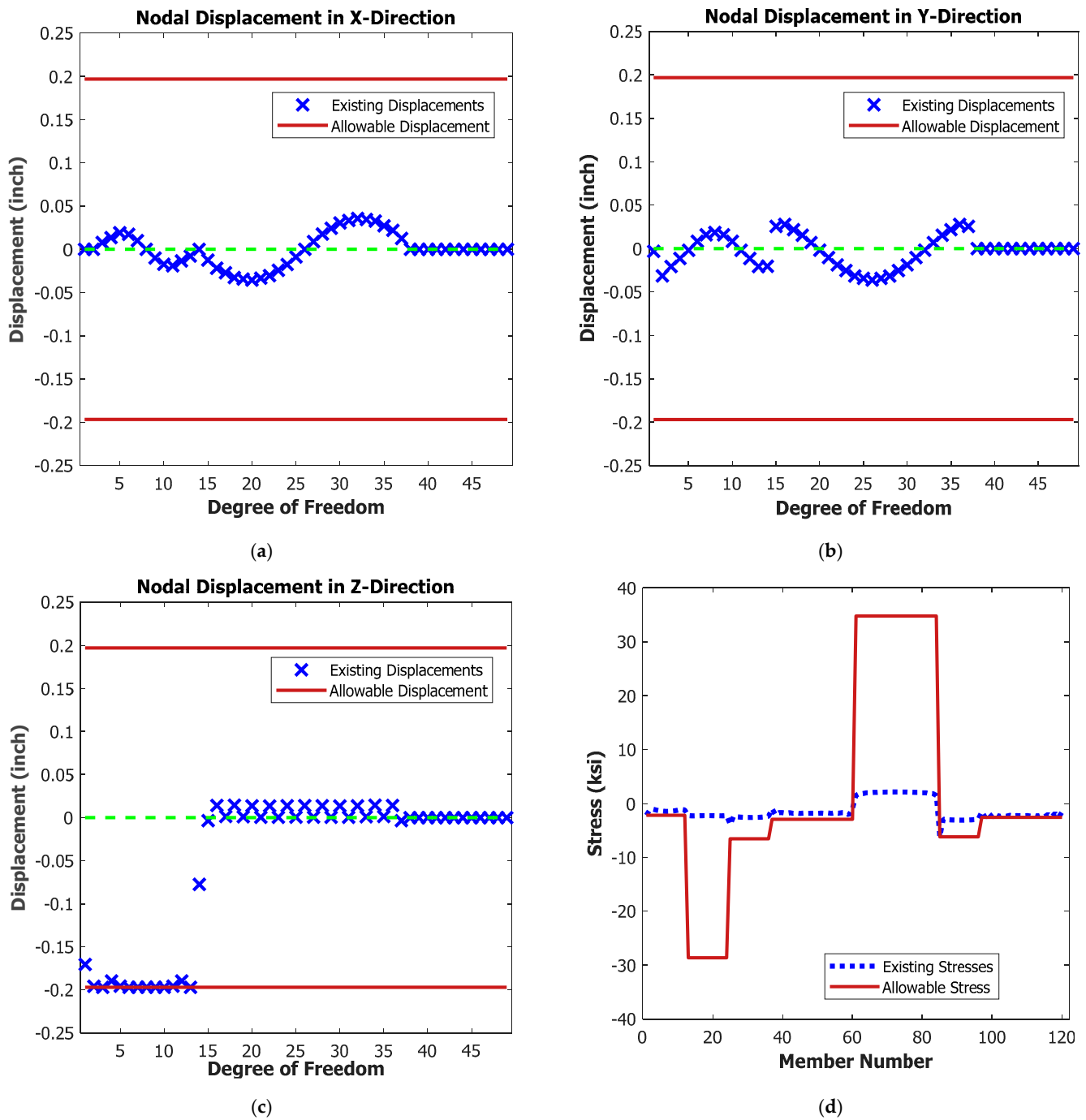


Figure 11. Comparison of permissible and existing restrictions for the 120-bar dome truss. (a) Displacement in the X-direction. (b) Displacement in the Y-direction. (c) Displacement in the Z-direction. (d) Element Stresses.

Table 10. Optimized result comparison for the 120-bar spatial dome truss.

Design Variable (in ^{2.4})	(CBO) [59]	(HPSSO) [51]	(WEO) [12]	(CPA) [52]	(PGO) [49]	(CSA) [53]	(ECSA) [53]	(ICA) [21]	(HHO) [21]	(ICHHO) [21]	This Study	
											(MPA) _{Best}	(MPA) _{Worst}
A ₁	3.0273	3.024139	3.0243	3.0250	3.0245	3.0232	3.0241	3.0278	3.1121	3.0241	3.0242452	3.0252388
A ₂	15.1724	14.78086	14.7943	14.6168	14.8288	14.6467	14.7998	14.8336	12.9862	14.8763	14.789522	14.521904
A ₃	5.2342	5.052164	5.0618	5.0201	5.0714	4.9639	5.0779	5.3251	4.9064	5.0091	5.0775395	5.0819089
A ₄	3.119	3.136943	3.1358	3.1404	3.1351	3.1426	3.1358	3.1231	3.2861	3.1324	3.1364955	3.1411414
A ₅	8.1038	8.500353	8.4870	8.5653	8.4556	8.6401	8.4737	8.1941	9.0996	8.4874	8.4769851	8.5270232
A ₆	3.4166	3.288777	3.2886	3.3648	3.2959	3.3005	3.2819	3.4141	3.8406	3.2991	3.2836077	3.4203555
A ₇	2.4918	2.49688	2.4967	2.4975	2.4965	2.4991	2.4964	2.4914	2.7655	2.4972	2.4964922	2.4946859
Weight (lb)	33,286.3	33,250.05	33,250.24	33,256.39	33,250.15	33,257.16	33,249.50	33,271.03	33,899.53	33,249.46	33,249.46	33,256.99
Avg. Weight (lb)	33,398.5	33,260.700	33,255.55	33,281.73	33,252.72	33,512.27	33,251.02	33,345.10	35,222.81	33,259.49	33,250.97	-
St. Deviation (lb)	67.09	10.49	8.07	16.45	2.76	255.23	3.22	66.30	1039.02	8.29	1.62	-
Con. Violation	None	None	None	None	None	None	None	None	None	None	None	None
No. of Evaluations	14,960	13,422	19,510	22,980	17,790	25,019	13,582	15,000	15,000	15,000	12,630	12,630
Avg. Exploration	–	–	–	–	–	–	–	–	–	–	7.74%	–
Avg. Exploitation	–	–	–	–	–	–	–	–	–	–	92.25%	–

Note: 1 in² = 6.452 cm²; 1 lb. = 0.4536 kg.

Table 11. Friedman rank test results for the 120-bar spatial dome truss.

Category	(CBO) [59]	(HPSSO) [51]	(WEO) [12]	(CPA) [52]	(PGO) [49]	(CSA) [53]	(ECSA) [53]	(ICA) [21]	(HHO) [21]	(ICHHO) [21]	This Study	
											(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Weight)	10	4	6	7	5	8	3	9	11	1.5	1.5	–
Friedman Rank (Avg.)	9	6	4	7	3	10	2	8	11	5	1	–
Friedman Rank (NFE)	4	2	9	10	8	11	3	6	6	6	1	–
Averaged Rank	7.6667	4	6.333	8	5.333	9.6667	2.6667	7.6667	9.333	4.16667	1.16667	–

6.4. A 272-Bar Transmission Tower

The 272-bar electric transmission tower, schematically depicted in Figure 12, is presented as the fourth optimization benchmark and the first large-scale structure investigated in this study. The truss was initially introduced by Kaveh and Massoudi [60] as a multi-objective optimization benchmark, but was later adopted for single-objective truss optimization problems by Kaveh and Zaerreza [61] and Sarjamei et al. [62] to minimize structural volume. Twelve loading conditions are considered to act simultaneously on the tower and are summarized in reference [49]. Moreover, the 272 bars comprising the structure were split into 28 distinct design variables by dint of structural symmetry (member groupings can be found in ref. [49]). Finally, Table 12 conveniently lists the structural parameters related to the benchmark, and 559 non-linear design constraints ($559 \times 12 = 6708$ design constraints for all 12 loading conditions) are considered.

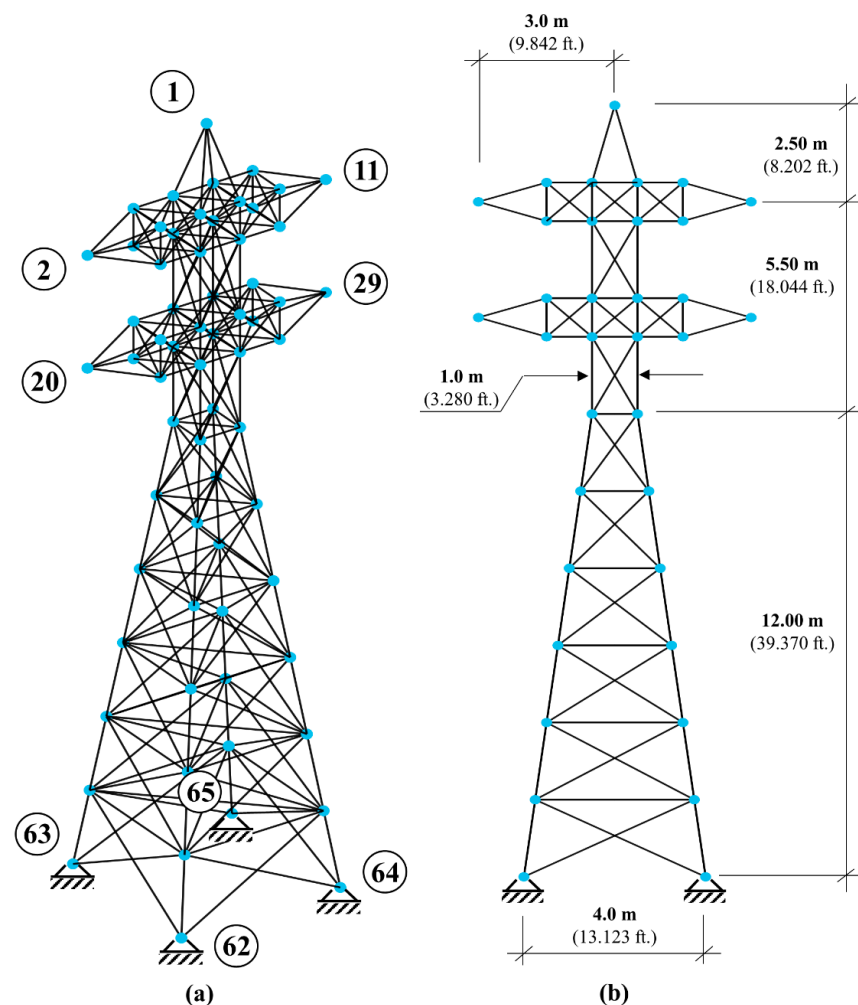


Figure 12. Schematic of the 272-bar electric transmission tower (a) Isometric View; (b) Front View.

Table 12. Structural parameters for the 272-bar transmission tower [49].

Property	Value
Material Density	N/A (assumed as 7850 kg/m^3)
Young's Modulus	$2 \times 10^8 \text{ kPa}$
Number of Area Groups	28
Maximum Normal Stresses	$\pm 275,000 \text{ kPa}$
Maximum Nodal Displacements (x, y)	$\pm 100 \text{ mm}$ in x and y; $\pm 20 \text{ mm}$ (Node 1, 2, 11, 20, and 29)
Range of Cross-sectional Areas	$1000\text{--}16,000 \text{ mm}^2$

Table 13 compares the results obtained by MPA with those obtained by three other metaheuristic optimizers (found after an extensive search through literature). A stopping criterion of 1000 iterations (30,000 function evaluations) was found suitable as a compromise between solution quality and computational speed (i.e., anything greater than 1000 iterations yielded no improvement to the MPA solution and only unnecessarily added further computational effort). Looking at the tabulated results, the proposed algorithm achieves the lightest structure with a final volume of 1,168,039.233 cm³ with no stress or displacement constraint violations. In terms of computational effort, MPA has fallen short in comparison to other algorithms. However, upon closer examination, it is clear that the lower computational effort reported by these algorithms is due to their premature convergence. MPA's excellent Lévy motion strategy prevented this outcome for itself. Figure 13a shows the convergence trend for the MPA optimizer. It can be seen that the improvement of solutions slows down in the second-third of iterations (i.e., Phase II). Possible explanations include the redundancy of having an “exploration” activity after Phase I of the algorithm (which is already entirely dedicated to exploring the search space). Table 14 provides the results of the Friedman rank test for the 272-bar truss, with values showing MPA ranking first in all metrics except for the “number of function evaluations” category. Overall, however, MPA ranked first on an average rank basis. This statistically showcases MPA's superiority over other algorithms that have attempted to solve the benchmark in the literature.

Finally, the exploration vs. exploitation values reported by MPA's best run revealed a drop in exploration activity from 7.74% for the 120-bar benchmark to just 5.68% for the 272-bar truss. This shows that as the size and complexity of the structure increase, MPA's reliance on exploitation increases. Furthermore, the Lévy flight/walk motion model, represented by the sharp spikes in the tradeoff curves shown in Figure 13b, served as an added diversification measure later in the optimization run. In that regard, promising areas of the search space were first identified through Brownian motion, and improved values were yielded after that through intense exploitation.

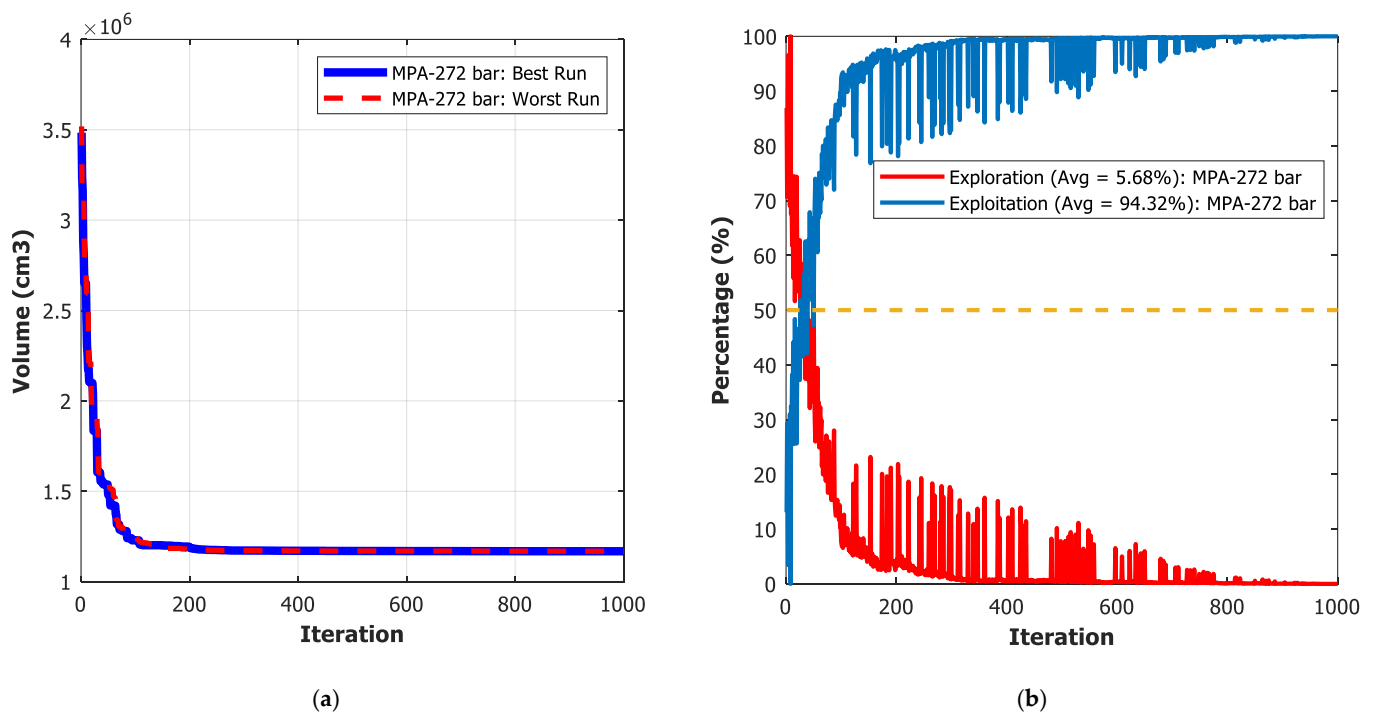


Figure 13. (a) Convergence history for the 272-bar truss tower. (b) Exploration to exploitation tradeoff trend for the 272-bar truss tower.

Table 13. Optimized result comparison for the 272-bar truss tower.

Design Variable (mm ²)	(SSOA) [61]	(PGO) [49]	(GRO) [62]	This Study	
				(MPA) _{Best}	(MPA) _{Worst}
A ₁	1000.5510	1000.6591	1000.0002	1000.0188	1000.0517
A ₂	1240.0130	1216.9322	1239.9505	1207.5918	1165.5132
A ₃	2491.8710	2445.0689	2491.8674	2467.4672	2454.4865
A ₄	1017.8290	1000.0000	1017.6729	1004.5014	1014.3719
A ₅	9618.8090	9580.2369	9618.8241	9561.3973	9563.4297
A ₆	1000.0000	1000.0184	1000.0000	1000.0000	1000.0386
A ₇	12,063.8160	12,259.5226	12,063.8293	12,269.641	11,820.495
A ₈	1001.7770	1001.5883	1001.1048	1000.0097	1001.0916
A ₉	1000.1880	1000.2804	1000.0001	1000.5054	1002.8344
A ₁₀	1000.4570	1000.0297	1000.0007	1000.0401	1000.0000
A ₁₁	10,217.0220	10,615.8809	10,217.0688	10,272.846	10,634.671
A ₁₂	1000.0640	1000.1339	1000.0000	1000.0000	1000.0000
A ₁₃	1000.0150	1000.0249	1000.0001	1000.0000	1000.0223
A ₁₄	1000.0050	1000.1262	1000.0001	1001.0886	1000.0908
A ₁₅	9320.5490	9118.9367	9321.0719	9313.4094	9567.7133
A ₁₆	1000.0280	1000.0014	1000.0000	1000.0044	1000.0000
A ₁₇	1000.3070	1000.0000	1000.0042	1000.0006	1000.0630
A ₁₈	1002.5180	1000.0000	1000.0004	1000.0032	1000.0152
A ₁₉	8389.8090	8541.0887	8389.6996	8602.1100	8808.2140
A ₂₀	1000.8140	1000.1291	1000.0000	1000.0000	1000.0001
A ₂₁	1000.0040	1000.0000	1000.0000	1000.0023	1000.0626
A ₂₂	1003.2880	1000.0000	1000.0000	1000.0035	1000.0688
A ₂₃	7982.2590	7988.3355	7982.2199	8009.0987	7773.3430
A ₂₄	1000.4450	1000.0001	1000.0000	1000.0000	1000.0023
A ₂₅	1000.5910	1000.0000	1000.0000	1000.0308	1000.0000
A ₂₆	1000.0530	1000.0307	1000.0009	1000.0006	1000.0410
A ₂₇	7504.2980	7656.6655	7504.2977	7559.0113	7585.2953
A ₂₈	1000.0760	1000.0000	1000.0007	1000.0000	1000.0000
Volume (cm ³)	1,168,200.62	1,168,113.56	1,168,069.326	1,168,039.233	1,168,459.394
Avg. Volume (cm ³)	1,168,668.720	1,168,793.420	1,168,701.004	1,168,209.147	-
St. Deviation (cm ³)	310.76	771.54	339.2781	117.1291	-
Con. Violation	None	None	None	None	None
No. of Evaluations	14,020	23,920	23,000	29,670	29,670
Avg. Exploration	-	-	-	5.68%	-
Avg. Exploitation	-	-	-	93.15%	-

SSOA = Shuffled Shepherd Optimization Algorithm, PGO = Plasma Generation Optimizer, GRO = Gold Rush Optimizer.

Table 14. Friedman rank test results for the 272-bar truss tower.

Category	(SSOA) [61]	(PGO) [49]	(GRO) [62]	This Study	
				(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Volume)	4	3	2	1	-
Friedman Rank (Avg.)	2	4	3	1	-
Friedman Rank (NFE)	1	3	2	4	-
Averaged Rank	2.333	3.333	2.333	2	-

6.5. A 26-Story, 942-Bar Truss Tower

The 942-bar spatial truss tower is schematically shown in Figure 14. This benchmark is widely regarded as a challenging high-dimensional optimization problem commonly used in studies investigating metaheuristic robustness. Table 15 conveniently summarizes the structural design parameters used for the truss, and references [17,63] detail the member groupings, loading conditions, and design constraints inherent to the structure. Finally, the benchmark has 1896 non-linear design constraints (1884 tension/compression and 12 displacement).

Table 15. Structural parameters for the 942-bar spatial truss tower [17,63].

Property	Value
Material Density	0.1 lb/in ³
Young's Modulus	10,000 ksi
Number of Area Groups	59
Maximum Normal Stresses	±25 ksi
Maximum Nodal Displacements (x, y, z)	±15.0 in. (4 top nodes)
Range of Cross-sectional Areas	0.1~200 in ²

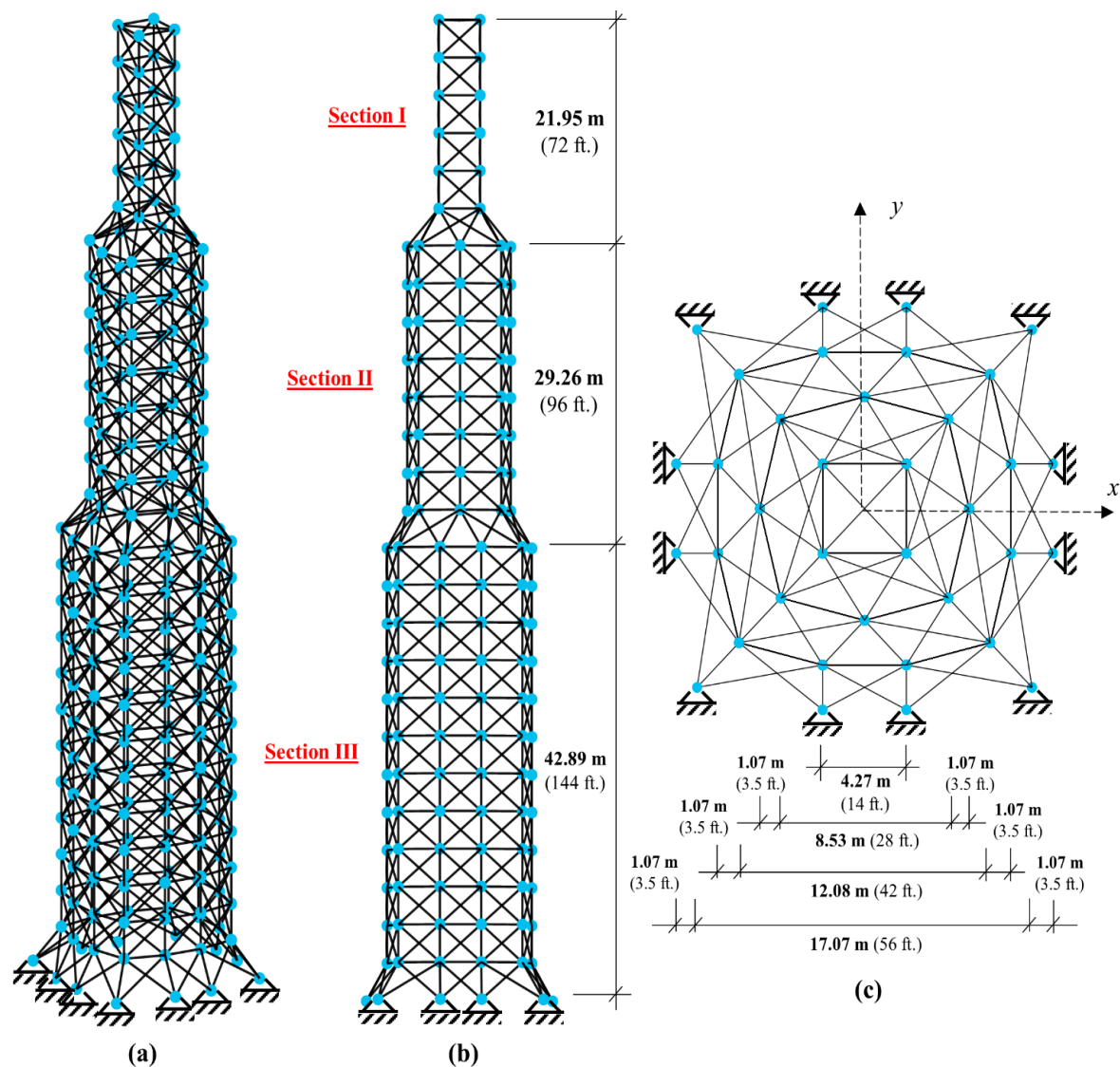


Figure 14. Schematic of the 942-bar truss tower (a) Isometric View; (b) Front View; (c) Top View.

Table 16 lists the results obtained by MPA along with the results of a sample of other previously published studies. For optimal MPA performance, 1600 iterations (48,000 FEs) were deemed suitable. Among the displayed weights, MPA yielded the lightest structural design (135,364.167 lb) with the least computational effort (38,880 function evaluations). Figure 15a shows the convergence history of the 942-bar truss. Numerically, MPA demonstrated a 30.2% reduction in computational effort over the Jaya Algorithm (JA) [64], a 27.21% reduction over IS-JA [64], a 22.24% reduction over FA [65], and a 74.08% decrease over ES [66]. A closer inspection of tabulated results, however, shows that the Improved Grey Wolf Optimizer (IGWO) proposed by Kaveh and Zakian [63] required only 28,000 function evaluations to converge (much less than the 38,880 required by MPA). Nonetheless, the final weight yielded by IGWO is heavier in comparison and therefore does not represent optimality. The Friedman rank test results statistically confirm the observations made earlier, where MPA ranks first in the “best weight” and “average weight” categories (Table 17). IGWO, however, ranks first in the “number of evaluations” category, and MPA obtains third place. Averaging the obtained ranks over all three categories, it becomes clear that MPA is the winner for this large-scale truss benchmark.

Studying the intensification vs. diversification tradeoff trend for this large-scale benchmark in Figure 15b clearly shows a metaheuristic that is exploitation-dominated. A decrease in exploration activity by 1% was noted over the previous 120-bar benchmark. It appears that as the size and complexity of the optimized structure increase, MPA’s reliance on Lévy operators (instead of pure exploration) to escape local optima increases.

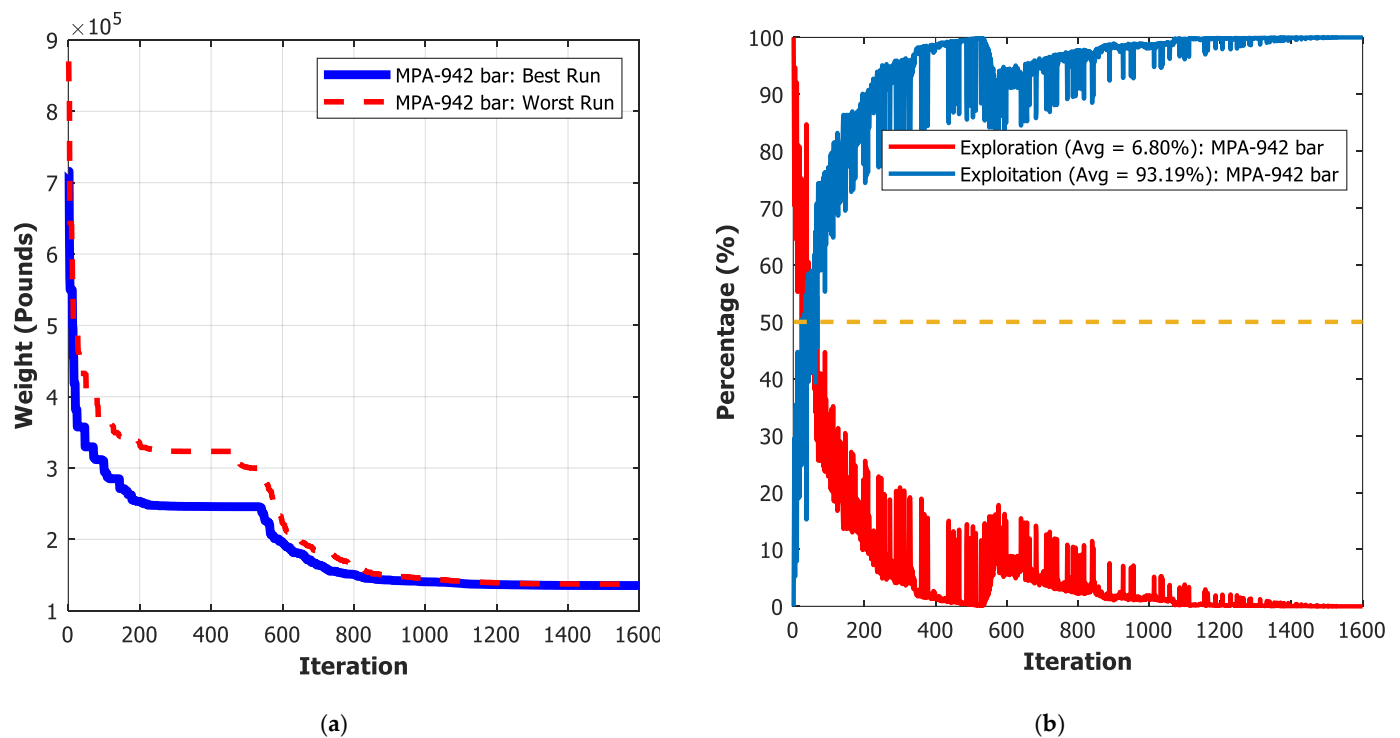


Figure 15. (a) Convergence history for the 942-bar truss tower. (b) Exploration to exploitation tradeoff trend for the 942-bar truss tower.

Table 16. Optimized result comparison for the 26-story, 942-bar truss tower.

Design Variable (in ²)	(SA) [67]	(ES) [66]	(GNMS) [68]	(FA) [65]	(GWO) [63]	(IGWO) [63]	(JA) [64]	(IS-JA) [64]	This Study	
									(MPa) _{Best}	(MPa) _{Worst}
A ₁	1	1.02	2.786	N/A	1.4245	4.2489	1	1	1.57037	2.1571
A ₂	1	1.037	1.357	N/A	2.1232	1.7702	1	1	1.12206	1.78726
A ₃	3	2.943	5.036	N/A	2.1749	1.5892	3	4	3.54765	1.61278
A ₄	1	1.92	2.24	N/A	2.3746	1.5235	2	2	2.60205	2.11236
A ₅	1	1.025	1.223	N/A	1.0000	1.0265	1	1	1.21885	1.43314
A ₆	17	14.961	14.958	N/A	17.5705	15.3979	16	15	13.6201	14.6095
A ₇	3	3.074	2.957	N/A	3.3655	2.8825	3	3	4.02917	2.75545
A ₈	7	6.78	10.904	N/A	19.1722	6.9912	8	6	1.21197	3.16720
A ₉	20	18.58	14.418	N/A	12.8837	11.2039	7	6	4.05446	8.59093
A ₁₀	1	2.415	3.709	N/A	2.6161	2.7262	17	28	10.5447	10.5975
A ₁₁	8	6.584	5.708	N/A	3.9268	8.1921	1	5	1.20188	1.44824
A ₁₂	7	6.291	4.926	N/A	4.7984	6.2178	6	7	5.56403	5.87840
A ₁₃	19	15.383	14.175	N/A	12.4939	16.5585	16	16	12.8569	13.1467
A ₁₄	2	2.1	1.904	N/A	1.0000	2.3668	2	2	2.68949	2.96611
A ₁₅	5	6.021	2.81	N/A	1.6022	4.1519	5	5	4.30342	4.43618
A ₁₆	1	1.022	1	N/A	1.0000	1.2370	1	1	2.60398	1.99889
A ₁₇	22	23.099	18.807	N/A	16.8974	22.3006	22	22	22.0078	20.4121
A ₁₈	3	2.889	2.615	N/A	2.5670	2.9996	3	3	3.03439	3.11877
A ₁₉	9	7.96	12.533	N/A	6.3981	7.7559	9	9	6.91035	11.5399
A ₂₀	1	1.008	1.131	N/A	1.1522	1.1283	1	1	1.01338	1.82290
A ₂₁	34	28.548	30.512	N/A	29.0131	28.2646	28	29	28.9839	28.5157
A ₂₂	3	3.349	3.346	N/A	3.5656	3.1924	4	5	3.22886	3.11393
A ₂₃	19	16.144	17.045	N/A	17.4563	16.3965	15	18	15.3133	17.4940
A ₂₄	27	24.822	18.079	N/A	21.3364	22.6095	29	26	21.6239	20.8702
A ₂₅	42	38.401	39.272	N/A	19.0983	40.0759	40	40	32.6892	27.2972
A ₂₆	1	3.787	2.606	N/A	11.6687	5.3549	3	3	8.29315	8.81768
A ₂₇	12	12.32	9.83	N/A	7.2854	9.2695	11	13	8.17984	8.60813
A ₂₈	16	17.036	13.113	N/A	13.2728	15.0911	16	15	13.0383	11.4355
A ₂₉	19	14.733	13.69	N/A	10.9616	14.0704	15	16	14.1649	16.3599
A ₃₀	14	15.031	16.978	N/A	16.9994	15.1962	18	17	16.6373	17.8619
A ₃₁	42	38.597	37.601	N/A	51.2551	37.1490	36	38	35.1591	33.5007
A ₃₂	4	3.511	3.06	N/A	3.5553	3.1643	4	3	3.68495	3.28520
A ₃₃	4	2.997	5.511	N/A	10.7749	3.4414	2	4	4.13725	3.22115
A ₃₄	4	3.06	1.801	N/A	2.2552	2.2813	3	3	3.37235	2.83088
A ₃₅	1	1.086	1.157	N/A	2.8847	1.0166	1	1	1.11879	1.67923
A ₃₆	1	1.462	1.242	N/A	1.4999	1.4089	1	1	1.00314	1.06030
A ₃₇	62	59.433	62.774	N/A	74.8387	59.6649	55	62	55.7636	52.5650
A ₃₈	3	3.632	3.328	N/A	4.4502	3.3173	3	3	3.29766	3.28016
A ₃₉	2	1.887	4.237	N/A	4.5565	2.0249	2	2	1.97339	1.74477
A ₄₀	4	4.072	1.72	N/A	1.6472	2.3953	3	3	3.15345	3.35289
A ₄₁	1	1.595	1.015	N/A	1.6962	1.0554	1	1	1.29984	2.45128
A ₄₂	2	3.671	5.643	N/A	1.0000	1.2294	3	8	1.15024	1.86427
A ₄₃	77	79.511	78.009	N/A	72.9916	79.5798	79	69	74.6142	72.6401
A ₄₄	3	3.394	3.221	N/A	3.3433	3.2875	3	5	3.12527	3.36496
A ₄₅	2	1.581	3.593	N/A	1.9913	1.9028	2	1	1.84162	1.52823
A ₄₆	3	4.204	4.767	N/A	2.3226	3.2460	3	5	3.51918	3.20781
A ₄₇	2	1.329	1.153	N/A	1.1452	1.0277	1	1	1.85418	2.33929
A ₄₈	3	2.242	2.17	N/A	1.0000	1.0898	1	1	2.84824	2.36750
A ₄₉	100	96.886	99.641	N/A	96.6037	93.8836	100	76	85.7870	85.9449
A ₅₀	4	3.71	4.147	N/A	4.0309	3.0634	3	3	3.27166	3.85777
A ₅₁	1	1.055	2.16	N/A	1.8735	1.7246	1	7	1.37781	3.54393
A ₅₂	4	4.566	4.15	N/A	4.7339	3.9313	4	5	3.94006	3.99825
A ₅₃	6	9.606	11.207	N/A	10.6370	8.1063	11	18	8.68285	14.3438
A ₅₄	3	2.984	11.09	N/A	3.4612	9.8391	3	16	9.78548	5.11418
A ₅₅	49	45.917	35.95	N/A	44.5447	42.7529	42	57	43.2339	44.0755
A ₅₆	1	1	2.194	N/A	1.2428	1.1219	1	1	1.34639	1.03085
A ₅₇	62	62.426	66.171	N/A	76.1124	63.0179	68	51	57.5432	53.3872
A ₅₈	1	2.977	3.34	N/A	11.4119	2.6542	5	7	5.47976	1.36417
A ₅₉	3	1	4.053	N/A	4.7082	1.6685	1	4	1.75849	9.18944
Weight (lb)	143,436	141,241	142,296	138,878	147,841.7416	136,311.1322	139,028.798	138,688.913	135,364.167	137,370.837
Avg. Weight (lb)	N/A	N/A	N/A	139,682	165,168.9424	137,453.6697	182,562.693	142,903.207	136,198.032	-
St. Deviation (lb)	N/A	N/A	N/A	1098	5392.7272	673.8566	42,037.068	3171.403	538.65	-
Con. Violation	None	None	None	N/A	None	None	None	None	None	None
No. of Evaluations	39,834	150,000	N/A	50,000	28,000	28,000	55,700	53,420	38,880	38,880
Avg. Exploration	-	-	-	-	-	-	-	-	6.808%	-
Avg. Exploitation	-	-	-	-	-	-	-	-	93.19%	-

Note: 1 in² = 6.452 cm²; 1 lb. = 0.4536 kg.

Table 17. Friedman rank test results for the 26-story, 942-bar truss tower.

Category	(SA) [67]	(ES) [66]	(GNMS) [68]	(FA) [65]	(GWO) [63]	(IGWO) [63]	(JA) [64]	(IS-JA) [64]	This Study	
									(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Volume)	8	6	7	4	9	2	5	3	1	–
Friedman Rank (Avg.)	8	8	8	3	5	2	6	4	1	–
Friedman Rank (NFE)	4	8	9	5	1.5	1.5	7	6	3	–
Averaged Rank	6.6667	7.333	8	4	5.16667	1.8333	6	4.333	1.6667	–

6.6. A 62-Story, 4666-Bar Spatial Truss Tower

The final benchmark considered in this study is the 4666-bar truss tower (shown in Figure 17). 238 design variables represent the 4666 members that makeup the structure (details available in ref. [69]). The lower and upper limits of sizing variables are set at 1 in² and 300 in², respectively. Allowable member stresses are limited to $\pm 25,000$ psi. Moreover, nodal displacement values are limited to ± 37.5 in (± 92.25 cm) for the top four nodes of the structure, and the elastic modulus of steel ($E = 10,000$ ksi) was used. Therefore, 9344 non-linear design constraints are imposed on the structure (9332 tension/compression and 12 displacement constraints). Three loading categories are considered for this structure:

- Load_1: A vertical load of 6 kips (26.69 kN) acting on all free nodes
- Load_2: Both left and right side nodes of the tower have a 1-kip (44.48 kN) load acting in the X-direction
- Load_3: Both front and back side nodes of the tower have a 1-kip (44.48 kN) load acting in the Y-direction

Using Load_1, Load_2, and Load_3 described earlier, three independent loading conditions are furthermore considered to act in the following manner:

- Load_1 acting alone
- Load_1 and Load_2 acting together
- Load_1, Load_2, and Load_3 all considered simultaneously

Table 18 compares the results provided by MPA against the few algorithms in the literature that attempted to solve the benchmark. After a careful investigation, an upper iteration limit of 1500 (45,000 function evaluations) was found suitable for yielding the best MPA optimization results with the least amount of computational effort. In full adherence to literary practice, the final volume of the optimized structure (in cubic inches) will be reported rather than structural weight. A look at tabulated results shows MPA yielding the best-optimized design with a total volume of 15,544,283 in³ (254.72 m³) in exactly 42,290 function evaluations. Figure 16a shows the MPA convergence history for the current benchmark. Although MPA required greater computational effort to arrive at its final solution (relative to the presented algorithms), the dramatic reduction in total truss volume more than justifies the algorithm's lack of computational speed. Simply put, previous algorithms that attempted to tackle the benchmark, although quicker than MPA, all prematurely converged to a local optimum. It is also worth pointing out that the worst run produced by MPA considerably outperformed the best runs of SAND [69], TLBO [70], iPSO [70], ISA [70], and IFSA [70]. The Friedman rank test results for the 4666-bar truss are shown in Table 19. Tabulated values confirm MPA's strength in finding the minimum truss volume with excellent solution robustness. MPA ranked first in the "optimized volume" and "average volume" categories, while 5th place was achieved for the function evaluation category. Overall, MPA obtained an average rank of 2.333 (second place) among all relevant statistical metrics, while IFSA obtained the first position. Table 20 reports the optimized 238 design variables produced by MPA.

A look at the exploration vs. exploitation tradeoff balance in Figure 16b shows a diversity trend similar to that depicted by the previous benchmark (942-bar truss). Average exploration and exploitation quantities are also similar, hovering around 7% for exploration and 93% for exploitation. It seems that robust exploitation capabilities ensure excellent results for large-scale truss structures.

Table 18. Optimized result comparison for the 4666-bar large-scale truss structure.

Result	(SAND) [69]	(TLBO) [70]	(iPSO) [70]	(ISA) [70]	(IFSA) [70]	This Study	
						(MPA) _{Best}	(MPA) _{Worst}
Volume (in ³)	21,387,963	26,008,542	25,917,009	25,310,333	21,378,918	15,544,283	19,874,604
Avg. Volume (in ³)	N/A	26,113,952	26,439,196	25,922,724	21,496,622	17,285,800	–
St. Deviation (in ³)	N/A	38,279	536,852	112,587	37,001	1,128,612	–
Con. Violation	None	None	None	None	None	None	None
No. of Evaluations	N/A	26,290	24,185	20,585	16,230	43,290	43,830
Avg. Exploration	–	–	–	–	–	6.73%	–
Avg. Exploitation	–	–	–	–	–	93.26%	–

Note: 1 in³ = 16.387 cm³.

Table 19. Friedman rank test results for the 4666-bar large-scale truss structure.

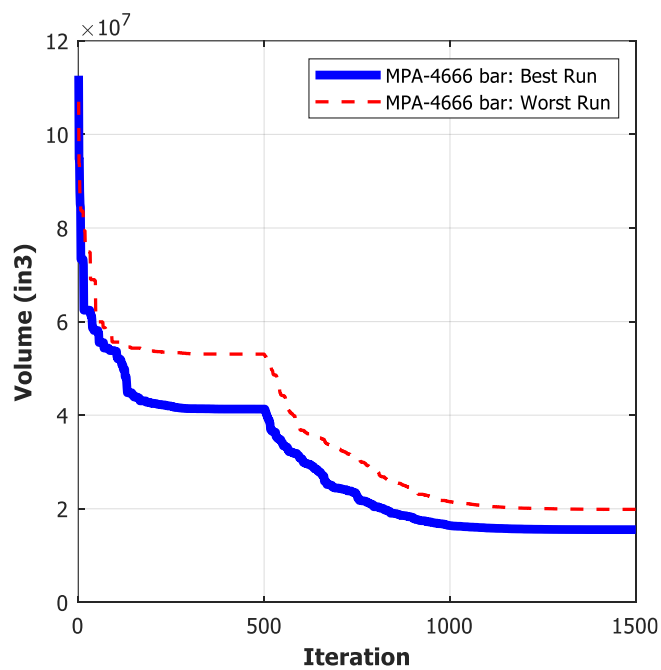
Category	(SAND) [69]	(TLBO) [70]	(iPSO) [70]	(ISA) [70]	(IFSA) [70]	This Study	
						(MPA) _{Best}	(MPA) _{Worst}
Friedman Rank (Volume)	3	6	5	4	2	1	–
Friedman Rank (Avg.)	6	4	5	3	2	1	–
Friedman Rank (NFE)	6	4	3	2	1	5	–
Averaged Rank	5	4.6667	4.333	3	1.6667	2.333	–

Table 20. Optimal member area values for the 62-story, 4666-bar truss tower (in²).

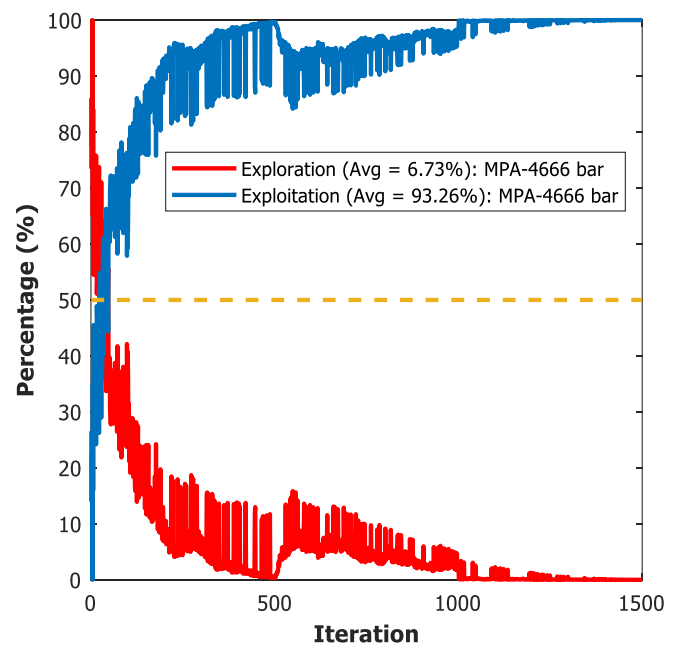
No.	Area	No.	Area	No.	Area	No.	Area	No.	Area	No.	Area	No.	Area
1	20.116	35	4.0431	69	4.6718	103	31.611	137	69.883	171	6.3133	205	10.028
2	1.1044	36	2.5941	70	37.776	104	92.135	138	3.5365	172	16.933	206	1.4531
3	38.866	37	45.979	71	164.90	105	3.5218	139	3.3469	173	4.6768	207	23.367
4	140.56	38	187.59	72	13.761	106	9.1029	140	1.9430	174	21.105	208	1.3975
5	3.9739	39	14.837	73	4.1036	107	2.5971	141	24.326	175	1.8693	209	2.9333
6	150.30	40	17.633	74	1.1231	108	37.163	142	2.2586	176	3.2983	210	7.4084
7	3.8956	41	4.3084	75	58.021	109	11.487	143	3.0357	177	4.2365	211	1.0309
8	30.660	42	53.899	76	4.9744	110	13.662	144	5.1336	178	5.8717	212	14.039
9	23.570	43	2.4716	77	7.6378	111	17.818	145	1.2681	179	7.2094	213	8.6079
10	77.925	44	4.2946	78	2.2080	112	1.3829	146	1.8152	180	12.048	214	3.9626
11	20.792	45	1.1999	79	38.300	113	2.7507	147	4.4062	181	22.809	215	9.5471
12	106.82	46	17.418	80	2.9000	114	14.723	148	48.337	182	1.5512	216	1.1346
13	16.210	47	20.569	81	52.340	115	82.270	149	3.5121	183	16.340	217	1.0745
14	1.7330	48	33.796	82	130.54	116	12.746	150	8.4996	184	5.1433	218	1.8914
15	177.18	49	194.11	83	16.638	117	1.9196	151	11.821	185	38.897	219	8.5074
16	166.84	50	4.3325	84	27.689	118	1.1833	152	7.6146	186	1.0967	220	4.7592
17	20.859	51	8.5119	85	1.3209	119	27.909	153	3.8642	187	11.433	221	1.0713
18	1.5684	52	1.0001	86	7.6958	120	1.0118	154	2.3644	188	1.1006	222	6.2481
19	14.149	53	25.585	87	10.040	121	1.1175	155	1.3693	189	2.9465	223	16.682
20	28.801	54	1.0897	88	10.579	122	3.0358	156	3.4809	190	1.0002	224	25.669

Table 20. Cont.

No.	Area	No.	Area	No.	Area	No.	Area	No.	Area	No.	Area	No.	Area
21	1.5801	55	11.126	89	34.231	123	3.3329	157	4.9570	191	11.338	225	16.837
22	42.627	56	29.010	90	3.4340	124	14.835	158	8.8991	192	23.480	226	2.9196
23	1.9921	57	7.9625	91	20.351	125	25.896	159	72.112	193	13.680	227	5.4858
24	1.0028	58	11.438	92	71.286	126	90.783	160	13.667	194	2.7432	228	1.7650
25	3.1229	59	5.7067	93	102.65	127	2.1534	161	6.6525	195	30.816	229	7.9808
26	134.74	60	178.61	94	2.6283	128	7.2467	162	1.0295	196	15.207	230	6.1468
27	183.90	61	13.874	95	18.130	129	11.357	163	16.426	197	17.854	231	3.4042
28	5.1044	62	3.0329	96	1.3578	130	65.825	164	3.4988	198	22.070	232	2.7263
29	11.801	63	19.366	97	9.9382	131	3.6184	165	25.416	199	3.0214	233	1.2352
30	4.5574	64	20.707	98	1.1636	132	6.8587	166	1.1646	200	1.1445	234	7.9640
31	44.651	65	8.8305	99	5.2479	133	2.9426	167	6.5469	201	19.085	235	7.0577
32	1.5231	66	16.572	100	1.7846	134	2.8983	168	15.840	202	37.360	236	2.5718
33	10.636	67	3.8128	101	5.0023	135	3.6839	169	4.1320	203	27.566	237	6.0137
34	1.2642	68	2.8722	102	7.6903	136	15.625	170	68.855	204	1.1956	238	34.052



(a)



(b)

Figure 16. (a) Convergence history for the 4666-bar truss structure. (b) Exploration to exploitation tradeoff trend for the 4666-bar truss tower.

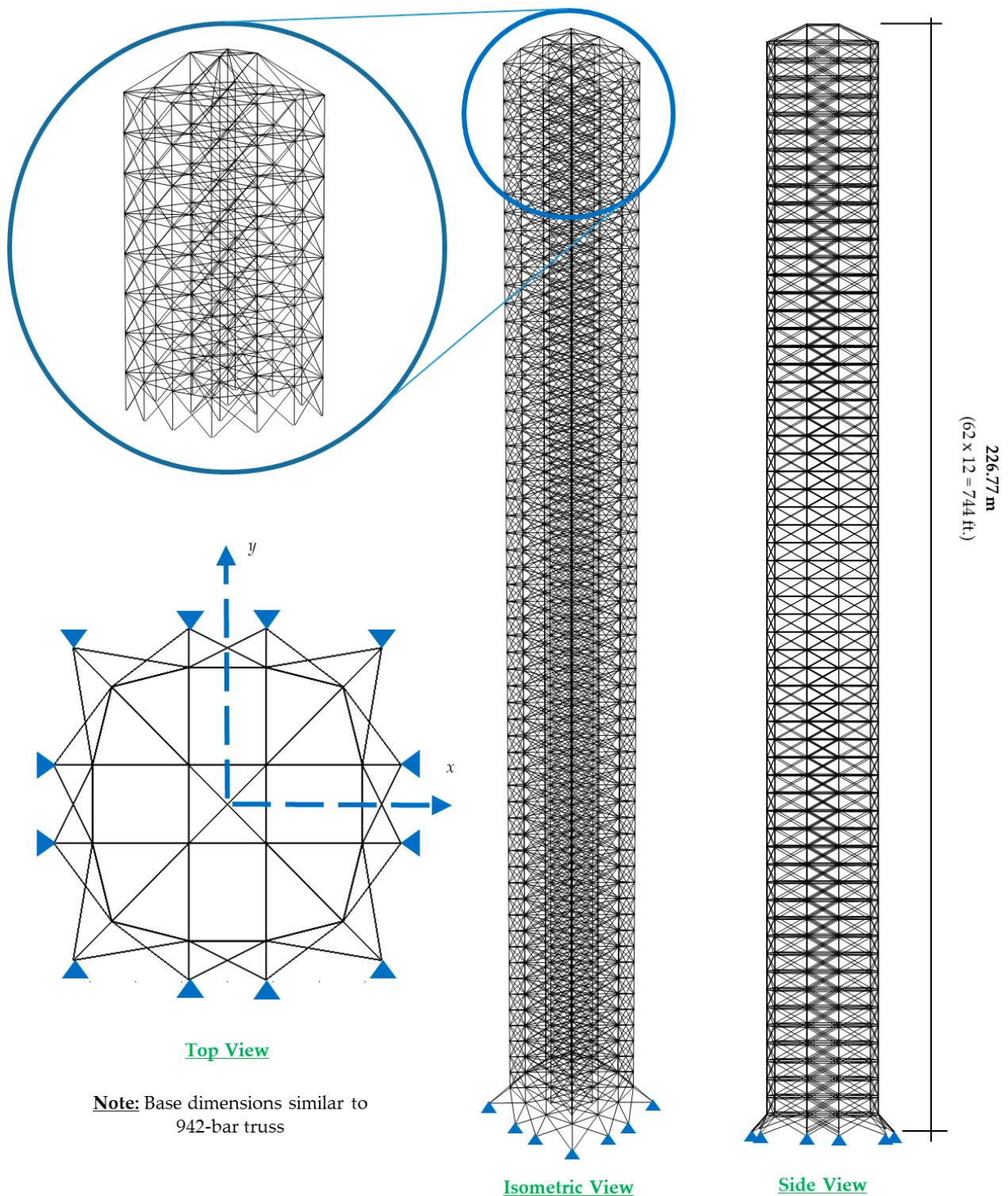


Figure 17. Schematic layout of the 62-story, 4666-bar truss tower.

7. Overview of MPA Structural Optimization Performance

This section summarizes MPA performance in relation to the diversity analysis and truss optimization results presented earlier. Overall, MPA showed excellent optimization capabilities when applied to structural optimization. However, some shortcomings were manifest, especially regarding large-scale truss optimization. The following points summarize the core findings of this study:

1. For normal-sized truss structures (10-bar, 60-bar, and 120-bar trusses), MPA yielded the lightest truss designs with the least amount of computational effort. Excellent solution robustness was also observed by dint of the lower average and standard deviation values for the 20 independent runs conducted. The Friedman ranking results numerically showcased this observation with the proposed algorithm achieving first place in all categories. Furthermore, in some instances, MPA's worst run outperformed the best runs reported by other recently proposed algorithms.
2. For the large-scale truss structures (272-bar, 942-bar, and 4666-bar structures), MPA still displayed excellent results. However, it was observed that more objective function evaluations were required for convergence. Nevertheless, the dramatically reduced weights produced by the algorithm fully compensates for it. Again, the Friedman rank results quantify this observation. In terms of algorithmic robustness, MPA showed either as-good-as or better performance than the other algorithms used to solve the benchmarks.
3. The proper combination of exploration and exploitation required to achieve MPA results varied by a number of degrees from one benchmark to another. However, the variation was mostly confined to a range of 6~9% for exploration and 90~93% for exploitation. *Ignoring these slight discrepancies nonetheless, it can be concluded that the correct amount of exploration and exploitation that any algorithm should possess to produce good results is around 10% exploration and 90 % exploitation.* Therefore, an *exploitation-dominated* metaheuristic is expected to perform better than those that are more exploration-oriented.
4. The incorporation of both Brownian and Lévy motion models as random number generators allowed MPA to both explore and exploit various areas of the design space thoroughly. The diversity spikes in the exploration vs. exploitation tradeoff figures visually show this. The incorporation of the Lévy flight/walk model in existing or new metaheuristic algorithms as an added exploitation tactic is, therefore, recommended.

Limitations of MPA Truss Optimization Performance

Any discussion concerning MPA performance would be lacking if the limitations experienced during its operation were not highlighted. For the most part (and based off the truss optimization results presented earlier), MPA was shown to experience slow convergence for large-scale truss benchmarks (high-dimensional problems). The author possibly attributes this to MPA's basic partitioning of its optimization sequence into three equal phases. The algorithm was seen to stagnate during the final iterations of Phase I. More specifically, little improvement in results was reported in the second half of Phase I. This can be clearly seen with the 942-bar and 4666-bar truss problems. Convergence curves show a pronounced stagnation period during the last portions of Phase I and the beginning of Phase II (see Figures 15a and 16a). Based on this observation, it can be surmised that a prolonged exploration activity for Phase I unnecessarily adds to the computational complexity experienced by the algorithm. This is in full conformity with the findings of the study showing the greater importance of "exploitation" rather than "exploration" for truss optimization problems. An alteration of the time-control mechanism inherent to MPA is subsequently recommended for greater convergence speeds. Dividing the three phases equally does not represent MPA optimality.

Furthermore, in some runs, the mathematical formulation for Phase III of MPA (pure exploitation) proved to be insufficient at improving final results (see Equation (11)). This was especially the case for the large-scale benchmark category. The exploitation formulation for Phase II of the algorithm can either be altered or incorporated into Phase III.

Finally, from the structural optimization experiments conducted in this study, it was found that MPA's weakness lies in its convergence speeds. Future studies are recommended to focus on rectifying the time-control mechanism and to try newer mathematical formulations for the "exploitation" phase of MPA.

8. Conclusions

In this study, the optimal design of truss structures is considered (for the first time) using the newly developed Marine Predators Algorithm (MPA). Six challenging structural benchmarks were selected from literature to investigate MPA performance.

Results show MPA is particularly suited for the medium-sized truss structure category, where optimal results were achieved with the least amount of computational effort. In contrast, for the large-scale benchmarks, MPA was shown to require a greater number of function evaluations to converge (relative to other methods). Nevertheless, the much lighter truss designs produced by the algorithm completely justified the slower performance. The experimental sections repeatedly emphasized this for the 272-bar, 942-bar, and 4666-bar truss towers. In addition, studying the diversity of MPA candidate solutions across the search space indicated the dominance of the Lévy motion strategy. This scheme allowed MPA to avoid being snarled into local optima. This was especially useful given the challenging multi-modal nature of the design search space associated with structural optimization problems. Therefore, the notion that Lévy-based MH algorithms are well suited for truss optimization problems was verified in this study. Similarly, owing to its robust Brownian-motion exploration mechanisms, the proposed algorithm successfully located promising regions of the search space, which only enhanced solution quality and stability. This observation was found to hold true for all the benchmarks considered.

Moreover, for the first time in the field, a numerical assessment of MPA's exploration to exploitation balance (in the context of truss optimization) was conducted. Results show that a combination of 10% exploration and 90% exploitation is favored for truss optimization problems. Therefore, exploitation is more important than exploration for structural optimization tasks.

In conclusion, the excellent performance of MPA along with its relatively few performance parameters make it a robust optimization metaheuristic when applied to truss sizing problems. Readers are encouraged to extend the application of MPA to other structural optimization tasks, such as frames, plates, shells, etc.

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