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# Mathematical Theory of Conflicts as a Cognitive Control Theory

**Ekaterina Antipova [1] and Sergey Rashkovskiy [2,*]**

[1] School of Information Systems, The State University of Management, 99 Ryazansky Prospekt, 109542 Moscow, Russia
[2] Ishlinsky Institute for Problems in Mechanics RAS, Vernadskogo Ave., 101/1, 119526 Moscow, Russia
* Correspondence: rash@ipmnet.ru; Tel.: +7-906-0318854

**Abstract:** We give a rigorous mathematical definition of conflict, on the basis of which we formulate the mathematical theory of conflicts as a problem of the theory of cognitive control. Possible ways of influencing the conflicting parties on each other are considered and analyzed. The analysis carried out shows that the control of a conflict situation is fundamentally different from the control of technical objects. So, when controlling technical objects, it is usually possible to directly influence the reason that causes error (deviation) in the system. In a conflict situation, there is often no opportunity to directly influence the opposite side of the conflict. However, each of the conflicting parties has the ability to change its own parameters and, thereby, create a conflict for the opposite side, which is forced to change its parameters to those necessary for the opponent in order to resolve its own conflict. Within the framework of the developed theory, the conflict between the worker and the employer is considered, and this conflict is analyzed from the point of view of the cognitive control theory.

**Keywords:** conflict; conflicting parties; mathematical theory of conflicts; theory of cognitive control; control actions



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## 1. Introduction

Currently, interdisciplinary research is actively developing, when the apparatus or ideas of one science are used to solve problems in another. An example is the use of physics methods to describe processes in social systems [1–9].

Conflicts are the driving force behind the development of society. The whole history of the development of human civilization and its individual components is a series of different conflicts: at each stage of the development of society, people are looking for ways to resolve existing conflicts, which leads to the evolution of society, and, at the same time, gives rise to new conflicts [10].

There are different types of conflicts: military, political, ethnolinguistic, social, industrial, family, etc., but they all have common patterns of development.

Conflictology [11,12], which arose at the junction of several scientific areas [13], is engaged in a systematic study of conflicts.

Currently, various approaches to the study of conflicts and their management are used [12–37]. Most often, conflicts from different points of view are studied within the framework of such sciences as philosophy, sociology, psychology, and political science [12,14,16]. Within these areas, methods have been developed for analyzing relevant conflict situations, as well as recommendations for resolving conflicts of various types [11,12,14–16].

Basically, these studies are of a descriptive, philosophical nature, laying the foundation for solving various applied problems. However, they do not have predictive power and do not allow, based on the available empirical data, to predict a possible conflict situation, describe its development over time, and calculate acceptable ways out of it.

From a predictive point of view, the mathematical modeling of conflicts is of the greatest interest.

Currently, various mathematical methods are used to model conflicts: game theory [17–20], dynamic systems theory [21–24], stochastic models [25–27], probabilistic and statistical methods [28–30], logical models [31–34], and algebraic models [35,36], etc. We also note the approach based on the analysis of hierarchies [37].

It should be noted that each model of conflicts, in fact, uses its own definition of a conflict and even its own idea of what to call a conflict. Currently, there are several definitions of conflict that consider this phenomenon from a philosophical or psychological point of view [12,15,38–42].

For example, according to [12], conflict is a relationship between two or more parties who believe they have incompatible goals or interests.

According to [15], conflict may be defined as a struggle or contest between people with opposing needs, ideas, beliefs, values, or goals.

In [41], conflict is defined as "the simultaneous occurrence of two or more mutually antagonistic impulses or motives".

In behavioral terms [42], conflict is viewed as an action, which prevents, obstructs, interferes with, injures, or renders ineffective another action with which it is incompatible.

Despite the absolute usefulness and legitimacy of these definitions, it should be noted that they do not allow formalizing the conflict and move on to its mathematical description.

In this paper, we propose a general formal definition of conflict, which allows us to proceed to the mathematical description of any conflict situations.

## 2. Formalization of the Conflict

First of all, we note that the presence of two or more mutually antagonistic impulses or motives (contradictory options, alternatives) [41] is not yet a conflict.

For example, a person has two mutually exclusive options (two alternatives) on how to spend the evening: stay at home or go to the theater. This is not yet a conflict. A conflict will arise when this person has a certain desire (for example, he wants to stay at home), while he is forced to do what he does not want (for example, to go to the theater), i.e., when his desire does not coincide with what he has or should do.

Let us give a definition of conflict, from which we will proceed further in our reasoning, and which allows us to proceed to a formal mathematical description of conflicts.

Conflict is a discrepancy between the desire of the person and his capabilities, i.e., between what he wants and what he has.

Thus, we believe that any situation in which the desire (goal) of the person does not coincide with what he currently has, can be called a conflict, regardless of the specific area of the person's activity in which this discrepancy arose.

According to this definition, any conflict is characterized by at least two parameters: what the person wants (i.e., his desire, goal) $W$, and what he currently has, $H$.

Thus, the cause (driving force) of the conflict is discrepancy:

$$u = \begin{cases} [W - H], \text{ if the goal is } H \geq W \\ [H - W], \text{ if the goal is } H \leq W \end{cases} \quad (1)$$

where $[x] = x$ for $x \geq 0$ and $[x] = 0$ for $x < 0$.

By definition, it is considered that in the absence of a conflict $u = 0$, while a conflict takes place if $u > 0$.

Obviously, according to this definition, any conflict can be considered as cognitive dissonance [43,44], and discrepancy (1) is a mathematical expression of the degree (intensity) of dissonance.

The subject of the conflict is objectively existing or non-existing subjects or objects with which the discrepancy (1) is associated.

In conflictology, when describing and analyzing conflicts, one usually speaks of a conflict between two or more persons [11,12,38–42]. It follows from the definition given above that it is more correct to say that these persons have a common subject of conflict, while each of the conflicting persons has its own conflict: each of them has its own discrepancy $u$,

and, therefore, its own the degree (intensity) of the conflict. In addition, different persons may have different attitudes towards the same conflict (discrepancy), and, consequently, for each of them the same conflict has a different significance. In other words, when talking about a conflict between two or more persons, it must be borne in mind that each of them has its own (in terms of significance and intensity) conflict associated with the same subject, and each of them tries to resolve its own conflict in a different way, taking into account the available opportunities, circumstances, and the reaction of other persons.

In this regard, further we will distinguish between a conflict and a conflict situation.

A conflict situation is a situation (state of the system, strategy) in which several persons participate that have a common subject of conflict, while different persons may have different conflict parameters $W$ and $H$, different discrepancies $u$, and, therefore, different reasons (driving forces) of the conflicts.

Conflicting parties (participants in the conflict) are the persons involved in this conflict situation.

Thus, any conflict situation is characterized by a full set of parameters $\{W_i,\ H_i\}$ and, accordingly, discrepancies $\{u_i\}$ of all participants in the conflict situation (conflicting parties).

The conflict parameters $W$ and $H$, and with them discrepancy $u$, can take both continuous and discrete values.

For this reason, one can speak of conflict with continuous discrepancy and conflict with discrete discrepancy. In the latter case, discrepancy $u$ is a boolean variable, and can only take two values, 0 or 1, depending on whether the person's desire is satisfied or not.

Conflict parameters can be internal and external. The internal parameters of the conflict are inherent in the person himself, and the person can always influence these parameters (i.e., he can change the corresponding parameter by applying various efforts—psychological, intellectual, physical, material, economic, political, etc.), although in some cases, this influence may be limited for various reasons. External parameters are inherent in other persons or objects, and this person either does not have the ability to directly influence these parameters (i.e., he does not have the ability to directly change them, but at the same time he can influence them indirectly, through other persons or objects), or his influence on these parameters is limited.

The parameter $W$ always belongs to the given person (i.e., it is his internal parameter), in the sense that this person can change it directly (i.e., the parameter $W$ is controlled directly by the person itself). At the same time, the parameter $H$ can either belong to the given person (being an internal parameter of the person) or not belong to him (being an external parameter for the given person). Depending on this, we will distinguish between internal and external conflicts.

Internal conflict (conflict with internal discrepancy)—when the $H$ parameter belongs to the same person as the $W$ parameter, i.e., this person has the ability to independently and directly change the parameter $H$. Example 1. A certain person is (or it seems to him that he is) overweight; this person has weight $H$, but wants to have weight $W < H$ (i.e., wants to lose weight); as a result, he has an internal conflict associated with his weight (the subject of the conflict is person's weight). In this case, he has two ways to resolve this conflict: (i) accept his own weight, i.e., increase your parameter $W$; (ii) make efforts to reduce your own weight $H$, such as sports, diet, etc.

External conflict (conflict with external discrepancy)—when this person does not have the ability to directly change parameter $H$. For example, when parameter $H$ belongs to another person or is controlled by another person that can directly change this parameter $H$. Example 2. A worker wants to receive salary $W$, while the employer pays him a salary $H < W$. As a result, the worker has a conflict associated with discrepancy (1). In this case, the worker cannot directly change his salary $H$; only the employer can do this, while the worker can exert various influences on the employer in order to force him to raise his salary. Note that as long as the worker does not influence the employer in order to force him to raise his salary, the employer has no conflict in this situation. Example 3. The child wants something ($W = 1$), but the parents do not give it to him or forbid it ($H = 0$). The child

himself cannot change the parameter $H$, but he can influence the parents (for example, by his behavior) to force them to change this parameter. Another type of external conflict is when the parameter $H$ is caused by external force majeure circumstances (for example, when what the person wants is limited or prohibited by the laws of nature). In this case, the only way for a person to resolve this conflict is adaptation, i.e., changing his parameter $W$ in order to bring it closer to the external parameter $H$ or even make it equal to the external parameter $H$.

## 3. Interaction of Subjects in a Conflict Situation

According to the definition given above, conflict arises only when a person has a goal that does not coincide with the existing state, and there is a desire to achieve this goal. Indeed, the mere presence of a goal that cannot be achieved without effort, or the presence of a contradiction (obstacle) that cannot be overcome without effort, is only a necessary but not sufficient condition for a conflict to arise. The second necessary condition for the emergence of a conflict is the desire of the person to achieve the goal or the desire of the person to overcome the contradiction that has arisen.

Thus, the conflict that has arisen in the person, by definition, causes him a reaction (effort, desire) aimed at resolving this conflict, i.e., to reduce the existing discrepancy.

According to (1), he can achieve this either by a corresponding change in the parameter $W$ (I want), or by a corresponding change in the parameter $H$ (I have). Thus, the search for ways to resolve any conflict ultimately comes down to the search for methods, ways, and means of appropriately changing the conflict parameters $W$ and $H$.

In particular, to resolve an external conflict at a constant $W$, the person must have such an impact on the opponent controlling the parameter $H$ that the opponent is forced (wanted, agreed) to change this parameter in the direction necessary for the person. There is only one way to influence the opponent: to create a conflict for him, resolving which the opponent will be forced to change the parameter $H$ controlled by him in the direction necessary for this person.

Figure 1 shows a conflict control scheme with two participants and two external and two internal conflicts.

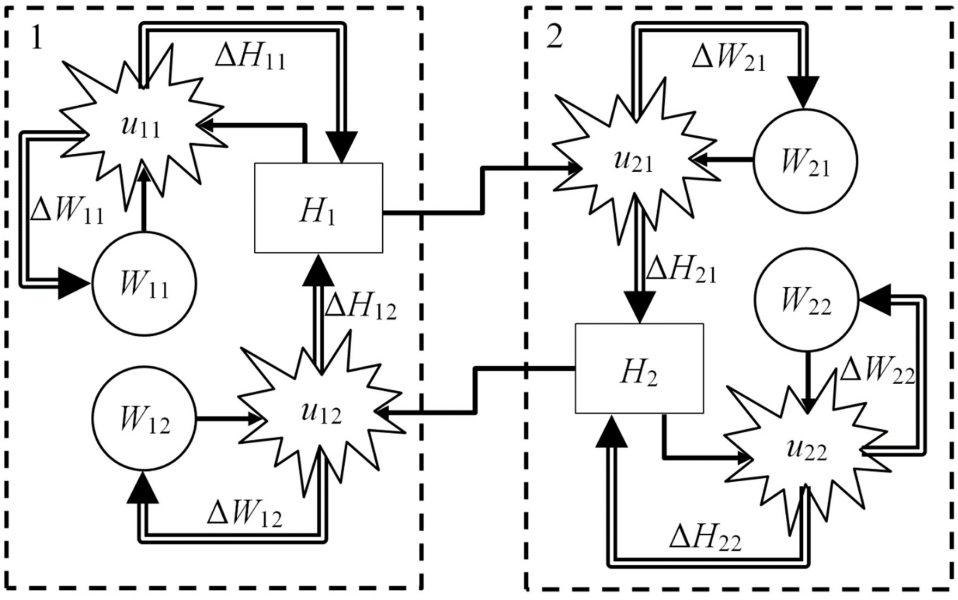


**Figure 1.** Scheme of control of the conflict situation with two participants and two external and two internal conflicts. The dashed frames indicate the persons (participants in the conflict situation). The stars show the conflicts (discrepancies); the double arrows show efforts made to change the parameters of the conflict. $\Delta W_{ij}$ and $\Delta H_{ij}$ are the control efforts applied by the person $i$ and aimed at changing its own parameters $W_{ij}$ and $H_i$.

Suppose that participant 1 controls parameter $H_1$ (i.e., he can directly change it), which affects participant 2, while participant 2 controls parameter $H_2$, which affects participant 1. For definiteness, we assume that participant 1 wants the parameter $H_2$ to be set to $H_2 \geq W_{12}$, while participant 2 wants parameter $H_1$ to be set to $H_1 \geq W_{21}$. If these conditions are not met, in the system under consideration, there are two external conflicts associated with discrepancies $u_{12}$ and $u_{21}$: participant 1 has discrepancy $u_{12} = [W_{12} - H_2]$, while participant 2 has discrepancy $u_{21} = [W_{21} - H_1]$. By changing the $H_1$ and $H_2$ parameters, the participants in this conflict situation can influence each other, forcing the opponent to change their $H_i$ parameter in the right direction: if you increase your $H$ parameter to reduce my discrepancy (conflict), then I can increase my $H$ parameter, to reduce your discrepancy (conflict), and vice versa, if you decrease your $H$ parameter, increasing my discrepancy (increasing my conflict), then I can decrease my $H$ parameter, increasing your discrepancy (increasing your conflict).

At the same time, each of the participants in this conflict situation has his own idea of what his $H_i$ parameter should be.

For example, consider participant 1.

Depending on the point of view of participant 1 and the relationship between parameters $W_{11}$ and $W_{21}$, the following situations are possible (Figure 2).

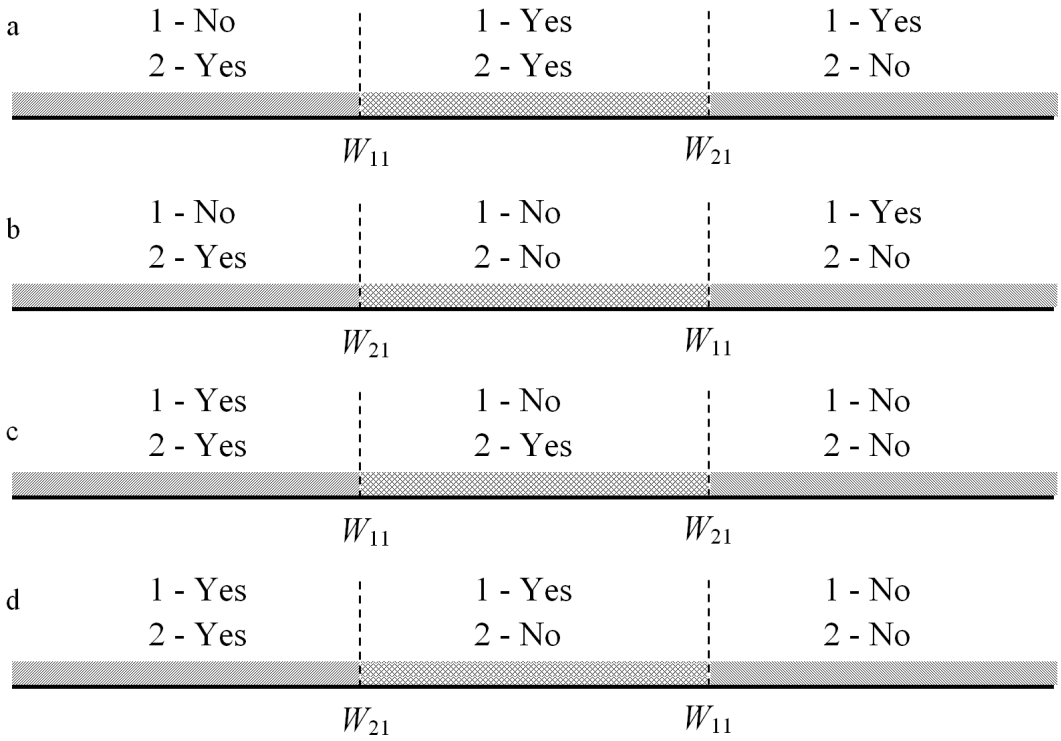


**Figure 2.** Scheme explaining the various options for the conflict situation shown in Figure 1, when parameter $H_1$ is changed by participant 1: (**a**)—situation $A_{11}$; (**b**)—situation $A_{12}$; (**c**)—situation $A_{21}$; (**d**)—situation $A_{22}$. In each area of change of the $H_1$ parameter for each participant ($i$ = 1,2), it is indicated whether it has a conflict associated with the $H_1$ parameter (Yes) or this conflict does not exist (No).

Option $A_1$: Let participant 1 want his parameter $H_1$ to take the values $H_1 \leq W_{11}$. In this case, participant 1 has an internal discrepancy $u_{11} = [H_1 - W_{11}]$ and an internal conflict associated with it.

Two situations are possible:

Situation $A_{11}$: $W_{11} < W_{21}$ (Figure 2a). In this case, whatever the parameter $H_1$ is, in the system under consideration there is always at least one conflict, while in the range

$W_{11} < H_1 < W_{21}$, there are both an internal conflict for participant 1 and an external conflict for participant 2.

Situation $A_{12}$: $W_{11} > W_{21}$ (Figure 2b). In this case, there is a limited range of variation of the $H_1$ parameter: $W_{11} < H_1 < W_{21}$, in which both the internal conflict of the participant 1 and the external conflict of the participant 2 are simultaneously absent. Thus, by controlling the $H_1$ parameter, the participant 1 can achieve a conflict-free situation associated with the parameter $H_1$.

Option $A_2$: Let participant 1 want his parameter $H_1$ to take the values $H_1 \geq W_{11}$. In this case, participant 1 has an internal discrepancy $u_{11} = [W_{11} - H_1]$ and an internal conflict associated with it.

Two situations are possible:

Situation $A_{21}$: $W_{11} < W_{21}$ (Figure 2c). In this case, by controlling the $H_1$ parameter, participant 1 can simultaneously resolve both its own internal conflict and the external conflict of participant 2 by making $H_1 > W_{21}$.

Situation $A_{22}$: $W_{11} > W_{21}$ (Figure 2d). In this case, by controlling the $H_1$ parameter, participant 1 can simultaneously resolve both its own internal conflict and the external conflict of participant 2 by making $H_1 > W_{11}$.

Thus, by controlling the $H_1$ parameter, participant 1 can influence participant 2, creating or eliminating an external conflict for him, and thereby forcing him to change his $H_2$ parameter in the direction necessary for participant 1. At the same time, such an impact is limited, because it can lead to internal conflict for participant 1.

Participant 2 can be considered similarly (situations $B_{11}$, $B_{12}$, $B_{21}$ and $B_{22}$).

The conflict situation shown in Figure 1, in the general case, is a combination of possible options for participants 1 and 2.

Thus, by changing their parameters $H_1$ and $H_2$, the participants in the conflict situation can influence the opponent, stimulating him to change the parameters he controls.

At the same time, this can lead to the emergence of an internal conflict for each participant in the conflict situation. At the same time, as can be seen from Figure 2, each of the participants cannot always achieve a conflict-free situation. In this case, each participant in the conflict situation must decide for himself what is more important for him: eliminate his internal conflict and, possibly, get an external conflict, or eliminate the external conflict, but at the same time get an internal conflict. Depending on this, each participant in the conflict situation chooses a behavior strategy for himself: to create an external conflict for the opponent (in order to force him to change the $H_i$ parameter that he controls) or not create it.

Thus, each participant in a conflict situation decides for himself which of the possible conflicts (internal or external) is more significant for him, and which is less.

This means that each conflict can be quantitatively characterized by a certain indicator of significance, showing how significant this conflict is for a given participant in the conflict situation. The higher the indicator of the significance of any conflict for a given participant, the more important it is for him to resolve this conflict.

In this case, for each participant $i$ of the conflict situation, one can introduce a function that characterizes the level of conflict (or dissatisfaction with the current situation as a whole), which for the conflict situation shown in Figure 1 has the form:

$$U_i = \mu_{i1} u_{i1} + \mu_{i2} u_{i2}, \;\; i = 1, 2 \tag{2}$$

where $\mu_{ij}$ is the coefficient (factor) of significance of the $j$-th conflict for the $i$-th participant in the conflict situation. For rational participants in the conflict situation, $\mu_{ij} > 0$. Parameters $\mu_{ij}$ are individual psychological (cognitive) characteristics of the $i$-th participant in this conflict situation. In the general case, these parameters are not constant even for the same person: they can change over time, as a result of which some conflicts that were previously considered insignificant for him may become significant over time (or in other circumstances), and on the contrary, other conflicts that were previously fundamental

for this person may become insignificant. In any case, each subject himself explicitly or implicitly sets for himself the scale of priorities, and, hence, the values of the parameters $\mu_{ij}$.

For all $\mu_{ij} \geq 0$, function (2) is non-negative, and its minimum $U_i = 0$ corresponds to the state when the $i$-th participant is completely satisfied with the existing situation, i.e., does not have any significant conflicts for him. The greater the value of the $U_i$ function, the greater the dissatisfaction of the $i$-th participant in this conflict situation, and, one can expect, the more he will make efforts to resolve existing conflicts.

Function (2) can be called the function of dissatisfaction with the conflict situation (dissatisfaction function, conflict function) for the $i$-th participant.

The goal of any person is to minimize his own dissatisfaction function by changing the parameters $W$ and $H$ controlled by him.

Thus, the problem of resolving a conflict situation is reduced to the variation of all conflict parameters $\{W_i,\ H_i\}$, taking into account the existing restrictions, and the search for such a combination of these parameters, in which the functions (2) for all participants in the conflict situation will be minimal (ideally, equal to zero).

In the general case, the conflicting parties fail to achieve a conflict-free situation, i.e., situations when $U_i = 0$ for all $U_i = 0$. In this case, one can talk about unresolvable conflicts. Otherwise, the conflicts are resolvable.

The conflict situation as a whole can be characterized by the total dissatisfaction function:

$$U = b_1 U_1 + b_2 U_2 \tag{3}$$

where $b_1,\ b_2 \geq 0$ are the significance coefficients of each participant in a given conflict situation, which can be normalized, for example, by taking $b_1 + b_2 = 1$.

For a conflict-free situation $U = 0$. The greater the value of the function $U$, the more conflicting this situation is. It is possible to set the problem to achieve the minimum value of the function $U$, however, this does not mean that all participants will be satisfied with this situation, because each person seeks to minimize its function $U_i$ as far as possible.

One can say that the minimization of function (3) is the point of view of an external observer who is not involved in this conflict situation, while the minimization of each function (2) is the point of view of each participant in this conflict situation.

## 4. Dynamics of the Conflict

It is of interest to consider the dynamics of the conflict, i.e., the development of the conflict in time in the process of interaction of the conflicting parties.

Consider an example of a conflict with continuous discrepancy (conflicts with discrete discrepancy will be considered in future papers).

Let a worker perform some work and receive a certain salary for this, however, this salary does not suit him, because he wants to get more for his work. In turn, the employer is not satisfied with the quality of the worker's work, or he wants the worker to work more. Each of them has discrepancy between what they want and what they have. Let the worker's discrepancy be $u_1 = u_{12} = [W_{12} - H_2]$, where $W_{12}$ is the worker's desired salary, $H_2$ is the worker's real salary paid by the employer. The discrepancy of the employer $u_2 = u_{21} = [W_{21} - H_1]$, where $W_{21}$ is the employer's desired quality or volume of work performed by the worker, $H_1$ is the actual quality or volume of work performed by the worker.

Note that this problem can be considered from an economic point of view, considering the profit received by the employer as the target function. However, we are not interested in an economic solution, but a solution from the point of view of conflict theory, when the goals $W_{12}$ and $W_{21}$ of the participants in the conflict situation are defined and do not change in the process of conflict development.

We will assume that contacts between subjects (worker and employer) occur periodically, and at each contact, each of them can change their parameters $H_i$ depending on the existing discrepancies. As a result, they approach the next contact with the parameters

$H_1 + \Delta H_1$ and $H_2 + \Delta H_2$, where $\Delta H_1$ and $\Delta H_2$ are changes in the corresponding conflict parameters per one contact.

In this case, taking into account (1), the dynamics of the conflict is described by the equations:

$$u_1^{(n)} = \left[u_1^{(n-1)} - \Delta H_2\right] \tag{4}$$

$$u_2^{(n)} = \left[u_2^{(n-1)} - \Delta H_1\right] \tag{5}$$

where the superscript means the contact number (cycle of interaction) of the persons.

In general, $\Delta H_1 = \Delta H_1(u_1,\ u_2)$; $\Delta H_2 = \Delta H_2(u_1,\ u_2)$, i.e., persons make decisions to change their $H_i$ parameters, taking into account not only their own discrepancy, but also the opponent's discrepancy. Depending on the type of functions $\Delta H_1(u_1, u_2)$ and $\Delta H_2(u_1, u_2)$, the conflict (discrepancy) for each person can either increase or decrease.

Consider the simplest model of the development of a conflict situation, when:

$$\Delta H_1(u_1\ , u_2) = k_{11}u_1 + k_{12}u_2;\ \Delta H_2(u_2, u_2) = k_{21}u_1 + k_{22}u_2 \tag{6}$$

where $k_{ij}$ are constant parameters characterizing the $i$-th person.

In this case, Equations (4) and (5) take the form:

$$u_1^{(n)} = \left[(1 - k_{21})u_1^{(n-1)} - k_{22}u_2^{(n-1)}\right] \tag{7}$$

$$u_2^{(n)} = \left[(1 - k_{12})u_2^{(n-1)} - k_{11}u_1^{(n-1)}\right] \tag{8}$$

From Definition (6), the meaning of the parameters $k_{ij}$, which characterize the psychology (behavior strategy) of persons in a given conflict situation, is clear. Parameters $k_{11}$ and $k_{22}$ characterize the intensity of the influence of the corresponding person on the opponent, taking into account the intensity of their own conflict (discrepancy). These parameters reflect the rationality of the respective person. The parameters $k_{12}$ and $k_{21}$ show that the corresponding person, when influencing the opponent, also takes into account the conflict (discrepancy) of the opponent. These parameters characterize the tolerance of the corresponding person in relation to his opponent (egoist, altruist, and misanthrope). Thus, the parameters $k_{12}$ and $k_{21}$ characterize the internal conflicts of the participants in the conflict situation.

When $k_{11} < 0$, the lower the salary the worker receives (the more $u_1$), the worse he works (the more $u_2$). In this case, the worker influences the employer, showing him that by raising salary, he will stimulate the worker to work better.

When $k_{22} < 0$, the worse the worker works (the more $u_2$), the less the employer pays him (the more $u_1$). In this case, the employer influences the worker, showing him that by working better, he will receive a higher salary.

When $k_{11} > 0$, the lower the salary the worker receives (the higher $u_1$), the better he works (the lower $u_2$). This case seems irrational, but it can occur when the worker hopes that if he will work better, the employer will notice this and increase his salary.

When $k_{22} > 0$, the worse the worker works (the more $u_2$), the more the employer pays him (the less $u_1$). This case also seems irrational, but it can occur when the employer hopes that if he pays the worker more, the worker will work better.

Thus, the parameters $k_{11}$ and $k_{22}$ describe (characterize) the strategy of the impact of one person on another when the personss are trying to resolve their own conflict (feedback).

When $k_{21} > 0$, the lower the worker's salary (the greater $u_1$), the more the employer raises his salary (reduces discrepancy $u_1$). That is, the employer goes to meet the worker, regardless of how the worker works.

When $k_{12} > 0$, the worse the worker works (the more $u_2$), the faster he improves the quality of his work (reduces discrepancy $u_2$). That is, the worker goes towards the employer, regardless of what salary he pays.

When $k_{21} < 0$, the employer reduces the worker's salary, for example, believing that he gets too much, or showing the worker that the employer is not interested in him.

When $k_{12} < 0$, the worker reduces the quality of work, for example, believing that he does too much anyway, or showing the employer that he does not hold on to this job.

There are several problems of modeling conflict situations:

(i) find the behavior strategies of the given person (i.e., the values of his $k_{ij}$ parameters) under which it can resolve his conflict if the behavior strategies (i.e., the $k_{ij}$ parameters) of the other participants in the conflict situation are given and are not changed;

(ii) find such behavioral strategies (i.e., parameters $k_{ij}$) of all participants in the conflict situation, under which all existing conflicts can be resolved;

(iii) find such behavioral strategies (i.e., parameters $k_{ij}$) of all participants in the conflict situation, in which it is possible to resolve certain conflicts, and at the same time strengthen the rest (for example, resolve own conflict, but at the same time create a conflict for the opponent).

## 5. Model Analysis

Let us consider at what values of the parameters $k_{ij}$ (i.e., at what strategies of the conflicting parties) the system will come to a stationary conflict-free state $u_1 = 0$ and $u_2 = 0$, and at what values of the parameters $k_{ij}$ conflicts existing in the system cannot be resolved, and how they will be in this case develop.

The solution of the system of Equations (7) and (8) in the general case has the form:

$$u_1^{(n)} = a\lambda^n;\ u_2^{(n)} = b\lambda^n \tag{9}$$

where $a$ and $b$ are the constants determined by the initial conditions, while the exponent $\lambda$ satisfies the characteristic equation:

$$(\lambda - 1)^2 + (k_{12} + k_{21})(\lambda - 1) - k_{11}k_{22} + k_{12}k_{21} = 0 \tag{10}$$

and takes the values:

$$\lambda_{1,2} = 1 - \frac{1}{2}(k_{12} + k_{21}) \pm \frac{1}{2}\sqrt{(k_{12} - k_{21})^2 + 4k_{11}k_{22}} \tag{11}$$

Consider first the case when $4k_{11}k_{22} \geq -(k_{12} - k_{21})^2$ and the parameters $\lambda$ are real-valued.

Taking into account that, by definition, discrepancies (1) can take only non-negative values, we conclude that for all $\lambda < 1$, the stationary (conflict-free) state $u_1 = 0$ and $u_2 = 0$ will be asymptotically stable. That is, while $u_1 > 0$ and $u_2 > 0$, the discrepancies $u_1$ and $u_2$ will decrease monotonically. Taking into account (11), in this case both conflicting parties will come to a conflict-free state if the conditions:

$$\sqrt{(k_{12} - k_{21})^2 + 4k_{11}k_{22}} < (k_{12} + k_{21}) \tag{12}$$

$$-\sqrt{(k_{12} - k_{21})^2 + 4k_{11}k_{22}} < (k_{12} + k_{21}) \tag{13}$$

If $k_{12} + k_{21} > 0$, then condition (13) is fulfilled automatically, and from condition (12) it follows that it should be:

$$k_{11}k_{22} < k_{12}k_{21} \tag{14}$$

If $k_{12} + k_{21} < 0$, then the stability condition (12) is not satisfied.

Due to the nonlinearity of Equations (7) and (8), it may turn out that one of the discrepancies has reached zero, while the other differs from zero. In this case, the conflict situation will be described by only one of the Equations (7) or (8). Let for definiteness be $u_2 = 0$. In this case, Equation (7) takes the form:

$$u_1^{(n)} = [1 - k_{21}]u_1^{(n-1)} \tag{15}$$

At the same time, the condition:

$$k_{11} > 0 \tag{16}$$

which follows from Equation (8) and ensuring the conservation $u_2 = 0$ must be satisfied.

In order for discrepancy $u_1$ to decrease monotonically in this case as well, from (15), it follows the condition:

$$k_{21} > 0 \tag{17}$$

Similarly, one obtains the conditions:

$$k_{22} > 0; k_{12} > 0 \tag{18}$$

Thus, for the real values of the parameters $\lambda$, the sufficient conditions for the stability of the conflict-free state of the system $(u_1 = 0,\ u_2 = 0)$ are $\lambda < 1$, that is, conditions:

$$\begin{cases} k_{12} + k_{21} > 0 \\ k_{11}k_{22} < k_{12}k_{21} \end{cases} \tag{19}$$

The system can come to a conflict-free state even when $\lambda \geq 1$. Let us consider this situation in more detail. The general solution is:

$$\begin{cases} u_1^{(n)} = a_1\lambda_1^{(n)} + a_2\lambda_2^{(n)} \\ u_2^{(n)} = b_1\lambda_1^{(n)} + b_2\lambda_2^{(n)} \end{cases} \tag{20}$$

with constants $a_1,\ a_2, b_1, b_2$.

The system of equations for initial discrepancies $u_1^{(0)},\ u_2^{(0)}$ and discrepancies at the first step $u_1^{(1)},\ u_2^{(1)}$ of the iterative process has the form:

$$\begin{cases} a_1\lambda_1 + a_2\lambda_2 = u_1^{(0)} - k_{21}u_1^{(0)} - k_{22}u_2^{(0)} \\ b_1\lambda_1 + b_2\lambda_2 = u_2^{(0)} - k_{11}u_1^{(0)} - k_{12}u_2^{(0)} \\ a_1 + a_2 = u_1^{(0)} \\ b_1 + b_2 = u_2^{(0)} \end{cases} \tag{21}$$

From (21) one obtains:

$$\begin{cases} a_1 = \frac{u_1^{(0)}(1-k_{21}) - k_{22}u_2^{(0)} - u_1^{(0)}\lambda_2}{\lambda_1 - \lambda_2} \\ b_1 = \frac{u_2^{(0)}(1-k_{12}) - k_{11}u_1^{(0)} - u_2^{(0)}\lambda_2}{\lambda_1 - \lambda_2} \\ a_2 = -\frac{u_1^{(0)}(1-k_{21}) - k_{22}u_2^{(0)} - u_1^{(0)}\lambda_1}{\lambda_1 - \lambda_2} \\ b_2 = -\frac{u_2^{(0)}(1-k_{12}) - k_{11}u_1^{(0)} - u_2^{(0)}\lambda_1}{\lambda_1 - \lambda_2} \end{cases} \tag{22}$$

For definiteness, we assume that $\lambda_1 > 1,\ \lambda_2 < 1$. Then, in (20) we can neglect the terms $a_2\lambda_2^{(n)},\ b_2\lambda_2^{(n)}$ as $n \to \infty$.

In order for the discrepancies to be damped or equal to zero, it is necessary that $a_1 < 0$, $b_1 < 0$ due to the non-negativity of the discrepancies $u_1,\ u_2$ and Equations (7) and (8).

From (22) one obtains the conditions under which $a_1,\ b_1$ will be negative:

$$\begin{cases} u_1^{(0)}(1 - k_{21} - \lambda_2) - k_{22}u_2^{(0)} < 0 \\ u_2^{(0)}(1 - k_{12} - \lambda_2) - k_{11}u_1^{(0)} < 0 \\ \lambda_1 - \lambda_2 > 0 \end{cases} \tag{23}$$

Thus, the criterion for the stability of the conflict-free state of the system for the real values of the parameters $\lambda$ are the conditions:

$$\begin{cases} 1 - k_{21} - \lambda_2 < k_{22} \frac{u_2^{(0)}}{u_1^{(0)}} \\ 1 - k_{12} - \lambda_2 < k_{11} \frac{u_1^{(0)}}{u_2^{(0)}} \end{cases} \tag{24}$$

In the case when one of the participants in the conflict has the conflict ended ($u = 0$), while for the second participant in the conflict, criterion (24) is not met, but conditions (16)–(18) are met, then its discrepancy will also tend to zero, and his conflict will end.

If $4k_{11}k_{22} < -(k_{12} - k_{21})^2$, then the parameter $\lambda$ (11) is complex-valued. In this case, one can write:

$$\lambda = |\lambda| \exp(\pm i\theta) \tag{25}$$

where $|\lambda| = \sqrt{(1 - k_{21})(1 - k_{12}) - k_{11}k_{22}}$, $\mathrm{tg}\theta = \frac{\sqrt{-(k_{12}-k_{21})^2 - 4k_{11}k_{22}}}{2 - k_{12} - k_{21}}$.

In this case, the function $\lambda^n$ will be sign-changing, and due to the non-negativity of discrepancies (1), the stationary conflict-free state will be stable. However, if one of the discrepancies has reached zero, then the sufficient conditions for a conflict-free state of the system are conditions (16)–(18).

A conflict situation can be represented by a point in the phase space $(u_1,\ u_2)$, and the conflict development dynamics can be represented by a phase trajectory.

Some results of calculations of conflict situations are shown in Figures 3–11. The open circle on the phase diagram (right) shows the initial state.

Figures 3–6 show a decrease in the discrepancies of both persons and, consequently, the attenuation (decreasing) of conflicts, i.e., the compromise end of the conflict: both the worker and the employer come to a compromise. For the parameters corresponding to Figures 3, 5 and 6, the conflicts (the discrepancies) of both persons decrease monotonously, while for the parameters corresponding to Figure 4, the worker's conflict monotonously decreases, but at the same time, the employer's conflict first intensifies, but then decreases to zero. For the parameters corresponding to Figures 3 and 4, first the conflict of the worker decreases to zero and only then the conflict of the employer decreases more slowly. This is due to the fact that with these parameters, the employer makes concessions to the worker faster than vice versa. For the data corresponding to Figures 5 and 6, the employer's conflict first decreases to zero, and only then the worker's conflict slowly tends to zero. This is due to the fact that in this case the worker makes concessions to the employer faster than vice versa. Figures 3–6 correspond to the real values of the parameter $\lambda$.

Figures 7–10 illustrate the attenuation of conflicts of both persons at the complex values of the parameter $\lambda$. In this case, the non-monotonic conflict dynamics occurs much more frequently than for the case of real values of the parameter $\lambda$. So, Figures 8 and 9 are similar to Figure 4 and correspond to cases when the worker's conflict decreases monotonously, while the employer's conflict develops non-monotonically and decreases more slowly than the worker's conflict. An interesting case is shown in Figure 10, when the conflicts of both the worker and the employer first develop non-monotonically, but then decrease zero. This corresponds to the case when both parties to the conflict adapt to each other and, after a certain period of intensification of mutual conflicts, eventually find a compromise.

Figure 11 shows a decrease in the discrepancy of one of the persons and an increase in the discrepancy of the second, i.e., when the conflict of one of the participants decreases, while the conflict of the other participant increases—the victorious conclusion of the conflict by one of the participants. This happens because for a given complex parameter $\lambda$, one of the discrepancies has reached zero, and the sufficient conditions for the stability of the conflict-free state of the system (16)–(18) are not met.

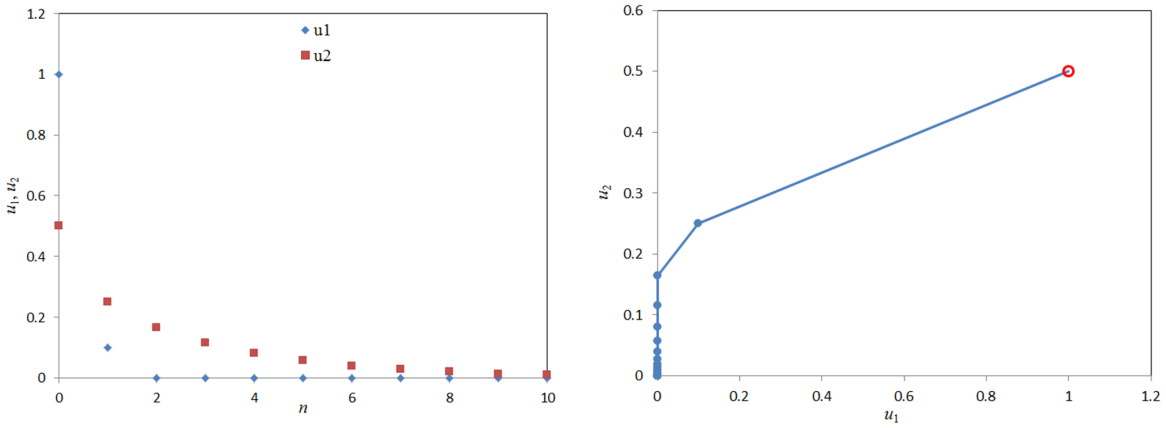


**Figure 3.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = 0.1$; $k_{22} = 0.2$; $k_{12} = 0.3$; and $k_{21} = 0.8$ ($\lambda_1 = 0.737$; $\lambda_2 = 0.163$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

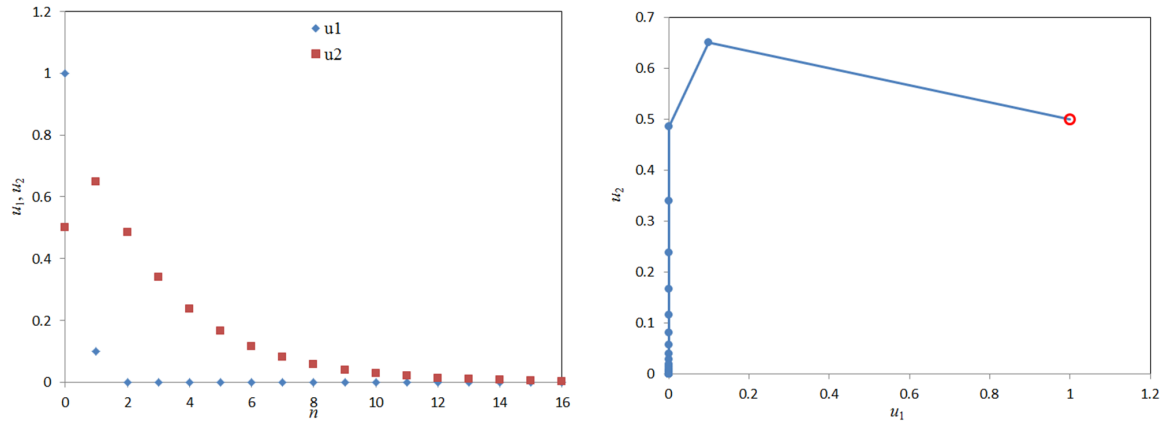


**Figure 4.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = -0.3$; $k_{22} = 0.2$; $k_{12} = 0.3$; and $k_{21} = 0.8$ ($\lambda_1 = 0.5$; $\lambda_2 = 0.4$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

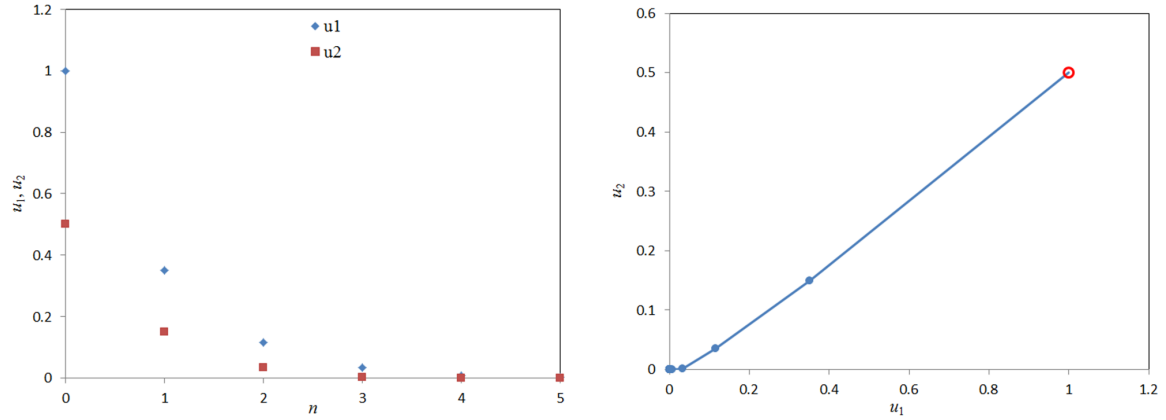


**Figure 5.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = 0.2$; $k_{22} = -0.3$; $k_{12} = 0.3$; and $k_{21} = 0.8$ ($\lambda_1 = 0.5$; $\lambda_2 = 0.4$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

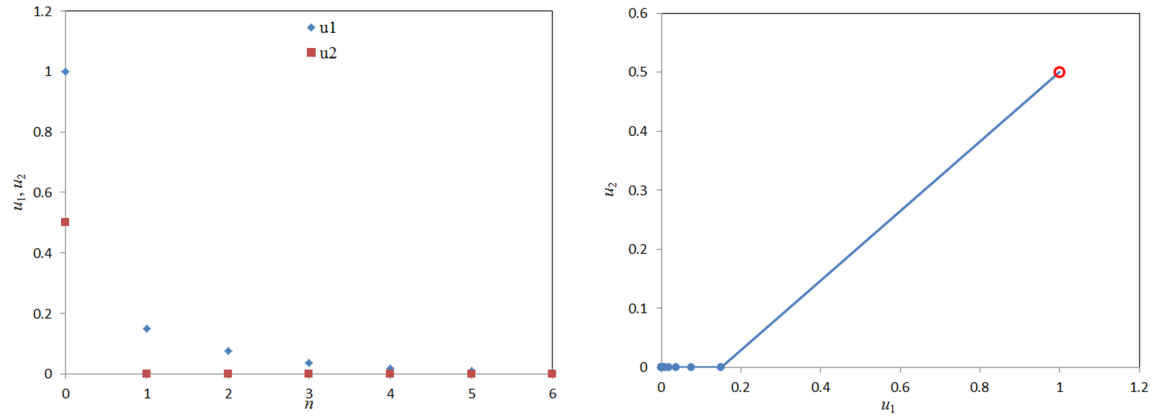


**Figure 6.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = 0.3$; $k_{22} = 0.7$; $k_{12} = 0.4$; and $k_{21} = 0.5$ ($\lambda_1 = 1.01$; $\lambda_2 = 0.089$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

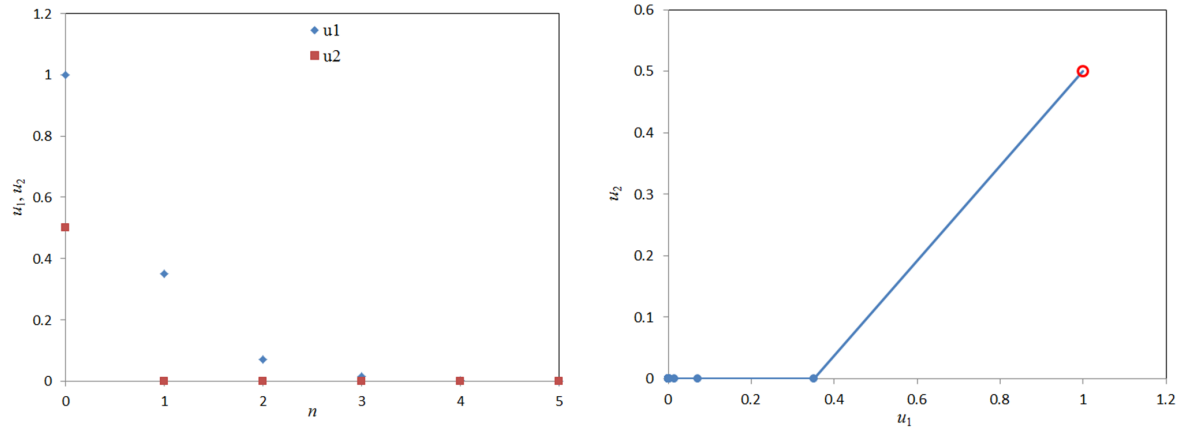


**Figure 7.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = 0.4$; $k_{22} = -0.3$; $k_{12} = 0.3$; and $k_{21} = 0.8$ ($|\lambda| = 0.51$; $\theta = 8.27$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

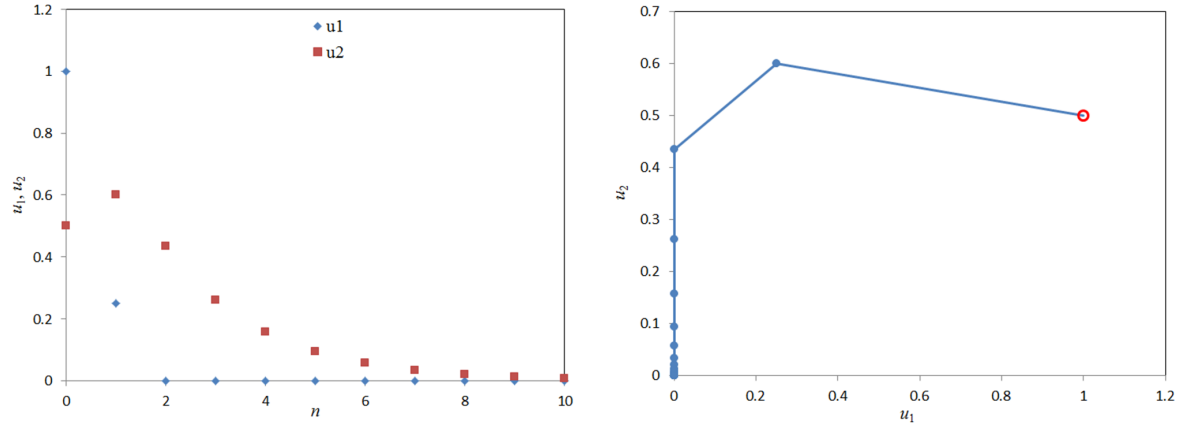


**Figure 8.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = -0.3$; $k_{22} = 0.5$; $k_{12} = 0.4$; and $k_{21} = 0.5$ ($|\lambda| = 0.671$; $\theta = 10.18$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

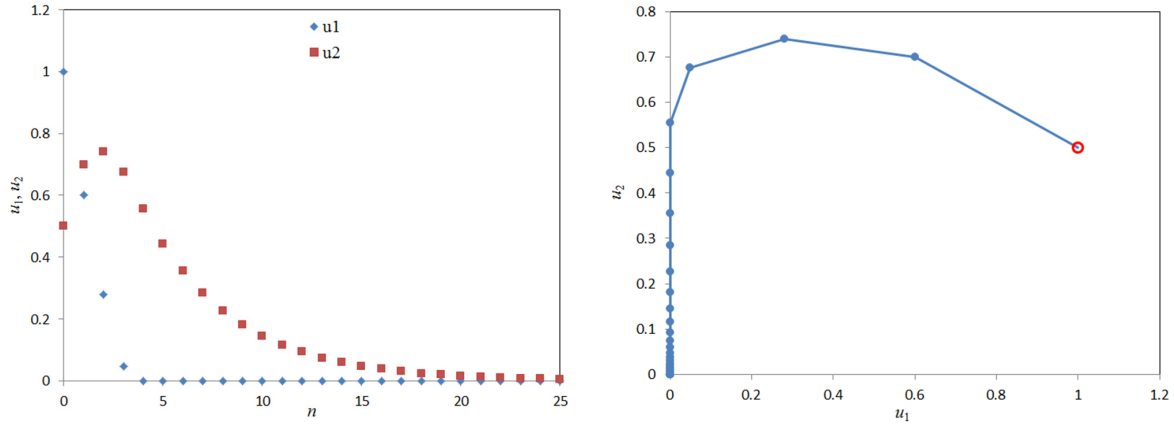


**Figure 9.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = -0.3$; $k_{22} = 0.2$; $k_{12} = 0.2$; and $k_{21} = 0.3$ ($|\lambda| = 0.787$; $\theta = 19.53$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

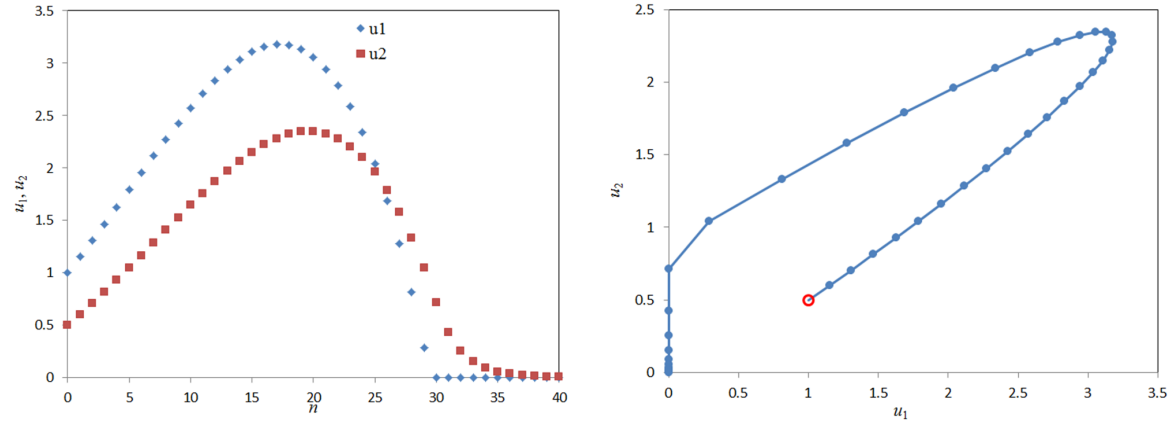


**Figure 10.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = -0.3$; $k_{22} = 0.7$; $k_{12} = 0.4$; and $k_{21} = -0.5$ ($|\lambda| = 1.05$; $\theta = 11.45$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

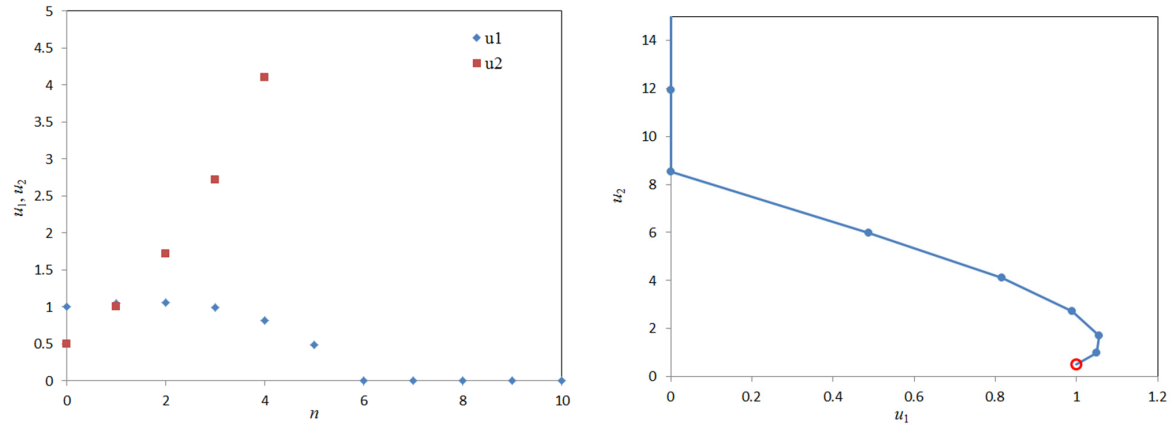


**Figure 11.** Dependencies $u_1(t)$ and $u_2(t)$ (**left**) and the phase trajectory of the worker-employer conflict (**right**) for $u_1^{(0)} = 1$; $u_2^{(0)} = 0.5$; $k_{11} = -0.3$; $k_{22} = 0.1$; $k_{12} = -0.4$; and $k_{21} = -0.1$ ($|\lambda| = 1.25$; $\theta = 34.66$). The discrepancy $u_1$ corresponds to the worker while the discrepancy $u_2$ corresponds to the employer.

Figure 6 shows a situation in which the conflict decay criterion (24) is not met for one of the conflict participants, but conditions (16)–(18) are met, while the conflict has already ended for the other participant.

## 6. Conclusions

Let us note that the conflict is not always a serious disagreement and not always protracted one. How serious and protracted the resulting conflict will be depends both on the magnitude of the discrepancy (for continuous conflict parameters, for example, the salary of a worker), and on the significance of this conflict for a particular person (see the significance coefficients in expression (2)). Consider, for example, a conflict between a child and a parent: the child wants to play a computer game, but the parent forbids it. One child will calmly react to the fact that the parent forbade him to play a computer game and find another activity for himself, the other one will be offended, but will not do anything, while the third child in the same situation may start to cry, behave poorly, etc., i.e., the latter will create conflict to the parent in the hope that he will reverse his decision. In all three cases we have a conflict, but this conflict will develop in these cases in different ways.

An analysis of various conflict situations shows that the cause of the conflict is always a discrepancy between what a person wants and what he has (or must do). It depends on the psychological state of a person how he will react to this conflict and how he will resolve this conflict: he will put up with this (i.e., change his "I want" parameter) or try to influence the opponent, creating a conflict for him (in order for him to change his "I have" parameter).

In the introduction, we noted that there are a large number of different definitions of conflict that characterize this phenomenon from different points of view, but they are mainly descriptive, philosophical in nature and do not allow formalizing all possible types of conflicts from a unified position and, moreover, constructing mathematical (quantitative) theory of conflicts. In our opinion, the most natural way to formalize conflicts is to consider the discrepancy between what a person wants and what he has. In this case, the theory of conflicts can be constructed as a control theory, however, not of conventional control, as is done in technical systems, but of cognitive control.

Therefore, from a mathematical point of view, conflict should be called precisely the discrepancy between what a person wants and what he has, but not the process itself. It is for this purpose that we separate the conflict and the conflict situation.

In this article, we have formulated a mathematical theory of conflicts, which allows quantitatively describing conflicts related to a wide variety of subject areas from a unified position. A feature of the theory is that in order to resolve their own conflict, persons cannot influence directly the parameters $(W,\ H)$ of the opponent; they can only change their own parameters $(W,\ H)$. For this reason, persons can influence the opponent indirectly by changing their own parameters $H$, thereby causing a conflict in him, and forcing the opponent to change his parameters in the direction necessary for this person to resolve his own conflict.

This is the fundamental difference between controlling a conflict situation (cognitive control) and controlling technical objects, when it is possible to directly influence the cause that causes discrepancy $u$.

We note that the parameter $W$ belongs to the person himself, and is determined by his psychological state, culture, interests, aspirations, etc. At the same time, the parameter $H$ is determined by other people, and it also depends on their psychological state, culture, interests, aspirations, etc. A person cannot directly influence the $H$ parameter; however, he can create a conflict to those people who define the $H$ parameter in order to force them to change this parameter in the direction that is necessary for him. This is the essence of any interaction between people. Moreover, we believe that any interaction between people ultimately comes down to the creation of conflicts and their resolution.

As an example, we considered a model of a conflict situation between two subjects and built a dynamic model for the development of a conflict situation in the worker-employer

system. The considered model allows reproducing all possible variants of the development of conflict situations depending on the psychological (cognitive) state of its participants.

From a practical point of view, by analyzing various conflict situations involving different opponents, for each person it is possible to determine the range of parameters $k_{ij}$ that characterize his psychology (behavior strategy), i.e., build a quantitative psychological portrait of the person. Assuming that the psychology of the person does not change in conflicts with other opponents, the coefficients $k_{ij}$ obtained in this way can be used to analyze other conflict situations involving this person.

This issue will be considered in future papers.

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