

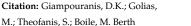
Article Berth Allocation at Passenger Terminals Using Auctions

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Abstract: In this paper, we compare a multi-round, second-price, sealed-bid bundle auction and a single-item, sequential, second-price, sealed-bid auction for berth slot leasing for vessels (roll-on/roll-off passenger vessels and/or cruise ships) at a public marine terminal. The bundle auction mechanism is designed to maximize port operator profits by auctioning berth (time) slots in groups. The framework is tested using simulation by varying: the number of roll-on/roll-off passenger/cruise ship operating companies; the number of slots they bid for; and the mechanism design with regards to the winner determination, slot valuation, and max to min slot bid ratio among the bidders. The results indicate that neither auction type is a clear winner, and depending on the assumptions, a terminal operators, given their knowledge and/or assumptions on slot valuations and demand, to select a winner-determination policy and the minimum number of slots they allow players to bid for when designing the auction of their berth capacity to maximize their profits.

Keywords: passenger marine terminals; berth scheduling; auctions; second price; multi-round



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1. Introduction

The importance of developing a profitable and reliable mechanism to assign quay slots (time and berth location) at seaports has long been recognized by the industry and academia. However, most scientific papers are concerned with container terminals, where berth allocation is mostly accomplished through a first-come-first-serve basis and/or based on 1 (that define limits on vessel wait times, departure times, and berth productivity [1-6]). Some of these approaches have been criticized for their limitations with regards to profitability for port operators and the preferential treatment of high-volume liner shipping companies, which lead to increased costs from fuel consumption and labor-related expenses while a vessel is waiting for the next available slot at the entrance of a port, increased traffic inside and nearby seaports, delayed deliveries, etc. Bulow and Klemperer [7] concluded that, provided certain assumptions are met, an auction will always lead to higher payoffs for the seller (terminal operator or port authority in the case of marine terminals) if it contains one more participant (vessel operators/owners) than a direct negotiation on the same object. That is, if a port operator engages in negotiations with a roll-on/roll-off passenger (RoPax)/cruise company, then an efficiently designed auction with at least two bidders will produce better payoffs for the port/terminal operator due to the enhanced level of competition.

Although the berth allocation problem is a very well-documented and researched topic, it appears that limited research has been published with regards to auctions as the main mechanism to efficiently distribute timeslots for berthing. Note that extensive research has been published on the strategic/tactical berth scheduling problem in public and private terminals, which resembles the problem studied herein [8–11]. In those papers, the problems deal with container terminals, and the terms public and private refer to the

port management and ownership model (i.e., a public terminal is operated by a public port authority, while in the private case, the terminal is operated by a private company—in most cases, a liner shipping company [12]). To the authors' knowledge, none of the proposed approaches in the literature used auctions as the mechanism to allocate berth capacity to vessels at passenger terminals (RoPax vessels and/or cruise ships). Gosh [1] tested the effects of a sequential English auction for queue management. This auction mechanism assumed that all bidders have the same preferences, and all the available slots are ranked in the same order by all bidders. Strandenes and Wolfstetter [13] argued that different vessels usually have different schedules, and the possibility of everyone wanting the same slots is highly unlikely in real-world problems. They also argued that single-slot auctioning is not efficient for the berth allocation problem and created a group auction mechanism where all bidders must report to the port operator all their expected profits from every feasible allocation. The port operator was responsible for choosing an allocation that maximized all the bidders' profits. One issue with the proposed approach by Strandenes and Wolfstetter [8] is that every bidder submits stated reported profits of every feasible allocation, which makes the problem intractable for the bidder for real-life situations. Our proposed approach addresses this issue by requiring the bidder to submit profits for one allocation in each round.

When focusing on passenger (cruise and roll-on/roll-off passenger or RoPax) terminals, most major ports' berthing policies are similar. The port authority determines the berth allocation and announces it for a prespecified time period (e.g., 6 months). New reservation requests must be submitted in advance to be taken under consideration (time varies by port from six months to four years). The major components for determining the berth allocation are vessel size and its nautical, operational, and commercial aspects [14–17]. Berth allocation is usually made by the port authority in its entirety and cruise/ferry lines have limited rights to claim a specific berth. This allows the opportunity to consider different berthing policies that maximize the port operators' profits by giving the option to cruise/ferry lines to bid for their desired berth slots against their competitors. Parameters such as proximity to public transportation, time of day, seasonality, and proximity to car parking lots play a significant role for ferry/cruise lines.

In this paper, we propose and compare a multi-round second price sealed-bid bundle auction and a single item multi-round sequential auction, as an alternative to the policies mentioned above, for the assignment of vessels at the berths of a public passenger/cruise terminal. The literature for berth scheduling suggests that using an auction yields positive results compared to negotiations when at least one extra bidder appears in the auction [7]. The question that remains is when is it more profitable to use a single item versus a bundle auction in the public berth allocation setting?

The rest of the paper is structured as follows: The next section presents the proposed multi-round, second-price, sealed-bid bundle auction (referred to as a bundle auction for the remaining of this paper) and the single-item, sequential, second-price, sealed-bid auction (referred to as a single auction for the remaining of this paper). The third section presents results from a simulation developed to evaluate the two proposed auction mechanisms. The final section concludes the paper and proposes future research directions.

2. Problem Description and Methodology

In this paper, we propose a multi-item (berth and time slot), second-price auction framework, which is compared against a single-item, sequential, second-price auction. Under the single-item sequential second-price auction (from now on referred to as singleitem auction), the port operator auctions each slot sequentially, where the winner for each slot is determined by their highest bid and the price paid to the port operator is equal to the second-highest bid. The sequence with which the slots become available for bidding is important for the outcome of the auction. For this paper, we assume that the port operator has some knowledge of demand and berth location preference (from experience) and can thus prioritize the auction with slots that they believe are of higher value and will increase the competition amongst the bidders. To start the auction, the port operator makes the first time-slot available for bidding. After the bidding ends, the winner and the price to be paid are determined. The first-round ends with the slot being removed from the pool of available slots. If the demand of the winner has been satisfied, the winner is removed from the pool of available bidders. The process continues until the demand has been met or the supply has been depleted. Figure 1 presents the steps of the single-item auction used in this paper.

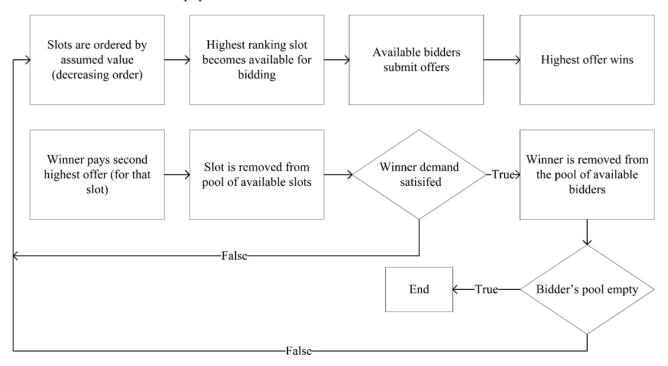


Figure 1. Single-Item Auction Flowchart.

2.1. Bundle Auction

The multi-round, second-price, sealed-bid bundle auction (from now on referred to as a bundle auction) proposed in this paper (as an alternative to the single-item auction) auctions all available slots at the same time, and each player bids for the slots they need, based on their valuation. Therefore, each bidder's offer includes only the slots they need with their highest valuation. At each round, the player with the highest bid wins the round and takes all the slots they bid for, paying the second-highest offer, regardless of what slots the second-highest offer was made for. In this paper, we evaluate two different winner determination policies (i.e., how the highest bid is estimated), described in more detail later in this section. The winner is removed from the pool of remaining players along with the slots they acquired, and the remaining bidders resubmit bids for the available slots. The procedure is terminated when all players have received slots or there are no more slots available. At each round, a reserve price is introduced for the slots that have not been awarded. A reserve price is defined as the minimum price that the auctioneer will accept as a winning bid. The reserve price for each slot is calculated based on the mean profit made from all the winning bids in all previous rounds. We adopt a reserve price to address situations where a second-best price for a slot (or bundle of slots) does not exist (i.e., at the last round where only one player exists). Thus, the reserve price, in the way the proposed auction is set up, is only used at the last round of the auction. In that case, the winner will pay a reserve price for the slots awarded, so that their profit equals the mean profit of all the previous winners. Note that in the proposed auction a reserve price does not alter the behavior of the bidders (i.e., it is still a dominant strategy to bid one's true valuation), since if the reserve price was higher than the bid, the bidder will not take the slot, and their expected payoffs will be zero. Figure 2 presents the steps of the bundle auction.

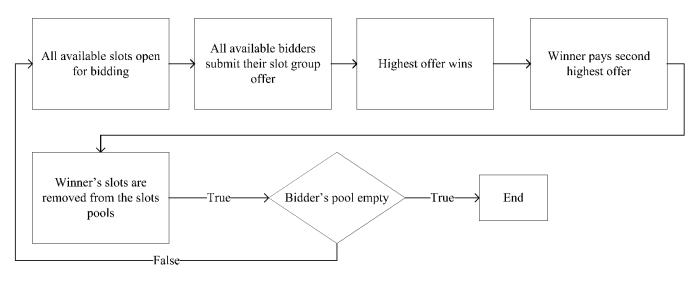


Figure 2. Bundle Auction Flowchart.

2.2. Rationale for Second-Price Bundle Auction

Standard auctions (e.g., English auction where each slot is auctioned sequentially) have been examined in the related literature, but they are generally inefficient [13]. In the auction scheme proposed in this paper, we utilize the advantages of a second-price bundle-auction. We require that the proposed mechanism satisfies the voluntary participation condition, where players are not charged if they lose, and their (expected) payoffs are at most the second-highest bid if they win [18]. In a second-price auction, it is a dominant strategy to bid the maximum valuation price of the object auctioned, even if other bidders are over-bidding, under-bidding, or colluding. This guarantees that players who bid truthfully always obtain non-negative (expected) profit, and as in other Vickrey–Clarke–Groves auctions, a second-price, sealed-bid auction guarantees truthfulness. As with many game theory applications, in real life, truthfulness might not be guaranteed due to the players irrational behavior. Since a second-price auction always guarantees that the player will end up with non-negative profits for the item(s) they are awarded, it encourages even irrational players to bid truthfully.

2.3. Auction Assumptions

To apply the proposed auction, the following assumptions are made:

- The supply of slots by the port operator is greater than (or at least equal to) the demand by the bidders (applied on three out of four simulations described in the next section).
- There is imperfect information, i.e., bidders are unaware of each other's valuation for each slot and for what slots they will bid.
- A possible situation where a bidder bids for more slots than they need to improve their position against the competition is allowed since we are interested in maximizing the port operator's profits and not in an equitable distribution of resources.

2.4. Slot Valuation Patterns

In this paper, we assumed two different slot valuation patterns for each slot and bidder to be used in the simulation experiments presented in the next section: (i) random and (ii) uniform. Next, we describe the assumptions made behind both slot valuation patterns.

2.4.1. Random Valuation Pattern (RVP)

Under the RVP, each bidder has their own valuation for each slot which has been assigned randomly. That means that a certain slot might be of high value for one player and of low value for another. This is a more generic way to determine each slot's valuation by every bidder.

2.4.2. Uniform Valuation Pattern (UVP)

Under the UVP, the value of each slot is based on a uniform probability distribution. That is, all bidders have similar valuations for each slot. This does not mean that the valuation for different slots is the same (i.e., players will value higher slots at peak demand period and lower slots at low demand periods, but their valuations at each period will be similar). This version of slot valuation allows for high competition between the bidders for the most desirable slots and, possibly, increases the port operator's profits since, in each round, the second-highest bid (i.e., price that the winner will pay) is close to the highest bid (i.e., winner's bid).

2.5. Winner Determination

To determine the winning bid of each round of the proposed auction, we propose two different policies, described next.

2.5.1. Winner Determination Policy Based on the Total Bid (MWD)

Under the MWD policy, the winner is determined based on highest total bid over all the slots, regardless of how many slots a player has bid for. Let S be the set of all available slots and $V(S_i)$ be the valuations of player *i* for the subset of slots $S_i \subseteq S$. Assume that players *i* and *j* bid for a subset of slots S_i and S_j (S_i , $S_j \subseteq S$). Then if $V(S_i) \ge V(S_j)$ player i will win the bid and be awarded the subset of slots S_i . This winner determination policy provides a significant advantage to players that bid for many slots at the same time and/or are willing to pay a higher amount. In case of a tie ($V(S_i)=V(S_j)$) the winner is the player with the lowest number of slots (i.e., the bidder with the highest average per slot bid). In case of a tie for both the bid ($V(S_i)=V(S_j)$) and number of slots ($|S_i|=|S_j|$), the bid price the winner is chosen randomly.

2.5.2. Winner Determination Policy Based on the Average Bid per Slot (AWD)

Under the AWD policy, the winner is determined based on the highest average per slot bid. For example, if bidders *i* and *j* offer $V(S_i)$ and $V(S_j)$ for slots S_i and S_j respectively, the player with the highest valuation to number of slots ratio (i.e., $(V(Si))/(|S_i|), (V(S_j))/(|S_j|))$ is the winner. In case of a tie, the winner is chosen based on the number of slots they bid for, but in this case, and in antithesis with MWD, the bidder with the highest number of slots wins..

3. Numerical Experiments

In this section, we present results from a set of numerical experiments, performed using Monte Carlo simulation, to compare the proposed multi-round second price sealedbid bundle auction to the single-item, sequential, second-price auction. We simulated four different demand/supply scenarios by varying assumptions on the slot valuation pattern, number of bidders, and slot demand by each bidder. For every scenario, we simulated 10,000 auctions for each winner determination policy (i.e., MWD and AWD) and slot valuation pattern (i.e., RVP and UVP) combination for both the single and bundle auctions. In all numerical experiments, the number of available slots has been set to 300. Next, we present a detailed description of each scenario:

a. *Base Scenario*: Under this scenario, valuations for each slot by each bidder were uniformly distributed between twenty and one hundred. We considered six distinct cases with respect to the number of bidders ranging from five to ten. We also considered seven distinct cases for the minimum number of slots a player will bid for ranging from three to nine. We also considered three distinct cases for the maximum number of slots a player would bid for. These were calculated based on a uniform distribution with a lower bound equal to the minimum number of slots a player would bid for and an upper bound equal to one, two, and three times that minimum (i.e., if the minimum number of slots between U[X, X], U[X,2X], and U[X, 3X], respectively,

where U[a, b] is the uniform distribution with bounds a and b). A total of 504 unique combinations of winner determination, slot valuation, and demand patterns were created. Under the base scenario, we assumed that supply was greater or equal to the demand (i.e., the total number of available slots is greater or equal to the total number of slots requested by the bidders).

- b. *Group Demand Scenario*: Under this scenario, we created three distinct groups of bidders based on the number of slots (i.e., demand) they request. Approximately one-third of bidders are considered 'weak', meaning they will only bid for the minimum number of slots allowed. The second third of the bidders have a medium competitiveness and bid for double the minimum number of slots allowed. The final subset of bidders bid for the maximum number of slots allowed and are considered as 'strong'. The rest of the assumptions remain the same as in the base scenario. This additional scenario is used to evaluate the case where players can be grouped into distinct subsets based on their strength as defined by the number of slots they are seeking.
- c. *Group Valuation Scenario*: Under this scenario, the available slots are divided into three distinct subsets based on the slot valuation of all bidders. For all bidders, one-third of the same available slots have a uniformly distributed valuation between twenty and forty, one-third between fifty and seventy, and one-third between eighty and one hundred. The rest of the assumptions remain the same as in the base scenario. This additional scenario is used to evaluate the case where slot valuations can be grouped in subsets for all players (e.g., AM period has a high, MD a medium, and PM a low valuation for all players).
- d. *Insufficient Supply Scenario*: Under this scenario, we assume that the number of bidders is equal to twenty. Under this scenario, and for the cases where the minimum slot bid for any bidder is greater or equal to eight, the available slots are not enough to satisfy the demand for all bidders. The rest of the assumptions remain the same as in the base scenario. This additional scenario is used to evaluate the effects of insufficient supply.

Next, we present and discuss results from the simulation of the four different scenarios. Figures 3–10 summarize the results from the simulation of the single and bundle auctions. The results are presented as the mean percentage change in profits for the port operator between the two different auction mechanisms for each simulation scenario. Figure 3 shows the terminal operator's mean profit change for the general simulation scenario for U[X,X], U[X,2X], and U[X,3X], respectively; Figure 5 for the demand group scenario; Figure 7 for the group valuation scenario; and Figure 9 for the insufficient supply scenario. For consistency purposes, we show results for the minimum slot bid of three through seven, even though under these minimum slot bids there is sufficient supply. Figure 4, Figure 6, Figure 8, and Figure 10 show the standard deviation for each scenario, respectively. The assumption of normality was tested visually through histograms and through the Anderson-Darling test [19]. In addition to the results presented in Figures 3-10, we also estimated the coefficient of variation (CoV) of the mean profit difference between the two auctions to evaluate the relative dispersion of the results around the mean. For the Base Scenario, the CoV was approximately 58% irrespective of the min/max slot bid ratio (i.e., U[X,X], U[X,2X], and U[X,3X]). For the Group Demand Scenario, the CoV was 58%. Finally, the Group Valuation Scenario had a CoV of 56%, and the Insufficient Supply Scenario had a CoV of 71%. Additionally, for Figures 3, 5, 7, and 9, the red color indicates that the port operator would be better off using the sequential auction and green that the port operator would be better off using the bundle approach.

							RAN	NDOM VA	LUATION	PATTER	N								
	numl	ber of slot	ts a playe	r would bi	id for U(X	,X)	numb	er of slots	a player	would bio	l for U(X	,2X)	numb	er of slots	a player	would bid	for U(X	,3X)	
Min Slot		1	Number o	f Bidders				1	Number of	f Bidders				Ň	umber of	f Bidders			
Bid	5	6	7	8	9	10	5	6	7	8	9	10	5	6	7	8	9	10	
									MV	VD			-						
													,	_		,			
3	12%	12%	12%	11%	11%	10%	4%	6%	6%	6%	6%	6%	0%	2%	2%	3%	3%	3%	
4	11%	11%	11%	11%	10%	10%	3%	4%	5%	5%	5%	5%	-1%	1%	2%	2%	2%	2%	
5	11%	11%	11%	10%	<u>10</u> % 9%	<u>9</u> % 9%	3%	4%	4%	5%	4%	4% 4%	-1%	1%	1%	2%	2%	2%	
6	10%	10%	10%	<u>10</u> % 9%			2%	3%	4%	4%	4%		-1%	0%	1%	2%		2%	
8	<u>10</u> % 9%	<u>10</u> % 10%	<u>10</u> % 9%		9%	8%	2%	3%	4%	4%	4%	4%	-2%	0%	1%	1%	2%	1%	
9	<u>9</u> % 9%	<u>10</u> % 9%	<u>9</u> % 9%	<u>9</u> % 9%	9% 8%	8% 8%	2% 1%	3% 3%	3% 3%	4% 3%	4% 3%	4%	-2% -2%	0% 0%	1%	1%	1%	1%	
,	9 70	9 70	9 70	9 70	0.70	0.70	170	370	370 AW		370	370	-2.70	070	070	170	170	1 70	
3	12%	12%	12%	11%	11%	10%	10%	11%	11%	10%	10%	9%	10%	10%	10%	10%	9%	9%	
4	11%	11%	11%	11%	10%	10%	10%	10%	10%	10%	9%	<u>9</u> %	9%	10%	10%	9%	9%	9%	
5	11%	11%	11%	10%	10%	9%	9%	10%	10%	9%	9%	8%	9%	9%	9%	9%	9%	8%	
6	10%	10%	10%	10%	9%	9%	9%	9%	9%	9%	8%	8%	8%	9%	9%	9%	8%	8%	
7	10%	10%	10%	9%	9%	8%	8%	9%	9%	9%	8%	8%	8%	8%	8%	8%	8%	7%	
8	9%	10%	9%	9%	9%	8%	8%	9%	9%	8%	8%	8%	8%	8%	8%	8%	8%	7%	
9	9%	9%	9%	9%	8%	8%	8%	8%	8%	8%	8%	7%	7%	8%	8%	8%	7%	7%	
							UNI	FORM VA	LUATION	N PATTEI	RN								
	numb			would bi	d for U(X	(,X)	numb	number of slots a player would bid for U(X,2X)						number of slots a player would bid for U(X,3X) Number of Bidders					
Min Slot			Number o						Number o				_				_		
Bid	5	6	7	8	9	10	5	6	7 MV	8	9	10	5	6	7	8	9	10	
							1		MV	VD									
3	-12%	-8%	-6%	-5%	104	-3 <mark>%</mark>	-20%	-15%	-11%	-9%	-8%	-7%	-25%	-19%	-15%	-13%	-11%	-10%	
4	-12%	-8%	-6%	4%	- <mark>4%</mark> - <mark>4%</mark> - 3%	-3%	-20%	-15%	-12%	-10%	-8%	-7%	-25%	-19%	-16%	-13%	-11%	-10%	
5	-12%	-8%	-6%	-4%	- 896	-3%	-20%	-15%	-12%	-10%	-8%	-7%	-25%	-19%	-15%	-13%	-11%	-10%	
6	-12%	-8%	-6%	-4%	- 3%	-3%	-20%	-15%	-12%	-10%	-8%	- <mark>7%</mark> - <mark>7%</mark>	-25%	-19%	-15%	-13%	-11%	-10%	
7	-12%	-8%	-6%	-4%	- 3%	-3%	-20%	-15%	-12%	-10%	-8%	-7%	-25%	-19%	-15%	-13%	-11%	-10%	
8	-12%	-8%	-6%	-4%	- <mark>3%</mark> - <mark>3%</mark>	-3%	-20%	-15%	-12%	-10%	-8%	-7%	-25%	-19%	-15%	-13%	-11%	-9%	
9	-12%	-8%	-6%	-4%	-3%	-3%	-20%	-15%	-12%	10%	-8%	-7%	-25%	-19%	-15%	-12%	-11%	-9%	
									AW	/D									
3	-12%	-8%	-6%	-5%	-4%	-3%	-12%	-8%	-6%	-5%	-4%	-3 <mark>%</mark>	-13%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>	
4	-12%	-8%	-6%	-4%	-4%	-3 <mark>%</mark>	-12%	-8%	-6%	-5%	-4%	-3%	-13%	-9%	-6%	-5%	-4%	-3%	
5	-12%	-8%	-6%	4%	-3%	-3 <mark>%</mark>	-12%	-8%	-6%	-5%	-4%	-3 <mark>%</mark>	-13%	-9%	-6%	-5%	-4%	-3%	
6	-12%	-8%	-6%	-4%	-3%	-3 <mark>%</mark>	-12%	-8%	-6%	4%	-3%	-3 <mark>%</mark>	-13%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>	
7	-12%	-8%	-6%	-4%	-3%	-3%	-12%	-8%	-6%	4%	-3%	-3 <mark>%</mark>	-13%	-9%	-6%	-5%	-4%	-3%	
8	-12%	-8%	-6%	-4%	- <mark>4%</mark> - <mark>4%</mark> - <u>8%</u> - <u>8%</u> - <u>8%</u> - <u>8%</u>	-3 <mark>%</mark> -3 <mark>%</mark>	-12%	-8%	-6%	4%	-3%	-3 <mark>%</mark>	-13%	-9%	-6%	-4%	-3%	-3 <mark>%</mark>	
9	-12%	-8%	-6%	-4%	-3%	-3 <mark>%</mark>	-12%	-8%	-6%	-4%	-3%	-3 <mark>%</mark>	-13%	-8%	-6%	-4%	-3%	-3%	

Figure 3. Base Scenario Mean Profit Difference (MPD): Bundle vs. Single-Item Auction.

							RA	NDOM VA	LUATION	N PATTER	N							
	num	ber of slot	ts a playe	r would bi	d for U(X,	X)	num	ber of slot	s a player	would bio	l for U(X,	2X)	numb	per of slots	a player	would bid	l for U(X,	3X)
Min Slot		1	Number o	f Bidders					Number o	of Bidders				1	Number of	f Bidders		
Bid	5	6	7	8	9	10	5	6	7	8	9	10	5	6	7	8	9	10
			Tabl	e 1a			MWD Table 1b						Table 1c					
3	6%	5%	4%	3%	3%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
4	6%	4%	3%	3%	2%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
5	5%	4%	3%	3%	2%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
6	5%	4%	3%	2%	2%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
7	5%	4%	3%	2%	2%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
8	5%	3%	3%	2%	2%	2%	5%	3%	2%	1%	1%	1%	6%	3%	2%	2%	1%	1%
9	5%	3%	3%	2%	2%	1%	5%	3%	2%	1%	1%	1%	5%	3%	2%	2%	1%	1%
									AV				1					
3	6%	5%	4%	3%	3%	2%	6%	4%	3%	3%	2%	2%	6%	4%	3%	3%	2%	2%
4 5	6%	4%	3%	3%	2%	2%	5%	4%	3%	2%	2%	2%	6%	4%	3%	2%	2%	2%
5	5%	4%	3%	3%	2%	2%	5%	4%	3%	2%	2%	2%	6%	4%	3%	2%	2%	2%
7	5% 5%	4% 4%	3% 3%	2% 2%	2% 2%	2% 2%	5% 5%	3% 3%	3% 3%	2% 2%	2% 2%	2% 1%	5% 5%	4% 4%	3% 3%	2% 2%	2% 2%	2% 1%
8	5%	4%	3%	2% 2%	2% 2%	2%	5%	3%	3% 2%	2%	2% 2%	1 76	5%	4%	3% 3%	2%	2%	1%
9	5%	3%	3%	2%	2%	1%	5%	3%	2%	2%	2%	1%	5%	4%	3%	2%	2%	1%
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							ii onui m	LUMIO	I I AI ILI	11								
	num	ber of slot	s a player	· would bi	d for U(X,	X)		ber of slot	s a player	would bio		2X)	numb	per of slots	a player	would bid	l for U(X,	3X)
Min Slot		1	Number o	f Bidders		·	numl	ber of slot	s a player Number o	would bio Bidders	l for U(X,			N	Number of	f Bidders		,
Min Slot Bid	num 5				d for U(X, 9	X) 10		ber of slot	s a player Number o 7	would bid Bidders 8		2X) 10	numt				l for U(X, 9	3X) 10
		1	Number o 7	f Bidders 8		·	numl	ber of slot	s a player Number o 7 MV	would bid of Bidders 8 WD	l for U(X,			N	Number of 7	f Bidders 8		,
Bid	5	6	Number o 7 Table	f Bidders 8 e 1d	9	10	numl 5	ber of slot	s a player Number o 7 MV Tabl	would bid f Bidders 8 WD le 1e	l for U(X,í	10	5	6	Number of 7 Table	f Bidders 8 e 1f	9	10
Bid 3	5	6 3%	Number o 7 Table 2%	f Bidders 8 e 1d 1%	9 1%	10	5 6%	6 4%	s a player Number o 7 MV Tabl 3%	would bid of Bidders 8 WD le 1e 2%	9 1%	10	5 8%	6 5%	Number of 7 Table 4%	e 1f 3%	9 2%	10 2%
Bid 3 4	5 5% 5%	6 3% 3%	Table 2% 2%	f Bidders 8 e 1d 1% 1%	9 1% 1%	10 1%	num 5 6% 6%	6 4% 4%	s a player Number o 7 MV Tabl 3% 3%	would bid f Bidders 8 WD le 1e 2% 2%	1 for U(X, 9 1% 2%	10 1% 1%	5 8% 8%	6 5% 5%	Number of 7 Table 4% 4%	Bidders 8 e 1f 3% 3%	9 2% 2%	10 2% 2%
Bid 3	5	6 3% 3% 3%	Number o 7 Table 2%	f Bidders 8 e 1d 1%	9 1% 1% 1%	10	5 6%	6 4%	s a player Number o 7 MV Tabl 3% 3% 3%	would bid of Bidders 8 WD le 1e 2%	9 1%	10	5 8%	6 5%	Number of 7 Table 4%	e 1f 3%	9 2%	10 2% 2% 2%
Bid 3 4 5	5 5% 5% 5%	6 3% 3%	Number o 7 Table 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1%	9 1% 1%	10 1% 1% 1%	num 5 6% 6% 6%	6 4% 4%	s a player Number o 7 MV Tabl 3% 3%	would big of Bidders 8 VD e 1e 2% 2% 2% 2%	9 1% 2% 2%	10 1% 1% 1%	5 8% 8% 8%	6 5% 5% 5%	Number of 7 7 Table 4% 4% 4% 4% 4%	Bidders 3% 3% 3% 3% 3%	9 2% 2% 2%	10 2% 2%
Bid 3 4 5 6	5 5% 5% 5%	6 3% 3% 3% 3%	Number o 7 Table 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1%	9 1% 1% 1% 1% 1%	10 1% 1% 1% 1%	numl	6 4% 4% 4% 4%	s a player Number o 7 MV Tabl 3% 3% 3% 3%	would big of Bidders 8 VD e 1e 2% 2% 2% 2% 2%	1 for U(X,) 9 1% 2% 2% 2%	10 1% 1% 1% 1%	5 8% 8% 8%	6 5% 5% 5% 5%	Number of 7 Table 4% 4% 4% 4%	f Bidders 8 e 1f 3% 3% 3% 3% 3%	9 2% 2% 2% 3%	10 2% 2% 2% 2%
Bid 3 4 5 6 7	5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3%	Table 7 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1%	num	6 4% 4% 4% 4% 4%	s a player Number o 7 MV Tabl 3% 3% 3% 3% 3% 3%	would bic f Bidders 8 VD 2% 2% 2% 2% 2% 2%	1 for U(X,) 9 1% 2% 2% 2% 2%	10 1% 1% 1% 1%	5 8% 8% 8% 8%	6 5% 5% 5% 5% 5% 5%	Table 4%	f Bidders 8 e 1f 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 2% 3% 3%	10 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9	5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3%	Table 7 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1%	num	6 4% 4% 4% 4% 4% 4%	s a player Number o 7 MV Tabl 3% 3% 3% 3% 3% 3% 3% 3%	would bid f Bidders 8 VD 2% 2% 2% 2% 2% 2% 2% 2% 2%	1 for U(X,) 9 1% 2% 2% 2% 2% 2%	10 1% 1% 1% 1% 1% 1%	5 8% 8% 8% 8% 8%	5% 5% 5% 5% 5% 5%	Table 7 Table 4%	Bidders 8 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 2% 3% 3% 3%	10 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9	5% 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3%	Number o 7 Table 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1%	numl 5 6% 6% 6% 6% 6% 6% 6% 6% 6% 7%	ber of slot: 1 6 4% 4% 4% 4% 4% 4% 4% 4% 4% 5%	s a player Number o 7 MV Tabl 3% 3% 3% 3% 3% 3% 3% 3% 4W 4%	would bic of Bidders 8 ND ie 1e 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 3% 3%	1 for U(X, 9 1% 2% 2% 2% 2% 2% 2% 2%	10 1% 1% 1% 1% 1% 1% 2%	5 8% 8% 8% 8% 8% 8% 8% 7%	6 5% 5% 5% 5% 5% 5% 5%	Table 7 Table 4%	Bidders 8 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3%	10 2% 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9 9	5% 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3%	Number o 7 7 2% 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1%	numi 5 6% 6% 6% 6% 6% 7% 6%	ber of slot: 6 4% 4% 4% 4% 4% 4% 4%	s a player Number of 7 MV Tabl 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 4W 3%	would bic of Bidders 8 VD ie 1e 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 3% 3%	9 1% 2% 2% 2% 2% 2% 2% 2% 2% 2%	10 1% 1% 1% 1% 1% 1% 1% 1% 2% 2%	5 8% 8% 8% 8% 8% 8% 7% 7%	5% 5% 5% 5% 5% 5% 5%	Number of 7 Table 4% 4% 4% 4% 4% 4% 4% 4%	f Bidders 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3%	10 2% 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9 3 4 5 5	5 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3% 3%	Number o 7 7 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1%	numi 5 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6%	ber of slot: 6 4% 4% 4% 4% 4% 4% 4%	s a player Number of 7 MV Tabl 3%	would bic of Bidders 8 VD le 1e 2% 2% 2% 2% 2% 2% 2% 3% 3% 3%	I for U(X,) 9 1% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	10 1% 1% 1% 1% 1% 1% 2% 2% 2%	5 8% 8% 8% 8% 8% 7% 7%	5% 5% 5% 5% 5% 5% 5% 5%	Number of 7 7 4%	Bidders 8 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3% 3% 2%	10 2% 2% 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9 3 4 5 6 6	5 5% 5% 5% 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3% 3% 3%	Number o 7 7 7 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	numl 5 6%	ber of slot: 6 4% 4% 4% 4% 4% 4% 4%	s a player Number O 7 MV Tabl 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	would bic of Bidders 8 VD ie 1e 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 3% 3% 3% 2%	1 for U(X,) 9 1% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2	10 1% 1% 1% 1% 1% 1% 2% 2% 2% 2%	5 8% 8% 8% 8% 8% 7% 7% 7% 7% 7%	6 5% 5% 5% 5% 5% 5% 5%	Number of 7 Table 4% 3%	Bidders 8 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3% 3% 2% 2%	10 2% 2% 2% 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9 3 4 5 6 7 7 8 9 9 1 1 1 1 1 1 1 1 1 1 1 1 1	5 5% 5% 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3% 3% 3%	Number o 7 7 Table 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 8 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1% 1% 1%	numl 5 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6% 6%	ber of slot: 6 4% 4% 4% 4% 4% 4% 4%	S a player Number of 7 MV Tabl 3%	would bic of Bidders 8 VD ie 1e 2% 2% 2% 2% 2% 2% 2% 3% 3% 3% 3% 2% 2% 2% 2% 2% 2% 2% 3% 3% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	I for U(X,) 9 1% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	10 1% 1% 1% 1% 1% 1% 2% 2% 2% 2% 2%	5 8% 8% 8% 8% 8% 8% 7% 7% 7% 7% 7%	6 5% 5% 5% 5% 5% 5% 5% 5%	Sumber of 7 Table 4% 4% 4% 4% 4% 4% 4% 4% 3% 3%	Bidders 8 e If 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3% 3% 3% 2% 2%	10 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%
Bid 3 4 5 6 7 8 9 3 4 5 6 6	5 5% 5% 5% 5% 5% 5% 5% 5% 5%	6 3% 3% 3% 3% 3% 3% 3% 3% 3%	Number o 7 7 7 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%	f Bidders 8 e 1d 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	9 1% 1% 1% 1% 1% 1% 1% 1%	10 1% 1% 1% 1% 1% 1% 1% 1% 1% 1%	numl 5 6%	ber of slot: 6 4% 4% 4% 4% 4% 4% 4%	s a player Number O 7 MV Tabl 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	would bic of Bidders 8 VD ie 1e 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2% 3% 3% 3% 2%	1 for U(X,) 9 1% 2% 2% 2% 2% 2% 2% 2% 2% 2% 2	10 1% 1% 1% 1% 1% 1% 2% 2% 2% 2%	5 8% 8% 8% 8% 8% 7% 7% 7% 7% 7%	6 5% 5% 5% 5% 5% 5% 5%	Number of 7 Table 4% 3%	Bidders 8 8 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3% 3%	9 2% 2% 3% 3% 3% 3% 3% 2% 2%	10 2% 2% 2% 2% 2% 2% 2% 2% 2% 2%

Figure 4. Base Scenario MPD Standard Deviation: Bundle vs. Single-Item Auction.

[MWD Number (- UVP of Bidders			AWD- UVP Number of Bidders						
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10	
3	-50%	-3 <mark>0%</mark>	-25%	-20%	-16%	-14%	-9%	-7%	-5%	-4%	-4%	-3%	
4	-49%	-2 <mark>9%</mark>	-25%	-20%	-15%	-14%	-9%	-7%	-5%	-4%	-4%	-3%	
5	-49%	-2 <mark>9%</mark>	-25%	-20%	-15%	-14%	-9%	-7%	-5%	-4%	-3%	-3%	
6	-48%	-2 <mark>9%</mark>	-25%	-20%	-15%	-14%	-8%	-7%	-5%	-4%	-3%	-3%	
7	-48%	-2 <mark>8%</mark>	-24%	-19%	-15%	-14%	-8%	-6%	-5 <mark>%</mark>	-4 <mark>%</mark>	-3%	-3%	
8	-48%	-2 <mark>8%</mark>	-24 <mark>%</mark>	-19%	-14%	-13%	-8%	-6%	-5 <mark>%</mark>	-4%	-3%	-3%	
9	-47%	-2 <mark>7%</mark>	-24 <mark>%</mark>	-19%	-14%	-13%	-7%	-6%	-5 <mark>%</mark>	-4%	-3%	-3%	
			MWD	- RVP			AWD- RVP						
		1	Number of	of Bidders	5		Number of Bidders						
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10	
3	-17%	-5%	-3%	-2%	0%	1%	16%	14%	14%	13%	11%	11%	
4	-17%	-5%	-4%	-2%	0%	0%	16%	15%	14%	12%	11%	10%	
5	-17%	-6%	-4%	-2%	0%	0%	16%	14%	14%	12%	10%	10%	
6	-18%	-6%	-4%	-2%	0%	0%	16%	15%	14%	12%	10%	10%	
7	-18%	-6%	-4%	-3%	0%	0%	16%	15%	14%	12%	10%	10%	
8	-18%	-6%	-4%	-3%	-1%	0%	16%	15%	14%	12%	10%	9%	
9	-18%	-6%	-4%	-3%	-1%	0%	16%	15%	14%	12%	9%	9%	

Figure 5. MPD for Group Demand Scenario: Bundle vs. Single-Item Auction.

				- UVP of Bidders	5			AWD- UVP Number of Bidders								
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10				
3	0.8%	0.6%	0.6%	0.6%	0.5%	0.5%	6%	4%	3%	2%	2%	1%				
4	0.9%	0.7%	0.7%	0.6%	0.6%	0.6%	6%	4%	3%	2%	2%	1%				
5	1.0%	0.8%	0.8%	0.7%	0.7%	0.6%	6%	4%	3%	2%	2%	1%				
6	1.0%	0.9%	0.8%	0.8%	0.7%	0.7%	6%	4%	3%	2%	2%	1%				
7	1.1%	0.9%	0.9%	0.8%	0.8%	0.7%	6%	4%	3%	2%	2%	1%				
8	1.2%	1.0%	1.0%	0.9%	0.8%	0.8%	6%	4%	3%	2%	2%	1%				
9	1.3%	1.1%	1.0%	0.9%	0.8%	0.8%	5%	4%	3%	2%	2%	1%				
			MWD	- RVP			AWD- RVP									
			Number (of Bidders	6		Number of Bidders									
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10				
3	6%	4%	4%	3%	3%	2%	7%	5%	4%	4%	3%	3%				
4	5%	4%	3%	3%	2%	2%	6%	5%	4%	3%	3%	2%				
5	5%	3%	3%	3%	2%	2%	6%	5%	4%	3%	3%	2%				
6	5%	3%	3%	2%	2%	2%	5%	5%	4%	3%	2%	2%				
7	4%	3%	3%	2%	2%	2%	5%	5%	4%	3%	2%	2%				
8	4%	3%	3%	2%	2%	2%	5%	5%	4%	3%	2%	2%				
9	4%	3%	2%	2%	2%	1%	5%	4%	4%	3%	2%	2%				

Figure 6. MPD Standard Deviation for Group Demand Scenario: Bundle vs. Single-Item Auction.

			MV	VD			AWD							
			Number o	of Bidders			Number of Bidders							
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10		
3	-26%	-20%	-16%	-13%	-11%	-10%	-13%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>		
4	-26%	-20%	-16%	-13%	-12%	-10%	-13%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>		
5	-26%	-20%	-16%	-14%	-12%	-10%	-13%	-9%	-7%	-5%	-4%	-3 <mark>%</mark>		
6	-26%	-20%	-16%	-14%	-12%	-10%	-13%	-9%	-7%	-5%	-4%	-3 <mark>%</mark>		
7	-26%	-20%	-16%	-14%	-12%	-10%	-13%	-9%	-7%	-5%	-4%	-3 <mark>%</mark>		
8	-26%	-20%	-16%	-13%	-12%	-10%	-14%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>		
9	-26%	-20%	-16%	-13%	-12%	-10%	-14%	-9%	-6%	-5%	-4%	-3 <mark>%</mark>		

Figure 7. MPD for Group Valuation Scenario: Bundle vs. Single-Item Auction.

			M	WD					AV	VD				
			Number of	of Bidders			Number of Bidders							
Min Slot Bid	5	6	7	8	9	10	5	6	7	8	9	10		
3	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
4	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
5	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
6	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
7	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
8	8%	5%	4%	3%	2%	2%	6%	3%	2%	2%	1%	1%		
9	8%	6%	4%	3%	3%	2%	6%	3%	2%	2%	1%	1%		

Figure 8. MPD Standard Deviation for Group Valuation Scenario: Bundle vs. Single-Item Auction.

		MV	VD		AWD					
Min Slot Bid	UVP		RVP			UVP	RVP			
3		-4.7		2.6		-0.8		6.1		
4		-4.9		2.2		-0.8		5.7		
5		-5.1		1.9		-0.8		5.2		
6		-5.2		1.6		-0.8		4.6		
7		-5.5		1.1		-0.9		3.3		
8		-6.3		-2.2		-2.0		-1.5		
9		-6.7		-4.8		-1.9		-3.5		

Figure 9. MPD for Insufficient Supply Scenario: Bundle vs. Single-Item Auction.

Min Slot Bid	UVP	RVP	UVP	RVP
3	0.5%	0.9%	0.3%	0.9%
4	0.6%	0.9%	0.3%	0.8%
5	0.6%	0.8%	0.3%	0.7%
6	0.6%	0.8%	0.4%	0.7%
7	0.7%	0.8%	0.4%	0.9%
8	0.8%	1.2%	1.0%	1.7%
9	0.9%	1.1%	1.1%	2.1%

Figure 10. MPD Standard Deviation for Insufficient Supply Scenario: Bundle vs. Single-Item Auction.

The results shown in Figures 3–10 reveal very distinctive patterns that can be summarized as follows:

- As the number of bidders and number of slots they bid for increases, the profit difference between the single and bundle auction reduces for the UVP cases. For the RVP cases, profit differences remain rather constant, favoring the bundle auction, except for the MWP with U(X,3X) (Figure 3).
- For the cases of uniform valuation pattern, and irrespective of the winner determination policy, the terminal operator is better off using the single-item auction with a decreasing profit difference as the number of bidders and slots they bid for increase (Figures 3, 5 and 7).
- For the cases of random valuation pattern (and again irrespective of the winner determination policy), the terminal operator is better off (except for six instances) using the bundle auction, especially under Winner Determination Policy 2 (Figures 3, 5, 7, and 9).
- Mean profit differences between the single and the bundle auction decrease, overall, with the increase in the number of bidders and slots they bid for (Figures 3, 5, 7, and 9).
- For the last scenario (Insufficient Supply Scenario, Figure 9), the mean profit change between the two auction mechanisms has the same patterns as the rest of the simulations, except for the cases where the minimum number of slot bids is greater than eight. These are cases where demand is greater than the supply, and a different pattern was expected. For these cases, when applying the bundle auction the port operator experiences a decrease in profits between 1.5% and 3.5% compared to the single auction, even for the random valuation pattern (where the port operator was better off using the bundle auction in all other cases).
- For most scenarios, the standard deviation of profit difference is either smaller or very close to the mean. The only exception is the base scenario, with a random valuation

with the number of slots a player would bid for drawn randomly from a U(X,3X), where X is the minimum number of slots a player will bid for.

4. Conclusions

In this paper, we proposed an auction framework for berth slot leasing in passenger marine terminals (roll-on/roll-off passenger vessels and/or cruise ships). Different variations of the proposed bundle and single auctions, with regards to winner determination, slot valuation, and the number of bidders, were tested and compared under four different simulation scenarios to determine which of the two proposed auction mechanisms is more profitable. This paper focuses on how to help port operators maximize their profits by choosing the correct auction for different circumstances.

The results from the computational experiments show that the proposed bundle auction outperforms the single-item auction when bidders have different valuations for each slot. Additionally, better results are reached when the winner of each round was determined based on their average per slot bid and not on their total bid. The single auction produced significantly higher profits when bidders have similar valuations for each slot (according to a probability distribution). For the cases where demand is higher than the supply (insufficient supply), the single auction produces more favorable results. The results from this study can be used by terminal operators, given their knowledge and/or assumptions on slot valuations and demand, to select a winner determination policy and the minimum number of slots they allow players to bid when designing the auction of their berth capacity to maximize their profits.

From this study, several future research opportunities arise as different winner determination policies, expected payoff strategies, and different slot valuation patterns, as well as a combination of valuation distribution patterns (i.e., random and uniform) that correspond to different seasonality factors, can be considered and tested for their effectiveness. For example, in this paper, we assume that the terminal operator has discretized the wharf both in space and time to allow for the largest vessels to be served at any time. Future research could further introduce a variable berth space and time assumption per auction round, thus increasing the number of available slots. Introducing variable berth space/time into the problem formulation would bring the proposed auction closer to reallife conditions. An optimization algorithm for determining the optimal sequence of slot availability for bidding can also be considered. The approach can be used in relation to different yield and customer satisfaction strategies and policies of the passenger terminal operations, reflecting decision dilemmas of busy passenger terminals, serving either RoPax or cruise vessels. Furthermore, the proposed auction framework can be applied in different types of terminals or even different transactions relating to the operation of intermodal freight terminal resource allocation, after necessary alterations have been made. Finally, future research should focus on including the environmental aspect of berth scheduling and port operations [20,21].

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