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Correction

Correction: Xing et al. An Investigation of Adaptive Radius for the Covariance Localization in Ensemble Data Assimilation. *J. Mar. Sci. Eng.* 2021, *9*, 1156

Xiang Xing [1,2], Bainian Liu [1,*], Weimin Zhang [1,3], Jianping Wu [1], Xiaoqun Cao [1] and Qunbo Huang [4]

[1] College of Meteorology and Oceanography, National University of Defense Technology, Changsha 410073, China; xingxiang14@nudt.edu.cn (X.X.); wmzhang104@139.com (W.Z.); wjp@nudt.edu.cn (J.W.); caoxiaoqun@nudt.edu.cn (X.C.)
[2] College of Computer, National University of Defense Technology, Changsha 410073, China
[3] Laboratory of Software Engineering for Complex Systems, Changsha 410073, China
[4] 93110 Troops of PLA, Beijing 100843, China; hqb09@163.com
* Correspondence: bnliu@nudt.edu.cn

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The authors wish to make the following corrections to this paper [1].

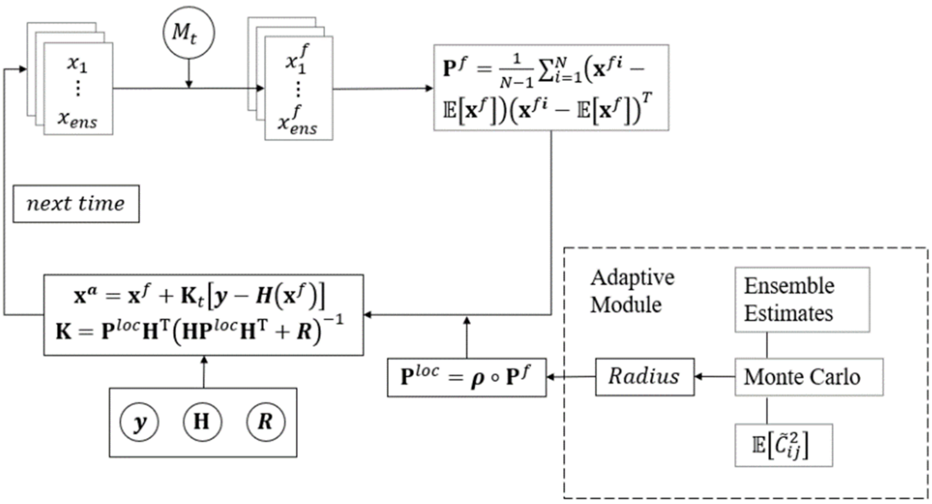
1. Figure Legend

In the original publication, there was a mistake in the legend for Figure 8. The original schematic diagram of calculating the correlation coefficient for the variables at a single grid point location with distance is prone to misinterpretation, so the expression was redesigned. The correct legend appears below:

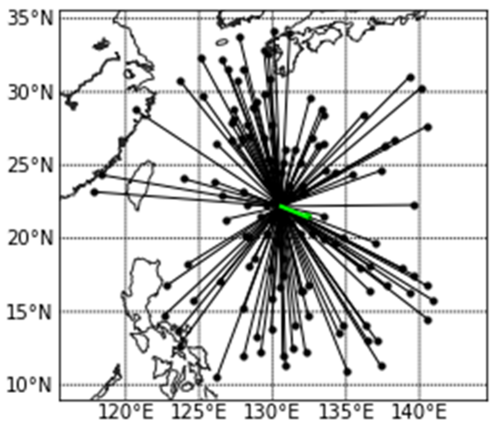
Figure 9. Shows the random sampling for calculating the correlation coefficients.

2. Error in Figure

In the original publication, there was a mistake in Figure 3 as published. The authors have discovered an error in the adaptive localization radius module in the figure. Figure 3 is newly added, and the following figures were renumbered accordingly. The corrected Figure 4 appears below.



In the original publication, there was a mistake in Figure 8 as published. The schematic diagram of calculating the correlation coefficient by using the correlation of the point with other grid points at different distances, and for example, the lime solid line is used to calculate the correlation coefficients at distances of 200 km. The corrected Figure 9 appears below.



3. Text Correction

(1) Following the correction of the derivation of the adaptive localization radius scheme, the text of *Section 3.2* was edited to reflect the localization radius is obtained adaptively through a certain criterion of correlations with the background ensembles in the adaptive localization scheme.

A correction has been made to *Section 3.2*:

We define the x_i are the ensemble members, $\overline{x}(\overline{x} = \mathbb{E}[x])$ is the estimate of the mean μ_x. Since ensemble members x are the random sampling using the Monte Carlo method in the EnKF, independent and uncorrelated. According to the Strong Law of Large Numbers, when the sample is infinite ($N \to \infty$) that we can get the estimate from the ensemble is exact:

$$P\left\{ \lim_{N\to\infty} \langle \overline{x} \rangle = \mu_x \right\} = 1$$

The estimated error is $\varepsilon = x - \mu_x$, we can get the estimation is unbiased with:

$$\mathbb{E}[\varepsilon] = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x) = \frac{1}{N-1}\sum_{i=1}^{N} x_i - \mu_x = 0$$

The variance $\mathrm{Var}[x]\left(\mathrm{Var}[x] = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2\right)$ is the estimate of the true variance σ_x^2. According to the Central Limit Theorem, the estimate values are normally distributed around the true, and the samples are independent, thus:

$$\mathrm{Var}\left[\frac{1}{N-1}\sum_{i=1}^{N} x_i\right] = \frac{1}{(N-1)^2}\sum_{i=1}^{N}\mathrm{Var}[x_i] = \frac{N}{(N-1)^2}{\sigma_x}^2$$

The standard error of the ensemble estimates approximates the square root of the ensemble size $\frac{1}{\sqrt{N-1}}(N \to \infty)$. The role of covariance localization is to cut-off spurious correlation between state variables that are spatially distant or not physically correlated. Since the estimation error of the ensemble members based on Monte Carlo sampling is $\frac{1}{\sqrt{N-1}}$, the spurious correlation in the correlation coefficient has seriously affected the accuracy of the estimated value when the correlation coefficient is less than $\frac{1}{\sqrt{N-1}}$. Therefore, it is considered that the corresponding distance at which the correlation coefficient is less than $\frac{1}{\sqrt{N-1}}$ is the radius of influence of the localization taper function that most of the spurious correlation can be cut-off with such radius. To avoid the effect of negative correlation coefficients, we use the square of the correlation coefficient for the determination of the localization radius $\mathbb{E}\left[C(r)^2\right] \geq \frac{1}{N-1}$.

Some studies have shown that there are strong links between localization and correlation, when the ensemble distribution is gaussian with the assumption of the sampling error is unbiased [49,50]:

$$\widetilde{\rho}_{ij} \approx \frac{(N-1)}{(N-2)(N+1)}\left((N-1) - \frac{1}{\mathbb{E}\left[\widetilde{C}_{ij}^2\right]}\right)$$

where $\widetilde{\rho}_{ij}$ is the first-order approximation of the localization coefficient ρ_{ij} and $\widetilde{C}_{ij}$ is the element of sample correlation. The threshold value of the localization radius for the Gaspari-Cohn function should make $\rho \geq 0$ in the adaptive localization scheme, so that spurious correlations can be accurately eliminated. Thus, the Equation (14) can be written as $\mathbb{E}\left[\widetilde{C}_{ij}^2\right] \geq \frac{1}{N-1}$.

The taper coefficient of the localization weighting function should be zero when $\mathbb{E}\left[C(r)^2\right]$ is less than $\frac{1}{N-1}$, in which the corresponding distance r is the localization radius threshold value. The localization radius can be computed adaptively with known statistical properties of sample covariances and updated with the real-time characteristics in the ensemble system. Figure 3a–d shows the square of the correlation coefficient in the Lorenz96 model with different ensemble members, and the dashed line corresponds to the position of the localization radius calculated by the adaptive localization scheme. It can be noticed that the spurious correlation weakens significantly with the increase of ensemble members (Figure 3e–h), and the adaptive radius reflects the range of spurious correlation well and gives a reasonable radius threshold. The larger ensemble has a larger radius than that of the small ensemble size, which can be explained that the accuracy of the estimated correlations of the small ensemble have been severely contaminated, but it is still accurate at the same distance in a larger ensemble.

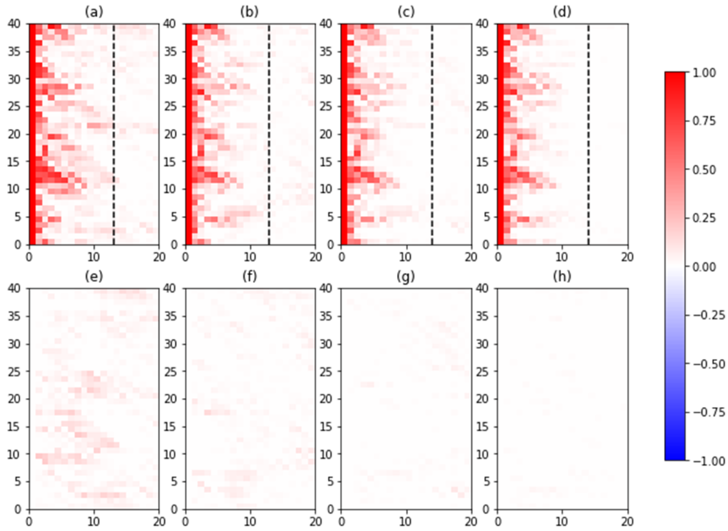


Figure 3. The square of the correlation coefficient in the Lorenz96 model with (**a**) 50 members, (**b**) 100 members, (**c**) 200 members and (**d**) 400 members. (**e**–**h**) represent the spurious correlation squares, respectively.

(2) Following the correction of the schematic diagram of calculating the correlation coefficient by using the correlation of the point with other grid points at different distances in Figure 8, the text of *Section 4.2* was edited to reflect the corrected schematic diagram.

A correction has been made to *Section 4.2*:

The expectations of correlation coefficients at different distances are calculated from the ensemble states by random sampling in the simulation area, which is shown in Figure 9 that the point pair connected by the lime solid line is used to calculate the correlation coefficients at distances of 200 km. The adaptive scheme is easy to implement online in the frameworks of the traditional covariance localization and has a low computational cost that the realizations of random sampling for expectations is roughly in the order of 10^2.

4. References Correction

A correction has been made to References, refs. [51,52] has been removed, The corrected refs. [49,50] appears below. The order of References will be automatically updated.

49. Ménétrier, B.; Montmerle, T.; Michel, Y.; Berre, L. Linear Filtering of Sample Covariances for Ensemble-Based Data Assimilation. Part II: Application to a Convective-Scale NWP Model. *Mon. Weather Rev.* **2015**, *143*, 1644–1664.
50. Sample Covariance Filtering. Available online: https://github.com/benjaminmenetrier/covariance_filtering/blob/master/covariance_filtering.pdf (accessed on 10 September 2020).

Reference

1. Xing, X.; Liu, B.; Zhang, W.; Wu, J.; Cao, X.; Huang, Q. An Investigation of Adaptive Radius for the Covariance Localization in Ensemble Data Assimilation. *J. Mar. Sci. Eng.* **2021**, *9*, 1156. [CrossRef]