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Collision Risk Index Calculation Based on an Improved Ship Domain Model

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Abstract: The traditional ship collision risk index model based on the distance at the closest point of approach (DCPA) and the time to the closest point of approach (TCPA) is insufficient for estimating ship collision risk and planning collision avoidance operations. This paper constructs an elliptical, dynamic ship domain that changes with ship speed and maneuverability parameters to overcome subjective human factors. Based on the constructed domain model, the concept of the ship domain proximity factor is introduced to improve the ship collision risk model based on DCPA and TCPA, and a risk calculation function model that considers the safety of ship navigation is constructed. The numerical calculation of the improved collision risk index calculation model confirms that the enhanced model has a higher rate of identification of risk between ships. The model is more compatible with the requirements of ship navigation decision-making and can provide theoretical support and a technical basis for research on ship collision avoidance decision-making.

Keywords: ship domain; collision risk index; index calculation

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1. Introduction

The domain of the ship is an imaginary water area that the ship's officer attempts to keep other ships from entering. This domain is vital for the safe navigation of any ship [1]. It is widely used in evaluating ship safety in the collision avoidance and path planning fields. The factors that affect the ship's domain's size are primarily ship-related and environmental. The characteristics of a ship include its size, maneuverability, and speed. Environmental factors include ship encounters, weather conditions, traffic conditions, the International Regulations for Preventing Collisions at Sea (COLREGs) requirements, and human factors [2]. The primary factors considered for the representative domain are summarized in Table 1. In some table cells, "N/A" shows that parameters are irrelevant or not involved. Most ship domain models are classified as static, dynamic, or fuzzy [2].

1.1. The Static Ship Domain Model

In the 1960s and 1970s, Fuji [1] first proposed the ship domain, which stipulates that other ships should avoid entering waters within a specific range of a ship in transit. Moreover, the statistical probability analysis method was used to obtain an elliptical ship domain expression suitable for the navigation environment at that time by analyzing the traffic situation using the questionnaire statistical method. During the same period, Goodwin [3] combined the provisions of the International Collision Regulations on ship encounters and collision avoidance, improved the Fuji ship domain, and established an inductive statistical ship domain model suitable for ships navigating open waters. The model uses three different sectors corresponding to the arcs of the sidelights and stern light of the ship according to the regulations. These three sectors are considered safe navigation for any ship. Subsequently, Davis et al. [4] established a ship domain that is an easy-to-use functional expression of the shortcomings of the Goodwin ship domain, such as a discontinuous boundary, and complex simulation and application. This resolved the problem of the boundary of the ship domain being difficult to express by function due to the three unequal sectors in Goodwin's ship domain model. In the proposed ship domain model, the ship deviates from the circle's center and advances to the lower left side of the domain. It can approximately divide the circular domain into four unequal regions, an innovation inherited from Goodwin's unequal area characteristic. Based on Fuji's elliptical ship domain, Davis's unequal area characteristic, and the COLREGs, Coldwell [5] established an elliptic ship domain in which ships move leftward along the minor axis of the ellipse by classifying the observed data by ship size and encounter situation.

Sun et al. [6,7] studied collision avoidance decisions through questionnaires and inquiries by investigating a ship officer's efforts to avoid collision with other ships. Simultaneously, the ship avoidance situation under various encounter situations was distinguished, and the ship domain under different navigation settings was established. These ship domain models had additional environmental impact factors compared to traditional ship domain models. Pietrzykowski Z et al. [8,9] discretized a ship based on the different headings and orientations of the target ship in a multiship encounter situation. Through statistical analysis of the collected data combined with the theory of the Collision Risk Index (CRI), a ship domain model surrounded by regular polygons was established. This ship domain model was affected by the ship officer's knowledge and the proper orientation of the target ship. Hansen M [10] used a large number of AIS data to study the ship domain after four years of observation and data statistics in the southern waters of Denmark and established a ship domain model for open waters.

1.2. The Dynamic Analytical Ship Domain Model

Jia [11] established a ship domain model suitable for congested waterways by analyzing the mapping relationship between ship domain size and ship speed and by having the captain control the ship domain size with a scaling factor obtained from a statistical method. Xiang [12] proposed a method for determining the ship domain in restricted waters using collected AIS data and the grid frequency method. Smierzchalski [13,14] established a hexagonal ship domain model by determining the size based on ship speed and cycle parameters; however, its physical meaning is vague and not convenient for practical application. In [15], considering the influence of speed in different directions around the ship, an empirical ship domain model for determining navigation safety in restricted waters was established. The model was corrected according to the ship navigation data collected in the Singapore Strait. Guo [16] analyzed the relationship between ship maneuverability and ship domain model size according to the motion characteristics of the ship steering process and proposed a method for calculating ship domain model size in various encounter situations. Wielgosz M [17] investigated the effect of ship speed on the shape and size of the ship domain in restricted waters. Dai [18] analyzed the yaw effect of water flow on ship navigation and modified the ship domain model.

1.3. The Fuzzy Ship Domain Model

Considering that the size of the ship domain is affected by the subjective factors of the officer, there are problems such as ambiguity and even wrong handling of the ship. Zhao et al. [19] used fuzzy mathematics theory to fuzzy the boundary of the ship domain model proposed by Goodwin so that obtaining the ship domain has a higher degree of freedom and flexibility, which is convenient for application in navigation practice. Zhou [20] used regression analysis to study the factors affecting the ship domain model, wavelet decomposition, and a neural network to analyze the mapping relationship between the ship domain and these factors, and to fuzzy the domain boundary through psychological function. Zhou established a new dynamic fuzzy ship domain model. Pietrzykowski [8,9,21,22] used empirical data to train the neural network, obtained the mapping relationship between the output collision risk and the ship collision avoidance parameter, and defined the ship domain model under different risks in open water and narrow water. Wang Ning [23,24] established the four-element ship domain model and the corresponding four-element fuzzy ship domain model and evaluated the subjectivity and objectivity of the ship domain from the perspective of humans, ships, and the environment. To improve the accuracy and operability of the ship domain model, a dynamic four-element model that changes with time was proposed.

Table 1. The factor taken into account in the representative domain, and the shape of the domain.

Domain By	Ship-Related Factor	Environment-Related Factors	Shape
Fuji [1]	Own ship's size and target ship's size	Weather conditions	Ellipse
Goodwin [3]	N/A	Weather conditions and COLREGs	Circular
Davis [4]	Own ship's size	COLREGs	Circular
Coldwell [5]	Own ship's size	Encounter situations and COLREGs	Ellipse
Sun [6]	N/A	Encounter situations, weather conditions, traffic conditions, and COLREGs	Ellipse
Pietrzykowski [8]	Own ship's size, speed, and maneuverability	Weather conditions and traffic conditions	Polygon
Hanse [10]	Own ship's size	N/A	Ellipse
Jia [11]	Own ship's size and speed	Encounter situations and weather conditions	Ellipse
Wang Y.Y [15]	Own ship's size and speed, and target ship's size and speed	COLREGs	Polygon
Pietrzykowski [22]	Own ship's size, speed, and maneuverability	Encounter situations, weather conditions, and traffic conditions	Ellipse
Wang Ning [23,24]	Own ship's size, speed, and maneuverability	Encounter situations, weather conditions, COLREGs, and human factors	QSD

The theory of the ship domain has been developed for over 50 years and can effectively support the study of ship behavior. The shapes of the representative domains are summarized in Table 1.

CRI is a crucial research topic for scholars and experts in the navigation field. In the 1970s, scholars began to study the calculation method of CRI. The scholar Zadeh proposed the theory of fuzzy mathematics in 1965, and its properties, particularly the dimensionless characteristics, agree with the concepts contained in the CRI. Therefore, researchers have started applying fuzzy mathematics to the study of ship CRI and expressing this concept by constructing functions. Liu et al. [25–28] used fuzzy mathematics theory to propose the CRI measurement model by using the distance at the closest point of approach (DCPA), the time to the closest point of approach (TCPA), and the distance between two ships as variables. Han-Jin Lee et al. [29–32] also established the fuzzy function model using the same idea and method. As with many parameters in these ships, CRI measurement models are determined by the ship officer's experience in handling; in some special cases, due to the influence of human factors such as the officer's psychology and experience in the ship's handling, the mathematical model may not work.

As the ship domain is affected by various uncertainties, the often-used elliptical or circular ship domains (such as Fuji, Goodwin, and Coldwell) are convenient to use but rarely consider the impacts of ship speed and maneuverability. The fixed size of the ship's domain does not correspond to the actual situation of navigation because an officer's execution of ship collision avoidance decisions is a dynamic process. In contrast, most ship domains that consider ship factors are irregular in shape and difficult to calculate and apply. This study utilizes the elliptical shape of the Fuji ship domain, the offset characteristics of the Coldwell ship domain, and the scaling characteristics of the quaternion ship to propose an elliptical dynamic ship domain that conforms to COLREGs requirements and varies with ship speed and maneuverability. Based on the newly constructed elliptical dynamic ship domain model, the collision risk model based on DCPA and TCPA is improved. Through the numerical calculation of the improved collision risk calculation model, it is verified that the improved model has a higher degree of risk identification between ships and is more in line with an actual navigation situation, where most officers prefer larger spacing and where the front and starboard sectors would probably be wider for head-on and crossing encounters.

2. Construction of Elliptical Dynamic Ship Domain Model

2.1. Elliptical Ship Domain Model

The scholars Fuji et al. proposed the concept of a ship domain in the 1960s and established a ship domain model for overtaking encounters in narrow waterways, that is, an ellipse centered on the ship, with the long axis along the bow and stern direction of the ship and the short axis along the abeam directions of the ship. According to long-term observation of the traffic situation in Japanese coastal waterways, an ellipse 8 times the ship's length on the long axis and 3.2 times the ship's length on the short axis was obtained. As shown in Figure 1, this model is the most convenient to use, so it is the most widely used domain model at present. However, this domain model does not consider the influence of speed on the size of the ship's domain.

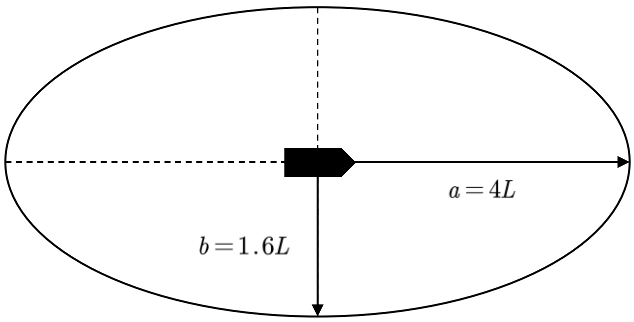

Figure 1. Ship domain of Fuji.

The Coldwell ship domain model was established by the scholar Coldwell, and considers the influence of COLREGs on the relevant regulations of ship navigation based on Fuji's model. It has an ellipse 12 times the ship's length on the long axis and 5 times the ship's length on the short axis, and the ship at the center is shifted to the left along the short axis by 1.75 times the ship's length, which was calculated by fitting the trajectory data of multiple ships. This is shown in Figure 2.

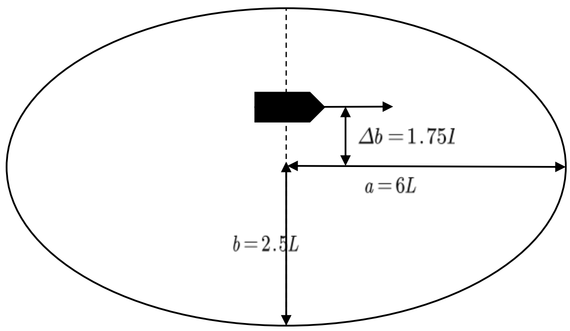

Figure 2. Ship domain of Coldwell.

2.2. Quaternion Ship Domain Model

The significant advantages of Fuji and Coldwell's elliptical ship domains are that they are easy to describe, use, and model. However, their disadvantages are that the size of the elliptical ship domain is only related to the ship size and does not involve the speed. They are static domains with a fixed size, which is inconsistent with the actual situation.

To solve the above problems, Wang et al. [23,24] proposed a new Quaternion Ship Domain (QSD) based on the method of Kijima [33,34]. The domain size of QSD is determined by four parameters (quaternions). These parameters are R_{fore}, R_{aft}, R_{starb}, and R_{port}, where R_{fore} and R_{aft} represent the longitudinal radii of the QSD in the bow and stern directions of the ship, respectively, and R_{starb} and R_{port} represent the transverse radii of the ship. The ship domain is divided into four unequal sectors, as shown in Figure 3. These four areas can fully consider the ship's maneuverability, speed, and course. Since the ship domain boundary can be either linear or nonlinear, this makes the QSD more flexible.

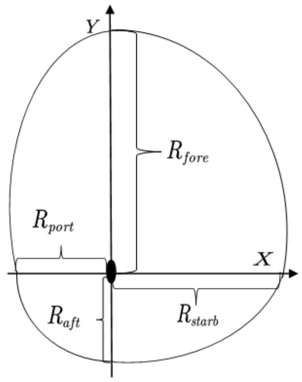

Figure 3. Quaternion ship domain.

The QSD model equation is as follows:

$$\text{QSD} = \left\{ (x,y) \middle| f(x,y,Q) \leq 1, Q = \left\{ R_{fore}, R_{aft}, R_{starb}, R_{port} \right\} \right\} \tag{1}$$

where $f(.)$ is a function defining the QSD model boundary, Q is a quaternion, which can more intuitively, effectively, and quickly establish the ship domain model. Without loss of generality, the QSD boundary can present a quadrilateral and a combined ellipse. As shown in Figure 3, the half-axis vertices of the combined ellipse are defined by the transverse and longitudinal radii in the quaternion, respectively. According to the blocking area estimation formula [34], the transverse and longitudinal radii in the QSD can be estimated via the following equations:

$$\begin{cases} R_{fore} = L + (1+s) \times 0.67\sqrt{k_{ad}^2 + \left(\frac{k_{dt}}{2}\right)^2} \\ R_{aft} = L + 0.67\sqrt{k_{ad}^2 + \left(\frac{k_{dt}}{2}\right)^2} \\ R_{starb} = (0.2 + k_{dt})/L \\ R_{port} = (0.2 + 0.75k_{dt})/L \end{cases} \tag{2}$$

In Equation (2), L is the owned ship's length; s is the coefficient considering the influence of the encounter situation; $k_{ad,}$ and k_{dt}, respectively, represent gains in advancement based on the distance from the position of the ship's center of gravity at the beginning of the rudder to the trimmed surface of the ship when the bow turns 90 degrees, and the tactical diameter based on the distance between the position of the longitudinal midship section when the ship turns 180 degrees from the original route to the bow. Additionally, s, k_{ad}, and k_{dt} can be calculated as follows:

$$\begin{cases} k_{ad} = L\exp(0.3591\log v + 0.0952) \\ k_{dt} = L\exp(0.5441\log v - 0.0795) \end{cases} \tag{3}$$

$$\begin{cases} s = 2 - \frac{(v - v_t)}{v}, & \text{head on} \\ s = 2 - \frac{\alpha}{\pi}, & \text{crossing} \\ s = 1, & \text{overtaking} \end{cases} \tag{4}$$

In Equations (3) and (4), v and v_t, respectively, represent the owned ship's speed and the target ship's speed in knots. The QSD model reasonably considers the ship's maneuverability, as determined by its advance and tactical diameter, and also uses the ship's speed to determine its instantaneous maneuverability.

2.3. Construction of Elliptic Dynamic Ship Domain Model

2.3.1. Construction Principles

Although the elliptic ship domains of Fuji and Coldwell are convenient to use, they are static domains and do not conform to the actual situation of navigation. Moreover, Wang Ning's QSD has an irregular shape that makes it difficult to calculate and apply, although it is dynamic.

This study further improves the ship domain based on a thorough analysis of the advantages and disadvantages of the above ship domains. The improvement principles are as follows:

(1) For the convenience of calculation and application, the elliptic ship domain is retained.
(2) Considering the COLREGs and ship maneuvering habits, the ship cannot be set at the center of the ship domain, but will be offset like in the Coldwell ship domain. This study intends to offset the ship from the center of the elliptical ship domain to its lower left.
(3) The size of the ship's domain is related to the speed and maneuvering parameters of the ship, and it should be a dynamic domain that varies with the speed and length of the ship.

2.3.2. Constructive Method

This study intends to establish an elliptical dynamic ship domain based on speed and length. To meet the COLREGs requirements for ship maneuvering, the elliptical ship domain constructed is expressed by four relevant parameters of the decentralized ellipse as follows: a: elliptical long semi-axis; b: elliptical short semi-axis; Δa: the offset of the ship from the center of the ellipse to the stern along the long axis of the ellipse; Δb: the offset of the ship along the elliptical short axis, as shown in Figure 4.

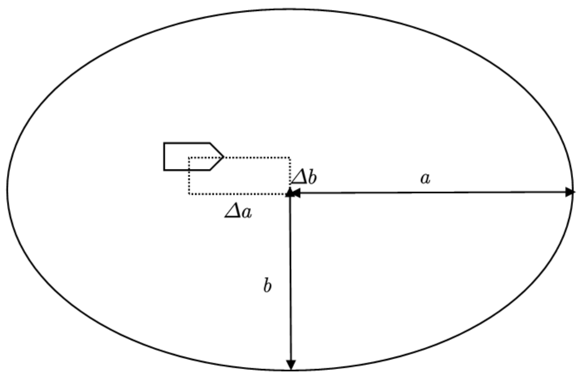

Figure 4. A decentralized elliptic ship domain.

The two variables a and b define the length and width of the ship domain, and the other two variables control the size of the four sectors of the ship. The bow and starboard areas in this ship domain are large, which can show navigators' perceptions of ship collision risk and meet the requirements of COLREGs for ship maneuvering. In addition, the elliptical ship domain is generally parallel to the ship's velocity vector, although the ellipse has a more complex geometry. The following elliptic parameter, Equation (5), is assumed to be analyzed to model the expression.

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1 \tag{5}$$

The influencing factors of ship speed are added to the ship domain model, and the region composed of four radii of the QSD is the polygon region surrounding the ship. The four radii (R_{fore}, R_{aft}, R_{starb}, and R_{port}) of the QSD divide the ship domain into four unequal sectors, and these four sectors fully consider the ship's maneuverability, speed, and course.

The ship's position is set at the Cartesian coordinate system's original center, and the coordinate axis is parallel to the speed vector of the ship. The quaternion of the QSD is taken as the four vertex coordinates of the ship quadrilateral: $(R_{fore}, 0)$, $(-R_{aft}, 0)$, $(0, R_{starb})$, $(0, -R_{port})$. The parameters of the elliptical ship domain are given by the following formulas:

$$a = \frac{R_{fore} + R_{aft}}{2}, b = \frac{R_{starb} + R_{port}}{2} \tag{6}$$

$$\Delta a = R_{fore} - a, \Delta b = R_{starb} - b \tag{7}$$

The elliptic parameter equation is as follows:

$$\frac{(x - \Delta a)^2}{a^2} + \frac{(y - \Delta b)^2}{b^2} = 1 \tag{8}$$

The long and short axes of the elliptical dynamic ship domain are determined by the four radii based on QSD. From Equations (2) and (3), the radius is determined by the shipping speed and ship length. Therefore, the variable of ship speed is integrated into the ship domain, and an elliptical dynamic ship domain is established that changes with ship speed and length, as shown in Figure 5.

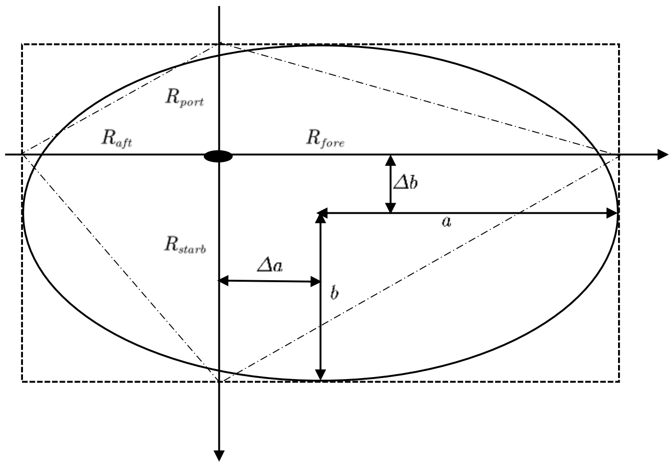

Figure 5. Elliptic dynamic ship domain model.

2.3.3. Ship Domain Model Calculation

To demonstrate the relationships between the radii of the elliptical dynamic ship domain and its ship speed, the corresponding numerical calculation was conducted using Equations (2)–(4), (6), and (7) and the results, which are expressed as a multiples of the ship length L, are illustrated in Table 2, where the test ship length is L, and the ship speed v ranges from 5 to 20 kn.

Table 2. The radius changes of dynamic Elliptical ship domain.

Speed (Knots)	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$R_{fore}(L)$	5.270	5.674	6.042	6.381	6.697	6.995	7.276	7.544	7.799	8.045	8.280	8.508	8.727	8.940	9.146	9.347
$R_{aft}(L)$	3.761	4.049	4.312	4.554	4.780	4.992	5.193	5.384	5.566	5.741	5.909	6.072	6.228	6.380	6.528	6.671
$R_{starb}(L)$	2.417	2.648	2.863	3.063	3.253	3.433	3.605	3.770	3.929	4.082	4.231	4.375	4.515	4.651	4.784	4.914
$R_{port}(L)$	1.863	2.036	2.197	2.347	2.489	2.625	2.754	2.877	2.997	3.112	3.223	3.331	3.436	3.538	3.638	3.735

As seen in the calculation results in Table 2, and as shown in Figures 5 and 6A, the size of the constructed dynamic ship domain model is not much different from the size of the QSD and increases with the increase in shipping speed. The four radii of the ship domain increase with the speed of the ship, and R_{fore} and R_{starb} are relatively larger. This meets the requirements of COLREGs which state that the ship on the right side must be more vigilant, and is also greatly in line with the actual situation of navigation wherein most officers prefer greater spacing, especially with a wider port and bow when meeting head-on and crossing. In addition, as shown in Figure 6B, compared with the Coldwell ship domain in the dashed line, the size of the elliptical dynamic ship domain in the solid red line is similar to that of the Coldwell domain when the shipping speed is about 10 kn, $a = 5.993L$, $b = 3.029L$ (Table 2 and Equation (6)). Meanwhile, the R_{aft} radius is much less than that of the Coldwell domain and the R_{starb} radius is slightly larger than that of the Coldwell domain since the ship maneuvering capability and encounter situations defined in the COLREGs are reasonably considered in the elliptical dynamic ship domain.

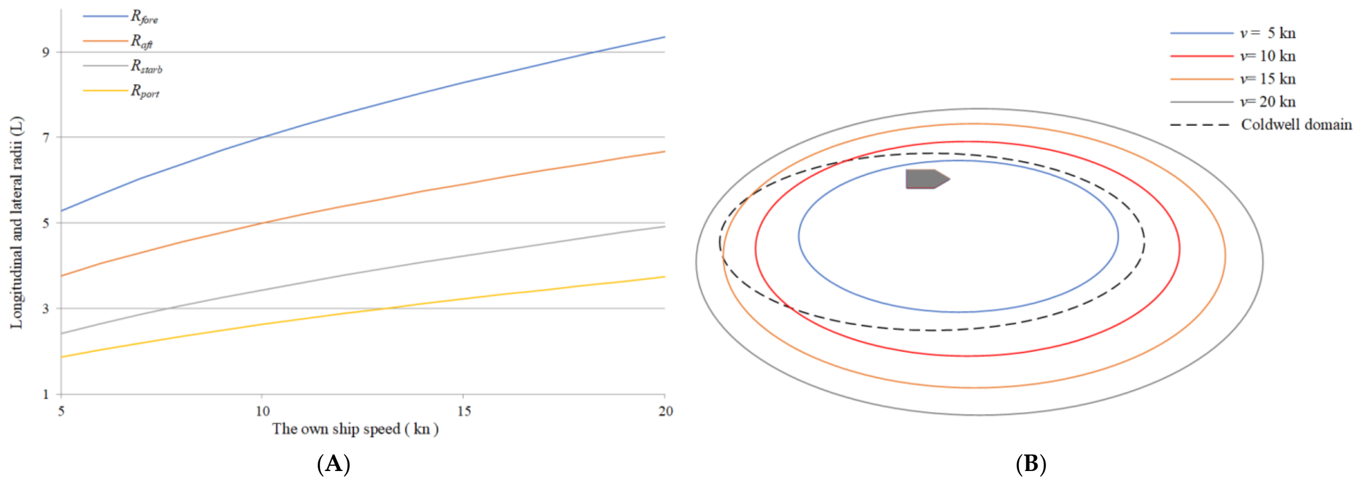

Figure 6. (**A**) The radii of the elliptical dynamic ship domain for various ship speeds; (**B**) combined elliptical dynamic ship domain and Coldwell domain.

3. Collision Risk Index Model Based on Improved Ship Domain

3.1. Collision Risk Index

The ship CRI is a risk coefficient used to measure the collision probability between ships. It is used between ships to measure the degree of collision risk between proceeding ships and static obstacles. Regarding the quantitative expression of CRI: there is no collision risk with 0; when there is an unavoidable collision risk, it is represented by 1; when the value is between 0 and 1, it indicates the degree to which a collision risk will occur. Therefore, calculating collision risk has practical application value. The crew can determine the avoidance time in encounter situations between two vessels and which vessel must be preferentially avoided in the case of multiple encounter situations by comparing the CRI value.

Lisowski J [35] proposed a collision risk model based on the DCPA and TCPA parameters, as shown in Equation (9):

$$r = \left(a_1\left(\frac{DCPA}{D_s}\right)^2 + a_2\left(\frac{TCPA}{T_s}\right)^2 + a_3\left(\frac{D}{D_s}\right)^2\right)^{-\frac{1}{2}} \tag{9}$$

where r is the sum of the collision risk between two ships; D_s is the minimum safety distance of the ship; T_s is the operating time from perceiving the collision risk to making avoidance decisions to steering; D is the distance between ships at any time; and a_1, a_2, and a_3 are the weight coefficients for the visibility of the navigation environment, the length and width of the ship, and the environment of the navigation waters. Under general conditions, take $a_1 = a_2 = a_3 = 1$. This coefficient is determined by the encounter situation of the ship.

If the visibility of the ships navigating is poor or the current situation requires focus on the avoidance distance between ships, a_1 can be greater than 1. If the ship's action to avoid collision by altering course is limited because of the ship's size, a_2 can be greater than 1. If the ship is navigating in a narrow channel, a traffic separation scheme, or other complex waters and needs to pay attention to the safe passing distance, then a_3 can be greater than 1.

3.2. CRI Model Based on Improved Ship Domain

3.2.1. Calculation of CRI Based on Ship Domain

Szlapczynski et al. [36] proposed an approaching factor (f) that can measure the collision risk between ships. Factor f is a proportional coefficient. If the target ship's trajectory is tangent to the ellipse scaled by f times the ship domain, it shows that the approaching factor between two ships is f; the smaller f, the higher the collision risk between ships, as shown in Figure 7.

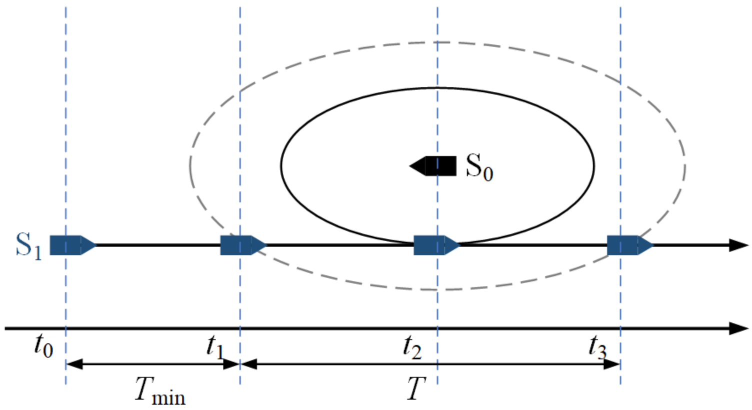

Figure 7. Schematic diagram of collision risk parameters.

The target ship (TS) is in danger of colliding with its own ship (OS) from t_0. At t_1, it invades OS's ship domain and reaches the elliptical boundary. At t_2, it is tangent to the ship domain with a scaling factor of f. At t_3, it leaves OS's domain. $f_{\min}$ represents the minimum value when TS is tangent to OS's domain boundary without changing the navigation state of the two ships when TS is located at time t_2. $T_{\min}$ represents the time TS requires to invade OS's domain from when the two ships remain in the same navigation state. T represents the invasion time of TS to the owned ship's domain from t_1 to t_3.

$f_{\min}$ is a coefficient between 0 and 1, and $f_{\min} = 1$ represents that TS is just tangent to OS's domain of the ship, and invasion of the domain of OS is narrowly avoided. As $f_{\min}$ decreases, the domain of OS is more deeply invaded. The following will explain how to use ship motion parameters to derive $f_{\min}$ with the above construction of the elliptical dynamic ship domain.

3.2.2. Elliptic Dynamic Ship Domain $f_{\min}$ Calculation

It is assumed that the course of OS is φ_0; the dimensions of the elliptical ship domain are a (semi-major axis of the ellipse), b (semi-minor axis of the ellipse); the speed of OS is v_0; and the position coordinates are X_0, Y_0, where X_{01} and Y_{01} are the initial position coordinates of OS. Because the coordinate system of the elliptical dynamic ship domain is parallel to the OS speed vector, for convenience in the calculation, the ship position coordinates ($X_0{}'$, $Y_0{}'$) under the new coordinates are parallel to the OS course in the Y axis direction. The course of TS is φ_t, the shipping speed is v_t, the ship position coordinates are (X_t, Y_t), and (X_{t1}, Y_{t1}) are the initial ship position coordinates of TS. The new ship position coordinates ($X_t{}'$, $Y_t{}'$) are shown in Figure 8.

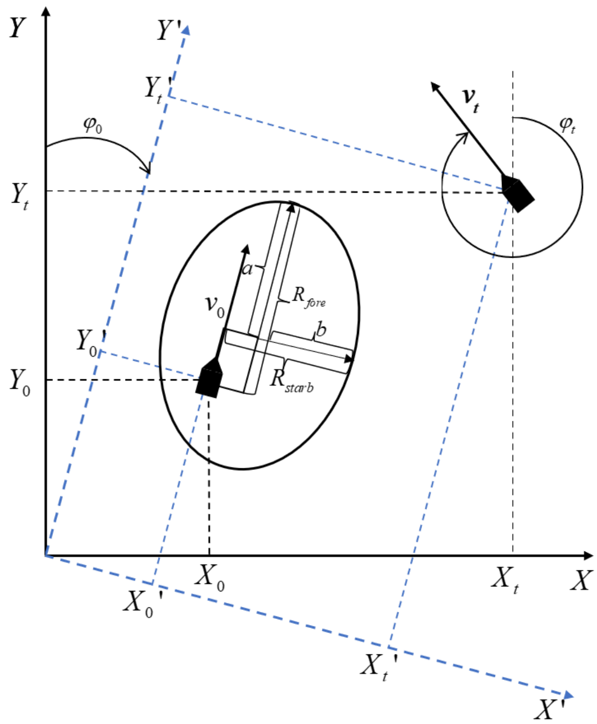

Figure 8. Schematic diagram of ship encounter parameters.

The relationship between the two coordinate systems is as follows:

$$\begin{aligned} X_0' &= X_0 \cos\varphi_0 - Y_0 \sin\varphi_0, Y_0' = X_0 \sin\varphi_0 + Y_0 \cos\varphi_0 \\ X_t' &= X_t \cos\varphi_0 - Y_t \sin\varphi_0, Y_t' = X_t \sin\varphi_0 + Y_t \cos\varphi_0 \end{aligned} \tag{10}$$

Since there is a ship position offset in the constructed elliptical dynamic ship domain, OS is not at the center point of the ellipse. Assuming that the center coordinates of the elliptical ship domain are (X_e, Y_e), then:

$$X_e = X_0' + \Delta a, Y_e = Y_0' + \Delta b \tag{11}$$

The parameter equation for constructing an elliptical dynamic ship domain is as follows:

$$\frac{(x - X_e)^2}{a^2} + \frac{(y - Y_e)^2}{b^2} = 1 \tag{12}$$

When TS is on the ship domain boundary scaled by a factor f of OS, or when the ship domain of the ship is multiplied by an approaching factor f such that TS is exactly on the ship domain boundary, it can be expressed by Equation (13):

$$\frac{(X_t' - X_e)^2}{(af)^2} + \frac{(Y_t' - Y_e)^2}{(bf)^2} = 1 \tag{13}$$

Upon simplification, it is equivalent to the following equation:

$$f^2 = \frac{(X_t' - X_e)^2}{a^2} + \frac{(Y_t' - Y_e)^2}{b^2} \tag{14}$$

It can be converted to the initial coordinate expression is as follows:

$$f^2 = \frac{((X_t - X_0)\cos\varphi_0 + (-Y_t + Y_0)\sin\varphi_0 - \Delta a)^2}{a^2} + \frac{((X_t - X_0)\cos\varphi_0 + (-Y_t + Y_0)\sin\varphi_0 - \Delta b)^2}{b^2} \tag{15}$$

When OS and TS keep their original courses, the coordinate change values of the two ships are as follows:

$$\begin{array}{c} X_0(t) = X_{01} + v\sin\varphi_0 t, Y_0(t) = Y_{01} + v\cos\varphi_0 t \\ X_t(t) = X_{t1} + v_t\sin\varphi_t t, Y_t(t) = Y_{t1} + v_t\cos\varphi_t t \end{array} \quad (16)$$

By substituting Equation (16) into Equation (15), the following Equation (17) can be obtained. Meanwhile, to facilitate writing, we will use A, B, and C instead of the corresponding coefficients of Equation (17), as follows:

$$f^2(t) = At^2 + Bt + C \quad (17)$$

$$\begin{aligned} A = {} & \left(\frac{(\sin\varphi_t\cos\varphi_0 - \sin\varphi_0\cos\varphi_t)^2}{a^2} + \frac{(\sin\varphi_t\cos\varphi_0 + \cos\varphi_t\sin\varphi_0)^2}{b^2}\right)v_t^2 \\ & - \frac{2(\sin\varphi_t\sin\varphi_0 + \cos\varphi_t\cos\varphi_0)}{b^2}vv_t + \frac{v^2}{b^2} \\ B = {} & 2\Big(\frac{(\sin\varphi_t\cos\varphi_0 - \sin\varphi_0\cos\varphi_t)((X_{t1}-X_{01})\cos\varphi_0) + (-Y_{t1}+Y_{01})\sin\varphi_0}{a^2} \\ & - \frac{(\sin\varphi_t\cos\varphi_0 + \cos\varphi_0\cos\varphi_t)((X_{t1}-X_{01})\sin\varphi_0) + (Y_{t1}-Y_{01})\cos\varphi_0}{b^2}\Big)v_t \\ & -2\frac{(X_{t1}-X_{01})\sin\varphi_0 + (Y_{t1}-Y_{01})\cos\varphi_0}{b^2}v \\ & -2\frac{((v_t\sin\varphi_t - v\sin\varphi_0)\cos\varphi_0 + (v\cos\varphi_0 - v_t\cos\varphi_t)\sin\varphi_0)\Delta a}{a^2} \\ & -2\frac{((v_t\sin\varphi_t - v\sin\varphi_0)\cos\varphi_0 - (v\cos\varphi_0 - v_t\cos\varphi_t)\sin\varphi_0)\Delta b}{b^2} \\ C = {} & \frac{((X_{t1}-X_{01})\cos\varphi_0 + (Y_{01}-Y_{t1})\sin\varphi_0)^2 - 2((X_{t1}-X_{01})\cos\varphi_0 + (Y_{01}-Y_{t1})\sin\varphi_0)\Delta a + (\Delta a)^2}{a^2} \\ & + \frac{((X_{t1}-X_{01})\sin\varphi_0 + (Y_{01}-Y_{t1})\cos\varphi_0)^2 - 2((X_{t1}-X_{01})\cos\varphi_0 - (Y_{01}-Y_{t1})\sin\varphi_0)\Delta b + (\Delta b)^2}{b^2} \end{aligned} \quad (18)$$

where $f(t)$ represents the f value at time t when the ship domain is scaled to the extent whereby TS is just on the domain boundary. Upon derivation of variable t from Equation (17), the following equation can be obtained:

$$\frac{f(t)}{dt} = \frac{2At + B}{2\sqrt{At^2 + Bt + C}} \quad (19)$$

Set Equation (19) equal to zero, and the corresponding $t_{\min}$ is solved as follows:

$$t = -\frac{B}{2A} \quad (20)$$

$$f_{\min} = f(t_{\min}) = \sqrt{-\frac{B^2}{4A} + C} \quad (21)$$

The time to domain violation is the time at which TS invades the domain of OS. Assuming that TS is just tangent to OS's domain of the ship that is not scaling, and invasion of the domain of OS is narrowly avoided, the value of f is equal to 1. Then, for Equation (17), set $f(t) = 1$:

$$At^2 + Bt + C - 1 = 0 \quad (22)$$

Then, Equation (23) is solved as follows:

$$t_{1,3} = \frac{-B \mp \sqrt{B^2 - 4A(C-1)}}{2A} \quad (23)$$

When $B^2 - 4A(C-1) < 0$, Equation (23) has no solution. At this time, $f_{\min} > 1$, TS does not invade OS's domain.

When $B^2 - 4A(C-1) = 0$, Equation (23) has a unique solution. At this time, $f_{\min} = 1$, TS just invades the boundary of OS's domain;

When $B^2 - 4A(C-1) > 0$, $f_{\min} < 1$, TS has the behavior of invading OS.

This is discussed and defined as follows:

$$t_1 = \min(\frac{-B - \sqrt{B^2 - 4A(C-1)}}{2A}, \frac{-B + \sqrt{B^2 - 4A(C-1)}}{2A}) \tag{24}$$

$$t_3 = \max(\frac{-B - \sqrt{B^2 - 4A(C-1)}}{2A}, \frac{-B + \sqrt{B^2 - 4A(C-1)}}{2A}) \tag{25}$$

(1) When $t_1 < 0$, $t_3 < 0$, it indicates that TS's invasion has ended.
(2) When $t_1 < 0$, $t_3 > 0$, it indicates that TS's invasion occurred at time t_1 and will end at time t_3.
(3) When $t_1 > 0$, $t_3 > 0$, it indicates that TS's invasion has not yet occurred; it will invade OS's domain at t_1 and leave the domain at t_3.

$T_{\min}$ indicates the time TS requires to enter OS's domain from the current time when two ships maintain their navigational states. T represents the duration of the invasion of TS in OS's domain while two ships remain in their current navigation states, as shown in Equations (26) and (27).

$$T_{\min} = t_1 - t_0 \tag{26}$$

$$T = t_3 - t_1 \tag{27}$$

3.2.3. The Improved CRI Model of the Ship Domain

$f_{\min}$ is the corresponding f value when TS is tangent to the ship domain boundary with a scaling factor f, as shown in Figure 9:

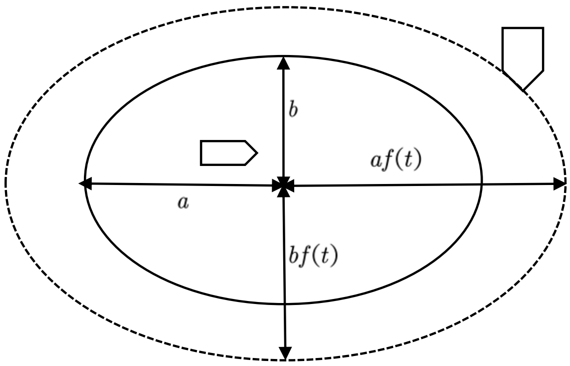

Figure 9. Schematic diagram of scaling factor of f.

The dashed ellipse in Figure 9 shows that TS is just on the boundary of the changed ship domain after the coefficient f is scaled. For Equation (9), $f_{\min}$ is denoted by $\frac{DCPA}{D_s}$, TCPA denotes $T_{\min}$, $\frac{D}{D_s}$ denotes $f(t)$, and CRI denotes. Then:

$$r_f = (a_1 f_{\min}^2 + a_2 (\frac{T_{\min}}{T_s})^2 + a_3 f^2(t))^{-\frac{1}{2}} \tag{28}$$

In practical application, when collision risk cannot be avoided between ships, CRI (r_f) is represented by 1. When there is no collision risk between ships, r_f is represented by 0. When r_f is between 0 and 1, it is necessary to make r_f equal to 0 through appropriate avoidance actions such as steering to avoid the collision.

4. Comparative Simulation of Ship CRI Models

To verify the accuracy and scientific validity of the improved collision risk model based on the dynamic ship domain, the CRI (r_f) calculation based on the improved ship domain is compared with the CRI (r) based on DCPA and TCPA in two-ship and multiship encounter situations using Microsoft Visual C++ 2019 software (Redmond, WA, USA).

4.1. Experimental Hypothesis

To simplify the experiment, this study makes the following assumptions about the simulation environment:

(1) Ships encounter open waters with good visibility, and the influence of external environmental factors such as wind, waves, and currents are not considered.
(2) OS size is only considered in the calculation of the ship domain, and OS length is ignored during other processing.
(3) The minimum safety distance of the ship is set to 0.5 n miles, that is, $D_s = 0.5$n miles. The process of perceiving the collision risk to making avoidance decisions to steering takes about 15 to 20 min. In this study, $T_s = 15$min.

4.2. Comparison Simulation of the Improved CRI (r_f) and CRI (r)

The course of OS is selected as north (0°). In the coordinate system with north as the Y axis direction, eight TSs are selected to compare the CRI parameters with OS in different encounter situations, as shown in Figure 10. Among them, the motion parameters of OS and TS are as follows: OS is S_0; OS speed $v = 10$ kn ; the ship length is $L = 400$ m; the ship's course is 0°; X and Y represent the ship coordinates, respectively; and $S_n(n = 1, 2, 3, \ldots)$ represents different TSs, as shown in Table 3.

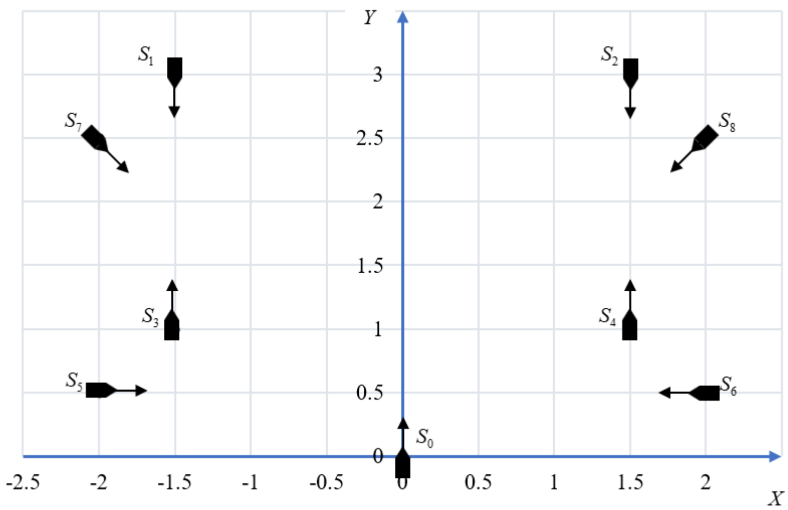

Figure 10. Ship encounter situation map.

Table 3. Ship motion parameters.

Parameters	**S$_0$**	**S$_1$**	**S$_2$**	**S$_3$**	**S$_4$**	**S$_5$**	**S$_6$**	**S$_7$**	**S$_8$**
v (kn)	10	10	10	8	8	10	10	10	10
course (°)	0	180	180	0	0	90	270	135	225
X (n mile)	0	−1.5	1.5	−1.5	1.5	−2	2	−2	2
Y (n mile)	0	3	3	1	1	0.5	0.5	2.5	2.5

Based on Equations (8) and (28), the weight coefficients of the visibility of the navigation environment, the length and width of the ship, and the navigation water environment are all taken as 1, $a_1 = a_2 = a_3 = 1$. The calculated improved ship collision risk is shown in Table 4, where TCPA and $T_{\min}$ are in minutes, and the other parameter values are dimensionless. In some table cells, "N/A" shows that the value cannot be calculated. The ship CRI line chart based on r_f and r is shown in Figure 11.

Table 4. Ship collision risk based on r_f and r.

Ship	DCPA/D_S	TCPA	D/D_S	r	f_{min}	T_{min}	$f(t)$	r_f
S_1	3.00	9.00	6.71	0.14	1.86	N/A	2.88	0
S_2	3.00	9.00	6.71	0.14	0.87	8.36	1.89	0.45
S_3	3.00	30.00	3.61	0.20	1.86	N/A	2.01	0
S_4	3.00	30.00	3.61	0.20	0.87	7.28	1.02	0.57
S_5	2.12	7.50	4.12	0.21	0.68	7.66	2.90	0.40
S_6	2.12	7.50	4.12	0.21	0.51	2.98	1.43	0.53
S_7	1.78	9.99	6.40	0.15	0.38	10.01	3.39	0.38
S_8	1.78	9.99	6.40	0.15	0.31	5.89	2.01	0.48

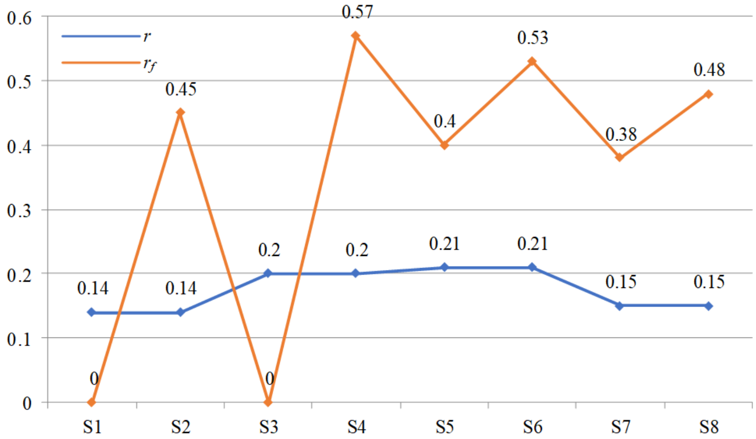

Figure 11. Ship collision risk index line chart based on r and r_f.

As shown in Figure 10 and Table 4, the ships S_1, S_2, S_3, S_4, and OS are left-and-right symmetrical. The CRI (r) obtained by the ship CRI model based on DCPA and TCPA is the same. The CRI (r_f) obtained by the improved CRI model is different. The improved model can identify that the right side of the ship has a higher risk under the same encounter situation, which is closer to the actual navigation decision-making situation. As shown in Figure 11, upon comparing the values of r and r_f, it can be seen that the change range of r is 0.14 to 0.21, and the change gradient is 0.07. The description of the risk is too compact, and the change is not significant. The variation range of r_f is 0 to 0.53, and the variation gradient is 0.53. The measurement of CRI is more accurate, which can reflect the difference in the ship collision risk degree.

In conclusion, the improved model has the following advantages: The precision in describing dangerous times and distances is higher. The ship on the right side is more vigilant, reflecting the requirements of COLREGs. The defined risk gradient changes significantly and is easier to observe in practical applications.

5. Conclusions

This study proposes an elliptical dynamic ship domain with size changes based on ship speed and maneuvering performance, combined with the Fuji elliptical ship domain, the offset characteristics of the Coldwell ship domain, and the scaling characteristics of the QSD, in response to the problem of the low accuracy of ship collision avoidance in the circular ship domain based on DCPA and TCPA. The ship domain model calculation and comparison with the Coldwell ship domain in Section 2.3.3 show that it is feasible to model and calculate the domain, as the R_{aft} radius is significantly less than that of the Coldwell domain and the R_{starb} radius is slightly larger under the same ship length, which conforms to the COLREGs and the officer's ship maneuvering habits. In addition, the ship's domain changes dynamically with the ship's speed, making ship navigation more realistic. Meanwhile, the CRI model is improved by introducing the ship domain risk identification parameter (proximity factor). By calculating the ship CRI of different encounter situations and comparing it with the calculation results of the ship CRI before the improvement, it is concluded that the improved geometric scaling ship domain risk index model is more accurate at measuring the risk of specific encounter situations. The model has higher accuracy, identification, safety, and reliability. The results of this study could provide a new idea and theoretical support for the operation of hazardous collision avoidance in ships in the current environment.

In the future, we will prepare to use the ship AIS data by: presenting the ship speed change situation as a function model that considers the navigation environment; optimizing the ship domain model and comparing it with other ship models; conducting collision risk calculation in a real scene; and researching ship autonomous collision avoidance decisions based on the improved collision risk model.

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