

Article

Application of Equivalent Single Layer Approach for Ultimate Strength Analyses of Ship Hull Girder

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Abstract: The objective of this paper is to present the application of equivalent single layer (ESL) approach for the ultimate strength assessment of ship hull girder in the context of numerical finite element (FE) simulations. In the ESL approach, the stiffened panel is replaced with a single plate, which has the equivalent stiffness of the original panel. Removal of tertiary stiffening elements from the numerical model facilitates time-savings in pre-processing and FE analysis stage. The applicability of ESL approach is demonstrated with two case studies, one compartment model and full-sized double hull tanker model in intact and damaged conditions. The damage extents are determined based on the international association of classification societies from common structural rules (IACS-CSR) for oil tanker. Ship hull girder is exposed to distributed pressure with the sinusoidal shape that bends the hull girder. This pressure load is applied separately to bottom and side structures to obtain the vertical and horizontal bending moments of the hull girder, respectively. Ultimate strength predictions obtained from ESL approach are compared to full three-dimensional finite element method (3D FEM) and IACS incremental-iterative method. The comparison between different methods is provided in terms of longitudinal bending moment and cross sectional stress distribution. Overall, ESL approach yields good agreement compared to the 3D FEM results in predicting the ultimate strength of ship hull girder while providing up to 3 times computational efficiency and ease of modeling.

Keywords: equivalent single layer; ultimate strength; hull girder; finite element method



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1. Introduction

Due to extreme and accidental loads, a ship's hull girder can reach its ultimate load-carrying capacity. One of the fatal consequences of structural failure is that the ship may suffer progressive collapse due to internal and external loads during seafaring. To minimize the risk of such accidents, rules stipulate longitudinal strength assessment for all ships [1,2]. The objective of this assessment is to determine the ultimate strength capacity of ship hull girder when the ship is subjected to bending loads. In case of ship grounding or collision, damaged hull section reduces the bending strength further. Therefore, the ultimate strength assessment must be performed both in intact and damaged conditions to ensure that hull girder has sufficient strength reserve.

Several studies have been conducted to evaluate the ultimate strength of ship hull girder using simplified methods and are currently applied in the classification society and commercial software. Caldwell [3] proposed an equivalent thickness approach to replace stiffened panel and used strength reduction coefficients to consider buckling. Smith [4,5] further refined the strength reduction coefficient method and considered that the ultimate strength of a hull girder is dependent on the strength of individual elements reaching their limit at different times. The Smith's method has been adopted in the Common

Structural Rules for Bulk Carrier and Oil Tanker, but the loading is limited to the vertical bending moment. In parallel, Ueda and Rashed [6] proposed the idealized structural unit method (ISUM) which considers more loading scenarios. For example, ISUM can model the buckling response due to all possible hull girder sectional load components (i.e., vertical bending, horizontal bending, vertical shearing force, horizontal shearing force, and torsion). Several advanced applications of ISUM approach have been developed by Ueda et al. [7], Masaoka et al. [8], Fujikubo et al. [9], Paik et al. [10], as well as Lindemann and Kaeding [11] for ultimate strength analyses of stiffened panel structures in different loading conditions.

The ISUM is recognized as one of the most time efficient methods for progressive collapse analysis of ship hull girder [12]. The method has been implemented in the ALPS/HULL program within the MAESTRO FEM analysis code. The ISUM can deal with interaction between local and global failures [13] for a short section of the ship structure. However, larger models offer a number of advantages. To capture the compartment level buckling relevant for lightweight ship structures [14], a large-scale ship model should be considered. Larger models permit inclusion of actual pressure distributions and various load combinations. Additionally, the model length influences the post-buckling behavior, which ultimately determines the bending moment capacity [15]. Furthermore, full ship model are advocated in [16,17] to minimize the boundary effect which often lead to more conservative, heavier scantlings.

The full three-dimensional finite element method (3D FEM) is an effective tool used for performing progressive collapse analysis to obtain structural strength capacity of ship hull girder. The analysis can reflect the local failure of structural members, e.g., local plate buckling and stiffener tripping, if they are modeled in detail. However, the 3D FE simulation of entire ship structure requires enormous modeling and computational effort. To reduce these while maintaining the accuracy of 3D FEM, we propose the use of equivalent single layer (ESL) approach. In the context of ship structures, ESL has been used for analysis of buckling response of panels [18,19], vibration response of sandwich panels [20,21], and ultimate strength of stiffened panels [22,23]. However, the application examples for entire hull girder analysis are missing which this paper aims to fulfill. In the traditional FE modeling, a stiffened panel is modeled in detail composed of longitudinal stiffeners with its attached plating. Using ESL methodology, a stiffened panel is modeled as a plate without the stiffeners, but with the same stiffness as the original panel. Consequently, simplification of stiffened panels enables consideration of design alternatives without changing the FE mesh, and thus more efficient exploration of design space. Therefore, the main benefits of the ESL approach compared with 3D FE analysis are: (1) reduced modeling effort, (2) reduction in degree of freedom (DOF), and (3) reduced computational effort.

This paper presents the application of ESL approach for the ultimate strength assessment of ship hull girder and one compartment models. In the ESL model, the stiffeners are removed and shell properties are defined with equivalent stiffnesses composed of 6×6 membrane and bending stiffness matrices. The stiffness matrices are calculated by the first derivative of membrane force and bending moment obtained from a unit cell under six loading conditions. Two different ship-scale case studies are presented, first focusing on the compartment level analysis (Benson et al. [14]) and second, the full-scale analysis of ship hull girder including structural damage (Tabri et al. [24]). The ultimate strength of ship hull girder is analyzed in intact and damaged conditions due to grounding or collision. For the damage conditions, the damage extents are determined from the IACS-CSR and the structural members located in the damage extent area are removed. The distributed pressures are applied separately along the bottom and side structures of the ship to obtain the vertical and horizontal bending moment curves, respectively. For comparative purposes, analysis are also conducted with the incremental-iterative method from the IACS-CSR. The ultimate strength analysis are validated with the detailed 3D FE simulations.

2. Methods

In this paper, the ESL approach is used to predict the ultimate strength of hull girder. The theoretical framework and assumptions used in the ESL approach are explained in the following section.

2.1. Overview of the ESL Approach

The ESL approach employs the first-order shear deformation theory (FSDT) in which the Kirchhoff hypothesis is relaxed by removing the assumption of transverse normal. In the FSDT, the transverse normals do not remain perpendicular to the middle of the surface after plate is deformed, as can be seen in Figure 1. In this manner, transverse shear strains (γ_{xz}) are considered in the FSDT. Additionally, the rotation in the z direction is assumed to be zero. In the deformed plates, the transverse normals are displaced by u_0 and are rotated by ϕ_x from the undeformed position. Under the assumptions and restrictions used in the FSDT, the non-linear strains are expressed in Equation (1):

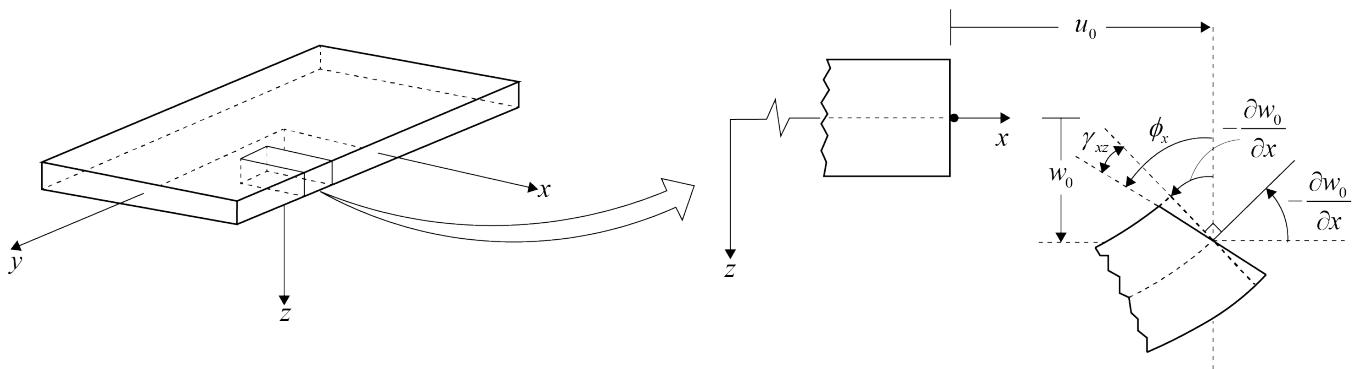


Figure 1. Undeformed and deformed geometries of an edge of a plate under the assumption of the first-order shear deformation theory (FSDT).

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^1 \\ \varepsilon_{yy}^1 \\ \gamma_{yz}^1 \\ \gamma_{xz}^1 \\ \gamma_{xy}^1 \end{Bmatrix}, \quad (1)$$

where

$$\{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \\ \frac{\partial w_0}{\partial y} + \phi_y \\ \frac{\partial w_0}{\partial x} + \phi_x \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \end{Bmatrix}, \quad (2)$$

$$\{\varepsilon^1\} = \begin{Bmatrix} \varepsilon_{xx}^1 \\ \varepsilon_{yy}^1 \\ \gamma_{yz}^1 \\ \gamma_{xz}^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}. \quad (3)$$

Note that the strains (ε_{xx} , ε_{yy} , γ_{xy}) are linear through the thickness, while the transverse shear strains (γ_{xz} , γ_{yz}) are constant through the thickness based on the FSDT. The strains (ε)

are composed of membrane (ϵ^0) and bending (ϵ^1) parts. Thus, the constitutive equations for the FSDT are obtained using the following relations:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & 0 \\ A_{21} & A_{22} & 0 & B_{21} & B_{22} & 0 \\ A_{31} & 0 & A_{33} & 0 & 0 & B_{33} \\ C_{11} & C_{12} & 0 & D_{11} & D_{12} & 0 \\ C_{21} & C_{22} & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & C_{33} & 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix}, \quad (4)$$

where N_{xx} , N_{yy} are the membrane forces, N_{xy} is the shear force, M_{xx} , M_{yy} are the bending moments, M_{xy} is the torsion, ϵ_{xx}^0 , ϵ_{yy}^0 , γ_{xy}^0 are the membrane strains, and ϵ_{xx}^1 , ϵ_{yy}^1 , γ_{xy}^1 are the curvatures. The **ABCD** stiffness matrices are obtained from the first derivative of membrane forces and bending moments of the unit cell (UC) simulations under six loading conditions, as can be seen in Figure 2. Compared to full 3D FEM, the generation of stiffness matrices requires an additional modeling and computational effort, which however can be made fairly automatic using programming.

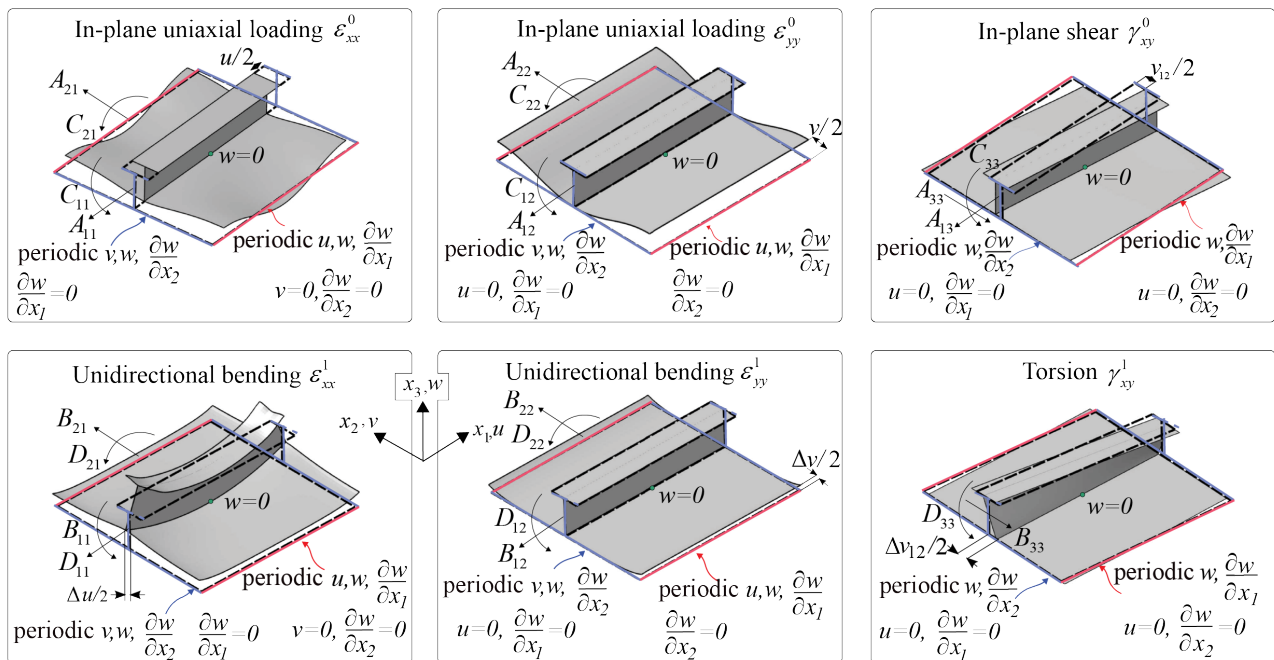


Figure 2. Six different unit cell (UC) configurations with boundary conditions needed for ABCD stiffness matrix definition. Forces and moments shown with arrows are associated with respective stiffness components. Boundary conditions for edges with the same color are identical.

The assumed UC is a periodic constituent of the full panel, thus periodic boundary conditions are imposed on UC, see Figure 2. The details of performing UC simulations are given in the previous papers by the authors [22,23]. Essentially, the same procedure was applied for each of the stiffened panel in case study analyses. Representative unit cell model was created for each panel with different topology (plate thickness and stiffener type) and 3D FE analyses were performed to obtain ABCD stiffness matrices. Compartment model (first case study) consisted only 1 stiffened panel configuration that was replaced by ESL. In a full ship hull (second case study), there were 45 different panel configurations, giving total of 270 (6×45) UC simulations. The entire procedure of UC analysis (pre- and post-processing) was made automatic using python scripts, which read the structural details from text file, prepare the models, and extract ABCD stiffness matrices.

Additionally, transverse shear strains are assumed to be constant based on the FSDT. The transverse shear force resultants (Q_x, Q_y) are calculated by multiplying the transverse

shear stiffnesses (D_{Q_x}, D_{Q_y}) and transverse shear strains (γ_{xz}, γ_{yz}), as expressed in the following equation:

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} D_{Q_x} & 0 \\ 0 & D_{Q_y} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (5)$$

$$D_{Q_x} = k_{xz} (G_p t_p + G_w h_w + G_f h_f) \quad (6)$$

$$D_{Q_y} = k_{yz} G_p t_p \quad (7)$$

where k_{xz} is the longitudinal shear correction factor calculated by dividing the average shear stress ($\tau_{xz(avg)}$) to the maximum shear stress ($\tau_{xz(max)}$), k_{yz} is the transverse shear correction factor as 5/6. The shear moduli of the web (G_w) and flange (G_f) are the function of plate (G_p), web thickness (t_w), flange width (b_f), and stiffener spacing (s):

$$G_w = G_p (t_w / s) \quad (8)$$

$$G_f = G_p (b_f / s) \quad (9)$$

2.2. Implementation of ESL in Abaqus

In ESL approach, stiffened panels are replaced by a single plate. An example is shown in Figure 3, where ship hull girder composed of stiffened panels is modeled using ESL approach. ESL properties are composed of 6×6 matrix considering membrane and bending stiffness components. In Abaqus software these ESL properties are given to elements by using shell general section option. This option allows reference to external Fortran-based VUGENS subroutine where the non-linear stiffness of ESL is calculated specific to loading stage. VUGENS subroutine is available starting from Abaqus 2022 and suitable for explicit analysis. To the best of authors knowledge, the analyses reported here are the first attempt to use VUGENS subroutine since in earlier versions of Abaqus only implicit version of shell general section definition could be used (UGENS). Note that in combination with explicit integration scheme the ESL approach could be potentially used in ship collision or grounding analysis. In non-ESL elements (frames, bracket, and other plates), standard isotropic properties are used by defining Young's modulus, Poisson's ratio, and material stress-strain curve.

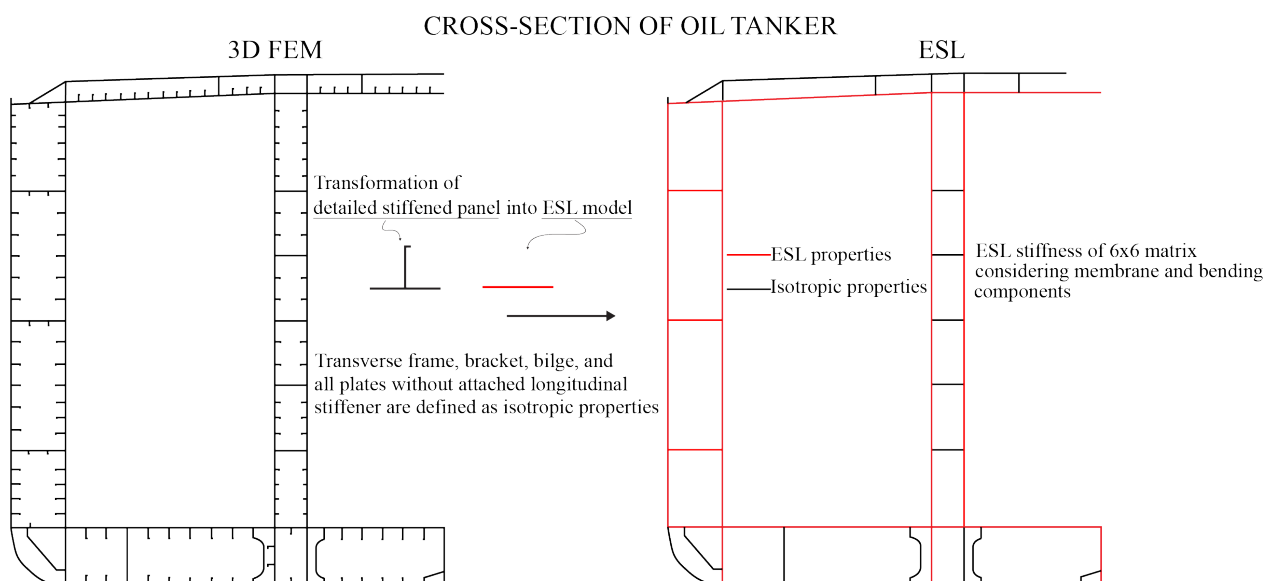


Figure 3. Application of ESL in a ship's hull girder.

3. Case Study of One Compartment Aluminium Box Girder

This paper examines the application of ESL to analysis of compartment level collapse, for details see Benson et al. [14]. The ultimate strength of one compartment box girder was analyzed and characterized by the bending moment versus curvature curve. The analyses were performed using the detailed 3D FEM and ESL approach. The detailed 3D FE analysis utilizes conventional modeling techniques with explicit modeling of stiffened plates further strengthened with larger transverse webframes with all parts given isotropic material properties. In the ESL model, longitudinally stiffened panels were represented with equivalent single layer plates having the same stiffness as the original plate with stiffeners while webframes were still explicitly modeled.

Modeled box girder with a 8.4 m length had a square cross section, stiffened on each side by 20 longitudinals spaced 400 mm apart. The panel configuration M1 was selected for analyses, see [14]. The longitudinal T-stiffeners had the web and flange dimensions of 120 × 55 mm and 55 × 7.7 mm, respectively. The transverse webframes were flat bars with the size of 180 × 10 mm spaced 1200 mm apart. There were six webframes between transverse bulkheads, which is sufficient to demonstrate buckling characteristics at the compartment level. To maintain the straightness of the compartment ends during bending, the bulkheads were modeled with a very large thickness. The mesh size used was 50 × 50 mm, which was determined by Benson et al. [14] through the mesh convergence analysis. The mesh size sensitivity study of ESL was performed using element sizes of 50 × 50 mm, 300 × 300 mm and 600 × 600 mm which are consisted of 24, 4 and 2 elements between webframes, respectively. The box girder was given aluminium 5083 properties with an elastic modulus of 70 GPa and a yield stress of 302 MPa. The non-linear stress–strain response of this material was characterized by the Ramberg–Osgood relationship, as expressed in the following equation:

$$\varepsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{\sigma_{0.2}} \right)^n \quad (10)$$

where ε is the strain, σ is the applied stress, E is the elastic modulus, $\sigma_{0.2}$ is the 0.2% offset proof stress, n is the exponent.

Boundary conditions applied in the box girder are explained in Figure 4. At the reference point, a moment to the z-axis was applied resulting in compression on the top panel and tension on the bottom panel. This reference point was connected to the nodes at the boundary using kinematic coupling so that the section remained flat during rotation. At the opposite section, the clamped boundary condition was imposed. Only half of the girder was modeled by imposing symmetry on the center line.

The response of the box girder is compared in terms of bending moment vs. curvature curves and overall deformations obtained at the maximum bending moment, see Figure 5. Current 3D FE analysis gives slightly softer response compared with 3D FE results from Benson et al. [14]. Curvature in current study was determined by tracking the rotation at the reference point. The way curvature was obtained in analysis of Benson was not detailed, which possibly explains the slightly softer response obtained with current 3D FE analysis, while the overall behavior is well captured. Proceeding to analyze differences between current 3D FEM and ESL we also note close agreement in maximum bending moment. Even the coarsest (and stiffest) ESL model of 600 × 600 mm shows the difference of mere 4.57% compared with 3D FEM result. All studied element sizes can capture accurately compartment level (overall) buckling behavior and for overall efficiency we advocate to use the largest mesh of 600 × 600 mm or two elements between webframes. However, the ESL accuracy decreases if the stiffened panel is subjected to local buckling of the stiffener web or plate [22,23]. Therefore, further investigation should be performed what is the suitable element size for such simulations. In the post-ultimate stage, there is a sudden decrease in bending moment due to transition from interframe to overall buckling mode. ESL model cannot accurately trace this structural response at the post-ultimate stage.

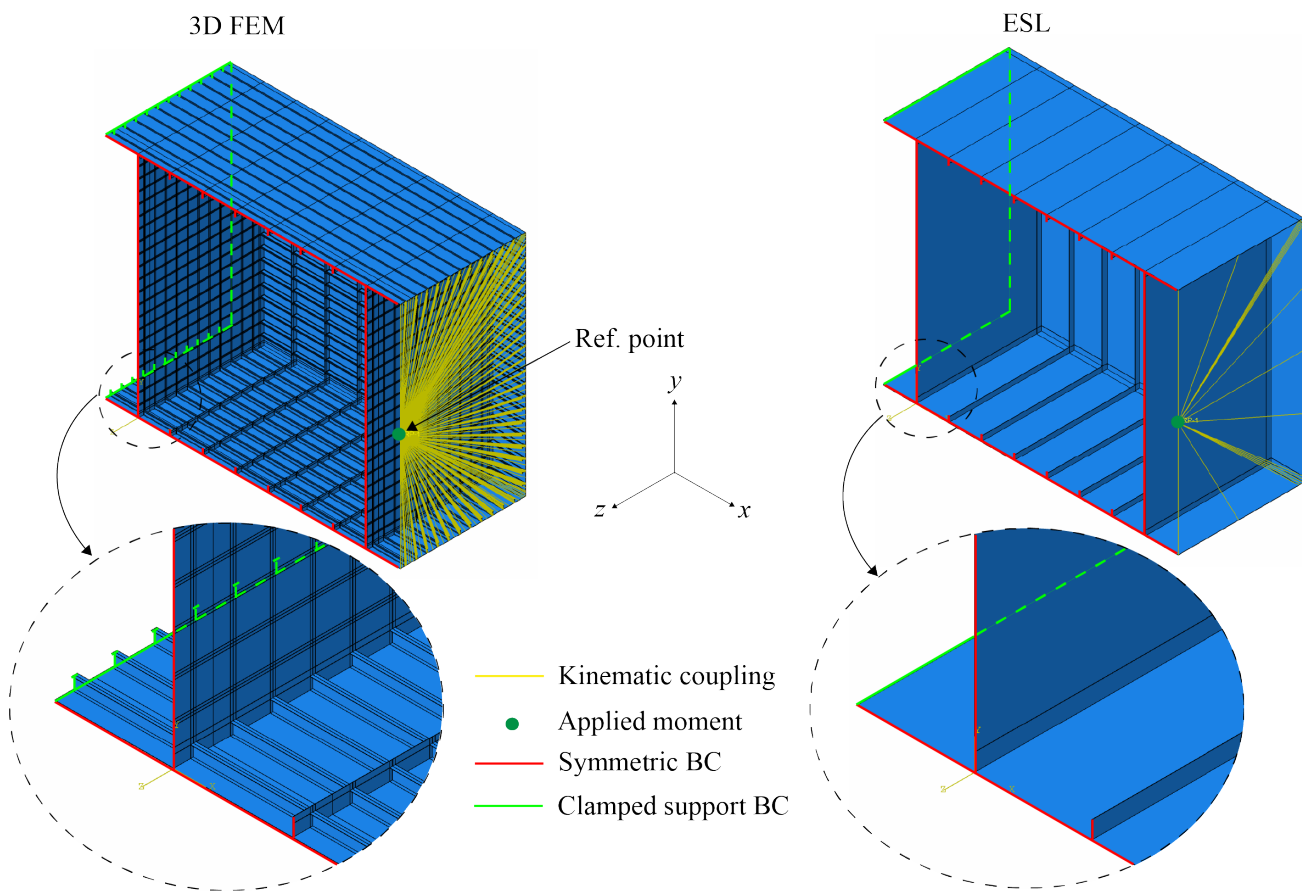


Figure 4. Boundary condition applied in 3D FEM and ESL of box girder.

In addition to providing a high level of accuracy with coarse elements of 600×600 mm, ESL approach also provides shorter computation times than 3D FEM. With ESL mesh of 600×600 mm the computation time is only 27.7% of full 3D model, see Figure 6. All simulations were ran with processor type of Intel(R) Xeon(R) CPU E5-2690 0 @ 2.90 GHz, 4 cores, 4 domains, and RAM of 24.0 GB.

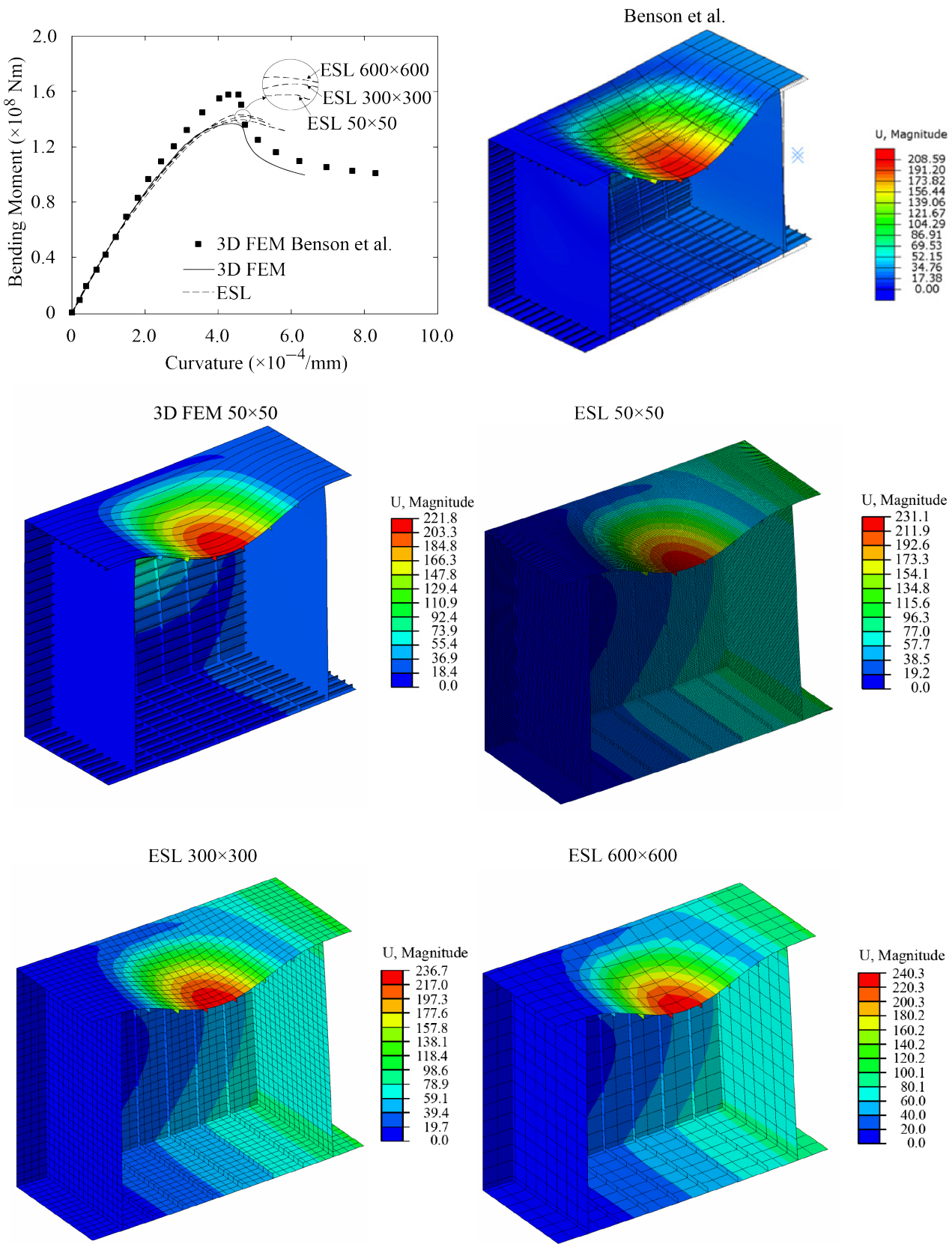


Figure 5. Bending moment–curvature curves and deformation shapes obtained from Benson et al. [14], 3D FEM and ESL models. U is in mm.

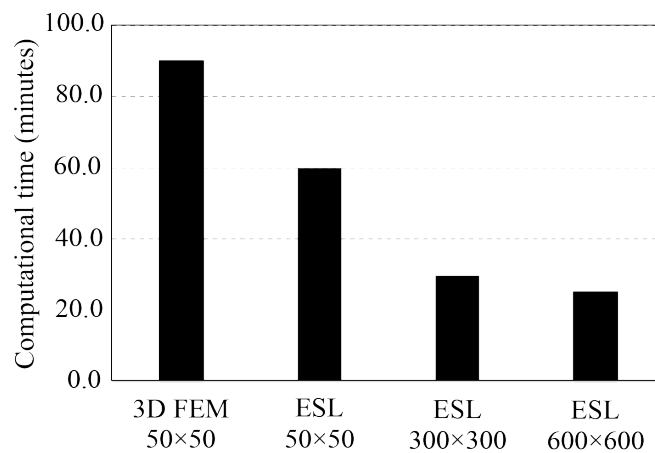


Figure 6. Computational time for 3D FEM and ESL with several mesh sizes.

4. Case Study of Full-Scale Steel Ship Structure

Ultimate strength analyses of ship hull girder in intact and damaged conditions were performed using the ESL approach, full 3D FEM, and incremental-iterative method from the IACS-CSR. A traditional ship hull can be considered as a light-weight thin-walled structure composed of an outer shell that is stiffened with framing members. In other words, ship hull girder is built from stiffened panels. From a design perspective, a detailed modeling of stiffened panel composed of plates and stiffeners is required for 3D FEM. However, ESL simplifies the modeling process as the plate elements are given stiffness properties representative of stiffened panels rendering explicit modeling of stiffeners unnecessary. In the IACS-CSR, ultimate strength is calculated based on a cross-section of hull girder between two adjacent transverse webframes. In Section 2, the theory and implementation of ESL approach was explained. In this section, the full 3D FEM and incremental-iterative method are described.

4.1. Ship Particulars

The case study structure is a chemical product tanker that was previously analyzed in Tabri et al. [24]. The midship section and bulkhead arrangement are given in Figure 7 and the main particulars in Table 1. The ship is designed from high strength steel (AH36) with Young's modulus of $E = 210$ GPa, Poisson ratio of $\nu = 0.3$, and yield stress of $\sigma_y = 355$ MPa.

Table 1. Main particulars of the chemical tanker.

Parameter	Symbol	Unit	Value
Overall length	L_{OA}	m	182.2
Length between perpendiculars	L_{PP}	m	175.3
Moulded breadth	B	m	32.2
Depth	H	m	15.0
Design draught	B	m	11.1
Displacement	∇	t	52,298
Double bottom height	H_{DB}	m	2.21

4.2. Loading and Boundary Conditions

Being a flexible thin-walled beam, the hull girder of the ship flexes globally when exposed to loads. The load components that act on the ship hull girder are the weight of the ship, its cargo, and the hydrostatic and hydrodynamic pressures (external load). The resultant of these load components can be treated with longitudinally distributed load applied on the hull girder, which can increase only by the external pressure due to waves. Therefore, global bending of the ship hull was achieved with longitudinally distributed

pressure, which amplitude was gradually increased until ultimate strength was reached. The distributed pressure was applied either on ship bottom or side, depending on whether vertical or horizontal bending moment was determined, respectively. The sinusoidal shape of distributed pressure was kept unchanged during loading, as shown in Figure 8b. Although, in realistic situations, the distribution of load can play an important role, here only the simplified sinusoidal shape was considered. The direction of bending was controlled by the sign of the pressure amplitude (A), see the Equation in Figure 8b. This resulted in sagging/hogging in vertical bending and starboard/portside bending in horizontal bending. In damaged condition, the pressure was not applied in the damage opening. In Abaqus, the VDL0AD subroutine was invoked to apply the distributed pressure.

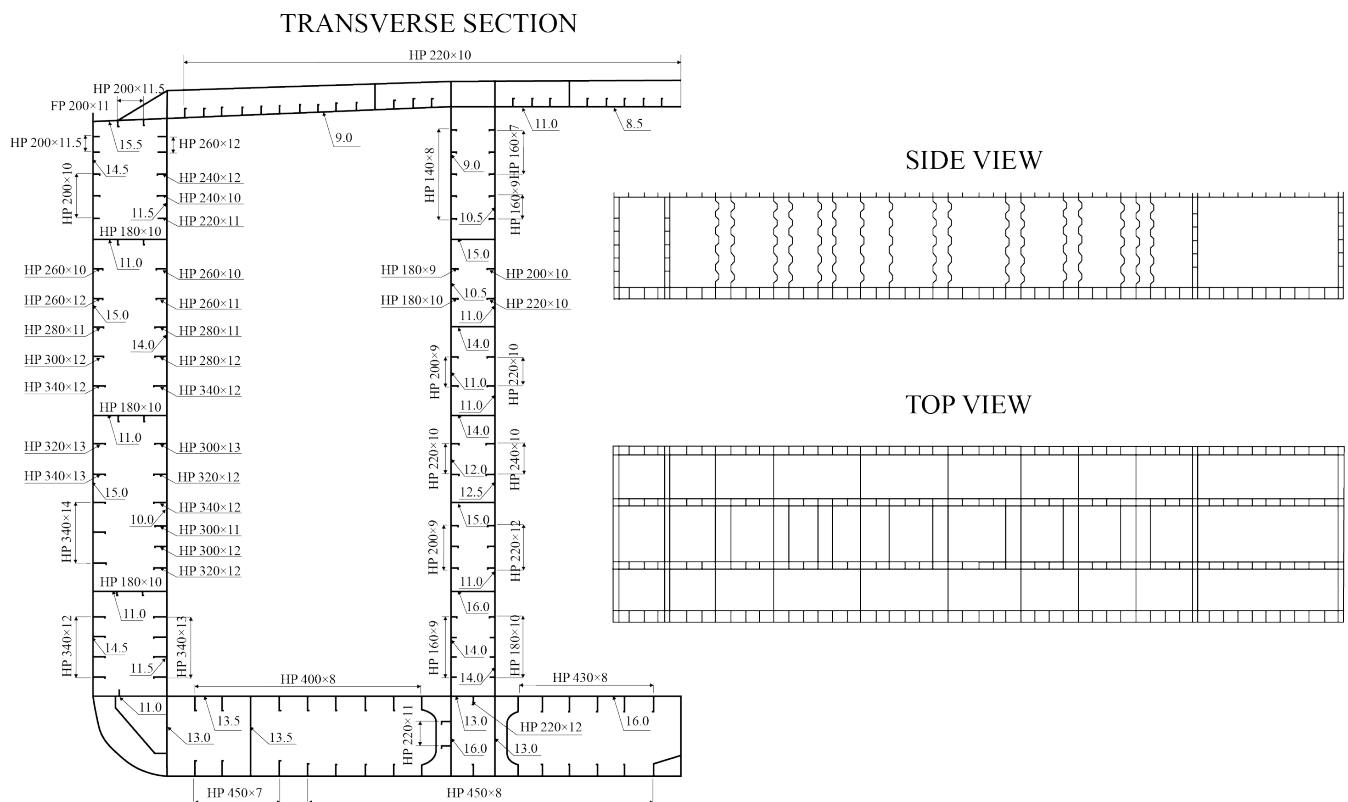


Figure 7. Design of hull structure for case study: midship section (**left**) and side view and top view (**right**).

A simply supported boundary condition was imposed on the ship at the mid nodes marked with $\bullet$, see Figure 8a. The aft of the ship was pin constrained with all translations fixed while rotations were free. In the fore part, the constraints were the same except translation in longitudinal direction which was free to avoid an excessive stress concentration during deflection. Furthermore, rigid beams were modeled through the constraint nodes to keep the ends of the beam straight under increasing load and, thus, prevent local buckling. The surfaces where pressure was applied were specified with rectangles ABCD (for vertical bending moment) and EFGH (for horizontal bending moment), see Figure 8a. The reaction forces on the support is assumed to be very small since the distributed pressures produce the resultant forces of zero.

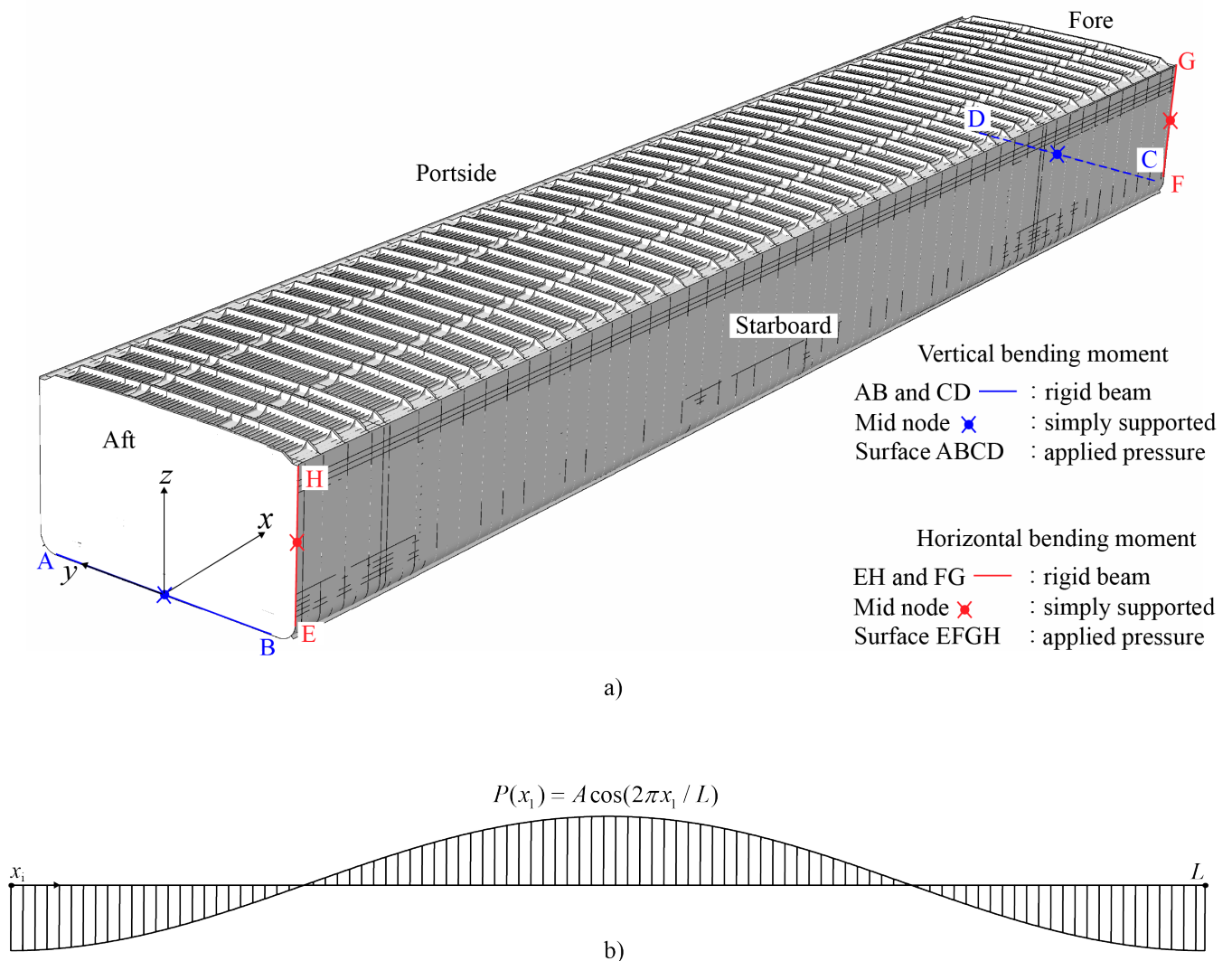


Figure 8. (a) Boundary condition applied in the tanker under vertical and horizontal bending moments. (b) Distributed pressure applied on the surface.

4.3. Damage Scenario

For post-accidental strength assessment of ship structure, both grounding and collision scenarios were considered. The extent of damage was determined based on the definition provided by the IACS-CSR. The damage was modeled by removing the plates, stiffeners, and frames that fall within the specified damage extent. The selected length of damage was 10 m with its center in the middle of the ship. This position was chosen since it significantly reduces the longitudinal bending strength of the ship. In case of collision damage, the transverse damage extent was equal to $B/16$ where B is the breadth of the ship. Therefore, respective structural elements in the parts of inner and outer skin and main deck were removed. The vertical damage extent was taken as $0.6D$ measured from the main deck, where D is the depth of the ship.

In case of grounding damage, the transverse damage extent was taken as $0.6B$, which is symmetric with respect to the centerline. The damage penetration height was equal to the depth of the double bottom (H_{DB}). Figure 9 shows the damage extents of the double hull oil tanker for grounding and collision scenario.

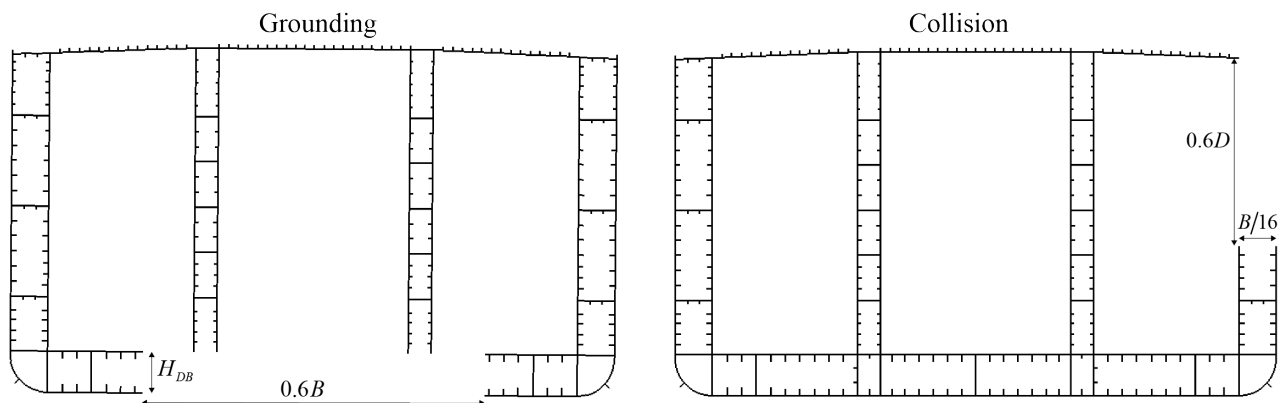


Figure 9. Assumed grounding and collision damage extents in tanker.

4.4. Full 3D Finite Element (3D FE) Model

Non-linear finite element method (NLFEM) is a sophisticated tool to solve the solid mechanics problem of complex engineering structures. Using NLFEM, the effect of material and geometrical non-linearities during progressive collapse of hull girder can be taken into account. A chemical tanker was modeled using Abaqus/Explicit software. All models were meshed using four-node shell elements (S4R), including plates, stiffener webs, frames, and girders. Stiffening was achieved with HP bulb profiles where the flanges were modeled using beam elements (B21). Stiffener webs were modeled using shell elements to capture the collapse mode, e.g., local buckling of stiffener web or stiffener tripping, expected during the progressive collapse of the hull girder. The mesh convergence study was conducted by Tabri et al. [24] to obtain the balance between numerical accuracy and simulation cost. Thus, the mesh density in the one compartment is illustrated in Figure 10 and can be summarized as follows: (1) plates between longitudinal stiffeners, 4×16 shells; (2) stiffener web plates, 1×16 (web height < 300 mm) shells and 2×16 (web height > 300 mm) shells; (3) side stringers and girders, 12×16 shells; and (4) corrugated bulkhead plates, 6×40 and 5×60 shells. The 3D FE model consisted of 3,160,000 nodes and 2,800,000 elements from which 2.5 M were shell and 0.3 M were beam elements. In the ESL model, stiffeners (shell elements related to web and beam element for flange) were removed so the total number of elements was reduced by 25% to 2,120,000 compared with the full 3D FE model.

4.5. Incremental-Iterative Method

The IACS-CSR provides the incremental-iterative method to calculate the ultimate strength of hull girder. Details of the method are given in CSR, which are briefly summarized here for entirety. The assumption is that hull girder collapse occurs in between two adjacent transverse webs. Accordingly, two-dimensional (2D) cross-section of the ship hull structure is divided into a series of structural elements, such as stiffeners, plates, and hard corners. The response of each of those elements is described with load-end-shortening curve compliant with prevalent collapse mode. Under compression, the stiffener element may experience the specific collapse mode, such as beam-column buckling, torsional buckling, or web buckling. For the plating element, the collapse mode of plate buckling may occur. Other element types under compression or tension may experience idealized elastic-plastic failure. Accordingly, the load-end-shortening curves for each structural element were obtained.

The calculation procedure of ultimate strength starts with estimation of neutral axis (NA) position. The iterative approach involves increasing the assumed curvature of the hull girder and calculating the new position of NA based on the moment equilibrium. Bending moment is obtained by integrating the load over the cross-section, while the load for each element is obtained from the load-end-shortening curves used as input. Once the updated

location of the NA is obtained, the curvature is further increased and procedure is repeated until the maximum bending moment is achieved.

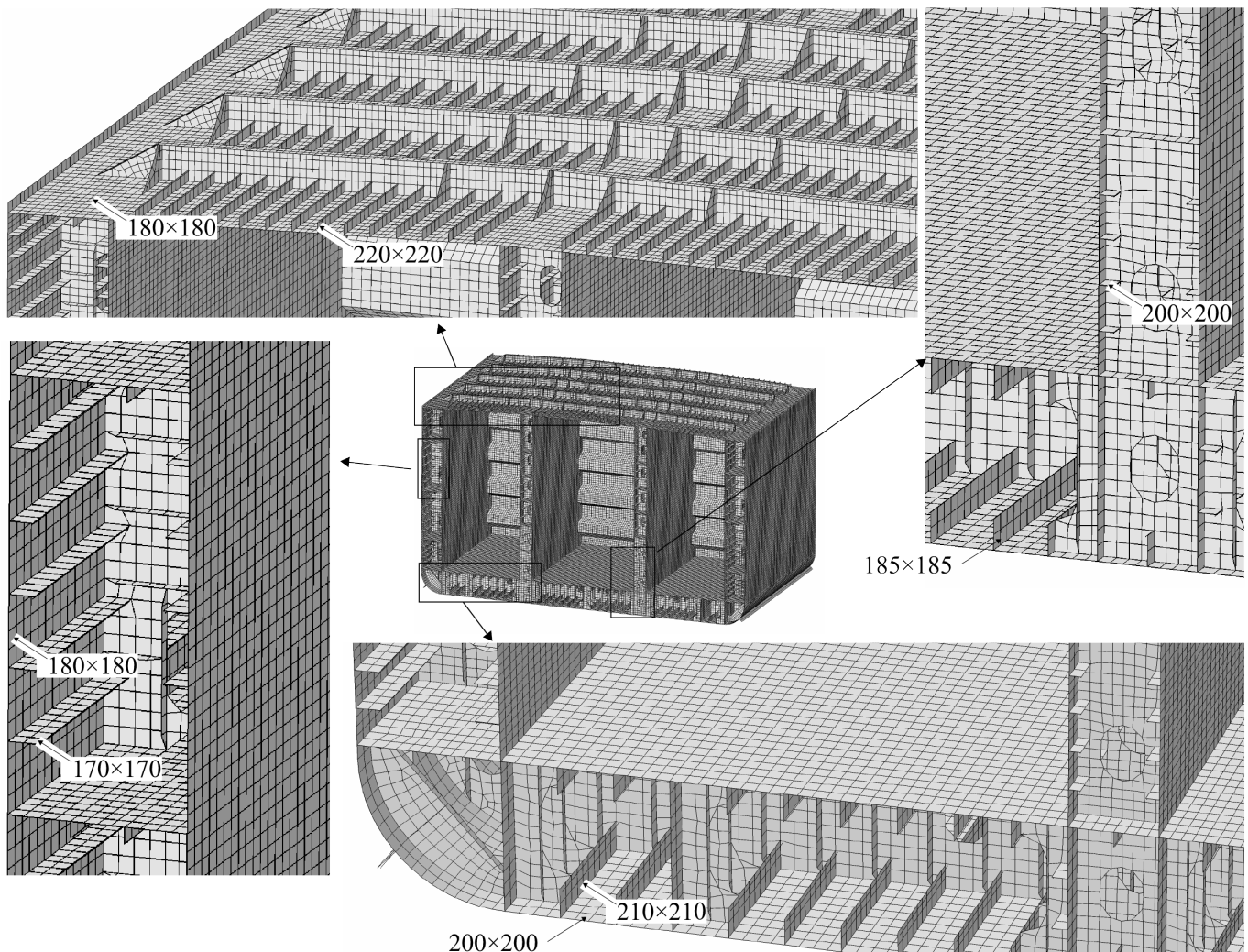


Figure 10. Mesh of the one compartment in the FE-model of chemical tanker.

5. Ultimate Strength Analysis

Ultimate strength of a chemical tanker was analyzed under vertical and horizontal bending moments. The results obtained from the ESL approach, 3D FEM, and incremental-iterative method are presented.

5.1. Vertical Bending Moment

Figure 11 shows the vertical deflection of intact chemical tanker along its length under hogging and sagging conditions. The horizontal bending moment is discussed in the next section. The deflections at the aft and fore are zero since the vertical translation of those parts is constrained. The maximum deflection occurs in the middle of the ship where the distributed pressure has maximum amplitude. In general, ESL deflection correlates well with the 3D FEM results.

The bending moment–deflection curves for 3D FEM, ESL, and incremental-iterative approach (hereinafter referred to as CSR method) for intact ship are presented in Figure 12a. The entire curve until maximum bending moment is accurately predicted by both, ESL and CSR method. Therefore, benefits of performing computationally more demanding ESL analyses are missing until more detailed information is desired. For instance, Figure 12b shows the longitudinal stress in vertical coordinate at the ship midsection under hogging

condition. The constant tensile stresses in the upper decks (above side stringer S2) indicate to fully plastic condition. In the compressive side below the NA (slightly above side stringer S1) stress distribution is more complex due to non-linear buckling of structural elements. The stress provided by ESL approach has better agreement with the 3D FEM than CSR method. Furthermore, with 3D FE model available, similar stress distributions could be obtained in different cross-sections along the length, which is not possible with CSR.

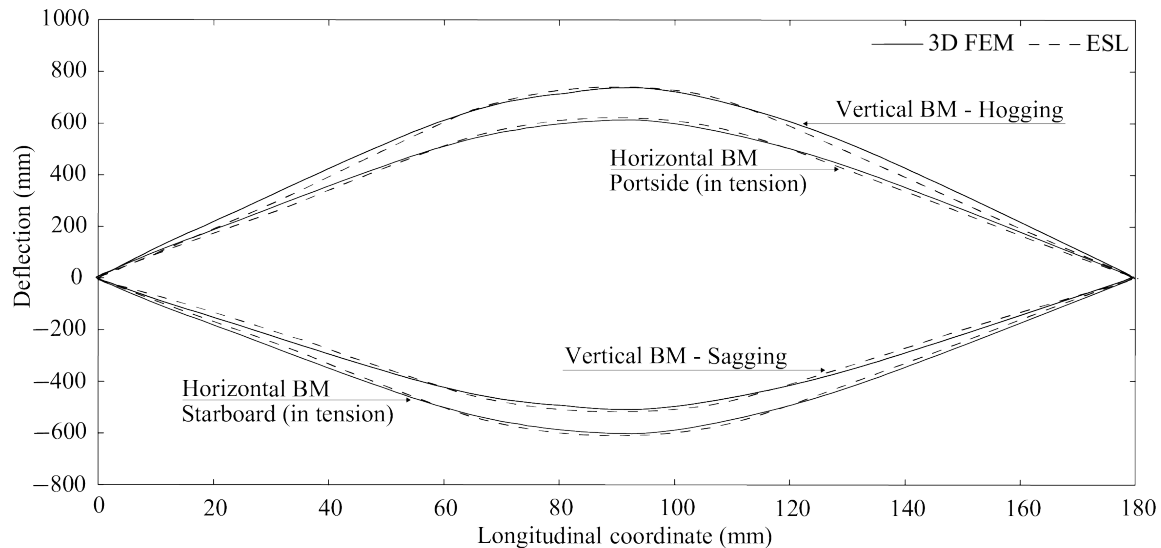


Figure 11. Deflection of ship's hull along its length at the ultimate stage under vertical and horizontal bending moments.

In the case of ship grounding, the bending moment–deflection curve is presented in Figure 13a. The initial stiffness is the same for all three curves. The bending moment increases linearly until gradual buckling of structural elements. The moment reduction under hogging condition is more significant since compressive loads are carried by damaged double bottom structure. The comparison of longitudinal stress distributions in Figure 13b shows that while ESL accuracy decreases compared with intact analyses, the stresses are still more accurately predicted compared with CSR.

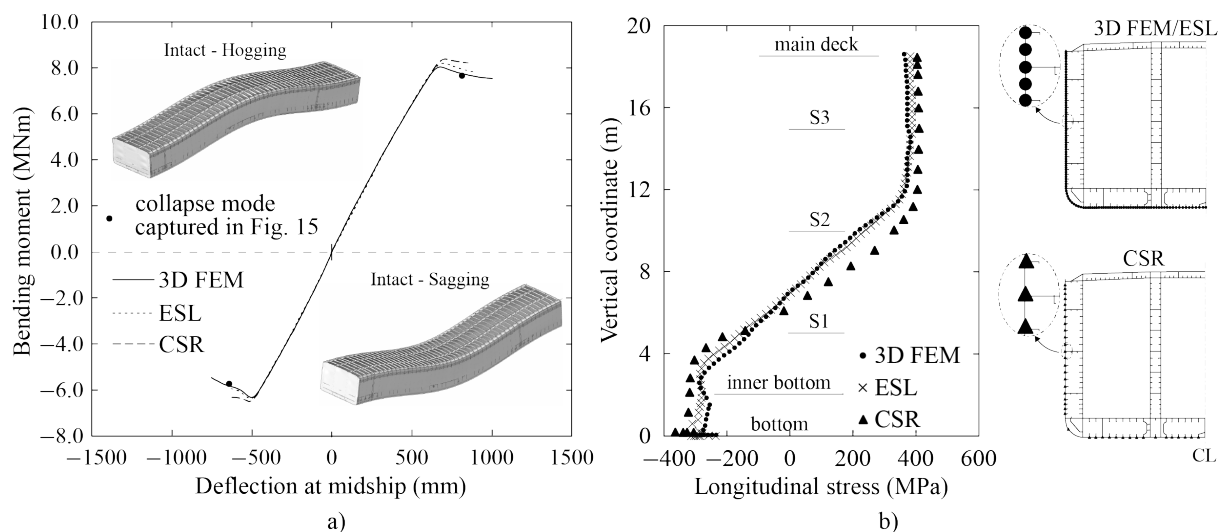


Figure 12. Response of intact ship under vertical bending moment. (a) Bending moment–deflection curves. (b) Longitudinal stress distribution to vertical coordinate at the midship under hogging condition.

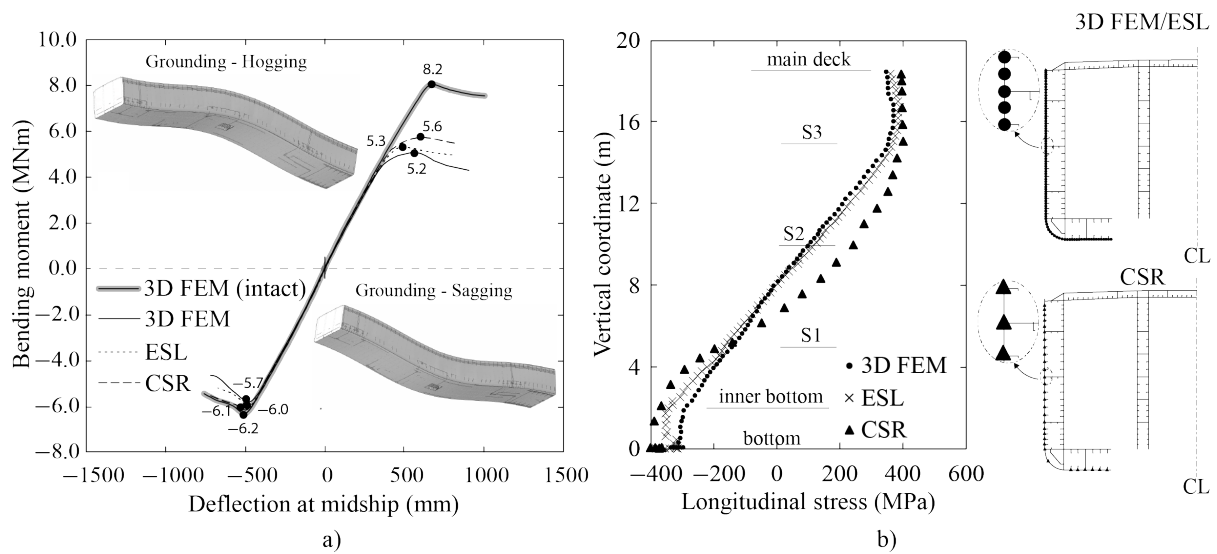


Figure 13. Response of ship grounding under vertical bending moment. (a) Bending moment–deflection curves. The maximum bending moments are used in calculating the histogram of moment reduction ratio. (b) Longitudinal stress distribution to vertical coordinate at the midship under hogging condition.

In the case of collision damage, the relationship between bending moment and deflection under sagging/hogging conditions is presented in Figure 14a. The longitudinal structure, especially the deck-side corner, greatly contributes to longitudinal strength. By removing the damaged side and deck structure, the ultimate bending moment decreases more than in case of grounding damage. However, this decrease in maximum bending moment is well captured by the ESL method which shows improved predictions compared with CSR. The reduced accuracy of CSR is explained by the effect of the removed deck-side corner. With respect to the longitudinal stresses in Figure 14b, the ESL method has similar accuracy as in grounding case, while maintaining the advantage over CSR.

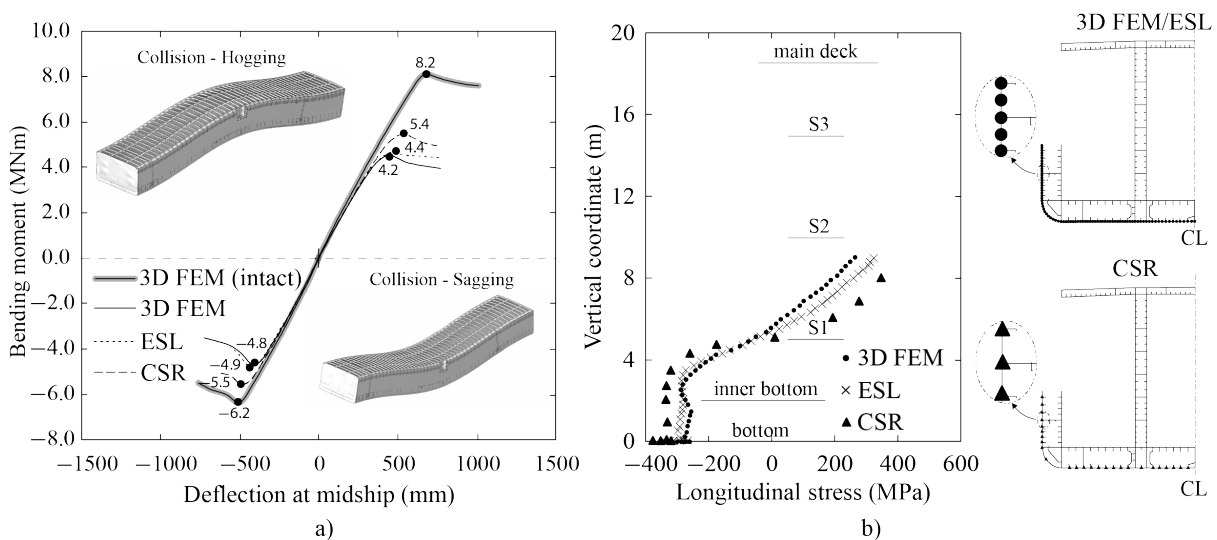


Figure 14. Response of ship collision under vertical bending moment. (a) The bending moment–deflection curves. The maximum bending moments are used in calculating the histogram of moment reduction ratio. (b) The longitudinal stress distribution to vertical coordinate at the midship under hogging condition.

The ESL approach captures the bending moment until ultimate stage with very good accuracy. In the post-ultimate stage bending moment is captured accurately under sagging, but not under hogging. Under hogging condition ESL fails to capture the local stiffener web buckling collapse in bottom structure as shown in Figure 15. It was also shown in analysis of [22,23] that beyond certain threshold slenderness ($\beta = 1.96$) the collapse mode becomes local which is not captured by ESL. In current analysis the plate slenderness in bottom structure is 2.12, while in deck structure slenderness is 1.55. In contrast, the slenderness of deck plates is much lower circumventing the local collapse under sagging condition, which ultimately leads to very good accuracy also in post-ultimate bending moment prediction, see Figure 14a.

The bending moment reduction with respect to intact condition obtained with each method (3D FEM, ESL, and CSR) are summarized in Figure 16. This reduction is calculated by dividing the maximum bending moment at damaged condition with the corresponding maximum from intact condition for each method concerned. Furthermore, the error in moment reduction obtained with ESL and CSR is calculated with respect to 3D FEM results and shown in the figure. The overall accuracy of ESL is excellent remaining in 5% in all cases. The compromise between accuracy and overall analyses cost favor CSR over ESL in grounding analyses, while accuracy of CSR reduces in collision analyses.

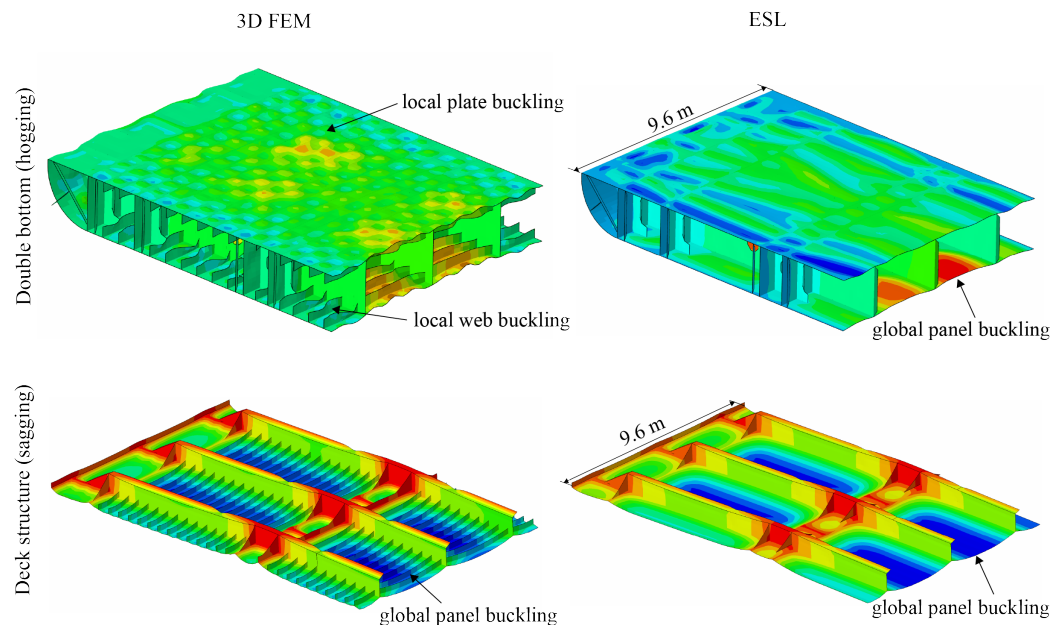


Figure 15. Collapse mode in double bottom and main deck structures under hogging and sagging conditions in post-ultimate stage, respectively. Deformation scaling factor is 5×.

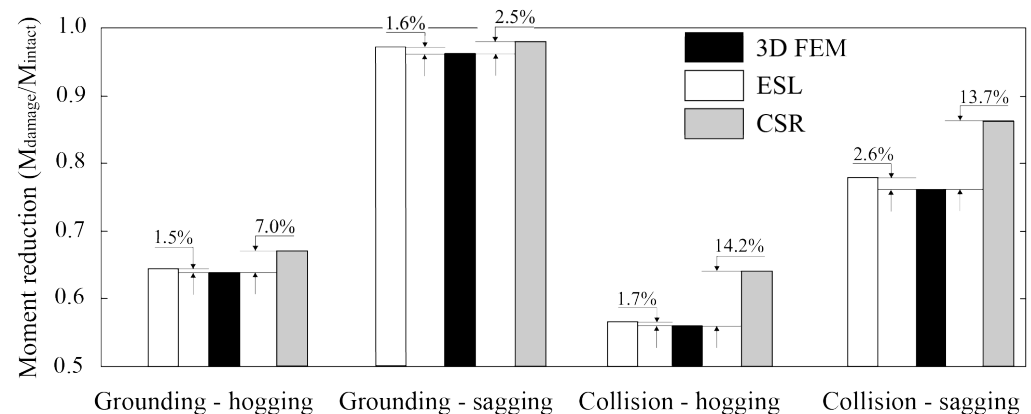


Figure 16. The moment reduction ratio under grounding and collision damage in the hogging and sagging conditions using the 3D FEM, ESL, and CSR methods.

5.2. Horizontal Bending Moment

The horizontal bending moment–deflection at midship curves are presented in Figure 17a–d for intact and damaged ship. Overall, a close agreement exists between the bending moment resulted by the ESL and 3D FEM until the ultimate stage. Since hull girder has greater depth than breadth, the ultimate horizontal bending moment is greater than vertical bending moment. For intact and grounding model, the cross-section of the ship is symmetric to the centerline so that the bending moment is independent whether the loading comes from port or starboard side. However, for collision model, the bending moment is dependent on the loading direction. For these cases, the ESL model can accurately capture the reduction in bending moment until the ultimate condition.

The moment reduction ratio under horizontal bending moment for the three methods are shown in Figure 18. The error percentage represents the difference between the analyzed method and 3D FEM. In all cases, the ESL results are consistently close to the 3D FEM.

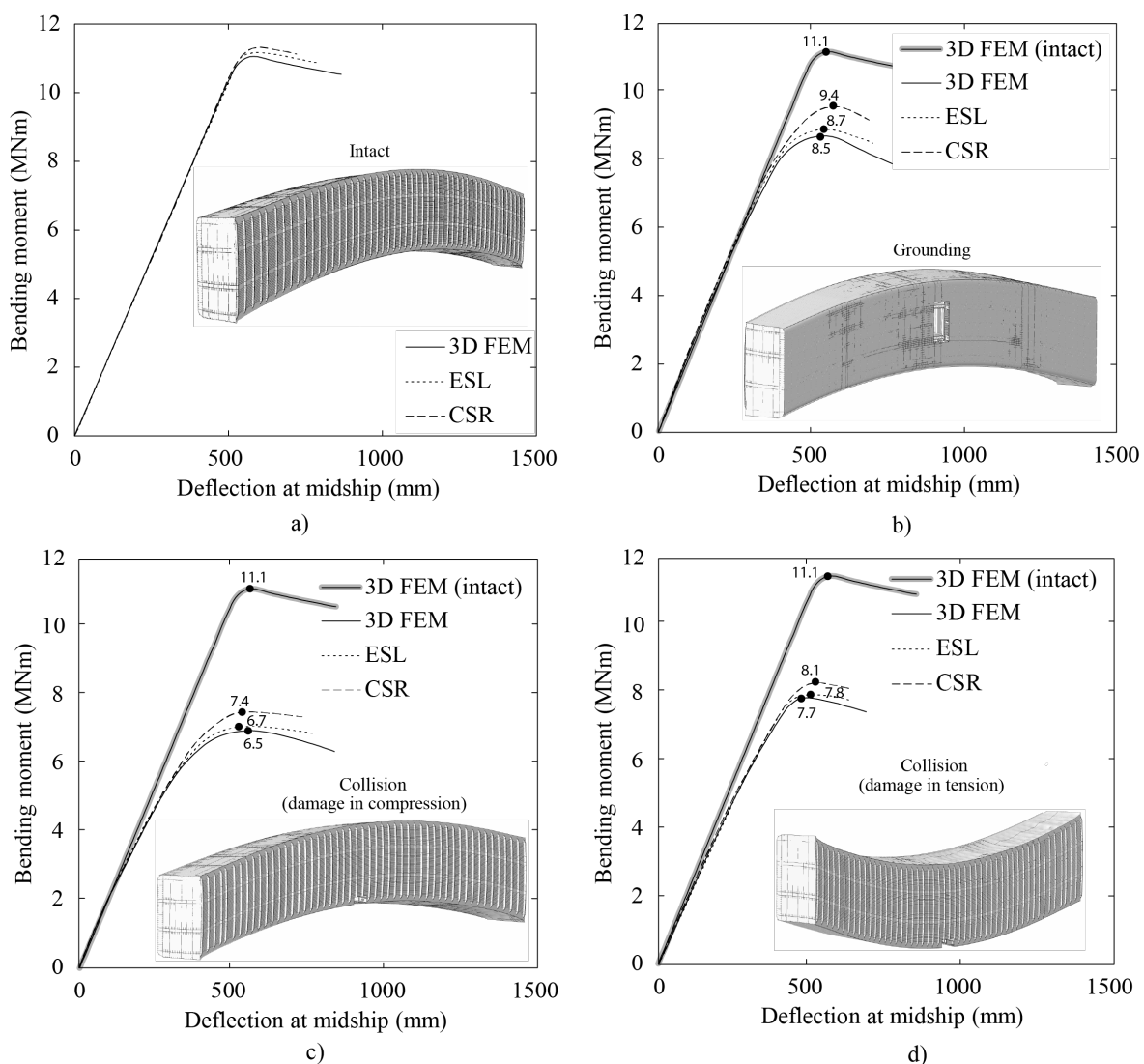


Figure 17. Horizontal bending moment-deflection relationships of tanker in the condition of (a) intact, (b) grounding, (c) collision with damage in compression, and (d) collision with damage in tension. The maximum bending moments are used in calculating the histogram in Figure 18.

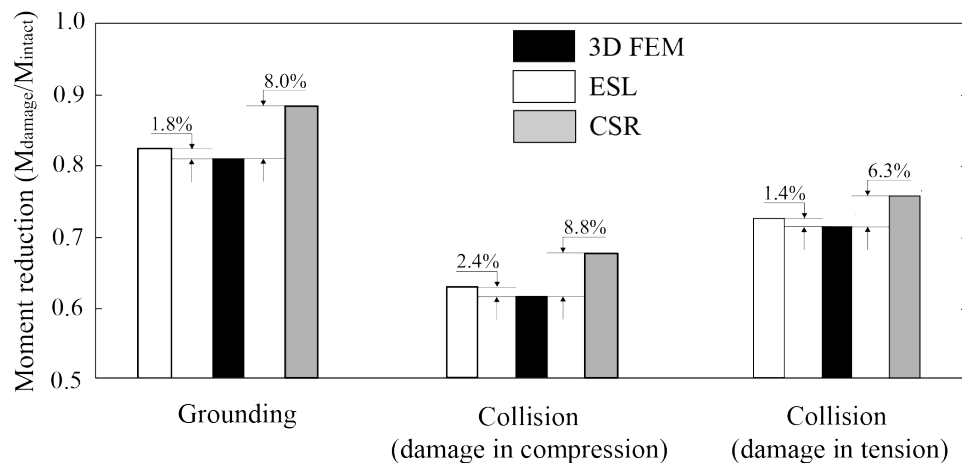


Figure 18. The moment reduction ratio of horizontal bending moment in grounding and collision cases for the 3D FEM, ESL, and CSR methods.

6. Conclusions

The ultimate strength analyses are carried out using the one compartment and full-scale ship models. The one compartment model is analyzed under vertical bending moment. The full-scale ship model in intact and damaged conditions is analyzed under vertical and horizontal bending moments. The analyses are performed with three methods, namely 3D FEM, ESL, and CSR. Overall, the ultimate strength predicted by the ESL approach give the results close to 3D FEM for all cases.

The ESL approach provides a more time-efficient way to analyze the ultimate strength compared to the detailed 3D FEM. In the ESL approach, plate with stiffeners is represented with an equivalent plate with equal stiffness so that stiffeners are not explicitly modeled. This reduces modeling effort as well as computational time and, thus, could be potentially used with great efficiency in structural optimization, which has seen increased popularity due to the advancements in computing power. Current analysis showed that in case of one compartment model, the analysis time was up to 3 times shorter when using the coarsest possible mesh. Moreover, the computational efficiency does not compromise the accuracy as the ultimate bending moment was captured with less than 5% error compared with 3D FEM in all analyzed cases. In contrast, the CSR results overestimate the collapse moment by up to 14.2% in collision damage scenario under hogging. This overestimation of bending moment obtained with CSR method is consistent with analysis in literature, see [25,26].

In addition to accurately visualizing the full-scale ship deflection, ESL model can capture the load-response of a ship structure until ultimate bending moment with very good accuracy. However, due to the interaction of structural members, the ESL approach cannot accurately account the local stiffener web buckling collapse in the post-ultimate stage. Furthermore, with ESL methodology one cannot visualize stresses in the post-processing stage. Both aspects need to be addressed in future investigations.

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