Article

Supplementary Material for "New Insights to Estimate Reproduction Numbers during an Epidemic"

We derive here Equation (12) of the main text. For any time $t = 1, \ldots, T$, we get

$$Y(t) = R_t(t)(Y * f_C)(t)$$

$$= R_t(t) \sum_{k=1}^{t} f_C(t-k)Y(k) + R_t(t) \sum_{k=-\infty}^{0} f_C(t-k)Y(k)$$

$$= R_t(t) \sum_{k=1}^{t} f_C(t-k)Y(k) + R_t(t) \sum_{k=-\infty}^{0} f_C(t-k)$$

$$= R_t(t) \sum_{k=1}^{t} f_C(t-k)Y(k) + R_t(t)\left[1 - \sum_{x=0}^{t-1} f_C(x)\right].$$

By solving with respect to $Y(t)$, we obtain

$$Y(t) = \frac{R_t(t) \sum_{k=1}^{t-1} f_C(t-k)Y(k)}{1 - R_t(t)f_C(0)} + \frac{R_t(t)\left[1 - \sum_{x=0}^{t-1} f_C(x)\right]}{1 - R_t(t)f_C(0)}. \tag{12}$$

We now provide the explicit computations to derive the standard estimator $\hat{R}_t(0)$ given in (13) of the main text.

$$\hat{R}_t(0) = \frac{\sum_{i=-7}^{6} R_t(i)(Y * f_C)(i)}{\sum_{i=-7}^{6} (Y * f_C)(i)}$$

$$= \frac{R_t(0) \sum_{i=-7}^{0} (Y * f_C)(i) + \sum_{i=1}^{6} R_t(i)(Y * f_C)(i) \pm R_t(0) \sum_{i=1}^{6} (Y * f_C)(i)}{\sum_{i=-7}^{6} (Y * f_C)(i)}$$

$$= \frac{R_t(0) \sum_{i=-7}^{6} (Y * f_C)(i) + \sum_{i=1}^{6} R_t(i)(Y * f_C)(i) - R_t(0) \sum_{i=1}^{6} (Y * f_C)(i)}{\sum_{i=-7}^{6} (Y * f_C)(i)}$$

$$= R_t(0) + \frac{\sum_{i=1}^{6} [R_t(i) - R_t(0)](Y * f_C)(i)}{\sum_{i=-7}^{6} (Y * f_C)(i)} = R_t(0) + \frac{\sum_{i=1}^{6} [R_t(i) - R_t(0)]\frac{Y(i)}{R_t(i)}}{\sum_{i=-7}^{6} \frac{Y(i)}{R_t(i)}}. \tag{13}$$