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Output-Only Parameters Identification of Earthquake-Excited Building Structures with Least Squares and Input Modification Process

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Abstract: Damage detection and system identification with output-only information is an important but challenging task for ensuring the safety and functionality of civil structures during their service life. In this paper, a relatively simple and efficient iteration identification method consisting of the least squares estimation (LSE) technique and an input modification process is proposed for the simultaneous identification of structural parameters and the unknown ground motion. The spatial distribution characteristics of ground acceleration on earthquake-excited building structures are considered as additional information for parameters identification in each iterative step. First, the unknown input is estimated using the measured responses and the initial guesses of the structural parameters. The estimated input is then modified on the basis of the property of its spatial distribution. This modified input is further employed for providing the updated estimation of structural parameters. The iterative procedure would continue until the preset convergence criterion is satisfied. The accuracy of the proposed approach is numerically validated via a shear building model under the El Centro earthquake. The effects of signal noise, the number of sample points, and the initial guesses of structural parameters are discussed. The results show that the proposed approach can satisfactorily identify the structural parameters and unknown earthquakes.

Keywords: output-only parameters identification; unknown ground motion; earthquake-excited building; least squares estimation; input modification process

1. Introduction

1.1. Background

Many civil structures in service may deteriorate or degrade due to many factors, such as environmental corrosion, long-term fatigue, strong winds, severe earthquakes, and so forth. The failure of these deficient structures could be catastrophic, not only in terms of loss of life and the economy, but also of subsequent social and psychological impacts [1]. It is therefore imperative to identify the parameters of the structure, detect the damage, and guide the maintenance to ensure structural safety and functionality. Moreover, since the loadings applied to the structures are of importance for structural evaluation and future life prediction, the development of loading identification techniques is also highly required.

1.2. Literature Survey

To date, a number of vibration-based system identification and damage detection methods have been developed, and review articles with different emphases can be also found [2–4]. In general, structural parameters identification can be performed in three different paradigms: (i) time domain (e.g., [5–10]), (ii) frequency domain (e.g., [11–14]), and (iii) time–frequency domain (e.g., [15–17]).

To identify structural parameters and their variations caused by damages, time domain analyses have been studied extensively. In particular, research on system identification based on least-squares estimation (LSE) has been carried out for a relatively long time, and much progress has been made. Caravani et al. [18] first proposed a time domain identification method by using recursive LSE in 1970s. By using the recursive LSE-based algorithm to upgrade the diagonal elements of the adaptation gain matrix at each time step, Lin et al. [19] proposed an on-line parametric identification algorithm for the identification of non-linear hysteretic structures. By using LSE for the coefficient estimation, Masri et al. [20] developed a system identification approach for non-linear restoring force identification. Based on the LSE approach, Yang and Lin [21] proposed an adaptive tracking technique to identify and track the system parameter variations due to damages. Yang et al. [22] further developed a sequential non-linear LSE approach for the on-line identification of structural parameters by employing the proposed adaptive tracking technique. Gui et al. [23] presented an adaptive system identification approach with the aid of the least mean square algorithm and least mean fourth algorithm. Kazemi and Arefi [24] proposed a fast iterative recursive LSE algorithm for identifying non-linear systems with output colored noise.

Although the aforementioned LSE-based approaches are capable of accurately identifying structural parameters, the external excitations in these approaches are all assumed to be available. However, in many cases, the excitations are unmeasurable or difficult to be precisely measured. Thus, the development of LSE-based algorithms for parameters identification under unknown loadings is required.

Based on recursive LSE and the extended Kalman filter (EKF), Wang and Haldar [25] proposed a system identification approach for the estimation of structural parameters under unknown excitations. Using the Taylor series approximation to transform a set of non-linear equations to linear equations, Ling and Haldar [26] developed a modified iterative LSE approach for the element-level system identification in consideration of Rayleigh damping under unknown input. Based on the LSE technique and partially known input information, Chen and Li [27] presented a time domain method for the simultaneous identification of the structural parameters and the time history of the unknown external excitations. A recursive least-squares estimation with unknown inputs (RLSE-UI) approach was proposed by Yang et al. [28] for the identification of the structural parameters and the unmeasured excitations. To reduce the number of sensors required, Yang and Huang [29] further proposed a sequential non-linear LSE method for damage identification of structures with unknown inputs and unknown outputs. With the aid of a weighted positive definite matrix and a learning coefficient, Xu et al. [30] proposed a weighted adaptive LSE approach for the identification of structural parameters and partially unknown loadings. This method was further extended for the identification of large-scale structures [31] and non-linear structures [32]. By sequentially using the EKF for the parameters identification of the concerned structure and LSE for its unknown external excitations, Lei et al. [33] proposed an approach for estimating structural parameters with limited input and output measurements. Based on the fluctuating properties of wind, Lei et al. [34] further extended their approach to identify the parameters of the high-rise wind-excited buildings. Based on recursive LSE with adaptive multiple forgetting factors, Askari et al. [35] proposed an online tracking technique for estimating the abrupt changes in structural parameters with unknown inputs.

1.3. Scope

The above-mentioned methods can all simultaneously identify the parameters of the structure and the excitation applied to it. The analytical solutions of most of these methods and their derivation, however, are rather complicated. In this paper, an iterative identification procedure based on the LSE technique and an input modification process is proposed for the simultaneous identification of the parameters of building structures and the unknown ground motion. The spatial distribution characteristics of the ground acceleration in earthquake-excited structures are used through the input modification process in each iterative step. The proposed iterative identification procedure can be described as follows. First, based on the equation of motion, the unknown ground motion is estimated using the measured responses and initial guesses of the structural parameters. The estimated input is then modified with the aid of the spatial distribution characteristics of the ground acceleration. This modified input is further employed to provide an updated estimation of the structural parameters. The iterative procedure would continue until the structural parameters satisfy the preset convergence criterion. The feasibility and accuracy of the proposed approach is numerically verified via a shear building model under the El Centro earthquake.

The framework of the paper is presented as follows: Section 2: description of the proposed iterative identification method; Section 3: investigation of the effect of some factors including the noise, the number of sample points, and the initial values of structural parameters; Section 4: concluding remarks.

2. Proposed Iterative Identification Method

The governing equation of motion of an n degree-of-freedom (DOF) linear structure under earthquake can be expressed as:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}1\ddot{x}_g(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ represent the $n \times n$ mass, damping, and stiffness matrices of the structure, respectively; $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ denote the structural acceleration, velocity, and displacement responses, respectively; **1** denotes an $n \times 1$ matrix with all the elements being equal to 1; and $\ddot{x}_g(t)$ is the time history of ground acceleration. In this study, the Rayleigh damping assumption is adopted:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2}$$

where α and β are mass-proportional and stiffness-proportional damping coefficients, respectively. Substituting Equation (2) into Equation (1) results in

$$\mathbf{M}\ddot{\mathbf{x}}(t) + (\alpha\mathbf{M} + \beta\mathbf{K})\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}1\ddot{x}_g(t) \tag{3}$$

Here, the mass matrix $\mathbf{M}$ is assumed to be known, and then Equation (3) can be rearranged as:

$$\alpha\mathbf{M}\dot{\mathbf{x}}(t) + \beta\mathbf{K}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}1\ddot{x}_g(t) - \mathbf{M}\ddot{\mathbf{x}}(t) \tag{4}$$

Notably, since both β and $\mathbf{K}$ are the parameters to be identified, their coupled products would introduce a source of non-linearity into the identification procedure when the LSE technique is used. To handle this situation, Equation (4) can be rewritten as:

$$\alpha\mathbf{M}\dot{\mathbf{x}}(t) + \mathbf{Q}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}1\ddot{x}_g(t) - \mathbf{M}\ddot{\mathbf{x}}(t) \tag{5}$$

where $\mathbf{Q}$ can be viewed as a transformation matrix ($\mathbf{Q} = \beta\mathbf{K}$) with the same dimension of $\mathbf{K}$.

By rearranging Equation (5), the identification equation can be expressed as follows:

$$\mathbf{H}(t)\boldsymbol{\theta} = \mathbf{P}(t) \tag{6}$$

in which $\mathbf{H}(t)$ is the response matrix with the dimension of $ns \times h$ ($ns = n \times s$); h and s are defined as the number of unknown coefficients and sample points to be used, respectively; $\mathbf{P}(t)$ is $ns \times 1$ inertia force vector; θ is the $h \times 1$ ($h = 2 \times n + 1$) unknown coefficients vector defined as:

$$\theta = [k_1, \ldots, k_n, \alpha, q_1, \ldots, q_n]^T \tag{7}$$

where k_i ($i = 1, \ldots, n$) denotes the stiffness of the i-th floor; $q_i = \beta \times k_i$ ($i = 1, \ldots, n$) is the element-wise coefficient in $\mathbf{Q}$ matrix.

The unknown parameters can then be estimated by LSE as follows:

$$\theta = \left[\mathbf{H}(t)^{\mathrm{T}}\mathbf{H}(t)\right]^{-1}\mathbf{H}(t)^{\mathrm{T}}\mathbf{P}(t) \tag{8}$$

It follows from Equation (8) that the input should be known for the identification. However, in some practical situations, for example an earthquake-excited building, the measurements of loadings are sometimes unavailable. Therefore, the parameters cannot be directly identified according to Equation (8) in this case. Additional information or supplementary equations are required for the identification.

Based on the spatial distribution characteristics of the ground motion in earthquake-excited structures, an input modification process is proposed in this study. A time-domain iterative identification procedure is then developed for the simultaneous identification of structural parameters and unknown ground motion. The details are given below:

Step 1: Assign the initial values of the unknown parameters named as $\widetilde{\theta}$.

Step 2: Estimate ground acceleration by using $\widetilde{\theta}$ and structural response measurements. Based on the Equation (5), the ground acceleration can be estimated as:

$$\widetilde{\ddot{\mathbf{X}}}_g(t) = -\mathbf{M}^{-1}(\mathbf{M}\dot{\mathbf{x}}(t) + \widetilde{\alpha}\mathbf{M}\dot{\mathbf{x}}(t) + \widetilde{\mathbf{Q}}\dot{\mathbf{x}}(t) + \widetilde{\mathbf{K}}\mathbf{x}(t)) \tag{9}$$

where $\widetilde{\alpha}$, $\widetilde{\mathbf{Q}}$ and $\widetilde{\mathbf{K}}$ are the estimated values of α, $\mathbf{Q}$ and $\mathbf{K}$, respectively. They can be determined with the aid of $\widetilde{\theta}$ and structural topology. Notably, the estimated ground acceleration $\widetilde{\ddot{\mathbf{X}}}_g$ obtained from Equation (9) is an $n \times s$ matrix in the form of

$$\widetilde{\ddot{\mathbf{X}}}_g(t) = \begin{bmatrix} \widetilde{\ddot{x}}_g^1(t_1) & \widetilde{\ddot{x}}_g^1(t_2) & \cdots & \widetilde{\ddot{x}}_g^1(t_s) \\ \vdots & \vdots & & \vdots \\ \widetilde{\ddot{x}}_g^n(t_1) & \widetilde{\ddot{x}}_g^n(t_2) & \cdots & \widetilde{\ddot{x}}_g^n(t_s) \end{bmatrix} \tag{10}$$

where $\widetilde{\ddot{x}}_g^i(t_j)$ ($i = 1, \ldots, n$; $j = 1, \ldots, s$) denotes the ground acceleration distributed on the i-th DOF at the time step of t_j.

Step 3: Modify the estimated ground acceleration and form the corresponding inertia force vector $\hat{\mathbf{P}}(t)$. For earthquake-excited building structures, the ground acceleration should be equally distributed on each floor of the building structure. It can be concluded that, if the unknown parameters are not stably converged to the actual ones, the estimated ground acceleration values shown in Equations (9,10) for different DOFs would not be close to each other. Based on this distribution characteristic, the following input modification process is presented in this study:

$$\hat{\ddot{x}}_g(t_j) = \frac{1}{n}\sum_{i=1}^{n}\widetilde{\ddot{x}}_g^i(t_j), (j = 1,\ 2,\ \ldots, s) \tag{11}$$

where $\hat{\ddot{x}}_g$ stands for the modified ground acceleration. Obviously, the mean values of the estimated ground acceleration are used as the modified ones, and the corresponding inertia force vector can be determined as:

$$\hat{\mathbf{P}}(t) = \begin{bmatrix} \hat{\mathbf{P}}(t_1)^{\mathrm{T}} & \hat{\mathbf{P}}(t_2)^{\mathrm{T}} & \cdots & \hat{\mathbf{P}}(t_s)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{12}$$

where

$$\hat{\mathbf{P}}(t_j) = -\mathbf{M}1\hat{\ddot{x}}_{\mathrm{g}}(t_j) - \mathbf{M}\ddot{\mathbf{x}}(t_j),\ (j = 1,\ 2,\ \ldots, s) \tag{13}$$

Step 4: Form the response matrix $\mathbf{H}(t)$, and obtain the updated parameters $\hat{\boldsymbol{\theta}}$ according to Equation (8).

$$\hat{\boldsymbol{\theta}} = \left[\mathbf{H}(t)^T\mathbf{H}(t)\right]^{-1}\mathbf{H}(t)^T\hat{\mathbf{P}}(t) \tag{14}$$

Step 5: Calculate the error between $\hat{\boldsymbol{\theta}}$ and $\widetilde{\boldsymbol{\theta}}$. Let $\hat{\boldsymbol{\theta}}$ be $\widetilde{\boldsymbol{\theta}}$, and repeat Steps 2–5 until the following convergence criterion is satisfied:

$$\left|\frac{\hat{\theta}_l - \widetilde{\theta}_l}{\widetilde{\theta}_l}\right| \le \varepsilon\ (l = 1,\ 2,\ \ldots,\ h) \tag{15}$$

where $\hat{\theta}_l$ and $\widetilde{\theta}_l$ are the l-th element of $\hat{\boldsymbol{\theta}}$ and $\widetilde{\boldsymbol{\theta}}$, respectively; ε is a preset value.

When the iteration procedure is complete, the ground acceleration can be directly found from Equation (11). Moreover, since both k_i and q_i are identified, the damping coefficient β can be determined as follows:

$$\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n}\frac{\hat{q}_i}{\hat{k}_i} \tag{16}$$

where $\hat{q}_i$ and $\hat{k}_i$ are the identified values of q_i and k_i, respectively.

The flowchart of the proposed approach is shown in Figure 1. As can be seen from Figure 1, the LSE technique is employed for parameter identification and the input modification process is used to force the estimated ground acceleration to comply with its spatial distribution along the structure. The key point of this approach is the input modification procedure as described in step 3, which converts the spatial distribution of ground motion into the additional information for parameters identification. It can also be observed that there are no complicated formulas and their derivations involved, implying the proposed approach is quite simple and can be easily realized. The accuracy of the proposed approach for the identification of structural parameters and unknown ground motion is numerically validated via a shear building model under the El Centro earthquake. More details can be found in the following section.

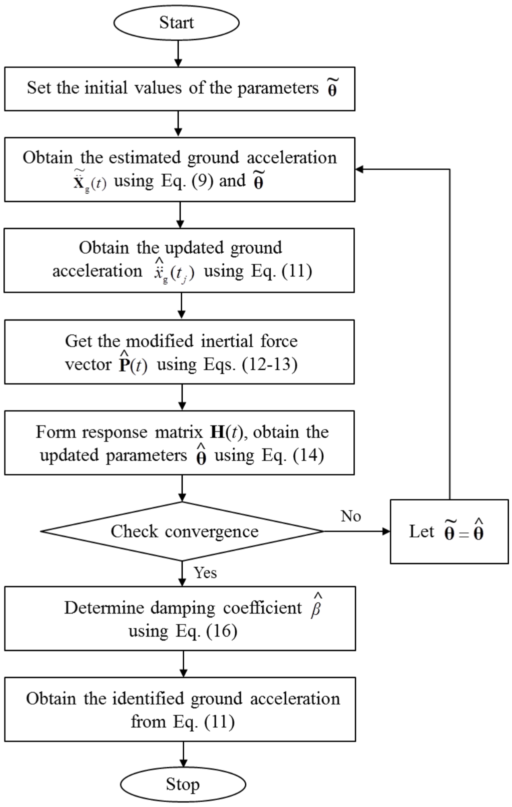

Figure 1. Flowchart of the proposed iterative identification procedure.

3. Numerical Investigation

3.1. General Description

A five-story shearing building model as shown in Figure 2 is employed in this study for the investigation of the effectiveness of the proposed iterative identification procedure. The structural parameters are set as m_i = 500 kg and k_i = 6.0×10^5 N/m (i = 1, 2, ... , 5). The Rayleigh damping coefficients are set as α = 0.6 and β = 8.0 × 10^{-5} providing approximately 3% damping ratio for the first mode of vibration. To simulate the structural damage, the stiffness of the first floor is assumed to be 80% of its designed value in the healthy state (i.e., k_1 = 4.8 × 10^5 N/m).

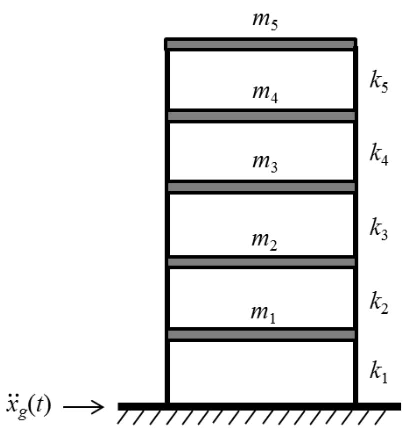

Figure 2. Five-story building model in the numerical example.

In this numerical example, the building structure is assumed to be subject to El-Centro earthquake with a peak ground acceleration (PGA) of 0.34 g. The corresponding structural responses are calculated using the Newmark method with a time interval of 0.02 s. Based on Equation (6), the response matrix $\mathbf{H}(t)$ used for the identification in this study can be expressed as

$$\mathbf{H}(t) = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{2,1} & \mathbf{H}_{3,1} \\ \vdots & \vdots & \vdots \\ \mathbf{H}_{1,k} & \mathbf{H}_{2,k} & \mathbf{H}_{3,k} \\ \vdots & \vdots & \vdots \\ \mathbf{H}_{1,s} & \mathbf{H}_{2,s} & \mathbf{H}_{3,s} \end{bmatrix} \tag{17}$$

where
$$\mathbf{H}_{1,k} = \begin{bmatrix} x_{1,k} & x_{1,k}-x_{2,k} & & & \\ & x_{2,k}-x_{1,k} & x_{2,k}-x_{3,k} & & \\ & & x_{3,k}-x_{2,k} & x_{3,k}-x_{4,k} & \\ & & & x_{4,k}-x_{3,k} & x_{4,k}-x_{5,k} \\ & & & & x_{5,k}-x_{4,k} \end{bmatrix}; \mathbf{H}_{2,k} = \begin{bmatrix} m_1 \cdot \dot{x}_{1,k} \\ m_2 \cdot \dot{x}_{2,k} \\ m_3 \cdot \dot{x}_{3,k} \\ m_4 \cdot \dot{x}_{4,k} \\ m_5 \cdot \dot{x}_{5,k} \end{bmatrix}; \mathbf{H}_{3,k} = \begin{bmatrix} \dot{x}_{1,k} & \dot{x}_{1,k}-\dot{x}_{2,k} & & & \\ & \dot{x}_{2,k}-\dot{x}_{1,k} & \dot{x}_{2,k}-\dot{x}_{3,k} & & \\ & & \dot{x}_{3,k}-\dot{x}_{2,k} & \dot{x}_{3,k}-\dot{x}_{4,k} & \\ & & & \dot{x}_{4,k}-\dot{x}_{3,k} & \dot{x}_{4,k}-\dot{x}_{5,k} \\ & & & & \dot{x}_{5,k}-\dot{x}_{4,k} \end{bmatrix}$$
; $x_{i,k}$ and $\dot{x}_{i,k}(i = 1, \ldots, 5; k = 1, \ldots, s)$ denote the displacement and velocity of the i-th floor at the k-th sampling point, respectively; the subscript 's' denotes the number of sample points as defined before.

The convergence threshold ε in Equation (15) is set to be 1%. The effects of signal noise, the number of sample points, and the initial values of the structural parameters are discussed in detail in the following subsections.

3.2. The Effect of Signal Noise

Here, the structural responses are first assumed to be noise-free. In practical situations, the measured signals are inevitably contaminated by noise. In this example, the noise level is set to be 5% and 10%, which means the ratios of the root mean square (RMS) of the noise to the RMS of the corresponding responses are 5% and 10%, respectively. All of the measured time duration from 0 s to 50 s with 2500 sample points is used for identification. The initial guess of the stiffness is set to be 3.0×10^5 N/m for each floor. The damping coefficients α and β are assumed to be 1.0 and 1.5×10^{-4}, respectively. The element-wise coefficient q_i is therefore initially equal to 45. By using the proposed approach, the identification results are given in Table 1.

Table 1. Identified structural parameters under different noise levels.

Structural Parameters	Noise-Free		5% Noise		10% Noise	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
k_1	4.800×10^5	0.00	4.795×10^5	0.10	4.781×10^5	0.41
k_2	6.000×10^5	0.00	5.995×10^5	0.08	5.979×10^5	0.35
k_3	6.000×10^5	0.00	5.994×10^5	0.10	5.979×10^5	0.35
k_4	6.000×10^5	0.00	5.995×10^5	0.08	5.981×10^5	0.32
k_5	6.000×10^5	0.00	5.996×10^5	0.07	5.983×10^5	0.28
α	0.615	2.50	0.617	2.83	0.626	4.33

Note: The error of 0.00% in this table means it is less than 10^{-4}%.

It can be observed from Table 1 that the stiffness can be precisely identified in the noise-free case. Although the identification errors increase with increasing noise intensity, it is still clear that the proposed approach is capable of satisfactorily identifying the structural parameters, including stiffness and damping coefficient α. The maximum identification errors for the stiffness in the case of 5% noise and 10% noise are 0.10% and 0.41%, respectively. The errors for identified damping coefficients are relatively large, but even with 10% noise the maximum error is only 4.33%. The comparison of the effect of noise intensity on the iteration process is given in Figure 3. Only the identified stiffness of the 5th floor (k_5) is plotted in Figure 3 as an example. Similar results can be found for the remaining parameters. It can be observed from Figure 3 that the larger the noise intensity involved, the more iterations are required. For the cases of noise-free, 5% noise, and 10% noise, the number of iterations is 147, 172, and 248, respectively.

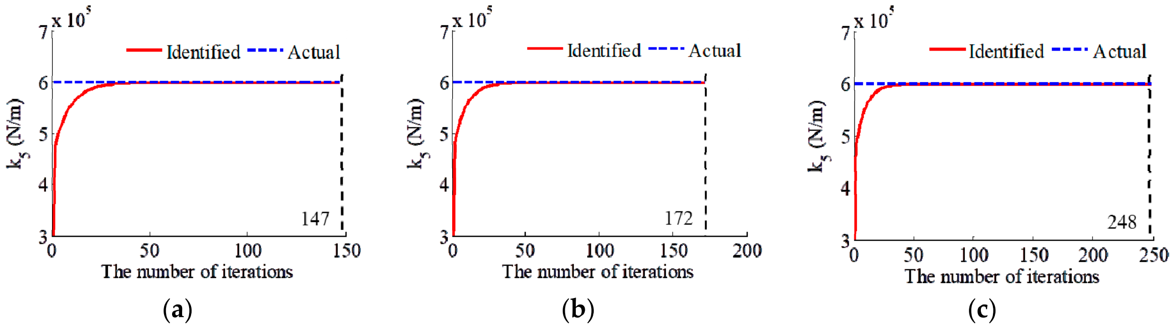

Figure 3. Comparison of the effect of noise intensity on the iteration process: (**a**) noise-free; (**b**) 5% noise; (**c**) 10% noise.

To further show the convergence process during the iteration, the identified stiffness in the case of 10% noise is given in Figure 4 as an example. Obviously, although the iteration is conducted for 248 times due to the relatively stringent tolerance defined before (i.e., ε = 1%), the identified stiffness parameters are stably converged to actual ones, the number of iterations being approximately 50. On the basis of the identified stiffness and the element-wise coefficient q_i in matrix $\mathbf{Q}$, the damping coefficient β can be calculated according to Equation (16) as 7.556×10^{-5}, 6.737×10^{-5}, and 4.103×10^{-5} for the cases of noise-free, 5% noise, and 10% noise, respectively. Since the value of β is extremely small and difficult to assess, these identified results for β, to some extent, are acceptable. Since only the standard LSE technique and the input modification process are involved, the computational time for the proposed approach is very short. In this example, only 10 seconds are required for the identification.

Besides the identification of structural parameters, the unknown ground motion can be estimated by the proposed approach as well. Similarly, taking the case of 10% noise as an example, the comparison of the identified ground acceleration with the actual one is given in Figure 5a. The dashed line represents the identified ground acceleration, whereas the solid line is its actual value. For clarity of comparison, the time segment from 4 s to 6 s is plotted in Figure 5b. Clearly, the proposed approach can identify the ground acceleration very well, even when the signals are contaminated by 10% noise. Based on these identified parameters and ground motion, the structural responses can also be calculated using the equation of motion. Figure 6 gives the comparison of the actual structural responses with the estimated ones. Due to the limitation of the length of this paper, only the responses on the top floor are shown as examples. It is obvious that the estimated responses are in good agreement with the actual ones.

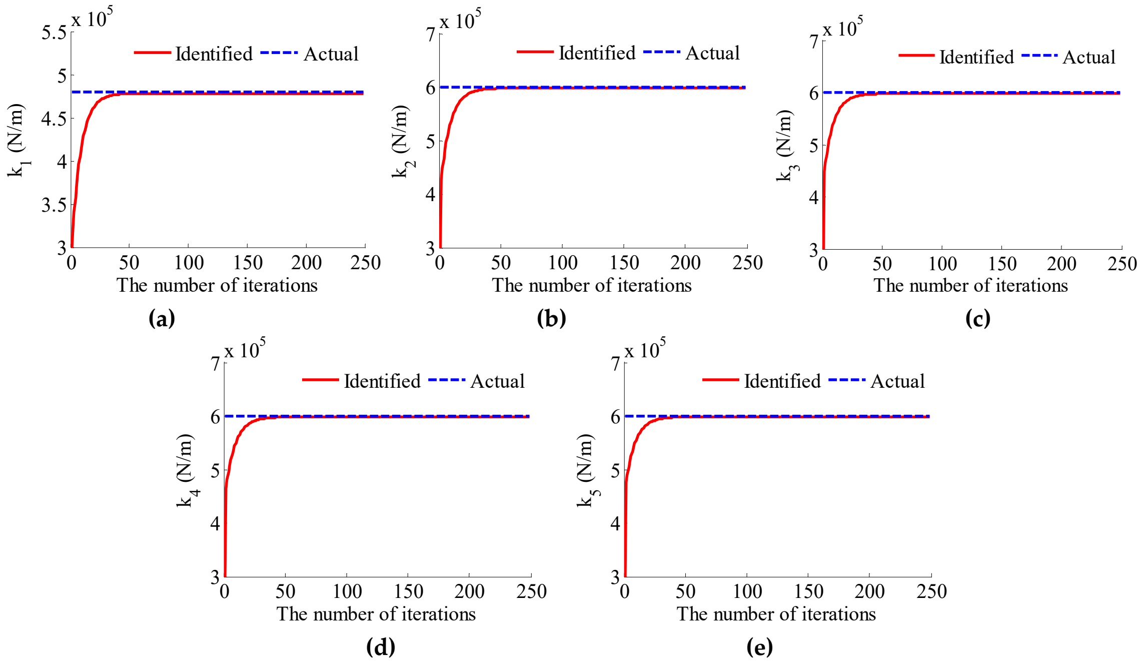

Figure 4. The identified structural parameters (10% noise): (**a**) k_1; (**b**) k_2; (**c**) k_3; (**d**) k_4; (**e**) k_5.

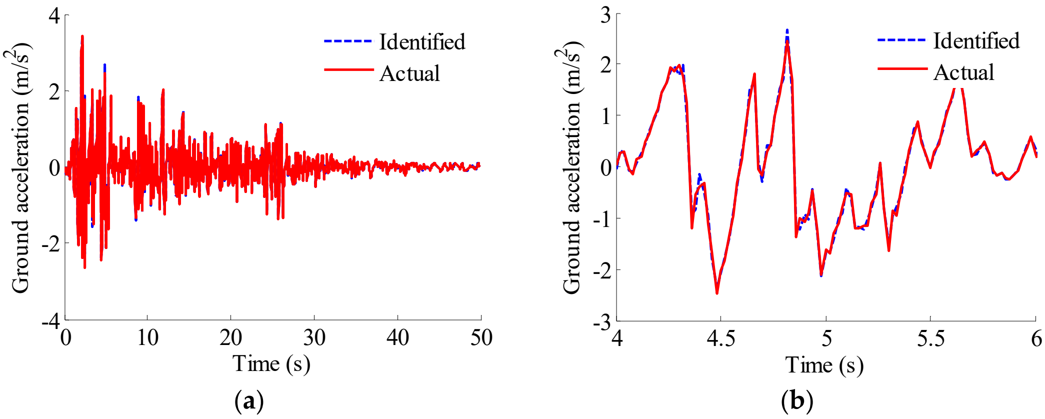

Figure 5. Identified results of the ground acceleration (10% noise): (**a**) the whole time history; (**b**) the time segment from 4 s to 6 s.

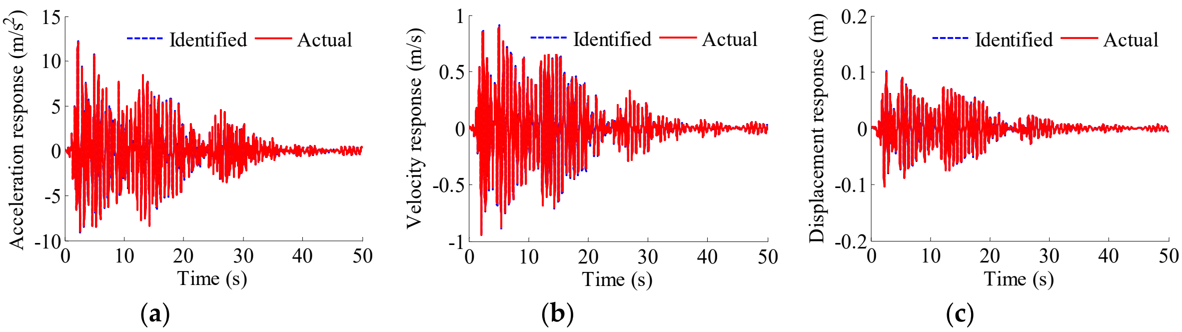

Figure 6. Comparison of the structural responses on the top floor (10% noise): (**a**) acceleration; (**b**) velocity; (**c**) displacement.

3.3. The Effect of the Number of Sample Points

In this section, the effect of the number of sample points is discussed. Similarly, the initial values of the stiffness and damping coefficients α and β are also assumed to be 3.0×10^5 N/m and 1.0 and

1.5×10^{-4}, respectively. The element-wise coefficient q_i is thus initially equal to 45. For practical consideration, noise of 10% intensity as mentioned before is introduced to the measured signals. Herein, the time durations of 6 s, 12 s, and 18 s with 300, 600, and 900 sample points, respectively, are considered. The identification results for these three cases are given in Table 2. It can be seen in Table 2 that as the number of sample points increases, the identification error decreases. By comparing the identified results using 600 sample points with those using 900 sample points, the results are improved slightly when even more sample points are being used. This means that the effect of the number of sample points on the identification results becomes small when sufficient sample points are used. To further explore the effect on the convergence procedure, the comparison of the identified k_5 for the cases of 300, 600, and 900 sample points is shown in Figure 7 as an example. It can be seen that the identified parameters can be stably converged to the actual ones. Moreover, the iterative process is conducted 415, 312, and 291 times for the cases of 300, 600, and 900 sample points, respectively, which means that more iterations are required if fewer sample points are employed.

Table 2. Identified structural parameters with different numbers of sample points.

Structural Parameters	300 Sample Points		600 Sample Points		900 Sample Points	
	Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
k_1	4.715×10^5	1.77	4.756×10^5	0.92	4.768×10^5	0.67
k_2	5.907×10^5	1.55	5.952×10^5	0.80	5.964×10^5	0.60
k_3	5.912×10^5	1.47	5.953×10^5	0.78	5.966×10^5	0.57
k_4	5.914×10^5	1.43	5.954×10^5	0.77	5.967×10^5	0.55
k_5	5.932×10^5	1.13	5.963×10^5	0.62	5.974×10^5	0.43
α	0.534	11.00	0.558	7.00	0.571	4.83

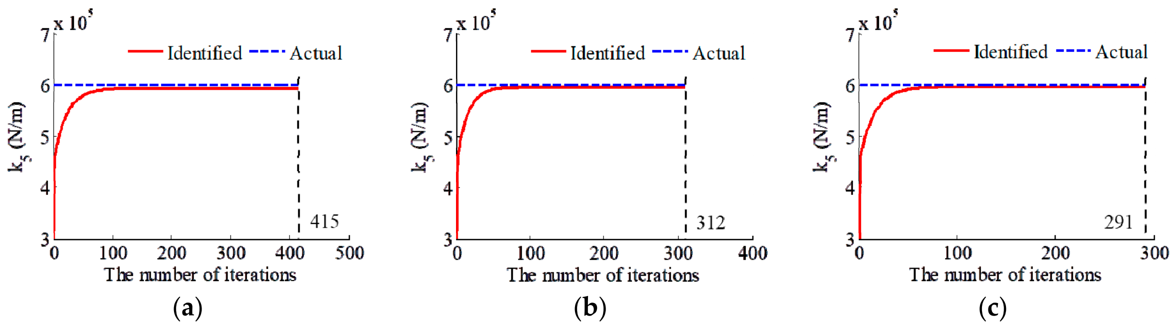

Figure 7. Comparison of the identified k_5 using different numbers of sample points: (**a**) using 300 sample points; (**b**) using 600 sample points; (**c**) using 900 sample points.

Moreover, by using the proposed approach, the ground acceleration can be identified as well. Taking the case of 900 sample points as an example, Figure 8 gives the identified excitation as a dashed line, whereas the actual one is plotted as a solid line. It is clear that the identified ground motion is close to the corresponding actual one.

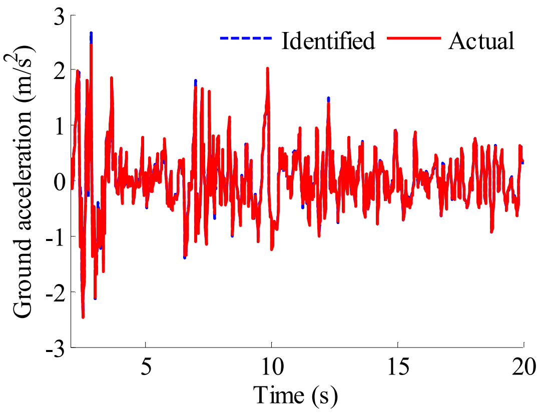

Figure 8. Identified ground acceleration using 900 sample points.

3.4. The Effect of Initial Values of Structural Parameters

The influence of the initial values of structural parameters on the convergence is investigated in this section. The structural responses are assumed to be contaminated by 10% noise. The time durations from 2 s to 20 s with 900 sample points are used for the identification. The initial values of the structural parameters are set to be 5 times, 10 times, and 15 times larger than the actual ones, respectively. The identification results are given in Table 3. It can be observed that the maximum identification error for the stiffness is only 1.08%, even though the initial guesses are 15 times larger than the actual ones. To further investigate the effect of the initial values on the iteration procedure, the comparison of the identified stiffness with different initial values is given in Figure 9. Similarly, only the identified k_5 is plotted in Figure 9 as an example. Obviously, the identified values are stably and accurately converged to the actual ones for the aforementioned three cases. Similar results can be obtained for the remaining parameters.

Table 3. The identified parameters with different initial values.

Structural Parameters	**5 Times**		**10 Times**		**15 Times**	
	Identified	**Error (%)**	**Identified**	**Error (%)**	**Identified**	**Error (%)**
k_1	4.758×10^5	0.88	4.751×10^5	1.02	4.748×10^5	1.08
k_2	5.954×10^5	0.66	5.948×10^5	0.87	5.944×10^5	0.93
k_3	5.956×10^5	0.73	5.950×10^5	0.83	5.946×10^5	0.90
k_4	5.957×10^5	0.71	5.951×10^5	0.81	5.947×10^5	0.88
k_5	5.964×10^5	0.60	5.960×10^5	0.67	5.954×10^5	0.77
α	0.638	6.33	0.642	7.00	0.648	8.00

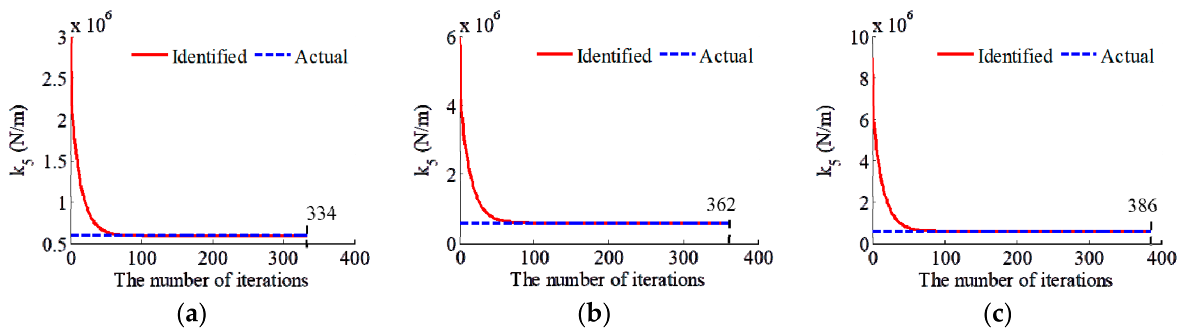

Figure 9. Comparison of the identified k_5 with different initial values: (**a**) 5 times; (**b**) 10 times; (**c**) 15 times.

Similarly, the unknown ground acceleration can be also identified. The comparison results for the case of 15 times initial values are plotted in Figure 10. It is clear that satisfactory identification results can be obtained.

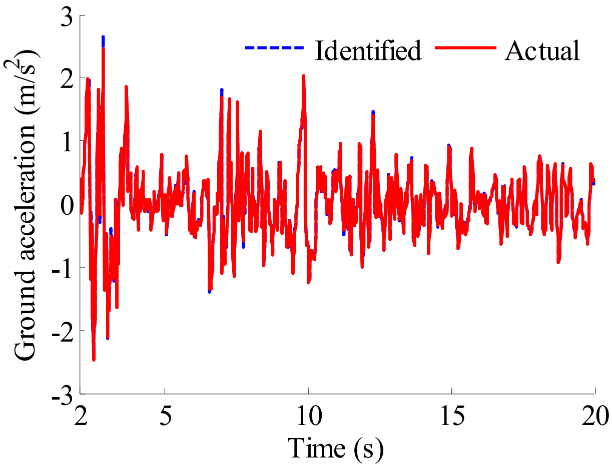

Figure 10. Identified results of the ground acceleration (15 times).

4. Conclusions

In this paper, a relatively simple and efficient iteration identification procedure is proposed for the simultaneous identification of the parameters of building structure and the unknown ground motion. There are no complicated formulas and their derivations involved, which implies the proposed iterative identification procedure can be easily understood and implemented. The standard LSE technique is employed for the estimation of unknown parameters, and an input modification process is developed to ensure that the parameters to be identified would be converged to actual ones. The feasibility and accuracy of the proposed approach is verified via numerical investigation of a shear building model under the El Centro earthquake. The effects of signal noise, the number of sample points, and the initial guesses of structural parameters are discussed. It can be concluded from the numerical results that the proposed approach provides an alternative way for simultaneously identifying the parameters of building structures and the unknown ground motion.

It should be noted that the proposed approach is performed under the condition of the structural responses all being available. Moreover, the effect of the bias caused by a faulty sensor or environmental changes is not considered in this study. Further research can be conducted with the consideration of limited output information and measurement bias.

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