

Article

Capacity Enhancement of Few-Mode Fiber Transmission Systems Impaired by Mode-Dependent Loss

El-Mehdi Amhoud *, Ghaya Rekaya-Ben Othman and Yves Jaouën

LTCI, Télécom ParisTech, 46 Rue Barrault, 75013 Paris, France; rekaya@telecom-paristech.fr (G.R.-B.O.); jaouen@telecom-paristech.fr (Y.J.)

* Correspondence: amhoud@telecom-paristech.fr

Received: 24 January 2018; Accepted: 17 February 2018; Published: 26 February 2018

Abstract: Space-division multiplexing over few-mode fibers is a promising solution to increase the capacity of the future generation of optical transmission systems. Mode-dependent loss (MDL) is known to have a detrimental impact on the capacity of few-mode fiber systems. In the presence of MDL, spatial modes experience different attenuations which results in capacity reduction. In this work, we propose a digital signal processing solution and an optical solution to mitigate the impact of MDL and improve the channel capacity. First, we show that statistical channel state information can be used for a better power allocation for spatial modes instead of equal launch power to increase the system capacity. Afterwards, we propose a deterministic mode scrambling strategy to efficiently reduces the impact of MDL and improves few-mode fiber systems capacity. This scrambling strategy can be efficiently combined with the optimal power allocation to further enhance the capacity. Through numerical simulations of the average and outage capacities, we show that the proposed techniques bring significant capacity gains.

Keywords: optical communications; space-division multiplexing; few-mode fibers; mode-dependent loss; multiple-input multiple-output (MIMO); channel state information; mode scrambling

1. Introduction

The last two decades have known an exponential growth in the demand for optical network bandwidth. Since single-mode fiber (SMF) systems are approaching the nonlinear Shannon limit, space-division multiplexing (SDM) holds the promise to increase the capacity of future optical transmission systems [1,2]. SDM can be realized either by few-mode fibers (FMFs) that allow the propagation of more than one spatial mode or multi-core fibers (MCFs) where each core can be single mode or few-mode [3]. In all schemes, cross-talk is inevitable especially if cores are close in MCFs or if the differential modal group delay (DMGD) is close to zero in FMFs. In both cases, it is compulsory to use multiple-input multiple-output (MIMO) approaches (already used in wireless communications) to recover the signals. In our work, we are interested in FMF systems. The impact of DMGD and unitary modal coupling in FMFs is very important to determine the MIMO equalizer complexity at the receiver [4,5]. Moreover, in FMF systems, propagating modes can also be affected by a non-unitary cross-talk known as mode-dependent loss (MDL) that can be either distributed through the fiber or local at the insertion of optical components. MDL arising from the fiber line is due to manufacturing imperfections such as splices or micro-bends. Accumulated splice losses as low as 0.03 dB may lead to system outage after only a few hundred of kilometers [6]. Another source of MDL comes from optical components such as multiplexers/de-multiplexers (MUX/DEMUX) [7,8] and few-mode optical amplifiers [9,10].

To mitigate the MDL effect, different optical and digital signal processing (DSP) solutions were proposed [11,12]. The availability of information about the FMF channel at the transmitter known as channel state information (CSI) is an important factor that governs the capacity of FMF systems impaired by MDL. In the absence of CSI, all modes are excited with equal launch power which results in capacity reduction when modes have different attenuations. However, with the availability of CSI at the transmitter, capacity can be improved by allocating optimal powers to spatial modes [12]. Additionally, strong mode coupling was reported to reduce the accumulated MDL and hence results in a better system capacity [11]. Moreover, modal coupling can be intentionally added at local points thanks to mode scramblers [13].

In this work, we investigate the FMF channel capacity impaired by MDL. First, we review our proposed statistical CSI that improves the channel capacity independently of channel variations [14]. Afterwards, we present a deterministic scrambling strategy to better average losses between spatial modes and reduce the accumulated MDL. Finally, we show that mode scrambling and the statistical CSI can be used together to reach higher capacities.

2. Few-Mode Fiber Channel Model

Optical devices such as few-mode amplifiers and multiplexers represent the main sources of MDL [7–10]. Nonetheless misaligned fiber splices also lead to an important end-to-end accumulated MDL [6]. In our work, we focus on distributed MDL arising from fiber splices and micro-bends. We consider graded-index FMFs (GI-FMFs) having parabolic profile with core radius r_c and a numerical aperture $NA = \sqrt{n_c^2 - n_{cl}^2} = 0.205$, where n_c (resp. n_{cl}) is the refractive index of the core of the fiber (resp. of the cladding). GI-FMFs have a low DMGD and more modal coupling is obtained than step index fibers [15]. The wavelength is set to $\lambda = 1.55 \mu\text{m}$, the number of propagating modes depends on the core radius r_c , the wavelength λ and the numerical aperture NA . In our work we fix λ and NA and we increase r_c to allow the propagation of higher order modes. In Table 1, we report the parameters of the GI-FMFs [6].

Table 1. Graded-index-few-mode fibers (GI-FMF) parameters for 6- and 10-mode fibers with $NA = 0.205$.

Number of Modes	Core Radius	Propagating Modes
6	8.7 μm	$LP_{01}, LP_{11a,b}, LP_{21a,b}, LP_{02}$
10	11 μm	$LP_{01}, LP_{11a,b}, LP_{21a,b}, LP_{02}, LP_{12a,b}, LP_{31a,b}$

We can express the transverse field distribution of the linearly-polarized (LP) modes with a good approximation by Laguerre-Gauss polynomials as in [16]:

$$E_{l,q}(r, \phi) = C_{l,q} \left(\frac{r}{w}\right)^l L_q^{(l)}\left(\frac{r^2}{w^2}\right) \exp\left(\frac{-r^2}{2w^2}\right) \begin{Bmatrix} \sin(l\phi) \\ \cos(l\phi) \end{Bmatrix} \quad (1)$$

The field distribution of the last equation is written in polar coordinates given by r and ϕ . l denotes the circumferential order and q is the radial order of the LP_{lq} mode. $L_q^{(l)}(x)$ is the generalized Laguerre polynomial of order l and degree q and $w = \sqrt{r_c / (k_0 NA)}$ is the spot size of the fundamental mode LP_{01} with $k_0 = 2\pi/\lambda$ is the free space wavenumber. $C_{l,q}$ is a normalization factor chosen to fulfill the orthogonal relation between spatial modes $\iint_A E_i E_j dA = \delta_{ij}$.

In our capacity analysis, we consider a physical description of the propagation through GI-FMFs with realistic non-unitary modal coupling generated by fiber misalignments. This modelization is based on the electrical field distributions of spatial modes as described before rather than a mathematical modeling based on random matrices as in [12,17]. Neglecting fiber non-linearities, the resulting linear MIMO transmission system is given by:

$$Y_{M \times 1} = H_{M \times M} X_{M \times 1} + N_{M \times 1} = \sqrt{L} \prod_{k=1}^K (T_k C_k) X_{M \times 1} + N_{M \times 1} \tag{2}$$

where $X_{M \times 1}$ (resp. $Y_{M \times 1}$) is the vector of transmitted (resp. received) symbols, $N_{M \times 1}$ is an additive white Gaussian noise vector with zero mean and variance $2\sigma^2$ per mode.

The matrix $H_{M \times M}$ represents the spliced FMF composed of K fiber sections. The factor L represents the mode averaged propagation loss. T_k is a diagonal matrix with random phase entries $\exp(i\phi_m)$ and $\phi_m \in [0, 2\pi]$. The dispersive effect of DMGD is not considered in our work since it does not impact the capacity of the system. The impact of DMGD only affects the complexity of the channel equalization. In the case of using OFDM modulation format, the size of the cyclic prefix should be longer enough to absorb the DMGD. Non-unitary modal coupling due to fiber misalignments at a splice point is given by an $M \times M$ coupling matrix C_k , with entries computed using an overlap integral of the electrical fields of propagating modes at fiber cross sections as in [6]:

$$c_{i,j} = \iint_A E_i^{(k-1)}(x,y) E_j^{(k)*}(x + \Delta x, y + \Delta y) dA \tag{3}$$

where $E_i^{(k-1)}(x,y)$ is the normalized complex amplitude of the i th mode before the splice and $E_j^{(k)*}(x + \Delta x, y + \Delta y)$ is the normalized complex amplitude of the j th mode after the splice. Δx and Δy are fiber misalignments assumed to be independent Gaussian distributed in x and y directions with zero mean and standard deviation (std) $\sigma_{x,y}$. In our channel model, a non-unitary coupling occurs between modes at each fiber misalignment, hence the effects of modal coupling and MDL cannot be separated. In [18], a more appropriate description of linear modal coupling of spliced FMFs was given, however MDL was not taken into account. In our simulations, we consider FMFs made of $K = 300$ concatenations of misaligned fiber sections. To emulate different MDL levels, we vary $\sigma_{x,y}$ as a percentage of the fiber core radius r_c . The accumulated effect from all misalignments results in an end-to-end MDL defined as the ratio in decibels (dB) of the maximum to the minimum eigenvalues of HH^\dagger .

$$\text{MDL(dB)} = 10 \log_{10} \left(\frac{\lambda_{max}}{\lambda_{min}} \right) \tag{4}$$

3. Capacity Enhancement by Channel State Information

Assuming that the channel matrix H is known to the receiver (e.g., through training sequences) but is unknown at the transmitter due to the long round-trip delays in optical transport networks. In this case, the transmitter sends uncorrelated signals of equal power on all modes. The average capacity of the FMF channel with equal power allocation C_{ep} is then given by [12,17]:

$$C_{ep} = \mathbb{E}_{\lambda_i} \left(\sum_{i=1}^M \log \left(1 + \frac{P_t \lambda_i}{M 2\sigma^2} \right) \right) \tag{5}$$

P_t is the total power at the transmitter. In the last equation, the average is taken over the eigenvalues of the channel H . The analytical computation of the average capacity requires the knowledge of the distributions of λ_i 's which is not always available, consequently we compute the capacity by extensive numerical simulations. Note that this is a linear capacity that increases with the amount of power, if nonlinearities are taken into account the capacity drops after a non-linear power threshold [19].

3.1. Channel State Information

By computing the singular value decomposition (SVD) of the channel matrix as $H = U\Lambda V^\dagger$, where $\Lambda = \text{diag}(\sqrt{\lambda_1}, \dots, \sqrt{\lambda_M})$ and U, V are unitary matrices. CSI refers to the matrices Λ and V that can be sent back from the receiver to the transmitter. In this case, V is used by the transmitter

to send correlated symbols vector VX_M . The matrix Λ is used to compute the optimal powers allocated to each mode. This can be done by the iterative water-filling algorithm known from MIMO wireless communications [20,21]. The average channel capacity with channel state information is given by [12,20,21]:

$$C_{CSI} = \mathbb{E}_{\lambda_i} \left(\sum_{i=1}^M \log \left(1 + p_i^* \frac{\lambda_i}{2\sigma^2} \right) \right) \quad (6)$$

p_i^* is the optimal power allocated to the mode i and is given by:

$$p_i^* = \mu - \frac{2\sigma^2}{\lambda_i} \quad (7)$$

μ is a constant that satisfies the total power constraint $\sum_{i=1}^M (\mu - \frac{2\sigma^2}{\lambda_i}) = P_t$. This power allocation strategy faces two major limitations. First, the CSI must be provided to the transmitter to send the data before the channel changes. Second, the optimal powers must be updated at the minimum as fast as the channel variations. These constraints cannot be satisfied for the moment for FMF optical systems where the channel dynamics change faster than round-trip delays. For example, in [22], the authors found that for a 26-km two-mode fiber, the channel variations at a wavelength $\lambda = 1550$ nm were of the order of 40 μ s. In this case, the round-trip delay, is equal to 250 μ s which is more than six-times longer than channel variations.

3.2. Statistical Channel State Information

To overcome the round-trip delay issue, we propose that the receiver computes an average of the channel that we note $\bar{H} = \bar{U} \bar{\Lambda} \bar{V}^\dagger$, with \bar{U}, \bar{V} are unitary matrices, $\bar{\Lambda} = \text{diag}(\sqrt{\bar{\lambda}_1}, \dots, \sqrt{\bar{\lambda}_M})$ contains the long term eigenvalues of the channel. \bar{V} and $\bar{\Lambda}$ represent the statistical channel state information. The matrix $\bar{\Lambda}$ is used to compute the optimal powers $\bar{p}_1^*, \dots, \bar{p}_M^*$ per excited modes only once, and the matrix \bar{V} will be used by the transmitter to send correlated symbols vector independently of the channel variations. We refer to this capacity by $C_{\bar{CSI}}$:

$$C_{\bar{CSI}} = \mathbb{E}_{\lambda_i} \left(\sum_{i=1}^M \log_2 \left(1 + \bar{p}_i^* \frac{\lambda_i}{2\sigma^2} \right) \right) \quad (8)$$

To have an insight on the FMF channel capacity behavior, we simulate the previous capacities for the 6-mode and 10-mode fibers. Plotted results are averaged over 10^5 channel realizations. Figure 1a shows the complementary cumulative distribution function (CCDF) of the capacities given by Equations (5), (6) and (8) for the 6-mode fiber for MDL = 15 dB. We notice that both C_{CSI} and $C_{\bar{CSI}}$ are higher than C_{ep} and as the SNR increases the gain brought by CSI decreases. In Figure 1b we plotted the CSI gain defined as the ratio of C_{CSI} (resp. $C_{\bar{CSI}}$) to C_{ep} as a function of the SNR for different MDL values. We notice that the CSI gain increases with MDL and for higher SNRs the CSI gain vanishes.

To focus on the proposed statistical CSI, we compare the average capacities of equal power allocation to the long term power allocation. The average channel \bar{H} is computed using 10^4 realizations of the instantaneous channel H . Figure 2a shows the average capacities C_{ep} and $C_{\bar{CSI}}$ for the 6-mode and 10-mode fibers as a function of SNR for MDL = 20 dB. We notice that the proposed statistical CSI increases the capacity for both fibers.

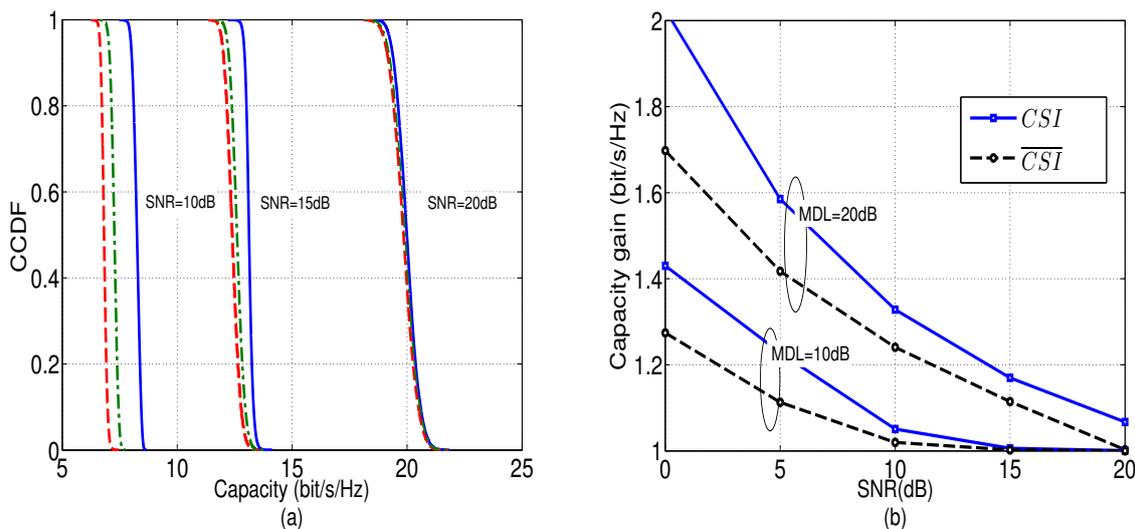


Figure 1. (a) Capacity complementary cumulative distribution function (CCDF) with channel state information (CSI) (solid), $\overline{C_{CSI}}$ (dash-dot) and C_{ep} (dashed) for SNR = 10, 15, 20 dB and mode-dependent loss (MDL) = 15 dB; (b) Capacity gain as a function of SNR for MDL = 10, 20 dB for a 6-mode fiber.

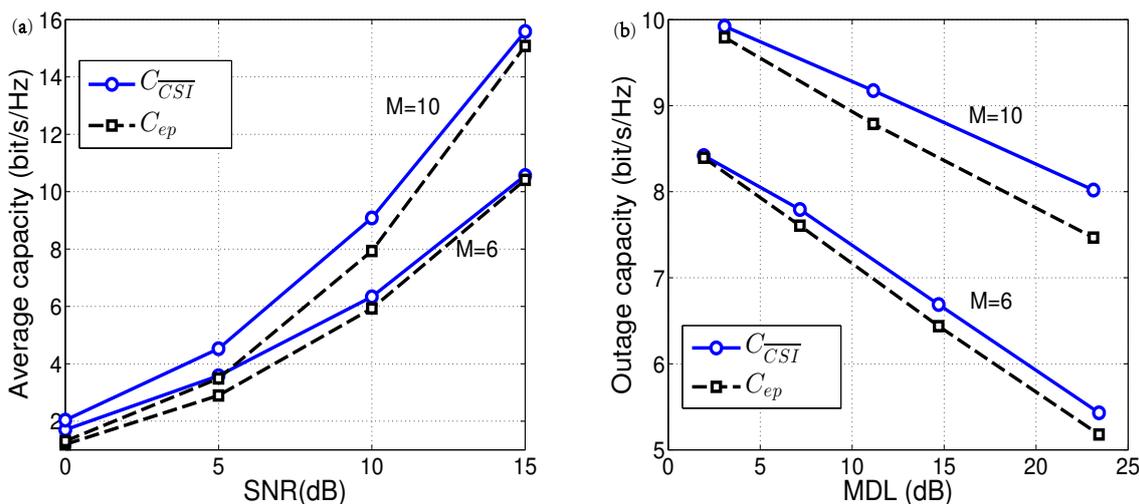


Figure 2. (a) Average capacities vs. SNR for $M = 6, 10$ modes for MDL = 20 dB; (b) Outage capacities vs. MDL for $M = 6, 10$ modes for SNR = 10 dB.

Outage considerations are very important for the design of FMF optical systems. Since the channel changes randomly, transmitters are configured to encode for a fixed capacity that is higher than the worst case capacity called outage capacity to avoid system outage. The outage capacity C_{out} is related to an outage probability p_{out} as:

$$\Pr(C_{inst}(H) \leq C_{out}) = p_{out} \tag{9}$$

where $C_{inst}(H)$ is the instantaneous capacity defined with the coherence time of the channel during which channel coefficients h_{ij} remain constants. In Figure 2b, we plotted the outage capacities for an outage probability of 10^{-3} for the 6-mode and 10-mode fibers as a function of MDL for SNR = 10 dB. We compare the capacity with equal launch power to the capacity when using the long term optimal powers based on the statistical CSI. We notice that both capacities are decreasing with MDL, however $C_{\overline{C_{CSI}}}$ is better than C_{ep} . At MDL = 20 dB we have 0.3 bit/s gain for the 6-mode fiber and 0.5 bit/s gain for the 10-mode fiber. Consequently, since the availability of instantaneous CSI at the transmitter

achieves the maximum capacity but is unfortunately unfeasible in practical systems, the proposed statistical CSI can be a solution to increase the capacity rather than using equal launch power at the transmitter. This method has a low-cost complexity since it works independently of the channel variations, thus it can be feasible for real implementation. Nonetheless, it is important to determine how many channel acquisitions are enough to properly use the average channel. Also, the time duration of the use of a given statistical CSI before having to compute a new one.

4. Mode Scrambling for Few-Mode Fibers

The principle of mode scrambling (MS) consists in introducing a random mode permutation after K_{scr} splices. The application of MS to reduce MDL and increase FMF systems capacity was first proposed in [13]. The authors showed that randomly scrambling all modes after each splice ($K_{scr} = 1$) leads to a completely uncorrelated modal coupling. This strategy can reduce the MDL of the link but a large number of scramblers is required. The MIMO transmission system including MS is given by:

$$Y_{M \times 1} = \sqrt{L} \prod_{k=1}^K (T_k C_k P_k) X_{M \times 1} + N_{M \times 1} \tag{10}$$

In the last equation, we added the scrambling matrix P_k corresponding to a random permutation matrix if k is a multiple of the scrambling period K_{scr} , or an identity matrix otherwise.

4.1. Deterministic Mode Scrambling

In our scrambling approach, we propose to use a deterministic mode scrambler that permutes modes having more received power with modes having less received power. To have an insight into this strategy, we consider the previous 6-mode and 10-mode fibers with a misalignment std $\sigma_{x,y} = 4\%r_c$. At the transmitter we launch all modes with unit energy $E_s = 1$ and we compute the received energy per mode at the receiver side averaged over 10^5 channel realizations.

From Figure 3a,b, we clearly notice that some modes arrive at the receiver with more power than the others. For the 6-mode fiber the less attenuated modes are the LP_{01} and $LP_{11a,b}$ and the most attenuated modes are the LP_{02} and $LP_{21a,b}$. Hence, we propose to permute the LP_{01} with the LP_{02} and the $LP_{11a,b}$ with the $LP_{21a,b}$. For the 10-mode fiber, modes arriving with more power are the LP_{01} , $LP_{11a,b}$ and $LP_{21a,b}$, these modes will be permuted respectively with modes arriving with less power which are the LP_{02} , $LP_{31a,b}$ and $LP_{12a,b}$. Our scrambling strategy, avoids permutations between modes arriving with close levels of power especially between degenerate modes that arrive with the same amount of power. However, this situations can occur if random mode scrambling is considered, which do not average power effectively between all modes.

To compare the different scrambling strategies, we add 6 scramblers ($K_{scr} = 50$) to the previous fibers (made of $K = 300$ sections). In Figure 4, we plot the PDF of MDL for 10^5 channel realizations and for a misalignment std $\sigma_{x,y} = 4\%r_c$. We can see that MS reduces the impact of MDL, and that the deterministic MS outperforms the random MS. For the 6-mode fiber, the average MDL without scramblers in the line is 25 dB, with random scrambling the MDL is reduced to 15 dB and for the proposed scrambling strategy is reduced to 9 dB. For the 10-mode fiber, the average MDL without scramblers in the line is 36 dB, with random scrambling the MDL is 15 dB and for the proposed scrambling strategy is reduced to 10 dB.

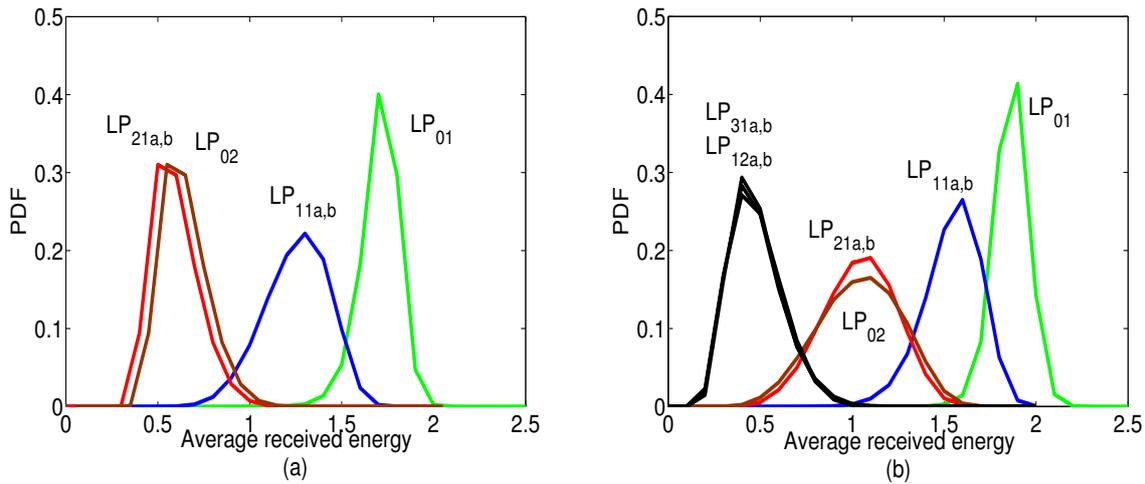


Figure 3. Probability distribution function (PDF) of the average received energy per mode. (a) 6-mode fiber; (b) 10-mode fiber.

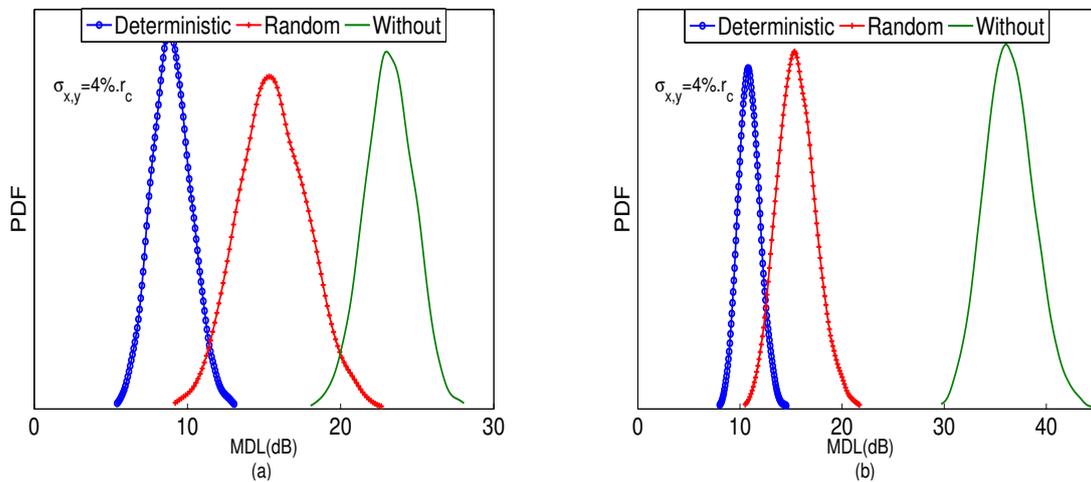


Figure 4. PDF of MDL for different scrambling strategies and a fiber misalignment std $\sigma_{x,y} = 4\%r_c$. (a) 6-mode fiber; (b) 10-mode fiber.

To study the impact of MS, we simulate the system average and outage capacities based on 10^5 channel realizations of the same previous fibers made of $K = 300$ sections and using 6 scramblers. In Figure 5, we plot the average capacities for the 6-mode and 10-mode fibers for different scrambling strategies. We notice that MS increases the system capacity and that the deterministic MS outperforms the random MS. For an SNR = 15 dB and a std $\sigma_{x,y} = 4\%r_c$, the capacity gain brought by using deterministic MS to a strategy where no scrambler is used is 4.5 bits/s for the 6-mode fiber and 3.5 bits/s for the 10-mode fiber. Consequently, deterministic MS averages power more efficiently between all modes, which reduces significantly MDL and results in a better average capacity. Furthermore, we also notice that for low SNRs the capacity gain provided by MS is very low. This is due to the impact of noise that dominates the effect of MDL (given by the different eigenvalues λ_i 's) in the capacity expression of Equation (5). However, for high SNRs the gain provided by MS becomes more important. This behavior is different from the CSI gain that is more important for low SNR levels.

In Figure 6a,b, we simulate the outage capacity as a function of the misalignment std $\sigma_{x,y}$ for the different scrambling strategies and for an outage probability of 10^{-3} . We notice that as std $\sigma_{x,y}$ increases which increases also MDL, the outage capacities are decreasing. However, the use of MS enhances these capacities significantly and the deterministic MS outperforms the random MS. For an std $\sigma_{x,y} = 4\%r_c$.

The capacity gain brought by using deterministic MS to a strategy where no scrambler is used is 7.8 bit/s for the 6-mode fiber and 5.2 bit/s for the 10-mode fiber.

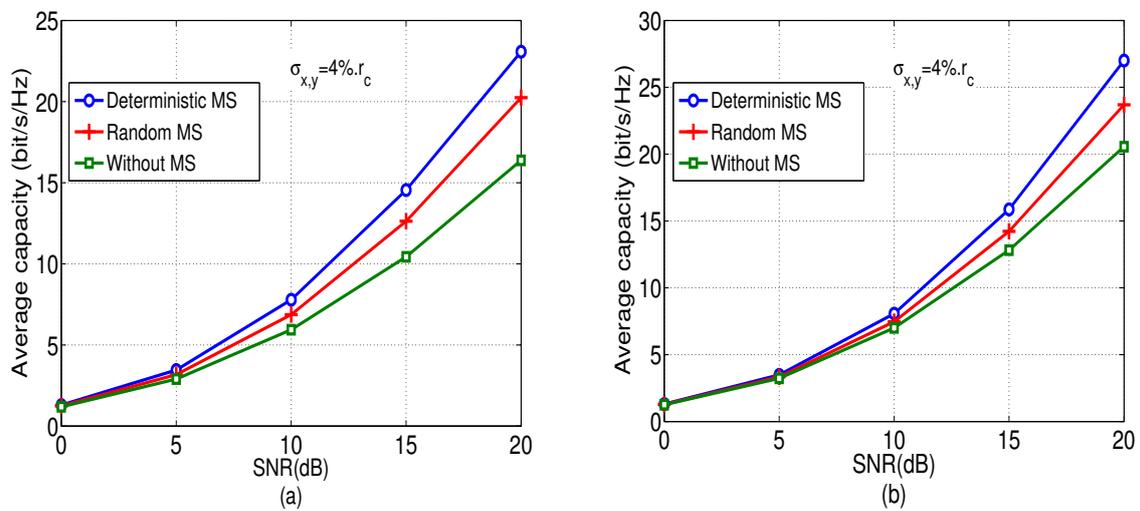


Figure 5. Average capacities vs. SNR for a misalignment std $\sigma_{x,y} = 4\%r_c$. (a) 6-mode fiber; (b) 10-mode fiber.

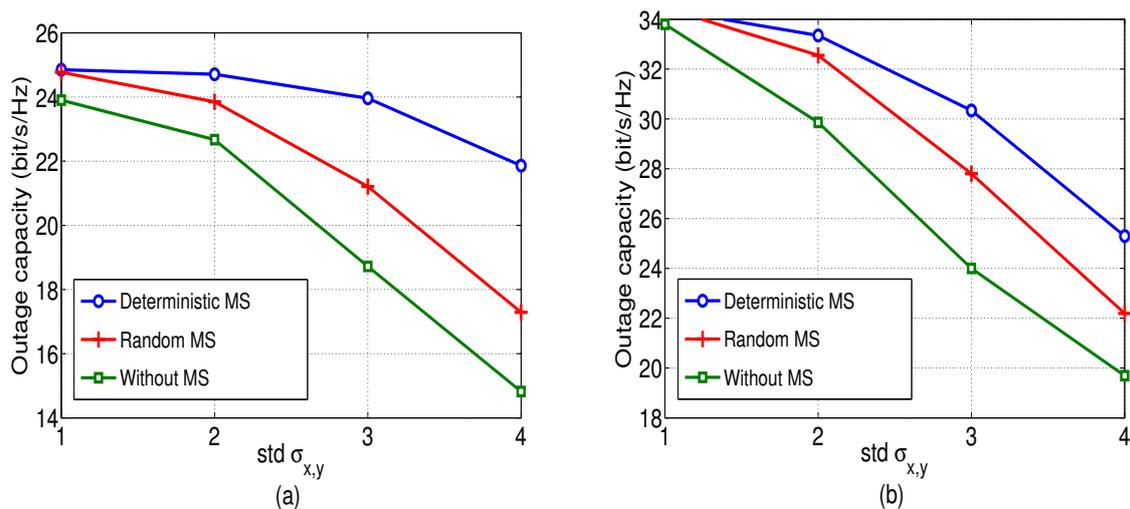


Figure 6. Outage capacities vs. misalignment std $\sigma_{x,y}$ for different scrambling strategies for SNR = 20 dB and an outage probability $p_{out} = 10^{-3}$. (a) 6-mode fiber; (b) 10-mode fiber.

4.2. Combined Statistical CSI and Deterministic MS

To further enhance the FMFs capacity, we combine the deterministic MS with the statistical CSI. In Figure 7a, we plotted the average capacity with different techniques for the 10-mode fiber and a misalignment std $\sigma_{x,y} = 4\%r_c$. From the figure, we notice that the statistical CSI provides an important capacity gain for low SNRs, however this gain decreases as the SNR increases. The deterministic MS increases the capacity as the SNR increases. The combination of both techniques enhances the average capacity for all levels of SNR. In Figure 6b, we simulate the outage capacities for the same previous techniques and an outage probability of 10^{-3} and SNR = 20 dB. From the figure, we notice that the capacity with deterministic MS is better than the capacity with statistical CSI, this is due the high SNR level of 20 dB. Furthermore, the outage capacity with the combination of both techniques is the highest one.

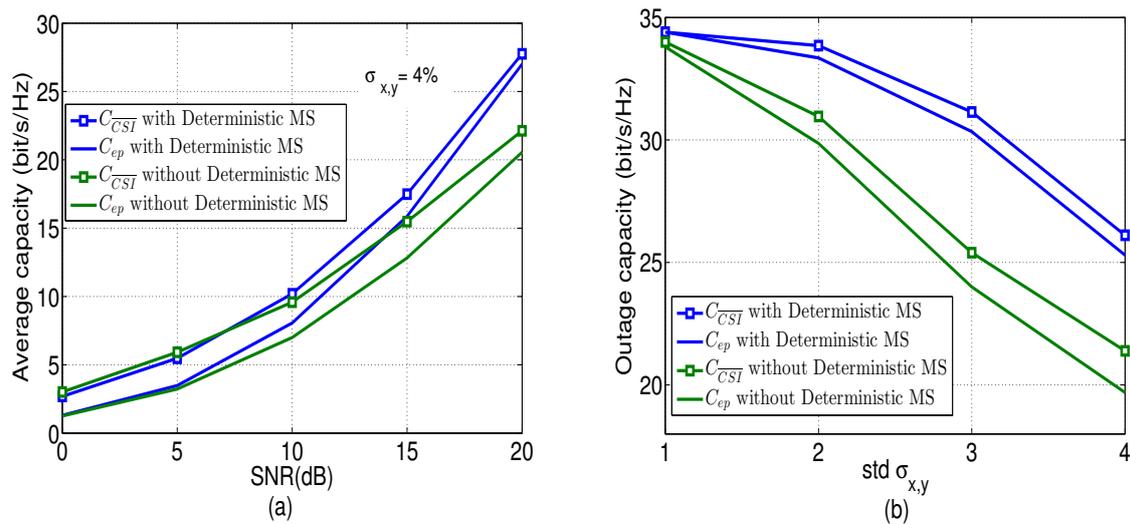


Figure 7. (a) Average capacities vs. SNR for the 10-mode fiber with a misalignment std $\sigma_{x,y} = 4\%r_c$; (b) Outage capacities vs. $\sigma_{x,y}$ for the 10-mode fiber for SNR = 20 dB and $p_{\text{out}} = 10^{-3}$.

5. Conclusions

Capacity of FMF systems impaired by MDL can be improved by appropriate techniques. A long term knowledge of the optical channel can help improve the capacity by allocating optimal powers to each mode. Depending on channel variations that are commonly longer than the round-trip delays, the statistics of the FMF channel can be used. Moreover, deterministic scramblers can be placed in the optical link to reduce the MDL penalty by averaging power between modes at local points. Permuting modes with more power with modes having low power results in a better power averaging at the link end. The two techniques can be complementary for a further capacity improvement.

Author Contributions: El-Mehdi Amhoud, Ghaya Rekaya-Ben Othman and Yves Jauouën proposed and adapted the power allocation method. El-Mehdi Amhoud proposed and developed the scrambling strategy and wrote the paper. All authors have read and approved the final paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

- Richardson, D.J.; Fini, J.M.; Nelson, L.E. Space-division multiplexing in optical fibres. *Nat. Photonics* **2013**, *7*, 354–362.
- Winzer, P.J. Spatial multiplexing in fiber optics: The 10x scaling of metro/core capacities. *Bell Labs Tech. J.* **2014**, *19*, 22–30.
- Essiambre, R.J.; Ryf, R.; Fontaine, N.K.; Randel, S. Breakthroughs in Photonics 2012: Space-Division Multiplexing in Multimode and Multicore Fibers for High-Capacity Optical Communication. *IEEE Photonics J.* **2013**, *5*, 0701307.
- Lobato, A.; Ferreira, F.; Inan, B.; Kuschnerov, M.; van den Borne, D.; Jansen, S.L.; Spinnler, B.; Lankl, B. The Impact of Differential Mode Delay on Mode-Division Multiplexed Coherent Optical OFDM Transmission. In Proceedings of the Optical Fiber Communication Conference, Los Angeles, CA, USA, 4–8 March 2012.
- Inan, B.; Spinnler, B.; Ferreira, F.; van den Borne, D.; Lobato, A.; Adhikari, S.; Sleiffer, V.A.J.M.; Kuschnerov, M.; Hanik, N.; Jansen, S.L. DSP complexity of mode-division multiplexed receivers. *Opt. Express* **2012**, *20*, 10859–10869.
- Warm, S.; Petermann, K. Splice loss requirements in multi-mode fiber mode-division-multiplex transmission links. *Opt. Express* **2013**, *19*, 519–532.
- Chang, S.H.; Moon, S.-R.; Chen, H.; Ryf, R.; Fontaine, N.K.; Park, K.J.; Kim, K.; Lee, J.K. All-fiber 6-mode multiplexers based on fiber mode selective couplers. *Opt. Express* **2017**, *25*, 5734–5741.

8. Li, A.; Ye, J.; Chen, X.; Shieh, W. Fabrication of a Low-Loss Fused Fiber Spatial-Mode Coupler for Few-Mode Transmission. *IEEE Photonics Technol. Lett.* **2013**, *25*, 1985–1988.
9. Le Cocq, G.; Quiquempois, Y.; Le Rouge, A.; Bouwmans, G.; El Hamzaoui, H.; Delplace, K.; Bouazaoui, M.; Bigot, L. Few mode Er³⁺ doped fiber with micro-structured core for mode division multiplexing in the C-band. *Opt. Express* **2013**, *21*, 31646–31659.
10. Ip, E.; Li, M.-J.; Bennett, K.; Huang, Y.K.; Tanaka, A.; Korolev, A.; Koreshkov, K.; Wood, W.; Mateo, E.; Hu, J.; et al. 146 × 6 × 19-Gbaud Wavelength-and Mode-Division Multiplexed Transmission Over 10 × 50-km Spans of Few-Mode Fiber With a Gain-Equalized Few-Mode EDFA. *J. Lightw. Technol.* **2014**, *21*, 790–797.
11. Lobato, A.; Ferreira, F.; Kuschnerov, M.; van den Borne, D.; Jansen, S.L.; Napoli, A.; Spinnler, B.; Lankl, B. Impact of mode coupling on the mode dependent loss tolerance in few-mode fiber transmission. *Opt. Express* **2012**, *20*, 29776–29783.
12. Ho, K.-P.; Kahn, J.M. Mode-dependent loss and gain: Statistics and effect on modedivision multiplexing. *Opt. Express* **2011**, *19*, 16612–16635.
13. Warm, S.; Petermann, K. Capacity increase in spliced mode-multiplexed transmission systems by using mode mixers. In Proceedings of the European conference on optical communications, Seattle, WA, USA, 9–11 July 2012; paper TuC3.3.
14. Amhoud, E.-M.; Jaouen, Y.; Rekaya Ben-Othman, G. Capacity enhancement of space-division multiplexed systems by using statistical channel state information. In Proceedings of the Asia Communications and Photonics Conference, Hong Kong, China, 19–23 November 2015; paper AS4E.2.
15. Gruner-Nielsen, L.; Sun, Y.; Nicholson, J.W.; Jakobsen, D.; Lingle, R.; Palsdottir, B. Few Mode Transmission Fiber With Low DGD, Low Mode Coupling, and Low Loss. *J. Lightw. Technol.* **2012**, *30*, 3693–3698.
16. Unger, H.-G. *Planar Optical Waveguides and Fibers*; Oxford University Press: Oxford, UK, 1977.
17. Winzer, P.J.; Foschini, G.J. MIMO capacities and outage probabilities in spatially multiplexed optical transport systems. *Opt. Express* **2011**, *19*, 16680–16696.
18. Ferreira, F.; Costa, C.S.; Sygletos, S.; Ellis, A.D. Semi-Analytical Modelling of Linear Mode Coupling in Few-Mode Fibers. *J. Lightw. Technol.* **2017**, *35*, 4011–4022.
19. Koebele, K.; Salsi, M.; Charlet, G.; Bigo, S. Nonlinear Effects in Mode-Division-Multiplexed Transmission Over Few-Mode Optical Fiber. *IEEE Photonics Technol. Lett.* **2011**, *23*, 1316–1318.
20. Caire, G.; Shamai, S. On the capacity of some channels with channel state information. *IEEE Trans. Inf. Theory* **1999**, *45*, 2007–2019.
21. Tse, D.; Viswanath, P. *Fundamentals of Wireless Communication*; Cambridge University Press: Cambridge, UK, 2005.
22. Chen, X.; He, J.; Li, A.; Ye, J.; Shieh, W. Characterization and Analysis of Few-Mode Fiber Channel Dynamics. *IEEE Photonics Technol. Lett.* **2013**, *25*, 1819–1822.



© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).