



Article Cloud Computing Network Empowered by Modern Topological Invariants

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Abstract: The cloud computing networks used in the IoT, and other themes of network architectures, can be investigated and improved by cheminformatics, which is a combination of chemistry, computer science, and mathematics. Cheminformatics involves graph theory and its tools. Any number that can be uniquely calculated by a graph is known as a graph invariant. In graph theory, networks are converted into graphs with workstations or routers or nodes as vertex and paths, or connections as edges. Many topological indices have been developed for the determination of the physical properties of networks involved in cloud computing. The study computed newly prepared topological invariants, K-Banhatti Sombor invariants (KBSO), Dharwad invariants, Quadratic-Contraharmonic invariants (QCI), and their reduced forms with other forms of cloud computing networks. These are used to explore and enhance their characteristics, such as scalability, efficiency, higher throughput, reduced latency, and best-fit topology. These attributes depend on the topology of the cloud, where different nodes, paths, and clouds are to be attached to achieve the best of the attributes mentioned before. The study only deals with a single parameter, which is a topology of the cloud network. The improvement of the topology improves the other characteristics as well, which is the main objective of this study. Its prime objective is to develop formulas so that it can check the topology and performance of certain cloud networks without doing or performing experiments, and also before developing them. The calculated results are valuable and helpful in understanding the deep physical behavior of the cloud's networks. These results will also be useful for researchers to understand how these networks can be constructed and improved with different physical characteristics for enhanced versions.

Keywords: topological invariants; K-Banhatti; Sombor indices; maple; network graph; cloud computing; scalability; latency; throughput; best-fit topology

1. Introduction

Cloud computing is the on-demand availability of workstation structure resources, especially data capacity limit (cloud limit) and computing power, without a direct unique organization by the client [1]. Gigantic clouds regularly have limits coursed over various regions, each region being a server ranch [2]. Cloud computing relies upon the sharing of resources to achieve clarity and regularly uses a "pay-all-the-more just as expenses emerge" model, which can help in diminishing capital expenses, yet may similarly provoke frightening working expenses for clueless clients [3]. It also means reducing costs by using



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Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). the best characteristics cloud computing network as it expanded effectively. A model for conveying the Web-based utility registering administrations, cloud computing has swiftly emerged. The rapid expansion of the cloud computing industry with its wide range of clients, from small businesses to major corporations, has made it difficult for cloud service providers to manage the vast amount of data and other resources in the cloud. Ineffective asset management can taint cloud computing's appearance. Therefore, resources should be distributed consistently to different partners without compromising the association's benefit or the satisfaction of clients. One of the most significant and constantly evolving cloud computing paradigms is Infrastructure as a Service (IaaS). The scalability, administration style, greatest utility, lower costs, increased throughput, decreased idle time, the particular environment, cost viability, and a softer connection point are several examples of the core cloud computing components. Additionally, modern information-focused organizations have noticed a rapid increase in the asset requirements of contemporary apps. Energy efficiency; heterogeneity; load balancing; task scheduling; resource management; quality of service; workload management; the enormous volume of data; provision of affordable, simple, and flexible services; scalability; dynamic resource allocation; quality of service; optimum utility; decreased overheads; and higher throughput are just a few of the issues that need to be addressed, as well as a reduction in costs, which is among the problems and challenges with cloud computing, flexibility, capacity, scalability, and dependability [4–9]. All of these attributes depend on the cloud's topology, which determines how different nodes, pathways, and clouds should be connected to maximize the previously described attributes or characteristics. The study only addresses the topology of the cloud network to achieve the best of all attributes and resolve all the issues and challenges to the possible extent with the help of topological invariants. The performance, efficiency, quality of service, security of cloud networks, flexibility, and cost-effectiveness are dependent on the topology of the cloud network to some extent. The study solves the cloud network topologically with the help of graph theory and cheminformatics, which is a combination of computer science, chemistry, and mathematics. The deduced results will be used for the scalability, modeling, capacity enhancement, and other challenges discussed above for cloud computing networks.

The review is likewise vital because of the vigorous nature of the issues regarding cloud computing. Therefore, the study discusses and resolves issues, such as scalability, higher throughput, reduced latency, efficient use, and adaptable cloud computing networks to fulfill the desired outcomes in the context of topology. For the formal reason, the study solves the existing cloud computing network with the help of certain freshly prepared topological invariants and gives the best-fit topology for the existing cloud computing network, and also gives the basis for the modeling of new scalable cloud computing networks with the best feasible characteristics [10].

On the other hand, a network's dynamic development is intimately related to its arrangement of internal connections [11]. The lack of periodic patterns in the great majority of computer networks, particularly recurrent neural networks, makes it difficult to demonstrate this link from a theoretical and formal standpoint, leaving only descriptive statistical characteristics to explain network dynamics [12]. The algebraic topology gives us invariants, which are a great tool for understanding the structure of abstract spaces and can also be defined for graphs. The study proposes that these invariants be used in network science. A topological index is a quantity derived from a graph that reflects the relevant structural properties of the underlying network. It is, in reality, a numerical number associated with the network used to correlate computer structures with specific physical qualities. A topological index is created by converting a computer network into a numerical value. It is a numeric number associated with a computer structure (graph) that characterizes the structure's topology and is invariant under a structure-preserving mapping. The study uses K-Banhatti Sombor (KBSO) invariants, Contraharmonic-Quadratic invariants (CQI), and Dharwad invariants for the solution of cloud computing networks [13].

Gutman defines the Sombor indices and their different forms. The vertex degree-based invariant graph, named the Sombor index, is utilized to catch the sharp lower and upper limits of the associated network and the attributes of the network arriving at the limits. There are two variations of KBSO indices, the first is the KBSO index and the second is its diminished adaptation means reduced form [14]. A KBSO index is a topological index that is a number related to a network graph that catches the symmetry of the network structure and gives a logical language to foresee the qualities of the network to enhance the cloud network topologically [15].

In 2021, VR Kulli presented some topological degree-based indices following Gutman's Sombor indices. These indices are called Dharwad indices. It has a couple of different forms such as Dharwad, Dharwad reduced remarkability, and the δ -Dharwad index which is utilized to tackle the geography of sweet-smelling compounds called aromatic compounds. Quantitatively predicting the hidden aspects of various network constructions, such as cloud networks, bridge networks, Sierpinski networks, and chemical component networks used in the production of various networks utilized in different product development, is made possible by CQIs. Dharwad indices can be used to solve the topology of cloud networks as these are already used for aromatic compounds [1]. These indices are used to solve the topology of a cloud network very effectively and efficiently and to find the lower bounds and upper bounds of a cloud network or graph [16,17]. It can also be used for the verification, improvement, and exploration of irregularities from the network under discussion [18].

1.1. Research Motivation

The investigation of cloud computing networks' topological invariants is the primary goal of this study. The study determines the seriousness and intensity of topological indices in particular cloud networks. The paper demonstrates the advantages of some topological invariants, such as KBSO, CQI, and Dharwad, as well as their reduced forms. Its main goal is to provide formulas that can be used to evaluate the topology and performance of certain cloud networks, both before they are manufactured, and without doing experiments. The research yielded mathematical conclusions that are used in the modeling of specific cloud networks.

Due to their incremental and quick character, it is also uncovering new and significant formulas or solutions for modeling and creating specific cloud networks, for which no acceptable solution has yet been identified.

The idea is to create new, highly effective cloud networks with the best features, while also enhancing the ones that already exist. This is because vendors and manufacturers require products that are reliable and effective. The study gives the ability to create the strongest, most reliable, and error-free specific networks.

1.2. Research Questions

Our research questions deal with the better cloud networks used in interconnection networks, parallel processing, power generation networks, bioinformatics, chemical compound development, and robotics. The study focuses on providing mathematical results for modeling purposes before the manufacturing of the above-mentioned products by avoiding compromised cloud networks.

The following questions arise from the said topic:

RQ1 How does the study solve the topology of the cloud networks involved in interconnection networks mathematically by graph theory?

RQ2 How does the study model the interconnection networks with the help of the deduced mathematical results?

RQ3 How did the study enhance the existing interconnection networks, reduce their irregularities, and find error-free, failure-free, and efficient advanced cloud networks as compared to existing networks?

2. Background and Literature Review

2.1. Background

Cloud computing is one of the most impressive creations that has received the interest of technologists from one side of the planet to the other. Cloud computing enjoys many benefits; however, it likewise has a large number of gambles in the context of security, scalability, latency delay, etc., which cannot be easily overlooked. For a fruitful cloud computing reception in an enterprise, legitimate preparation and familiarity with the arising chances, dangers, weaknesses, and potential arrangements are essential. Accordingly, deciding the best arrangement guidelines to increment cloud security has become significant for all cloud tasks. In this examination, we are exploring and surveying the most vital organization security and information security gambles on cloud frameworks because of a writing audit. Since numerous organizations have advanced and promoted virtual conditions as the answer to current security concerns, a more profound look finds that virtualization adds the product to the organization framework, which might impact security whenever inadequately constructed and conveyed [19–21]. In previous studies, cloud networks have some issues regarding security, latency delay, etc. These issues, along with some other issues, linger on in this research theme.

2.2. Literature Review

An analysis in light of the big data and cloud computing innovation is suggested for meeting the correspondence requirements of the photoelectric hybrid network design. The main focus of this study is the examination of big data and cloud computing innovation. To conduct this, it investigates specialized attributes, makes use of topological optical connections, sidesteps data structures and other techniques, and, in the end, develops an exploration methodology for big data and cloud computing. The results of the exploratory work indicate that loads of the optical connections are 60, 50, and 20, respectively. Hub B reaches the paths of the six objective nodes at the point where loads of optical connections start to become more modest. The photoelectric hybrid network structure makes it possible to communicate in more ways by using optical connections and controlling the scope of nearby connections. The challenges with the photoelectric hybrid network structure in communication can be examined in the context of cloud computing and big data innovation [22–24].

The overall reception of cloud server farms (CDCs) has brought about an omnipresent interest in facilitating application administrations on the cloud. Additionally, modern information-driven enterprises have noticed a rapid increase in the asset requirements of current apps. Due to this, more cloud servers have been made available, which has increased energy consumption and, as a result, raised supportability issues. The customary heuristics and the support of learning-based calculations for energy-efficient cloud assets help CEOs to partially overcome the challenges associated with adaptability and flexibility. The existing work frequently neglects to catch conditions across the warm attributes of hosts, asset utilization of undertakings, and the relating planning choices. This prompts the unfortunate adaptability and an expansion in the figure asset necessities, especially in conditions with non-fixed asset requests. To address these constraints, the man-made brainpower (AI)-based, all-encompassing, asset-the-board strategy for maintainable cloud computing called HUNTER has been proposed by S. Tuli and his team. The suggested model considers three important models-energy, warmth, and cooling-when planning the goal of increasing the energy efficiency in server farms as a multi-objective planning issue. As a replacement model, the tracker uses a gated graph convolutional network to generate the best planning decisions and approximate the Quality of Service (QoS) for a framework state. The probes reproduced, and actual cloud conditions utilizing the CloudSim tool stash and the COSCO system, show that HUNTER outflanks cutting-edge baselines as far as the energy utilization, SLA infringement, planning time, cost, and temperature by up to 12, 35, 43, 54, and 3 percent individually [25].

To modify the load in the context of cloud computing, this study presents a crossover meta-heuristic-based asset designation system called RAFL. The objective is to proactively reduce the heap lopsidedness among dynamic actual machines and in their asset limit-thinking (e.g., CPU and RAM). This avoids overloading or underloading dynamic physical machines and makes fair use of their asset limit consideration. In the proposed system, a phasor molecule swarm improvement and dragonfly calculation-based, half-breed stream-lining calculation named PPSO-DA are utilized to create an ideal asset portion plan for adjusting the heap. Recreation tests are performed utilizing the CloudSim test system to quantify the measurements of burden unevenness across dynamic actual machines and among their thought about asset limits. Results show that the proposed PPSO-DA calculation beats the phasor molecule swarm streamlining, dragonfly calculation, exhaustive learning molecule swarm enhancement, memory-based half-breed dragonfly calculation, sine cosine calculation, and elephant grouping advancement in tracking down an ideal asset assignment for adjusting the heap. The measurable examination and benchmark testing likewise approve the general predominance of PPSO-DA [26].

The quick development of the cloud computing climate, with numerous clients going from individual clients to huge corporate or business houses, has turned into a test for cloud associations to deal with the gigantic volume of information and different assets in the cloud. The wasteful administration of assets can corrupt the exhibition of cloud computing. Consequently, assets should be uniformly distributed to various partners without compromising the association's benefit as well as the clients' fulfillment. Because the necessary assets are not available for free on the board, a client's solicitation cannot be held indefinitely. To address these concerns, the RATS-HM technique, which combines asset distribution security with expert task planning for cloud computing, is proposed [27]. The following are the suggested RATS-HM procedures: The make range time is first limited, and the throughput is increased by a better feline multitude streamlining, calculation-based, short scheduler for task booking (ICS-TS). Second, a gathering improvement based on profound brain organization (GO-DNN) for the effective asset portion utilizing different plan requirements incorporates the data transfer capacity and asset load. Third, a lightweight confirmation conspires, i.e., SUPREME is proposed for information encryption to give security to the information capacity. At last, the proposed RATS-HM procedure is reproduced with an alternate recreation arrangement and the outcomes are contrasted with condition of-workmanship methods to demonstrate the viability. The outcomes regarding asset usage, energy utilization, reaction time, and so forth, show that the proposed strategy is better than the current one [28].

The review is additionally fundamental because of the dynamic assignment of assets lately; organizations have utilized the cloud computing worldview to run different computing and stockpiling responsibilities. The cloud offers a quicker and more beneficial administration. In any case, the issue of asset designation is difficult for cloud suppliers. The extreme utilization of assets has raised the requirements for better administration of them. What is more, the assets required may surpass those accessible in the cloud as the interests and limits differ after some time. Hence, dynamic asset designation procedures permit the utilization of the accessible limit all the more productively. The researchers give a functional dynamic resource allocation (DRA) to concentrate on a cloud computing climate. It represents the unique part of the cloud computing climate and how it is tended in the writing. Additionally, it gives the scientific categorizations of approaches, planning types, and streamlining measurements. Their study assists researchers in understanding the powerful part of asset distribution in the cloud, and consequently further developing its presentation [29].

Cloud computing is a plan of action where clients and suppliers are contending to sell and purchase administrations. Because of the assorted programming applications and the dynamic cloud market, clients' prerequisites are truly evolving. Dynamic closeout-based asset portion models, or, to be specific, the combinatorial, twofold sale asset assignment (CDARA) model, stood out to the analysts. In the existing work, administrations are distributed to clients in light of less of a cost, which might prompt assistance level arrangement (SLA) infringements and increment the clients' disappointment in the cloud commercial center. The choice of the single property cost might make certified and questionable suppliers take issue, as the suppliers will attempt to offer bad quality types of assistance. To settle these issues, they proposed a versatile, market-situated combinatorial twofold sale asset portion (AMO-CDARA) model that designates administrations to clients given different boundaries, such as less value, QoS, and supplier positioning. In the proposed model, offers are received by a salesperson from different suppliers and clients. The broker works out the client's and supplier's bid densities and distributes administrations to the most proficient clients from the most productive suppliers, as per their expected solicitation. After the fruitful running of undertakings, the barker requests criticism of the pre-owned administrations from clients and computes the last positioning of suppliers for future sales. The reproduction results show that the proposed model ensured SLA infringement up to almost 100% because of two times the discipline punishment and positioning of suppliers. The surefire QoS clients/representatives will give additional installment to that, and the additional installment will be from 1% to 10% of the absolute value, as per the last cost.

costs by adding QoS costs [30]. These days, cloud computing is pulled into wider consideration as it can convey IT administrations and assets on an interesting premise over the internet. Load balancing is a vital test in cloud computing. Because of the perplexing structure of cloud computing, it is troublesome and expensive to assess the way of behaving of load-balancing procedures on various cloud assets in light of the QoS boundaries in a genuine cloud climate. Subsequently, to beat what was going on, cloud computing devices are utilized for reproduction to test the way of behaving of load balancing strategies in the cloud framework under various circumstances in a rehashed way by changing different boundaries. Today, a variety of tools are available, including CloudSim, WorkflowSim, CloudSim4DWf, GreenCloud, and CloudAnalyst. Each tool has a different trademark, engineering, boundary, and outcome evaluation. Therefore, it is crucial to choose a capable, load-balancing device that complies with the QoS requirements. The researchers focus on important cloud load-balancing tools and provide a comparative analysis of the important, recently suggested, and existing load-balancing tools. In addition, they will look at load-balancing plans divided into three categories [31].

Moreover, we tackled the bidder drop issue by up to 10% by expanding administration

Giving wise task execution and responsible resource usage, however, is essential. Due to the nature of the administration, the executives' assets, task planning, burden adjustment, and board duty, a few solutions have been considered in the writing to improve the execution and asset utilization. The ability of server farms to avoid overloading or underloading virtual machines, thanks to the load adjustment in the cloud, is in and of itself a test for cloud computing [32]. Engineers and scientists must, therefore, build and implement an appropriate load balancer for equal and distributed cloud situations. This study presents a cutting-edge survey of the issues and difficulties related to the existing burden-adjusting methods for specialists to foster more Efficient methods [33].

2.3. Expected Contributions

The following are the expected contributions of said research:

- 1. The expected contribution of this research is to analyze how existing cloud networks can be improved by optimizing their adaptability.
- 2. During the said research, certain cloud networks were modeled through deduced results by topological invariants. These results will be graphically developed over the solution of networks by freshly prepared topological indices.
- 3. Existing networks will be studied for topological perspectives, and the QSPR and QSAR models will be developed and analyzed.
- 4. The relation between the lower bounds and upper bounds of the network or graph will be discovered. Further, these relationships will be defined through optimization.

- 5. Cloud networks and other certain computer networks are solved and evaluated with the help of topological invariants.
- 6. The outcomes of the research will provide design guidelines for advanced cloud networks and their applications in interconnection networks.

2.4. Scope

The research work concentrates on the topological properties and solutions of cloud networks for interconnection networks, power generation networks, chemical compounds, robotics, etc., through topological invariants. The topological properties include lower bounds, upper bounds, and prediction qualities of deduced mathematical results. As cloud networks are modeled through these solved results, engineers and manufacturers foresee concerning products before manufacturing or developing them.

3. Research Methodology

This systematic study will take an existing cloud computing network and associate it with a graph and solve the topology of the graph with the help of the KBSO indices, QCI, Dharwad index, and their reduced forms. The concerning results in the form of formulas will compare with existing results. These deduced results will be used for the modeling and development of a best-fit network having the best feasible characteristics. This model is very concerning as it solved the topology of cloud computing networks in numeric and graphical form and gives accurate results. After the analysis, a simulation tool maple is used for the verification and validation of the results [34]. A ' $C_{r,s}$ ' is an existing cloud network that is under investigation, and the study finds vertices and edges of the given network, defines certain modern topological invariants KBSO, CQI, Dharwad, and their reduced forms, then converts the cloud network into the graph after mapping, afterward solving the mapped network graph through the given topological invariants, as mentioned in Figure 1. In the end, validation and optimization have been performed by ML-based mathematical tools. Mapping and the predicted graphical results based on proven mathematical results are portrayed with the help of the ML-based mathematical tool Maple. The $C_{r,s}$ represents a graph of a cloud network, where 'C' is the name of the cloud graph, 'r' and 's' are their parameters which represent the r_{th} number of big clouds and the sth number of small clouds. As Figure 2 shows with one cloud network with one big cloud consisting of s number of times small clouds, the study generates results and equations for any number of small or large clouds.

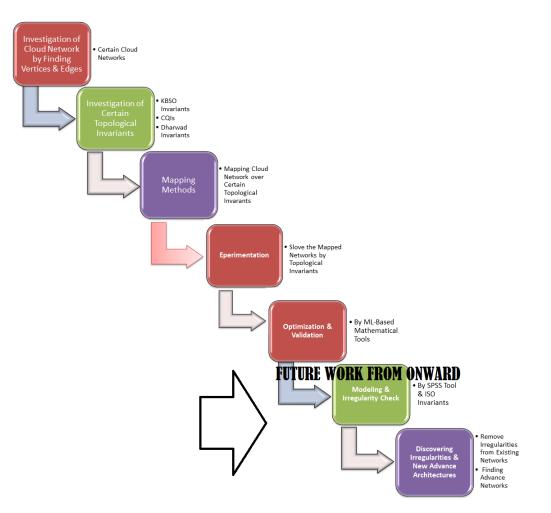


Figure 1. Methodology flow diagram.

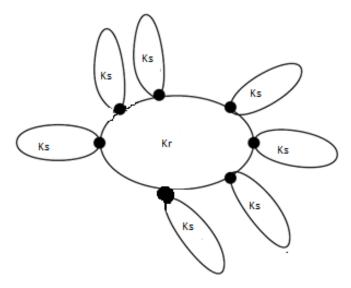


Figure 2. Cloud network graph $C_{r,s}$.

4. Experimentation and Results

The ' $C_{r,s}$ ' cloud network was mapped, converted into the graph, solved, validated, and the results optimized according to the steps and methodology discussed in the methodology section, with the help of topological invariants mentioned in Equations (1)–(6).

$$KBSO(G) = \sum_{ue} \sqrt{d_u^2 + d_\rho^2}$$
(1)

$$KBSO_{r\rho} \underline{d}_{-}(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + (d_{\rho} - 1)^2}$$
(2)

Equations (1) and (2) show the KBSO index and its reduced form, which will be used for the solution of the cloud computing network. In the above equations, d_u and d_e are showing edge partitions, where 'u' and 'e' are the vertices of the graph $C_{r,s}$ under discussion.

$$CQI(G) = \sum_{uv \in E(G)} \frac{\sqrt{2(d_G(u)^2 + d_G(v)^2)}}{d_G(u) + d_G(v)}$$
(3)

$$QCI(G) = \sum_{uv \in E(G)} \cdot \frac{(d_G(u) + d_G(v))}{\sqrt{2(d_G(u)^2 + d_G(v)^2)}}$$
(4)

Equations (3) and (4) show the CQI and QCI, which will be used for the solution of the cloud computing network. In the above equations, d_u and d_v are showing the edge partitions, where the vertices of the graph $C_{r,s}$ are the 'u' and 'v' under discussion.

$$D(G) = \sum_{ue} \sqrt{du^3 + dv^3}$$
(5)

The cloud computing network is converted into graphical form first and then associated with the graph. The graph is solved through the KBSO index, CQI, Dharwad index, and their other forms.

RD (G) =
$$\sum_{ue} \sqrt{(du-1)^3 + (dv-1)^3}$$
 (6)

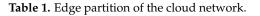
Equations (5) and (6) show the Dharwad index and its reduced form will also be used for the solution of the cloud computing network. In the above equations, d_u and d_v are showing the edge partitions where the vertices of the graph $C_{r,s}$ are the 'u' and 'v' under discussion.

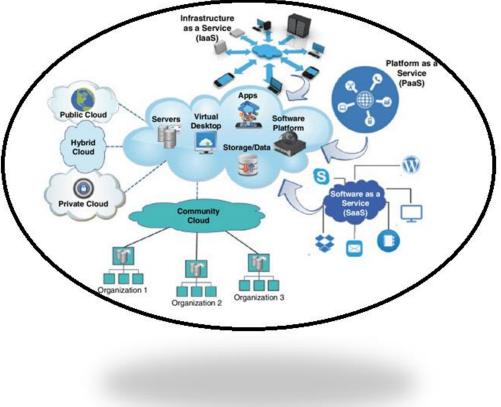
Table 1 describes the edge partitions of the graph of the cloud computing network $C_{r,s}$, as given in Figure 1. Where ' ε ' represents the edge, the d_u and d_v are showing the edge partitions, and 'u' and 'v' are the vertices of the graph $C_{r,s}$ under discussion. Recurrence means the number of edges attached to a particular vertex, also called frequency. By dividing a graph's collection of nodes into mutually exclusive groups, a cloud graph partition reduces the network to a smaller graph. The partitioned graph will have edges made up of the original graph edges that cross over into the groups. The partitioned graph may be more useful for analysis and problem-solving than the original if the total number of edges is lower than in the case of the original graph.

4.1. Main Results of Cloud Computing Graph

The graph $C_{r,s}$ is obtained from the 'Kr' and 'r' duplicates of 'Ks' by distinguishing each vertex of 'Kr' with a vertex of one 'Ks'. Here, we figure out the KBSO, CQI, and Dharwad invariants of the graph $C_{r,s}$, and infer their decreased and other topological structures from it. Graph $C_{r,s}$ is portrayed in graph-wise vision in Figure 2, and cloud-wise vision and graph-wise vision in Figure 3 [35–40].

Е	ε (du, dv)	De	ε(du, de)	Recurrence
ε1	$\epsilon(s - 1, s - 1)$	2s - 4	$\varepsilon(s-1, 2s-4)$	(r(s-1)(s-2))/2
ε2	$\varepsilon(s-1, r+s-2)$	r + 2s - 5	$\epsilon(s - 1, r + 2s - 5)$	r(s – 1)
ε3	$\varepsilon(r+s-2, r+s-2)$	2r + 2s - 6	$\varepsilon(r+s-2,2r+2s-6)$	R(s-1)/2
de = du + dv - 2.				





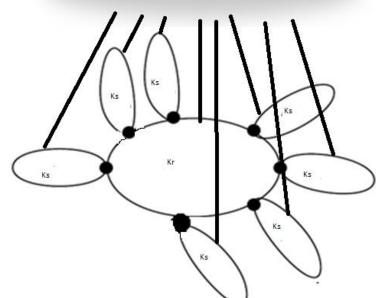


Figure 3. A cloud graph is extracted from a piece of a cloud computing network.

Figure 2 shows the cloud graph but Figure 3 shows the extraction of the cloud graph from one of the cloud computing networks.

Figure 3 shows a cloud computing network in which different clouds are involved, from which the extraction of the cloud graph is carried out. A bigger centralized cloud is represented by ' K_r ' and a smaller cloud attached to a bigger one is represented by ' K_s '.

4.1.1. Cloud Computing Graph

Let $C_{r,s} = G$ be a graph of the cloud computing network with edge partitions mentioned in Table 1.

4.1.2. Theorem 1

Let $C_{r,s} = G$ be a graph of the cloud computing network, then the KBSO and KBSO_{red} indices are

KBSO (G) =
$$\frac{1}{2}\sqrt{2}\sqrt{(s-1)^2 + (2s-4)^2}r(s-1)(s-2) + \sqrt{(s-1)^2 + (r+2s-5)^2}$$

 $r(s-1) + \frac{1}{2}\sqrt{(r+s-2)^2 + (2r+2s-6)^2}r(s-1)$
(7)

KBSO_{red} (G) =
$$\frac{1}{2}\sqrt{(s-2)^2 + (2s-5)^2} r(s-1)(s-2) + \sqrt{(s-2)^2 + (r+2s-6)^2}$$

r(s-1) + $\sqrt{(r+s-2)^2(2r+2s-7)^2} r(s-1)$ (8)

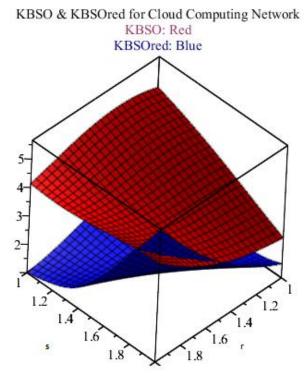
Equations (7) and (8) represent the proven results of the graph of $C_{r,s}$ of the cloud computing network mentioned in Figure 2. These are the proven results used for modeling the means scalability and reducing latency because these modeling results have the best characteristics for the development of enhanced cloud networks, with the help of the KBSO invariants.

4.1.3. Investigation of C_{r,s} of the Cloud Computing Network Graph by KBSO Indices

Proof. KBSO (G) =
$$\sum_{ue} \sqrt{d_u^2 + d_\rho^2}$$

KBSO (G) = $\sqrt{(s-1)^2 + (2s-4)^2 \frac{(r(s-1)(s-2))}{2}} + \sqrt{(s-1)^2 + (r+2s-5)^2}$
(r(s-1)) + $\sqrt{(r+s-2)^2 + (2r+2s-6)^2 \frac{(r(s-1))}{2}}$
KBSO (G) = $\frac{1}{2}\sqrt{2}\sqrt{(s-1)^2 + (2s-4)^2} r(s-1)(s-2) + \sqrt{(s-1)^2 + (r+2s-5)^2} r(s-1) + \frac{1}{2}\sqrt{(r+s-2)^2 + (2r+2s-6)^2} r(s-1)$
KBSO red (G) = $\sum_{ue} \sqrt{(d_u-1)^2 + (d_\rho-1)^2}$
KBSO_{red} (G) = $\sqrt{(s-1-1)^2 + (2s-4-1)^2} \left(\frac{(r(s-1)(s-2))}{2}\right) + 5\sqrt{(s-1-1)^2 + (r+2s-5-1)^2} (r(s-1)) + 4\sqrt{(r+s-2-1)^2 + (2r+2s-6-1)^2 \frac{(r(s-1))}{2}}$
KBSO_{red} (G) = $\frac{1}{2}\sqrt{(s-2)^2 + (2s-5)^2} r(s-1)(s-2) + \sqrt{(s-2)^2 + (r+2s-6)^2} r(s-1) + \sqrt{(r+s-2)^2 + (2r+2s-7)^2} r(s-1).$

Figure 4 shows the results (Equations (7) and (8)) of the KBSO and KBSO_{red} invariants in the red and blue colors, respectively, in the 3D version, which shows the upper and lower bounds of a graph of the cloud network. These are not fully separated and do not show sharp bounds. There are also some irregularities present in the graph of the cloud



network because the lower and upper bounds are not quite separate and straight lines. These irregularities can be found with the help of irregularity indices in future work.

Figure 4. Results of KBSO and $\ensuremath{\mathsf{KBSO}_{\mathrm{red}}}$ invariants for cloud computing network.

4.1.4. Theorem 2

Let C_{r.s} = G be a graph of the cloud computing network, then the CQI and QCI indices are

$$CQI(G) = \frac{\sqrt{((s-1)^2)}r(s-1)(s-2)}{2s-2} + \frac{\sqrt{2((s-1)^2+2(r+s-2)^2)}}{2s-3+r}r(s-1) + \frac{\sqrt{((r+s-2)^2)}r(s-1)}{2r+2s-4}$$
(9)

$$QCI(G) = \frac{\frac{1}{4} (2s-2)r(s-1)(s-2)}{\sqrt{(s-1)^2}} + \frac{(2s-3+r)r(s-1)}{\sqrt{2(s-1)^2+2(r+s-2)^2}} + \frac{1}{4} \frac{(2r+2s-4)r(s-1)}{\sqrt{(r+s-2)^2}}$$
(10)

Equations (9) and (10) represent the proven results of the graph of the cloud computing network mentioned in Figure 2. These are the results that have been validated and are utilized for modeling the methods that reduce latency. The results are validated because Theorem 2 is proven, and the graphical results also show quite sharp upper and lower bounds of the graph of the cloud network by the CQI and QCI. So, these modeling outcomes exhibit the best qualities for the creation of improved cloud networks using the CQI invariants.

$$\begin{split} CQI(G) &= \sum_{uv \in E(G)} \frac{\sqrt{2 \Big(d_G(u)^2 + d_G(v)^2 \Big)}}{d_G(u) + d_G(v)} \\ CQI(G) &= \frac{\sqrt{2 \big((s-1)^2 + (s-1)^2 \big)}}{s^{-1 + s - 1}} \frac{(r(s-1)(s-2))}{2}}{s^{-1 + r + s - 2}} \\ &+ \frac{\sqrt{2 \big((s-1)^2 + (r + s - 2)^2 \big)}}{s^{-1 + r + s - 2}} r(s-1) \\ &+ \frac{\sqrt{2 \big((r + s - 2)^2 + (r + s - 2)^2 \big)}}{r + s - 2 + r + s - 2} \frac{(r(s-1))}{2} \end{split}$$

$$\begin{split} \text{CQI}(\text{G}) &= \frac{\sqrt{\left((\text{s}-1)^2\right)} \, \text{r}(\text{s}-1)(\text{s}-2)}{2\text{s}-2} + \frac{\sqrt{2\left((\text{s}-1)^2+2(\text{r}+\text{s}-2)^2\right)}}{2\text{s}-3+\text{r}} \text{r}(\text{s}-1)}{2\text{s}-3+\text{r}} \\ &+ \frac{\sqrt{\left((\text{r}+\text{s}-2)^2\right)} \, \text{r}(\text{s}-1)}{2\text{r}+2\text{s}-4} \\ \text{QCI}\left(\text{G}\right) &= \sum_{\text{uv}\in \text{E}(\text{G})} \frac{\left(\text{d}_{\text{G}}(\textbf{u}) + \text{d}_{\text{G}}(\textbf{v})\right)}{\sqrt{2\left(\text{d}_{\text{G}}(\textbf{u})^2 + \text{d}_{\text{G}}(\textbf{v})^2\right)}} \\ \text{QCI}(\text{G}) &= \frac{\frac{\text{s}-1+\text{s}-1}{\sqrt{2\left((\text{s}-1)^2+(\text{s}-1)^2\right)}} \frac{\left(\text{r}(\text{s}-1)(\text{s}-2)\right)}{2}}{\sqrt{2\left((\text{s}-1)^2+(\text{r}+\text{s}-2)^2\right)}} \text{r}(\text{s}-1) \\ &+ \frac{\frac{\text{s}-1+\text{r}+\text{s}-2}{\sqrt{2\left((\text{s}-1)^2+(\text{r}+\text{s}-2)^2\right)}} \text{r}(\text{s}-1)}{\sqrt{2\left((\text{r}+\text{s}-2)^2+(\text{r}+\text{s}-2)^2\right)}} \\ \text{QCI}(\text{G}) &= \frac{\frac{1}{4} \left(2\text{s}-2\right)\text{r}(\text{s}-1)(\text{s}-2)}{\sqrt{(\text{s}-1)^2}} + \frac{\left(2\text{s}-3+\text{r}\right)\text{r}(\text{s}-1)}{\sqrt{2(\text{s}-1)^2+2(\text{r}+\text{s}-2)^2}} \\ &+ \frac{1}{4} \frac{\left(2\text{r}+2\text{s}-4\right)\text{r}(\text{s}-1)}{\sqrt{(\text{r}+\text{s}-2)^2}} \end{split}$$

Figure 5 shows the results with clear upper and lower bounds Equations (9) and (10) of the CQI and QCI in red and blue colors, respectively, in the 3D version. As seen in Figure 5, the separation and straight line started from one to three for the 'r' and 's' parameters. This would be used accordingly during the construction of cloud computing networks.

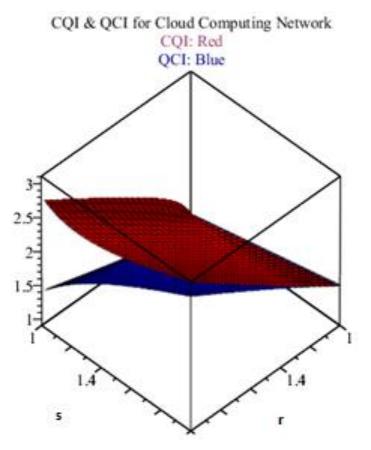


Figure 5. CQI and QCI for cloud network.

4.1.5. Theorem 3

Let $C_{r,s} = G$ be a graph of the cloud computing network, then, the *Dharwad* and *Dharwad*_{red} indices are

$$D(G) = \frac{1}{2}\sqrt{2}\sqrt{(s-1)^3} r(s-1)(s-2) + \sqrt{(s-1)^3 + (r+s-2)^3} r(s-1) + \frac{1}{2}\sqrt{2}\sqrt{(r+s-2)^3} r(s-1)$$
(11)

RD (G) =
$$\frac{1}{2}\sqrt{2}\sqrt{(s-2)^3}r(s-1)(s-2) + \sqrt{(s-2)^3 + (r+s-2)^3}r(s-1) + \frac{1}{2}\sqrt{2}\sqrt{(r+s-2)^3}r(s-1)$$
 (12)

Equations (11) and (12) represent the proven results of the Dharwad invariants of the graph of the cloud computing network mentioned in Figure 2.

4.1.6. Investigation of Cloud Computing Graph by Dharwad Indices

$$\begin{aligned} & \operatorname{Proof.} D(G) = \sum_{ue} \sqrt{du^3 + dv^3} \\ & D(G) = \sqrt{(s-1)^3 + (s-1)^3} \frac{(r(s-1)(s-2))}{2} + \sqrt{(s-1)^3 + (r+s-2)^3} r(s-1) + \\ & \sqrt{(r+s-2)^3 + (r+s-2)^3} \frac{(r(s-1))}{2} \\ & D(G) = \frac{1}{2} \sqrt{2} \sqrt{(s-1)^3} r(s-1)(s-2) + \sqrt{(s-1)^3 + (r+s-2)^3} r(s-1) + \\ & \frac{1}{2} \sqrt{2} \sqrt{(r+s-2)^3} r(s-1) \\ & \operatorname{RD} (G) = \sum_{ue} \sqrt{(du-1)^3 + (dv-1)^3} \\ & \operatorname{RD} (G) = \sqrt{((s-1)-1)^3 + ((s-1)-1)^3} \frac{(r(s-1)(s-2))}{2} + \\ & \sqrt{((s-1)-1)^3 + ((r+s-2)-1)^3} \frac{(r(s-1))(s-2)}{2} + \\ & \sqrt{((s-1)-1)^3 + ((r+s-2)-1)^3} \frac{(r(s-1))}{2} \\ & \operatorname{RD} (G) = \frac{1}{2} \sqrt{2} \sqrt{(s-2)^3} r(s-1)(s-2) + \sqrt{(s-2)^3 + (r+s-2)^3} r(s-1) + \\ & \frac{1}{2} \sqrt{2} \sqrt{(r+s-2)^3} r(s-1). \end{aligned}$$

Figure 6 shows the results Equations (9) and (10) of the Dharwad and Dharwad_{red} in the red and blue colors, respectively, in the 3D version, which shows the sharp upper and lowers bounds of a cloud network. It is quite a straight line for high values of parameters 'r' and 's', which means that a large number of clouds can be attached for the best characteristics.

4.2. Discussion

According to the aforementioned findings, topological invariants, such as K-Banhatti Sombor invariants, Contraharmonic-Quadratic invariants, Dharwad invariants, and their reduced forms, allow us to gather information about cloud networks in the form of algebraic structures and provide us with a mathematical technique to infer the hidden properties of various structures, such as the particular networks. The degree-based topological indices and the distance-based topological indices are the two primary classes of topological indices that clash, but in the present study, the networks were solved using the degreebased topological indices, and the best results are displayed in the graphs. Through the application of topological invariants, the research focuses on the topological characteristics and solutions of cloud networks for interconnection networks used on the internet, powergenerating networks, chemical compounds, robotics, etc. The lower bounds, higher bounds, and the ability to predict the outcomes of the derived mathematical operations are among the topological traits. Although these solved solutions serve to model interconnection networks, engineers and product developers anticipate issues with items before creating or developing them. The efficiency, load balancing, and latency delay are also dependent on the topology of the network [41,42]. This is the reason why the study solved the topology of a cloud network. The deduced mathematical results are used for the construction of the new, best characteristic cloud networks, including efficiency, load balancing, and less latency delay.

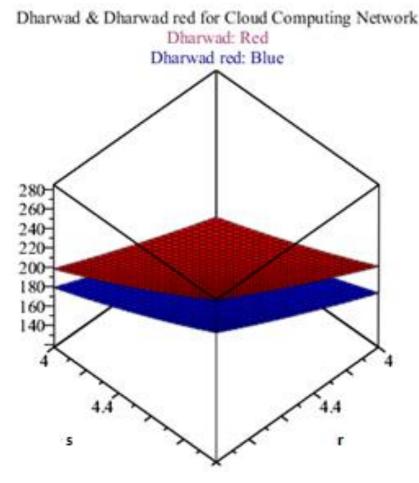


Figure 6. Dharwad and Dharwad_{red} invariants for cloud network.

In answer to the expected contribution, the study provides the mathematical results for the modeling of the cloud networks, developed the model in the Methodology section, and provided the graphical predicted results and the lower and upper bounds of the cloud graph. Equations (7)–(12) are the mathematical solutions of the cloud networks, with the help of topological invariants and graphical and mathematical results, providing the guidelines for the network engineers and architectural engineers during the modeling of the cloud network and all other networks in which the cloud network is used, and also during actual constructions of these networks.

5. Conclusions

TIs have lots of uses and implementations in many fields, including computer science, chemistry, biology, informatics, arithmetic, material sciences, and many more, and especially in cloud networks and other network architectures. However, the application with the utmost significant is in the non-exact QSPR and QSAR. TIs are associated with the structure of cloud networks used in cloud computing. The study discusses the KBSO invariants, CQIs, and Dharwad invariants and their reduced forms, which are freshly presented and

have numerous prediction qualities for different variants of cloud computing networks for improvements in the context of scalability, efficiency, higher throughput, best-fit topology, and latency in context to their topology. The study achieves improvements in all the mentioned characteristics through the best-fit topology of the cloud network. For this purpose, the study solves the existing network by converting it into a graph through topological invariants and gets the solution in mathematical and graphical form. The graphical results show the irregularities in cloud networks as mentioned by the KBSO, CQI, and its reduced forms. Equations (7)–(12) are the mathematical solutions of the cloud networks, with the help of topological invariants, and they provide modeling tools and instructions for network engineers. The study established the model and provided the anticipated results and lower and upper bounds graphically.

6. Future Work

Future work is to deal with these irregularities. Mathematically deduced results from Equations (3)–(5) will be used for the modeling and improvements of cloud networks used in cloud computing, as well as in different chemical structure developments with the best characteristics.

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