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Monitoring, Evaluation, and Improvement Model for Process Precision and Accuracy

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Abstract: Process Capability Indices (PCIs) are devices widely used in the industry to evaluate process quality. The commonly used process capability indices all contain accuracy indices and precision indices. As the accuracy index is closer to zero, the process accuracy is higher. The precision index mainly represents the extent of process variation. As the value is smaller, the process variation is smaller, that is, the precision is higher. In fact, process capability indices are the functions of accuracy indices and precision indices. Obviously, as long as accuracy indices and precision indices are controlled, the process capability indices can be controlled as well. Therefore, this study first derived accuracy and precision control charts to observe not only process accuracy but also process precision. Then, this study adopted in-control data to acquire a 100 $(1-\alpha)$% confidence region of an accuracy index and a precision index, with which statistical tests were performed. Subsequently, according to the definition of the six sigma quality level, both indices were examined. Furthermore, based on the testing results, suggestions for process improvement were proposed, including correcting the direction of process deviation and deciding whether to reduce process variation. Finally, this study demonstrated the applicability of the proposed model using an empirical example.

Keywords: process capability indices; accuracy index; precision index; six sigma quality level; accuracy and precision control charts

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1. Introduction

Process Capability Indices (PCIs) are measurements frequently adopted by the industry for process quality evaluation [1–3]. They are not just communication tools between sales and customers; for process engineers, they are useful means of evaluating, analyzing, and improving processes [4–6]. Industries that often employ process capability indices include various machine tools and machining industries, semiconductor manufacturing processes, and packaging processes [7–10]. The process capability index C_{pk} proposed by Kane [11] and the index C_{pm} suggested by Chan et al. [12] are two process quality evaluation tools which are most commonly adopted in the industry. According to numerous studies, as the process capability index is relatively large, it is guaranteed that the process yield is high while the process loss is low. The process capability index incorporates two important tools for evaluating the pros and cons of a process—process yield and process loss [13]. Let the random variable (RV) X follow $N(\mu,\sigma^2)$, which represents a normal distribution with process mean μ and variance σ^2. Then, indices C_{pk} and C_{pm} are denoted below:

$$C_{pk}=\frac{d-|\mu-T|}{3\sigma} \quad (1)$$

$$C_{pm} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} \quad (2)$$

where d is the half-length from the lower specification limit (LSL) to the upper specification limit (USL), that is $d = (USL - LSL)/2$; $T = (USL + LSL)/2$ refers to the target value. In fact, the denominator of process capability index C_{pm} means the expected value of the Taguchi loss function. Clearly, process capability has two important factors: (1) as the process mean μ gets closer to target value T, the process accuracy gets higher; (2) as the process standard deviation σ gets smaller, the process precision gets better [14]. Boosting the process accuracy and precision will cut down process loss as well as level up process capability. In addition, the target values and tolerances of various processes are all different. To facilitate the process evaluation, the tolerance can be standardized by variable transformation. Let RV $Y = (X - T)/d$. When $X = LSL$, then $Y = -1$. When $X = T$, then $Y = 0$. When $X = USL$, then $Y = 1$. Since different quality characteristics have different specification limits, this variable transformation can help all quality characteristics convert their different specification limits from (LSL, T, USL) to $(-1, 0, 1)$. In fact, δ, which stands for the mean of RV Y, is regarded as an accuracy index; γ, which refers to the standard deviation of RV Y, is viewed as a precision index. These two indices are written as

$$\text{accuracy index}: \ \delta = \frac{\mu - T}{d} \quad (3)$$

$$\text{precision index}: \ \gamma = \frac{\sigma}{d} \quad (4)$$

Based on the aforementioned, $C_{pk} = (1 - |\delta|)/3\gamma$ and $C_{pm} = 1/3\sqrt{\delta^2 + \gamma^2}$ stand for the functions of δ and γ. Accuracy index δ is mainly applied to measure the degree to which the process mean μ deviates from the process target value T. When the value is closer to 0, then the process is more accurate. The greater the positive number of accuracy index δ is, the more the process is shifted to the right. Conversely, as the negative number of accuracy index δ is greater, the process is shifted to the left more. As to the precision index γ, it mainly represents the extent of process variation. As the index is smaller, the process variation is smaller; that is to say, the process is more precise. Apart from process capability indices, the six sigma method, initiated by Motorola in 1986 [15–17], is also applied by this study. The six sigma method is also a tool prevalently employed by the industry to assess and enhance process quality [18–20]. According to the research of Chen and Chang [14], when the standard deviation of the process is $\sigma = d/k$ and the process mean shifted from the target value falls within 1.5σ, it means that the process has reached the k-sigma quality level. When the quality level of the process reaches six sigma, then

$$\gamma = \frac{\sigma}{k\sigma} = \frac{1}{k} \quad (5)$$

$$|\delta| = \frac{|\mu - T|}{d} \leq \frac{1.5\sigma}{k\sigma} = \frac{1.5}{k} \quad (6)$$

Subsequently, the statistical testing method is adopted to evaluate whether the process standard deviation is $1/k$ and the process mean shifted from the target value falls within 1.5σ (i.e., $\gamma = 1/k$ and $|\delta| \leq 1.5/k$). When $\gamma = 1/k$ and $|\delta| \leq 1.5/k$, it means that the process meets the k-sigma quality level. Numerous researchers have addressed that the evaluation of process capability is usually performed when the statistical process is in control [7]. Hence, in this paper, the $\delta - \gamma$ control charts are derived to control the process accuracy and precision. Also, in-control data are used to obtain a $100\,(1 - \alpha)\%$ confidence region of (δ, γ), which is employed to conduct statistical tests for accuracy index δ and precision index γ. The statistical testing results of these two indices can be concluded as follows: if the process cannot reach the six-sigma quality level, then the deviation direction of the process can be seen from the testing result of accuracy index δ, which can be provided to the industry for improvement reference; meanwhile, according to the testing result of precision index γ, we can decide whether to reduce the process variation. It is clear that this study aims to monitor the process mean and try to make it fall within the target value using two important parameters of the normal process, namely accuracy index δ and precision index γ. In the meantime, the process variation can be supervised so that the process can meet the requirement of the k-sigma quality level. In order to decrease the risk of misjudgment incurred by sampling errors, a statistical testing model of these two indices is developed to individually evaluate whether the process accuracy and process precision can reach the required level, as well as decide whether to make improvements at the same time, in order to raise the process quality level of products.

Concerning the rest of the paper, it is arranged in the following sections. In Section 2, expected values and standard deviations of the estimators for the accuracy index and the precision index are first derived from normal approximation rules. Then, the control limits between the accuracy index and the precision index are established based on the principles of three-sigma control charts, so as to provide the basis for the industry to monitor the process. In Section 3, the in-control data are retrieved to gain a 100 (1 − α)% confidence region of (δ, γ), and then statistical tests are conducted with this confidence region for accuracy index δ and precision index γ. The statistical testing results of these two indices are regarded as the basis of determining whether to improve the process. In Section 4, based on the testing results of accuracy index δ and precision index γ, suggestions about process improvement are made, including how to correct process deviation directions and whether to reduce process variation. In Section 5, an empirical example is presented to demonstrate the application of the model proposed by this study. In Section 6, conclusions are made.

2. Monitoring Process Precision and Accuracy

The evaluation of process capability must take place in a stable statistical process. This study first constructs $\delta - \gamma$ control charts to oversee process accuracy and precision. Next, this study derives a 100 (1 − α)% confidence region of (δ, γ) from the in-control data. Let $\left(Y_{i,1}, \ldots, Y_{1,j}, \ldots, Y_{1,n}\right)$ be the ith subsample, i = 1, 2, …, m. According to the concept of Montgomery [21], to estimate the unknown parameters $-\delta$ and γ, at least 20 to 25 subsamples should be taken. Then, the jth subsample and its observation values are shown below:

$$\left(X_{j,1}, \ldots, X_{j,i}, \ldots, X_{j,n}\right) = \left(x_{j,1}, \ldots, x_{j,i}, \ldots, x_{j,n}\right) \tag{7}$$

where $j = 1, 2, \ldots, m$. Since $Y_{j,i} = \left(X_{j,i} - T\right)/d$, then the jth subsample and its observation values processed by the variable transformation are written as follows:

$$\left(Y_{j,1}, \ldots, Y_{j,i}, \ldots, Y_{j,n}\right) = \left(y_{j,1}, \ldots, y_{j,i}, \ldots, y_{j,n}\right) \tag{8}$$

Accordingly, for the jth subsample, $j = 1, 2, \ldots, m$, the subsample mean and the subsample variance are separately represented below:

$$\hat{\delta}_j = \overline{Y}_j = \frac{1}{n}\sum_{i=1}^{n} Y_{j,i} \tag{9}$$

$$\hat{\gamma}_j = S_j = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_{j,i} - \overline{Y}_j\right)} \tag{10}$$

As normality is assumed, the jth subsample mean, $\hat{\delta}_j$, is denoted as a normal distribution with mean δ and standard deviation $\gamma/\sqrt{n}$. The expected value of $\hat{\delta}_j$ is δ and the standard deviation of $\hat{\delta}_j$ is $\gamma/\sqrt{n}$. Obviously, $\hat{\delta}_j$ is the unbiased estimator of accuracy index

δ. Let $K = n\hat{\gamma}_j^2/\gamma^2$, then K is regarded as a chi-square distribution with $n-1$ degrees of freedom. The expected value of $\hat{\gamma}_j$ is

$$\begin{aligned} E\left[\hat{\gamma}_j\right] &= E\left[\frac{\gamma}{\sqrt{n}}K^{1/2}\right] = \frac{\gamma}{\sqrt{n}}\int_0^\infty k^{1/2}\frac{1}{\Gamma((n-1)/2)2^{(n-1)/2}}k^{(n-1)/2-1}e^{-k/2}d_k \\ &= \frac{\gamma}{\sqrt{n}}\frac{\Gamma(n/2)2^{n/2}}{\Gamma((n-1)/2)2^{(n-1)/2}}\int_0^\infty \frac{1}{\Gamma(n/2)2^{n/2}}k^{n/2-1}e^{-k/2}d_k \\ &= b_n\gamma \end{aligned} \tag{11}$$

where

$$b_n = \left(\frac{\sqrt{2}\Gamma(n/2)}{\sqrt{n}\Gamma((n-1)/2)}\right) \tag{12}$$

Obviously, $b_n^{-1}\hat{\gamma}_j$ is the unbiased estimator of precision index γ. Furthermore, since $E\left[\hat{\gamma}_j^2\right] = (n-1)/n\gamma^2$, then the standard deviation of $\hat{\gamma}_j$ is

$$\sigma_{\hat{\gamma}_j} = \sqrt{E\left[\hat{\gamma}_j^2\right] - E^2\left[\hat{\gamma}_j\right]} = \sqrt{(n-1)/n - b_n^2}\gamma. \tag{13}$$

Consequently, on the three-sigma control chart of accuracy index δ, the upper control limit, the center limit, and the lower control limit are defined as follows:

$$UCL_\delta = \delta + \frac{3}{\sqrt{n}}\gamma \tag{14}$$

$$CL_\delta = \delta \tag{15}$$

$$LCL_\delta = \delta - \frac{3}{\sqrt{n}}\gamma. \tag{16}$$

Similarly, on the three-sigma control chart of precision index γ, the upper control limit, the center limit, and the lower control limit are represented as follows:

$$UCL_\gamma = \left(b_n + 3\sqrt{(n-1)/n - b_n^2}\right)\gamma \tag{17}$$

$$CL_\gamma = b_n\gamma \tag{18}$$

$$LCL_\gamma = \left(b_n - 3\sqrt{(n-1)/n - b_n^2}\right)\gamma. \tag{19}$$

Since the three control limits on the two control charts contain unknown parameters $-\delta$ and γ. Then, based on the control chart data in statistical process control, the observed values of these two indices are defined as follows:

$$\overline{\delta} = \overline{\overline{y}} = \frac{1}{m}\sum_{j=1}^{m}\overline{y}_j \tag{20}$$

$$\overline{\gamma} = \frac{1}{m}\sum_{j=1}^{m}b_n^{-1}s_j = b_n^{-1}\frac{1}{m}\sum_{j=1}^{m}s_j = b_n^{-1}\overline{s}. \tag{21}$$

Therefore, the limits of the δ control chart can be shown below:

$$UCL_\delta = \overline{\delta} + A_n\overline{\gamma} \tag{22}$$

$$CL_\delta = \overline{\delta} \tag{23}$$

$$LCL_\delta = \overline{\delta} - A_n\overline{\gamma}. \tag{24}$$

where $A_n = \frac{3\Gamma((n-1)/2)}{\sqrt{2}\Gamma(n/2)}$.

Similarly, the limits of the γ control chart can be displayed below:

$$UCL_\gamma = B_n\overline{\gamma} \tag{25}$$

$$CL_\gamma = \overline{\gamma} \tag{26}$$

$$LCL_\gamma = B'_n\overline{\gamma}. \tag{27}$$

where $B_n = 1 + 3\sqrt{(n-1)/nb_n^{-2} - 1}$ and $B'_n = 1 - 3\sqrt{(n-1)/nb_n^{-2} - 1}$. Based on Montgomery [21], when the value of sample size n of each subsample is larger, it is more effective to estimate precision index γ using the sample standard deviation than to estimate the precision index γ based on the sample range. The average and range control chart is usually used when the subsample size n is smaller than 6. However, the average and standard deviation control chart developed by this study is suitable for the larger subsample size n. Thus, Table 1 shows the values of b_n, A_n, B_n, and B'_n for subsample size n = 6(1)11 as follows:

Table 1. The values of b_n, A_n, B_n, and B'_n for subsample size n = 6(1)11.

Subsample Size n	b_n	A_n	B_n	B'_n
6	0.869	1.410	1.970	0.030
7	0.888	1.277	1.882	0.118
8	0.903	1.175	1.815	0.185
9	0.914	1.094	1.761	0.239
10	0.923	1.028	1.716	0.284
11	0.930	0.973	1.679	0.321

Next, according to the values of b_n, A_n, B_n, and B'_n received from the above table and the control chart data obtained in the statistical process control, the limits of the $\delta - \gamma$ control charts can be completed as shown in Equations (22)–(27) and used for monitoring process accuracy and precision.

3. Evaluation of Process Precision and Accuracy

As mentioned earlier, this study used the $\delta - \gamma$ control charts derived in Section 2 to monitor the process precision and accuracy. As the process was statistically controlled, this study used the statistical testing method as well as applied the control chart data in statistical process control, in order to propose a model to evaluate process precision and accuracy. Plenty of studies have suggested that as the process quality attains the k-sigma quality level, the required conditions are (1) $|\delta| \leq 1.5/k$ and (2) $\gamma \leq 1/k$ [20]. That is, when the accuracy index is bigger than $1.5/k$ ($\delta > 1.5/k$), it is learned that the process is excessively shifted to the right of tolerance, so the process must be improved. On the other hand, when the accuracy index is smaller than $-1.5/k$ ($\delta < -1.5/k$), it indicates that the process is excessively shifted to the left of tolerance, so the process must make some improvements. Likewise, when the precision index is bigger than $1/k$ ($\gamma > 1/k$), it shows that the process variation is too big and must be modified. Therefore, if the process quality level is required to reach k-sigma, then the null hypothesis (H_0) and the alternative hypothesis (H_1) for statistical tests are expressed as follows:

$$H_0 : \delta \in A_\delta \; and \; \gamma \in A_\gamma \tag{28}$$

$$H_1 : \delta \notin A_\delta \text{ or } \gamma \notin A_\gamma \tag{29}$$

where $A_\delta = \{-1.5/k \le \delta \le 1.5/k\}$ and $A_\gamma = \{\gamma \le 1/k\}$. Then, the 100 $(1-\alpha)$% confidence region of (δ, γ) is received to establish testing rules.

In statistical process control, the unbiased estimators of δ and γ^2, respectively, are expressed as follows:

$$\hat{\delta} = \frac{1}{m}\sum_{j=1}^{m}\overline{Y}_j = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n}Y_{j,i} \tag{30}$$

$$\hat{\gamma}^2 = \frac{1}{m(n-1)}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(Y_{j,i} - \overline{Y}_j\right)^2 \tag{31}$$

Furthermore, let random variables be $Z_{mn} = \sqrt{mn}\left(\hat{\delta} - \delta\right)/\gamma$ and $K_{mn} = m(n-1)\hat{\gamma}^2/\gamma^2$. Given the assumption of normality, Z_{mn} and K_{mn} are, respectively, distributed as $N(0,1)$ and χ^2_{N-m}. Therefore,

$$p\{-Z_{\alpha'/2} \le Z_{mn} \le Z_{\alpha'/2}\} = \sqrt{1-\alpha} \tag{32}$$

$$p\left\{\chi^2_{\alpha'/2;N-m} \le K_{mn} \le \chi^2_{1-(\alpha'/2);N-m}\right\} = \sqrt{1-\alpha} \tag{33}$$

where $N = mn$, $Z_{\alpha'/2}$ is the upper $\alpha'/2$ quantile for a standard normal distribution, $\chi^2_{\alpha'/2;N-m}$ is the upper $\alpha'/2$ quantile of χ^2_{N-m}, and $\alpha' = 1 - \sqrt{1-\alpha}$. $\hat{\delta}$ and $\hat{\gamma}^2$ are mutually independent, and so are Z_{mn} and K_{mn}. Inferring from their relationships, we have the following equation:

$$p\left\{-Z_{\alpha'/2} \le Z_{mn} \le Z_{\alpha'/2}, \chi^2_{\alpha'/2;N-m} \le K_{mn} \le \chi^2_{1-(\alpha'/2);N-m}\right\} = 1-\alpha \tag{34}$$

Equivalently,

$$p\left\{\hat{\delta} - Z_{\alpha'/2} \times \left(\frac{\gamma}{\sqrt{N}}\right) \le \delta \le \hat{\delta} + Z_{\alpha'/2} \times \left(\frac{\gamma}{\sqrt{N}}\right), \sqrt{\frac{N-m}{\chi^2_{1-(\alpha'/2);N-m}}}\hat{\gamma} \le \gamma \le \sqrt{\frac{N-m}{\chi^2_{\alpha'/2;N-m}}}\hat{\gamma}\right\} = 1-\alpha \tag{35}$$

According to Equations (20) and (30), the observation value of $\hat{\delta}$ is $\overline{\delta}$ and the observation value of $\hat{\gamma}$ is $\overline{\gamma}$, where $\overline{\delta} = \sum_{j=1}^{m}\overline{y}_j$ and $\overline{\gamma} = b_{11}^{-1}\overline{s}$. Obviously, the 100 $(1-\alpha)$% confidence region of (δ, γ) is similar to a trapezoid, wide at the top and narrow at the bottom. Therefore, this study defines $CR_\delta = [\delta_L, \delta_R]$ and $CR_\gamma = [\gamma_L, \gamma_R]$, where

$$\delta_L = \overline{\delta} - Z_{0.5-\sqrt{1-\alpha}/2} \times \sqrt{\frac{N-m}{N\chi^2_{0.5-\sqrt{1-\alpha}/2;N-m}}}\overline{\gamma} \tag{36}$$

$$\delta_R = \overline{\delta} + Z_{0.5-\sqrt{1-\alpha}/2} \times \sqrt{\frac{N-m}{N\chi^2_{0.5-\sqrt{1-\alpha}/2;N-m}}}\overline{\gamma} \tag{37}$$

$$\gamma_L = \sqrt{\frac{N-m}{\chi^2_{0.5+\sqrt{1-\alpha}/2;N-m}}}\overline{\gamma} \tag{38}$$

$$\gamma_R = \sqrt{\frac{N-m}{\chi^2_{0.5-\sqrt{1-\alpha}/2;N-m}}}\overline{\gamma} \tag{39}$$

This study makes the testing rules based on CR_δ and CR_γ as follows:

1. If $[\delta_L, \delta_R] \cap A_\delta = \phi$, then reject H_0 and conclude that the process accuracy needs to be improved.

2. If $\gamma_L > 1/k$, then reject H_0 and conclude that the process precision needs to be improved.
3. If $[\delta_L, \delta_R] \cap A_\delta \neq \phi$ and $\gamma_L \leq 1/k$, then do not reject H_0 and conclude that the process reaches the k-sigma quality level.

4. Improvement Decision on Process Precision and Accuracy

As mentioned above, when the testing result revealed that the process did not meet the six-sigma quality level ($CR \cap A = \phi$), its quality must be improved. Then, the statistical test of precision index γ was employed to determine whether the process required an improvement plan to reduce variation. As noted above, when the value of precision index was bigger than $1/k$, it meant that the process variation was enormous and must make some adjustment. Therefore, for the statistical test of precision index γ, the null hypothesis (H_0') and the alternative hypothesis (H_1') are described as follows:

$$H_0' : \gamma \leq 1/k \tag{40}$$

$$H_1' : \gamma > 1/k \tag{41}$$

Then, the 100 $(1 - \alpha)$% lower confidence limit γ_L is adopted to establish the following testing rules:

1. If $\gamma_L > 1/k$, then H_0 is rejected. It is concluded that precision index γ is bigger than $1/k$, indicating that the process variation is so huge that the process must be improved.
2. If $\gamma_L \leq 1/k$, then H_0 is not rejected. It is concluded that precision index γ is smaller than or equal to $1/k$, showing that the process variation does not need to reduce.

When the accuracy index is bigger than $1.5/k$, then it is learned that the process is overly shifted to the right of tolerance, so it must be improved. In contrast, as the value of accuracy index is smaller than $-1.5/k$, it is known that the process is overly shifted to the left of tolerance, so it must be bettered as well. Therefore, for the statistical test of accuracy index δ, the null hypothesis (H_0'') and the alternative hypothesis (H'') are depicted as follows:

$$H_0'' : -1.5/k \leq \delta \leq 1.5/k \tag{42}$$

$$H_1'' : \delta < -1.5/k \; or \; \delta > 1.5/k \tag{43}$$

Next, the 100 $(1 - \alpha)$% lower confidence limit δ_L and upper confidence limit δ_R are applied to the testing rules as follows:

1. If $[\delta_L, \delta_R] \cap [-1.5/k, 1.5/k] \neq \phi$, then do not reject H_0, and the process accuracy does not need to be improved.
2. If $\delta_L > 1.5/k$, indicating that the right deviation of the process exceeds 1.5 sigma, then it is necessary to make some improvements to help the process mean move toward the target value. Also, the process deviation must be controlled and fall within 1.5 sigma or less.
3. If $\delta_R < -1.5/k$, meaning that the left deviation of the process exceeds 1.5 sigma, then it is necessary to make some adjustments to help the process mean move toward the target value. Also, the process deviation must be controlled and fall within 1.5 sigma or less.

5. An Empirical Example

As mentioned above, Process Capability Indices (PCIs) are tools for process quality evaluations that are widely applied in the industry, including various machine tools and machining industries, semiconductor processes, packaging processes, etc. Many studies have indicated that Taiwan's output value for machine tools ranks number seven worldwide, and its export volume ranks number five worldwide [22]. Central Taiwan is an important stronghold of the machine tool industry. About 70% of manufacturers of machine tools, precision machinery, and their components are situated in central Taiwan. Aiming to cut operating costs and concentrate resources on the core technologies, the machine tool industry outsources some non-core businesses to specialized manufacturers. Meanwhile, the industry focuses on improving its specialized machining processes in order to elevate the entire quality level, efficiency, and competitiveness of the machine tool industry and the entire industry chain [23–25]. Additionally, the machine tool industry also combines the aerospace technology and intelligent machinery industries. Driven by the high clustering effect, the central region of Taiwan has turned into a complete industry chain of machine tools and plays a significant role in the machine tool industry worldwide [22].

According to various studies, a machine tool contains many important components, including axles, bearings, and gears. These components usually have nominal-the-best quality characteristics (QCs), such as diameter [7]. As mentioned earlier, enhancing the process accuracy index and the process precision index would not only make the process loss lower but also make the process capability higher. This study took the outer diameter of an axle as an example to demonstrate the application of the proposed model, like converting the tolerance into a target value of zero (T = 0), the lower specification limit into -1, and the upper specification limit into 1 by means of variable transformation. Then, the $\delta - \gamma$ control charts developed by this study were employed to monitor the process accuracy and precision. When the process precision and the process accuracy became stable, the statistical testing method recommended by this study was adopted to evaluate the process capability. Furthermore, the statistical testing method of accuracy and precision indices was also used as the decision-making basis of process improvement.

In the example, the outer diameter tolerance of the axle is 2.8 ± 0.03, and its 25 groups of statistical process control data are displayed as follows

$$x_{j,1},\ x_{j,2}, \ldots,\ x_{j,i}, \ldots,\ x_{j,11}$$

where $j = 1, 2, \ldots, 25$ and $i = 1, 2, \ldots, 11$. Based on the above developed model, we started monitoring, evaluating, and improving the machining process quality for the outer diameter of the axle by Excel software 2016 and offered manufacturers a direction of decision making on improvement for reference.

5.1. Process Quality Monitoring

According to the variable transformation formula, $y_{j,i} = \left(x_{j,i} - 2.8\right)/0.03$, the subsample data of $y_{j,i}$, subsample mean, and subsample standard deviation are depicted below:

$$y_{1,1}, y_{1,2}, \ldots,\ y_{1,i}, \ldots,\ y_{1,11},\ \overline{y}_1 = \frac{1}{11}\sum_{i=1}^{25} y_{1,i} = 0.415,\ s_1 = \sqrt{\frac{1}{n}\sum_{i=1}^{11}(y_{1,i} - \overline{y}_1)^2} = 0.172$$

$$y_{j,1},\ y_{j,2}, \ldots,\ y_{j,i} \ldots,\ y_{j,11},\ \overline{y}_j = \frac{1}{11}\sum_{i=1}^{25} y_{j,i} = 0.452,\ s_j = \sqrt{\frac{1}{11}\sum_{i=1}^{11}\left(y_{j,i} - \overline{y}_j\right)^2} = 0.187$$

$$y_{25,1},\ y_{25,2}, \ldots,\ y_{25,i}, \ldots,\ y_{25,11}, \overline{y}_{25} = \frac{1}{11}\sum_{i=1}^{25} y_{25,i} = 0.444,\ s_{25} = \sqrt{\frac{1}{11}\sum_{i=1}^{11}(y_{25,i} - \overline{y}_{25})^2} = 0.181$$

where $j = 1, 2, \ldots, 25$ and $i = 1, 2, \ldots, 11$. According to Equations (20) and (21), then

$$\overline{\delta} = \overline{\overline{y}} = \frac{1}{25}\sum_{j=1}^{25} \overline{y}_j = 0.443$$

$$\overline{\gamma} = \frac{1}{25}\sum_{j=1}^{25} b_{11}^{-1} s_j = b_{11}^{-1}\overline{s} = 1.075 \times 0.182 = 0.196$$

Based on Table 1, the values of b_{11} and A_n are received, and the limits of the δ control chart can be calculated below:

$$UCL_{\delta} = \overline{\delta} + A_{11}\overline{\gamma} = 0.443 + 0.973 \times 0.196 = 0.643;$$

$$CL_{\delta} = \overline{\delta} = 0.443;$$

$$LCL_{\delta} = \overline{\delta} - A_{11}\overline{\gamma} = 0.443 - 0.973 \times 0.196 = 0.252.$$

Similarly, the values of B_{11} and B'_{11} obtained from Table 1, and the limits of the γ control chart, can be computed as follows:

$$UCL_{\gamma} = B_{11}\overline{\gamma} = 1.679 \times 0.196 = 0.329$$

$$CL_{\gamma} = \overline{\gamma} = 0.196$$

$$LCL_{\gamma} = B'_{11}\overline{\gamma} = 0.321 \times 0.196 = 0.063$$

The above control limits of accuracy and precision indices can be used to supervise the process accuracy and precision.

5.2. Process Quality Evaluation

Then, the process quality is evaluated based on 25 groups of subsample data in statistical process control. Since $Z_{0.0025} = 2.807$, $\chi^2_{0.0025;250} = 191.802$, $\chi^2_{0.9975;250} = 317.362$ and based on the above data in the control charts, we have

$$\delta_L = \overline{\delta} - \frac{Z_{0.0025}}{\sqrt{275}} \times \sqrt{\frac{250}{\chi^2_{0.0025;250}}}\overline{\gamma} = 0.443 - \frac{2.807}{\sqrt{275}} \times \sqrt{\frac{250}{191.802}} \times 0.196 = 0.405$$

$$\delta_R = \overline{\delta} + \frac{Z_{0.0025}}{\sqrt{275}} \times \sqrt{\frac{250}{\chi^2_{0.0025;250}}}\overline{\gamma} = 0.443 + \frac{2.807}{\sqrt{275}} \times \sqrt{\frac{250}{191.802}} \times 0.196 = 0.481$$

$$\gamma_L = \sqrt{\frac{250}{\chi^2_{0.9975;250}}}\overline{\gamma} = \sqrt{\frac{250}{317.362}} \times 0.196 = 0.174$$

$$\gamma_R = \sqrt{\frac{250}{\chi^2_{0.0025;250}}}\overline{\gamma} = \sqrt{\frac{250}{191.802}} \times 0.196 = 0.224$$

According to Section 3, the process was required to reach a quality level of six-sigma, the accuracy index was required to be $-0.25/k \le \delta \le 0.25/k$, and the required precision index was $\gamma \le 1/6$. Thus, on the statistical test, the null hypothesis (H_0) and the alternative hypothesis (H_1) are expressed as follows:

$$H_0 : \delta \in A_{\delta} \text{ and } \gamma \in A_{\gamma} \tag{44}$$

$$H_1 : \delta \notin A_{\delta} \text{ or } \gamma \notin A_{\gamma} \tag{45}$$

where $A_{\delta} = \{-0.25 \le \delta \le 0.25\}$ and $A_{\gamma} = \{\gamma \le 1/6\}$. According to the testing rules,

1. If $[\delta_L, \delta_R] \cap A_{\delta} = \phi$, then reject H_0. It is concluded that the process accuracy needs to be improved.

2. If $\gamma_L = 0.174 > 1/6$, then reject H_0. It is concluded that the process precision needs to be improved.

5.3. Decision Making on the Direction of Process Quality Improvement

Based on the above-stated statistical testing results, there are two statistical tests that need to be performed to determine the direction of process improvement. The first is the test of process accuracy. Since the required quality level is six-sigma, the null hypothesis (H_0'') and the alternative hypothesis (H'') on the statistical test of precision index δ are depicted as follows:

$$H_0' : \gamma \le 1/6 \quad (46)$$

$$H_1' : \gamma > 1/6 \quad (47)$$

If the value of γ_L equal to 0.174 is bigger than 1/6, it is learned that the process variation is so large that the process must be improved. For the same reason, the required quality level is six-sigma. Then, the null hypothesis (H_0'') and the alternative hypothesis (H'') on the statistical test of accuracy index δ are denoted as follows:

$$H_0'' : -0.25 \le \delta \le 0.25 \quad (48)$$

$$H_1'' : \delta < -0.25 \; or \; \delta > 0.25 \quad (49)$$

If the value of $[\delta_L, \delta_R] = [0.405, 0.481]$, then 0.405 is bigger than 0.25, indicating that the process is shifted to the right by more than 1.5 sigma. Therefore, some improvements must be made to help the process mean move toward the target value, and the process deviation must be controlled and fall within 1.5 sigma or less.

6. Conclusions

Process Capability Indices (PCIs, which are commonly adopted by the industry, are functions of accuracy index δ and precision index γ. Accuracy index δ focuses on measuring the degree to which process mean μ deviates from process target value T. As the value is closer to 0, the process is more accurate. Precision index γ mainly represents the size of process variation. As the value is smaller, the process variation is smaller. That is to say, the process Is more precise. It is obvious that accuracy index δ and precision index γ are two important parameters of the process capability index, as well as two important indices for evaluating the process quality level. For example, when $|\delta| \le 1.5/k$ and $\gamma \le 1/k$, then the process quality reaches the k-sigma quality level [20]. Raising the process precision and process accuracy can increase the values of process capability indices as well.

Therefore, the $\delta - \gamma$ control charts were derived to monitor process accuracy and precision based on in-control data. This study used the unbiased estimator of accuracy index δ to derive the three-sigma control chart of index δ and discovered the upper control limit, the center limit, and the lower control limit. Similarly, this study employed the unbiased estimator of precision index γ to derive the three-sigma control chart and found the upper control limit, the center limit, and the lower control limit. Table 1 provides the values of items b_n, A_n, B_n, and B_n' to help the quality control engineer figure out the control limits of these two control charts, so as to monitor the process accuracy and precision. When the process precision and accuracy became stable, the statistical testing method proposed by this study was applied to the evaluation of process capability. Then, according to the normal process, it was learned that the sample mean and the sample variation were independent. In this paper, the $100(1-\alpha)\%$ confidence region of (δ, γ) was first derived, and then confidence interval CR_δ for index δ and confidence interval CR_γ for index γ were defined based on this confidence region. Subsequently, this study made the testing rules based on CR_δ and CR_γ. First, considering the definition of the six-sigma quality level, we examined whether the process could reach the six-sigma quality level. Second, when the process failed to reach the six-sigma quality level, the statistical testing method of accuracy and precision indices was adopted to come up with a direction for process improvement, such as correcting the direction of process deviation and deciding whether to reduce process variation. Finally, this study presented an empirical case to prove the feasibility of the proposed model.

To sum up, we have provided a mechanism for monitoring, evaluating, and improving the normal process quality through the statistical testing method, which has the following advantages and functions:

1. The accuracy and precision control charts developed by this study can assist quality control engineers with the calculation of the control limits of these two control charts, so that they can supervise the accuracy and precision of the process.
2. The statistical testing method and the definition of the six-sigma quality level can help evaluate whether the precision and accuracy of the process meet the required level and decide whether to make improvements.
3. Accuracy index δ and precision index γ are not only two important parameters of the process capability index but also two important indices for evaluating the process quality level. Therefore, when the process accuracy and precision reach the required level, then both the process capability and the six-sigma quality level can meet requirements.

The monitoring, evaluation, and improvement model of process precision and accuracy proposed by this study is suitable for manufacturers who have large quantities of production and use control charts. However, the batches produced by many original equipment manufacturers (OEMs) are not large, belonging to the high-mix low-volume manufacturing process, which cannot apply to the model proposed by this study. In addition, monitoring, evaluating, and improving the risk assessment of control and decision making is another important research topic [26], which is not included in the model. Clearly, the above two issues, including the high-mix low-volume manufacturing process and monitoring, evaluating, and improving the risk assessment of control and decision-making, are not only the research limitations of this paper but also significant directions for future research.

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