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# Performance of Iterative Coded CDMA Receivers with APP Feedback: A Use of a Weighted Delay Filter

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**Abstract:** The prohibitive computational complexity of optimal coded multiuser detection necessitates using suboptimal detectors in practical implementations. The filter is very computationally simple and is also demonstrated to provide faster convergence and superior bit error rate (BER) performance. Further investigation of the weighted delay filter concept produces a second filter—derived via the joint likelihood function. It is analytically demonstrated that extrinsic feedback systems will not benefit from weighted delay filtering. A system model is provided that introduces the notion of feedback 'residue', which is shown to be the key difference between a-posterior probability (APP) and extrinsic systems when determining the parallel interference cancellation (PIC) output statistics. It is analytically shown that the weighted delay filter derived via a maximum signal-to-noise ratio (SNR) approach is identical to a weighted delay filter derived via the joint likelihood function. It is analytically shown that when extrinsic feedback is used in a coded-code division multiple access (C-CDMA) system, no benefit will be realised by weighted delay filtering, as soft outputs from previous cycles are a merely scaled, noisy version of the most recent data. The notion of a 'feedback residue' for systems with APP feedback is introduced, and it is empirically shown that this residue term is a key consideration when determining the PIC output statistics. Using the 'residual feedback' model, it is shown that when APP feedback is utilised, data from previous cycles is not simply "a scaled, noisy version" of the current data. For this reason, benefits may be realised by APP feedback use. The simulation results shows that the residue may be trivial at small loads, the residue builds to the substantial value of nearly 0.4 at a reasonably modest load of $K/N = 15/10$, and continues to grow as the load increases.

**Keywords:** data mining; technology management; multiuser detector; parallel interference cancellation; statistical signal processing; low density parity code

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## 1. Introduction

The importance of coding techniques has achieved much attention in modern technology. In the 90s, the Turbo coding technique was proposed to improve the performance of communication systems. Then the era of many modern iterative decoding techniques starts which includes the iterative, turbo, convolutional and repeat accumulate (RA) [1–4]. Such error correcting techniques were utilized in many applications specially in mutli-user detection, forward error control, data estimation and control signals. The reader is referred to [5–9] for more reading and application dependent techniques.

Multi-input and user detection is closely related to the detection of data over the multiple transmission in a network, many users are sending the information over a channel in one time and access it simultaneously. In such situation, there is another issue arised known as multiple access interference (MAI). The major objective behind using the MUD is to utilized the inherent structure of MAI via joint detection of many transmitter [10–16].

To discuss the coded code division multiple access (C-CDMA) system, it was explained in reference [17] that the Viterbi decoder is the optimal maximum-likelihood decoders. These are operational on the joint trellis of multi-access links. But the bad things for such systems is the greater complexity which is the major hindrance of its implementation.

The prohibitive computational complexity of optimal coded MUD necessitates using suboptimal detectors in practical implementations. Even when functionality at the receiver is split into the roles of MUD and single user decoding, using an optimal multiuser detector is still computationally costly. For this reason, using suboptimal detectors is common in iterative systems, which incurs a performance loss in terms of possible system load, output bit error rate (BER) and speed to convergence [18,19]. The tables of symbols and acronyms are given in Tables 1 and 2.

**Table 1.** Table of Acronyms.

| Accronyms | Description | Accronyms | Description |
|---|---|---|---|
| APP | A-Posterior Probability | CDMA | Code Division Multiple Access |
| C-CDMA | Coded-CDMA | BER | Bit Error Rate |
| SNR | Signal-to-noise Ratio | MUD | Multi User Detection |
| PIC | Parallel Interference Cancellation | LLR | Log-likelihood Ratio |
| MAI | Multiple Access Interference | SISO | Single Input Single Output |

**Table 2.** Table of Symbol.

| Symbol | Description | Symbol | Description |
|---|---|---|---|
| $\Lambda$ | LLR | $\rho$ | Correlation Factor |
| $\mu$ | Mean | $\sigma^2$ | Variance |
| $W_1, W_2$ | Normal RV | $E[.]$ | Expected Value |
| $\Gamma$ | Received Signal | $X$ | Symbol |
| $\delta$ | Scaled value of LLR | $K/N$ | System load |
| $N$ | Gain | $K$ | No. of users |
| $L$ | Data Sequence Length | $x$ | BPSK sybmol |
| $\tilde{x}$ | Estimated of $x$ | $\Lambda_1$ | LLR at output of PIC |
| $\Lambda_2$ | LLR from previous iteration | $\Upsilon_\Gamma$ | SNR of $\Gamma$ |

In iterative systems with optimal components, the sum-product algorithm tells us one should pass extrinsic information between system components. However, in systems with sub-optimal building blocks, the most suitable soft information to exchange within the system remains an open problem; however, other schemes, such as passing a-posteriori probability (APP) information, have been demonstrated and can yield better results [20,21]. This work considers a coded multiuser receiver that utilizes a (suboptimal) parallel interference cancellation (PIC) component for its MUD stage. A recursive log-likelihood ratio (LLR) filter is derived, considering the LLR streams of the previous and current iterative cycles. The filter produces an output that maximizes the output LLR sequences based on knowledge of the statistics (mean and variance) of the two input LLR streams and the noise correlation between the two LLR streams [22–25].

In this paper, we focus on the design of a novel filter and compare its performance in different scenarios. The proposed work is an extension of the maximum ratio combining (MRC) filter. In contrast, in the previous work, it has been acknowledged that: *combining over iterations introduces correlations, which lead to sub-optimal performance*. Here we ponder the question: 'Knowing that correlations have been introduced, can we utilize this knowledge and perhaps derive a filter that provides better results despite these correlations?' This idea is investigated in the proposed research work. The key contributions were established upon which the research element of this work is built:

- Extrinsic feedback is used in a coded CDMA system.

- The notion of a 'feedback residue' for systems with APP feedback is introduced, and it is empirically shown that this residue term is a key consideration when determining the PIC output statistics.
- Using the 'residual feedback' model, it is shown that when APP feedback is utilised, data from previous cycles is not simply "a scaled, noisy version" of the current data. For this reason, benefits may be realised by APP feedback use.
- LLRs with Extrinsic Feedback are derived.
- It is demonstrated that extrinsic data, in the case of iterative receivers with suboptimal components, may not always yield the best results.
- It is analytically shown that when extrinsic feedback is used in a coded CDMA system, no benefit will be realised by weighted delay filtering, as soft outputs from previous cycles are a merely scaled, noisy version of the most recent data.

## 2. Coded CDMA

In the case of multiple access systems, it is common to use Forward Error Correction (FEC) techniques to combat the effects of multiple access interference (MAI) and channel noise. In such a case, a user would pre-code their data, before interleaving and spreading with their unique spreading code. A diagram of such a scenario is shown in Figure 1.

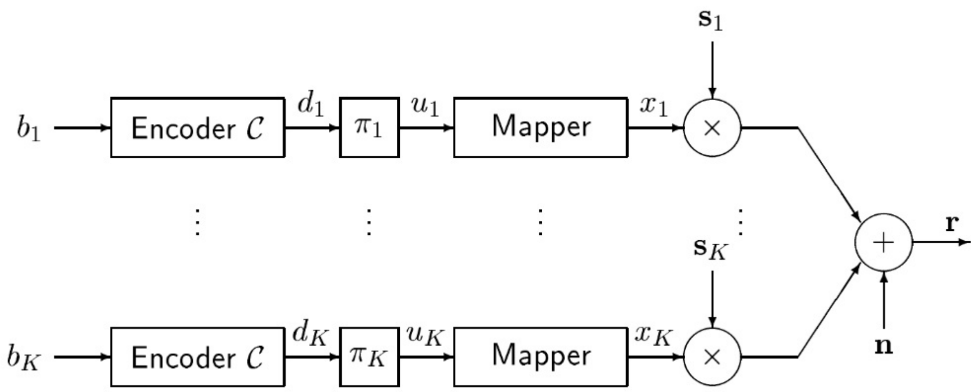


**Figure 1.** Coded Code Division Multiple Access transmission.

In Figure 1 binary data of length $L$ for user $k$, denoted $b_k$, is presented to encoder $C$, where $b_k \in 0, 1$ and $k \in 1 \ldots K$. The encoder produces ary coded data $d_k$ of length $L/R$, where $R$ is the rate of code. The data is randomly interleaved through the interleaving block $\pi$ to produce $\mu_k$ which is then mapped onto the binary phase shift keying (BPSK) constellation to produce coded, interleaved BPSK data $x_k$. The BPSK data is then multiplied by the unique spreading vector $s_k$ and transmitted across the additive white Gaussian noise (AWGN) channel.

### 2.1. Optimal Detectors in Coded CDMA

In reference [17] an optimal maximum likelihood detector for convolutionally coded CDMA was demonstrated, and was shown to have a computational complexity of the order $O(2^{K\nu})$. Such a processing requirement is prohibitive for large CDMA systems. Another approach, considered in reference [26], was to view the multiuser spreading stage of the coded CDMA transmitter to be the inner code of a serially concatenated encoder. In reference [5], an iterative decoder was realised based on this premise, which exchanged information between an optimal multiuser SISO and a bank of SU APP decoders. This separation of functionality reduced the complexity to $O(2^K + K2^\nu)$; still exponential in $K$ and hence still too complex for large systems.

### 2.2. Suboptimal Detectors in Coded CDMA

As optimal detectors are too computationally expensive for implementation in practical systems, there has been significant interest in linear multiuser detectors in coded multiuser receivers [27–29]. It is in the field of coded multiuser receivers with sub-optimal multiuser detectors that this thesis expresses an interest. Specifically, the primary focus of this work involves the reception of coded CDMA with a receiver which utilises Interference Cancellation (IC) techniques. A typical diagram is shown in Figure 2. Operation of the receiver of Figure 2 is as follows:

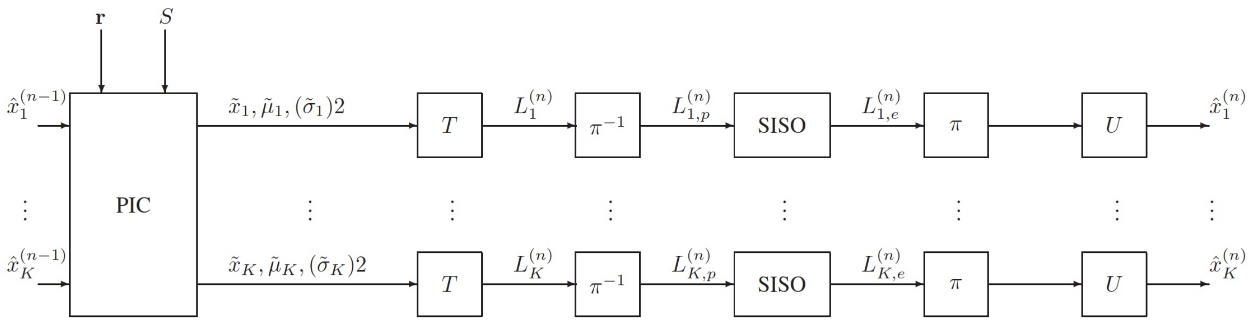


**Figure 2.** Coded Code Division Multiple Access Reception with Parallel Interference Cancellation (PIC).

At iteration $n$, the PIC takes as input the received signal $r$ and the signal estimates $x_k^{(n-1)}$, from the previous iteration $(n-1)$. Based on these inputs, the PIC provides an updated signal estimate $\tilde{x}_k^{(n)}$. The output signal is considered Gaussian, $\tilde{x} \sim \mathcal{N}(\tilde{\mu}_k^{(n)}, (\tilde{\sigma}_k^{(n)})^2)$, and the output statistics $\tilde{\mu}_k^{(n)}$, $((\tilde{\sigma}_k^{(n)})^2)$ are also determined. The output signal $\tilde{x}_k^{(n)}$ is then converted to LLR format through the function block $T$, which is then de-interleaved to form the prior LLR input $L_{k,p}^{(n)}$ for the SU SISO decoders. Based on the inputs $L_{k,p}^{(n)}$ and knowledge of the code trellis, the SISO generates extrinsic LLR outputs $L_{k,e}^{(n)}$, which are then interleaved, and converted back to signal form through the function block $U$ to form a-priori signal estimates $\hat{x}_k^{(n)}$ for the next iteration. An iterative cycle is now complete.

## 3. Materials and Methods

### *3.1. Extrinsic Feedback, a Study*

Results in Section 4 strongly suggest that no improvement is to be realised by using a weighted delay filter with extrinsic feedback at the SISO, a somewhat surprising result given that the weighted delay filter maximises the output LLR. We now investigate this finding based on [29]. The expressions for the data at the output of the PIC:

$$\tilde{x}_k^{(n)} = x_k + \sum_{j \neq k} c_{kj} \left( x_j - \hat{x}_{j,e}^{(n-1)} \right) + z_k. \tag{1}$$

The conditional mean and variance of $\tilde{x}_k^{(n)}$, given that $X = +1$ are;

$$\tilde{\mu}_k^{(n)} = 1 \tag{2}$$

$$\left( \tilde{\sigma}_k^{(n)} \right)^2 = \frac{1}{N} \sum_{j \neq k} \left( 1 - \left( \hat{x}_{j,e}^{(n-1)} \right)^2 \right) + \sigma_n^2. \tag{3}$$

We investigate the circuit of Figure 3, and seek to gain expressions for the *per-user* scaling factors $\delta_{1,k}^{(n)}$ and $\delta_{2,k}^{(n)}$. In order to find these parameters, we start at the output of the PIC and work our way around the circuit. At this point of the investigation, we know the data at the output of the PIC, we now gain an expression for the LLR conversion, $\Lambda$.

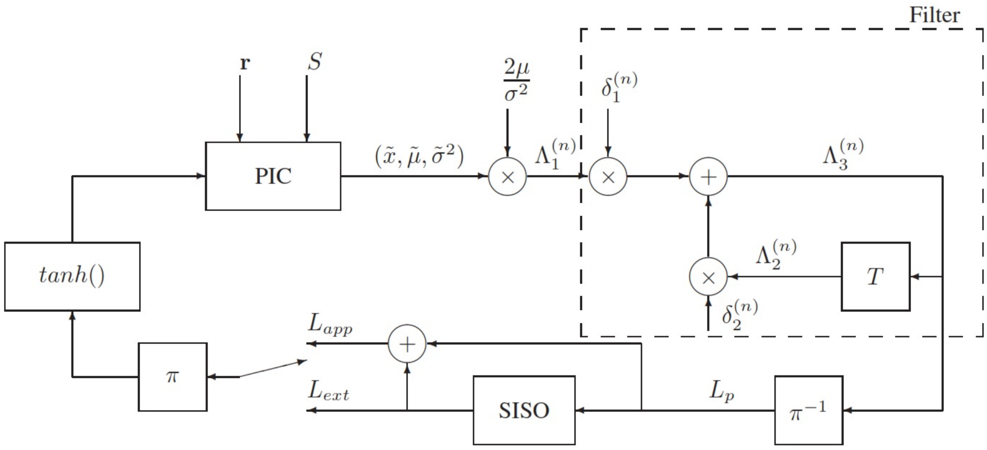


**Figure 3.** Receive Circuit with Joint Likelihood Filter.

$$\begin{aligned}
\Lambda_{1,k}^{(n)} &= \tilde{x}_k^{(n)} \frac{2\tilde{\mu}_k^{(n)}}{\left(\tilde{\sigma}_k^{(n)}\right)^2} \\
&= \frac{2\tilde{\mu}_k^{(n)}}{\left(\tilde{\sigma}_k^{(n)}\right)^2} x_k + \frac{2\tilde{\mu}_k^{(n)}}{\left(\tilde{\sigma}_k^{(n)}\right)^2} \left( \sum_{j \neq k} c_{kj} \left( x_j - \hat{x}_{j,e}^{(n-1)} \right) + z_k \right) \\
&= \frac{\left(\lambda_{1,k}^{(n)}\right)^2}{2} x_k + \lambda_{1,k}^{(n)} \omega_k^{(n)}
\end{aligned} \tag{4}$$

where $\Lambda_z = \frac{\lambda_z^2}{2} X + \lambda_z W_z$ and $\lambda_z = \frac{2\mu_z}{\sigma_z}$. The expression (4) is due to the assumption that the LLR output will be a consistent LLR. Now group like terms and conclude:

$$\lambda_{1,k}^{(n)} = \frac{2}{\tilde{\sigma}_k^{(n)}} \tag{5}$$

$$\omega_k^{(n)} = \frac{\sum_{j \neq k} c_{kj} \left( x_j - \hat{x}_{j,e}^{(n-1)} \right) + z_k}{\tilde{\sigma}_k^{(n)}}. \tag{6}$$

The next stage of the receiver circuit is the weighted delay filter. By inspection of Figure 3, one can determine the output of the filter:

$$\Lambda_{3,k}^{(n)} = \delta_{1,k}^{(n)} \Lambda_{1,k}^{(n)} + \delta_{2,k}^{(n)} \Lambda_{2,k}^{(n)}. \tag{7}$$

Now seek a recursive expression, and start by investigating the pattern:

$$\begin{aligned}
\Lambda_{3,k}^{(1)} &= \Lambda_{1,k}^{(1)} \delta_{1,k}^{(1)} \\
\Lambda_{3,k}^{(2)} &= \Lambda_{1,k}^{(2)} \delta_{1,k}^{(2)} + \delta_{2,k}^{(2)} \Lambda_{3,k}^{(1)} \\
&= \Lambda_{1,k}^{(2)} \delta_{1,k}^{(2)} + \delta_{2,k}^{(2)} \Lambda_{1,k}^{(1)} \delta_{1,k}^{(1)} \\
\Lambda_{3,k}^{(3)} &= \Lambda_{1,k}^{(3)} \delta_{1,k}^{(3)} + \delta_{2,k}^{(3)} \Lambda_{3,k}^{(2)} \\
&= \Lambda_{1,k}^{(3)} \delta_{1,k}^{(3)} + \delta_{2,k}^{(3)} \Lambda_{1,k}^{(2)} \delta_{1,k}^{(2)} + \delta_{2,k}^{(3)} \delta_{2,k}^{(2)} \Lambda_{1,k}^{(1)} \delta_{1,k}^{(1)} \\
\Lambda_{3,k}^{(4)} &= \delta_{1,k}^{(4)} \Lambda_{1,k}^{(4)} + \delta_{2,k}^{(4)} \Lambda_{3,k}^{(3)} \\
&= \Lambda_{1,k}^{(4)} \delta_{1,k}^{(4)} + \delta_{2,k}^{(4)} \Lambda_{1,k}^{(3)} \delta_{1,k}^{(3)} + \delta_{2,k}^{(4)} \delta_{2,k}^{(3)} \Lambda_{1,k}^{(2)} \delta_{1,k}^{(2)} + \delta_{2,k}^{(4)} \delta_{2,k}^{(3)} \delta_{2,k}^{(2)} \Lambda_{1,k}^{(1)} \delta_{1,k}^{(1)}.
\end{aligned}$$

A pattern can be seen to emerge, it can be expressed in a general term:

$$\Lambda_{3,k}^{(n)} = \Lambda_{1,k}^{(n)} \delta_{1,k}^{(n)} + \sum_{i=1}^{n-1} \prod_{j=i+1}^{n} \Lambda_{1,k}^{(i)} \delta_{1,k}^{(i)} \delta_{2,k}^{(j)} \tag{8}$$

the *per-user* scaling factors $\delta_{1,k}^{(n)}$ and $\delta_{2,k}^{(n)}$ are dependent on the *per-user* correlation, $\rho_k^{(n)}$, where:

$$\rho_k^{(n)} = \mathcal{E}\left[\omega_k^{(n)} \varphi_k^{(n-1)}\right] \tag{9}$$

$$\Lambda_{1,k}^{(1)} = \frac{\left(\lambda_{1,k}^{(n)}\right)^2}{2} x_k + \lambda_{1,k}^{(n)} \omega_k^{(n)} \tag{10}$$

$$\Lambda_{3,k}^{(n)} = \frac{\left(\lambda_{3,k}^{(n)}\right)^2}{2} x_k + \lambda_{3,k}^{(n)} \varphi_k^{(n)}. \tag{11}$$

The expression (11) is due to the fact that the joint likelihood output is a consistent LLR. In order to determine $\rho_k^{(n)}$, we may substitute (11) into (8) and simplify to get,

$$\Lambda_{3,k}^{(n)} = x_k \left[ \frac{\left(\lambda_{1,k}^{(n)}\right)^2}{2} \delta_{1,k}^{(n)} + \sum_{i=1}^{n-1} \prod_{j=i+1}^{n} \frac{\left(\lambda_{1,k}^{(i)}\right)^2}{2} \delta_{1,k}^{(i)} \delta_{2,k}^{(j)} \right] + \lambda_{1,k}^{(n)} \omega_k^{(n)} \delta_{1,k}^{(n)} + \sum_{i=1}^{n-1} \prod_{j=i+1}^{n} \lambda_{1,k}^{(i)} \omega_k^{(i)} \delta_{1,k}^{(i)} \delta_{2,k}^{(j)}. \tag{12}$$

Grouping like terms gives us expressions for $\lambda_{3,k}^{(n)}$ and $\varphi_k^{(n)}$,

$$\lambda_{3,k}^{(n)} = \sqrt{\left(\lambda_{1,k}^{(n)}\right)^2 \delta_{1,k}^{(n)} + \sum_{i=1}^{n-1} \prod_{j=i+1}^{n} \left(\lambda_{1,k}^{(i)}\right)^2 \delta_{1,k}^{(i)} \delta_{2,k}^{(j)}} \tag{13}$$

$$\varphi_k^{(n)} = \frac{\lambda_{1,k}^{(n)} \omega_k^{(n)} \delta_{1,k}^{(n)} + \sum_{i=1}^{n-1} \prod_{j=i+1}^{n} \lambda_{1,k}^{(i)} \omega_k^{(i)} \delta_{1,k}^{(i)} \delta_{2,k}^{(j)}}{\lambda_{3,k}^{(n)}}. \tag{14}$$

Now that we have an expression for $\varphi_k^{(n)}$, it is possible to determine the correlation $\rho_k^{(n)}$ as,

$$\rho_k^{(n)} = \mathcal{E}\left[ \frac{\lambda_{1,k}^{(n-1)} \omega_k^{(n-1)} \delta_{1,k}^{(n-1)} + \sum_{i=1}^{n-2} \prod_{j=i+1}^{n-1} \lambda_{1,k}^{(i)} \omega_k^{(i)} \delta_{1,k}^{(i)} \delta_{2,k}^{(j)}}{\lambda_{3,k}^{(n-1)}} \right] \tag{15}$$

$$= \frac{\lambda_{1,k}^{(n-1)} \mathcal{E}\left[\omega_k^{(n)} \omega_k^{(n-1)}\right] \delta_{1,k}^{(n-1)} + \sum_{i=1}^{n-2} \prod_{j=i+1}^{n-1} \lambda_{1,k}^{(i)} \mathcal{E}\left[\omega_k^{(n)} \omega_k^{(j)}\right] \delta_{1,k}^{(i)} \delta_{2,k}^{(j)}}{\lambda_{3,k}^{(n-1)}} \tag{16}$$

We therefore need to solve $\mathcal{E}\left[\omega_k^{(n)} \omega_k^{(j)}\right]$, we make use of (6) and simplify to get,

$$\mathcal{E}\left[\omega_k^{(n)} \omega_k^{(j)}\right] = \frac{1}{\tilde{\sigma}_k^{(n)} \tilde{\sigma}_k^{(j)}} \mathcal{E}\left[ \sum_{i \neq k} \sum_{q \neq k} c_{ki} c_{kq} f_{\omega\omega}(x, \hat{x}) + z^2 \right] \tag{17}$$

where $f_{\omega\omega}(x, \hat{x}) = x_i \left(x_q - \hat{x}_{q,e}^{(j-1)}\right) + \hat{x}_{i,e}^{(n-1)} \left(\hat{x}_{q,e}^{(j-1)} - x_q\right)$

We now consider the expectation $\mathcal{E}\left[\sum_{i\neq k}\sum_{q\neq k} c_{ki}c_{kq}f_{\omega\omega}(x,\hat{x})\right]$ for the case where $i=q$ and the case where $i\neq q$. For the case $i=q$, we assume $\mathcal{E}[x_i]=\hat{x}_{i,e}^{(n-1)}$, after simplification, we get,

$$\mathcal{E}\left[\sum_{i\neq k}\sum_{q\neq k} c_{ki}c_{kq}f_{\omega\omega}(x,\hat{x})|_{i=q}\right]=\frac{1}{N}\sum_{i\neq k}\left(1-(\hat{x}_{i,e}^{(n-1)})^2\right)+\sigma_n^2. \quad (18)$$

For the case where $i\neq q$, we get $\mathcal{E}\left[\sum_{i\neq k}\sum_{q\neq k} c_{ki}c_{kq}f_{\omega\omega}(x,\hat{x})|_{i\neq q}\right]=\sigma_n^2$
We therefore conclude that,

$$\mathcal{E}\left[\omega_k^{(n)}\omega_k^{(j)}\right]=\frac{1}{\tilde{\sigma}_k^{(n)}\tilde{\sigma}_k^{(j)}}\left(\frac{1}{N}\sum_{i\neq k}\left(1-(\hat{x}_{i,e}^{(n-1)})^2\right)+\sigma_n^2\right) \quad (19)$$

Utilizing (3) for the $\frac{1}{N}\sum_{i\neq k}\left(1-(\hat{x}_{i,e}^{(n-1)})^2\right)+\sigma_n^2$, we ended up to,

$$\mathcal{E}\left[\omega_k^{(n)}\omega_k^{(j)}\right]=\frac{\tilde{\sigma}_k^{(n)}\lambda_{1,k}^{(j)}}{2}. \quad (20)$$

To reach the final step above, (2) was utilized. To determine $\rho$, we substitute (20) into (15) and utilizing (13) to get,

$$\rho_k^{(n)}=\frac{\lambda_{3,k}^{(n-1)}}{\lambda_{1,k}^{(n)}}. \quad (21)$$

Utilizing the values of $\rho_k^{(n)}$, we can compute $\delta_1$ and $\delta_2$,

$$\delta_{1,k}^{(n)}=\frac{\lambda_{1,k}^{(n)}-\rho_k^{(n)}\lambda_{2,k}^{(n)}}{\lambda_{1,k}^{(n)}\left(1-(\rho_k^{(n)})^2\right)}=1 \quad (22)$$

$$\delta_{2,k}^{(n)}=\frac{\lambda_{2,k}^{(n)}-\rho_k^{(n)}\lambda_{1,k}^{(n)}}{\lambda_{2,k}^{(n)}\left(1-(\rho_k^{(n)})^2\right)}=0. \quad (23)$$

This is a very interesting finding. At this point the reader is reminded that $\delta_1$ is the current data scaling factor, and that $\delta_2$ is the delayed data scaling factor: in the case of extrinsic feedback, we should never use a weighted delay filter and should only ever consider the most recent data. This result verifies the simulation results for extrinsic feedback.

### *3.2. LLRs with Extrinsic Feedback*

The theory in Section 2.1 shows that when extrinsic feedback is being used, a weighted delay filter should not be utilized. At first sighting, these conclusions may seem counter intuitive given the significant performance gains that have been seen for the APP feedback case. We now present an insight into the reasons behind this finding, based on [29]. Consider the two LLR sequences:

$$L_k^{(n)}=\frac{\left(l_k^{(n)}\right)^2}{2}x_k+l_k^{(n)}\omega_k^{(n)} \quad (24)$$

$$\Lambda_k^{(n-1)}=\frac{\left(\lambda_k^{(n-1)}\right)^2}{2}x_k+\lambda_k^{(n-1)}\varphi_k^{(n-1)}. \quad (25)$$

We may model $\varphi_k^{(n-1)}$ as follows:

$$\varphi_k^{(n-1)} = \rho\omega_k^{(n)} + \sqrt{1-\rho^2 n} \quad (26)$$

where $n \sim N(0,1)$, $\mathcal{E}\left[\varphi_k^{(n-1)}\right] = 0$, $\mathcal{E}\left[(\varphi_k^{(n-1)})^2\right] = 1$ and $\mathcal{E}\left[\omega_k^{(n)}\varphi_k^{(n-1)}\right] = \rho$, then we have the following;

$$\Lambda_k^{(n-1)} = \frac{\left(\lambda_k^{(n-1)}\right)^2}{(l_k^{(n)})^2}\left(\frac{\left(l_k^{(n)}\right)^2}{2}x_k + \frac{\lambda_k^{(n-1)}\rho\omega_k^{(n)}\left(l_k^{(n)}\right)^2}{\left(\lambda_k^{(n-1)}\right)^2}\right) + \lambda_k^{(n-1)}\sqrt{1-\rho^2 n}. \quad (27)$$

Now by utilizing (21),

$$\Lambda_k^{(n-1)} = \frac{\left(\lambda_k^{(n-1)}\right)^2}{(l_k^{(n)})^2}L_k^{(n)} + \lambda_k^{(n-1)}\sqrt{1-\rho^2 n}. \quad (28)$$

In (28), we see that the signal $\Lambda_k^{(n-1)}$, is simply a scaled noisy version of $L_k^{(n)}$. $\Lambda_k^{(n-1)}$ is not providing any extra information. Hence, there is nothing to be gained by combining the current signal with the signal from the previous iteration.

### *3.3. The APP Feedback Case*

It is seen in Section 2.2 that in the case of extrinsic feedback, there is nothing to be gained from the use of a weighted delay filter. However a large performance gains is observed when a weighted delay filter is used with APP feedback. The following questions are raised:

- What are the significant differences between the extrinsic feedback case and the APP feedback case?
- How does the analysis in Section 2.2 differ when APP feedback is used?

In the case where APP feedback is used, we have $\mathcal{E}[x_k\hat{x}_j^{(n-1)}] \neq 0$; as correlations have been allowed to build. As a result, the expressions for the mean and variance at the PIC output, no longer hold. The analysis provided is therefore invalid for APP data. To investigate this further, it is necessary to first determine the statistics at the PIC output.

#### 3.3.1. The PIC Output Mean with APP Feedback

Before we begin, it is necessary to clarify some terms relating to the receiver in Figure 4. $L_a$ is an APP LLR output from the SISO, $L_e$ is the extrinsic output from the SISO decoder, and $L_p$ is the LLR input to the SISO decoder. $L_a$, $L_e$ and $L_p$ are related as:

$$L_a = L_e + L_p \quad (29)$$

The above LLRs may all be converted to signal estimates via use of the tanh() function. Therefore, for user $k$ at iteration $n$ we have:

$$\hat{x}_{k,e}^{(n)} = \tanh\left(\frac{L_{k,e}^{(n)}}{2}\right) \quad (30)$$

$$\hat{x}_{k,a}^{(n)} = \tanh\left(\frac{L_{k,a}^{(n)}}{2}\right) \quad (31)$$

$$\hat{x}_{k,p}^{(n)} = \tanh\left(\frac{L_{k,p}^{(n)}}{2}\right) \tag{32}$$

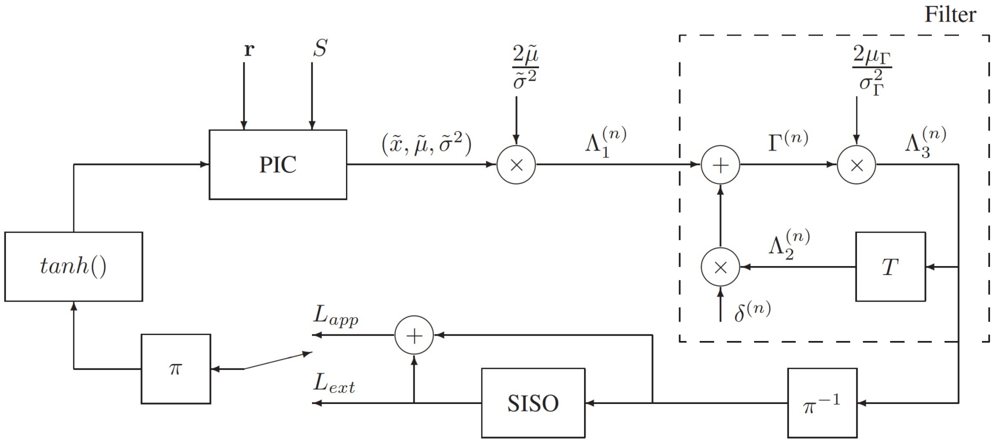


**Figure 4.** Receive circuit diagram.

**Another estimate, the residue**: Consider $\hat{x}_{k,a}^{(n)}$, statistical parameters for the case of extrinsic data have already been determined, so it would be advantageous to express the APP estimate in terms of extrinsic feedback as follows:

$$\begin{aligned}
\hat{x}_{k,a}^{(n)} &= \tanh\left(\frac{L_{k,ext} + L_{k,p}}{2}\right) \\
&= \frac{\tanh\left(\frac{L_{k,ext}}{2}\right) + \tanh\left(\frac{L_{k,p}}{2}\right)}{1 + \tanh\left(\frac{L_{k,ext}}{2}\right)\tanh\left(\frac{L_{k,p}}{2}\right)} \\
&= \hat{x}_{k,e} + \frac{\left(1 - \tanh^2\left(\frac{L_{k,ext}}{2}\right)\right)\tanh\left(\frac{L_{k,p}}{2}\right)}{1 + \tanh\left(\frac{L_{k,ext}}{2}\right)\tanh\left(\frac{L_{k,p}}{2}\right)} \\
&= \hat{x}_{k,e} + \frac{\left(1 - \hat{x}_{k,e}^2\right)\hat{x}_{k,p}}{1 + \hat{x}_{k,e}\hat{x}_{k,p}}.
\end{aligned} \tag{33}$$

The APP feedback is the sum of an extrinsic feedback term $\hat{x}_{k,e}$ and a *residue* term $\hat{x}_{k,R}$.

$$\hat{x}_{k,a} = \hat{x}_{k,p} + \hat{x}_{k,R} \tag{34}$$

$$\hat{x}_{k,R} = \frac{\left(1 - \hat{x}_{k,e}^2\right)\hat{x}_{k,p}}{1 + \hat{x}_{k,e}\hat{x}_{k,p}}. \tag{35}$$

The PIC output conditional mean is evaluated as follows:

$$\begin{aligned}
\tilde{\mu}_{k,a}^{(n)} &= \mathcal{E}\left[\hat{x}_{k,a}^{(n)}|_{x_k=1}, \hat{x}_{j,e}^{(n-1)}, \hat{x}_{j,a}^{(n-1)}, \hat{x}_{k,p}^{(n-1)} \forall j\right] \\
&= 1 + \mathcal{E}\left[\sum_{j \neq k} c_{jk}\left(x_j - \hat{x}_{j,e}^{(n-1)} - \hat{x}_{j,R}^{(n-1)}\right)|_{x_k=1}, \hat{x}_{j,e}^{(n-1)}, \hat{x}_{j,a}^{(n-1)}, \hat{x}_{k,p}^{(n-1)} \forall j\right] \\
&= 1 - \mathcal{E}\left[\sum_{j \neq k} c_{jk}\hat{x}_{j,R}^{(n-1)}|_{x_k=1}\right]
\end{aligned} \tag{36}$$

We can see from (36) that the conditional mean is no longer unity. In addition, calculation of the conditional mean is not straight-forward, as one is needed to determine an expectation where there are correlated terms in both the numerator and denominator, since:

$$\mathcal{E}\left[x_k \sum_{j\neq k} c_{jk}\hat{x}_{j,R}^{(n-1)}|_{x_k=1}\right] = \mathcal{E}\left[x_k \sum_{j\neq k} c_{jk}\frac{\left(1-(\hat{x}_{j,e}^{(n-1)})^2\right)\hat{x}_{j,p}^{(n-1)}}{1+\hat{x}_{j,e}^{(n-1)}\hat{x}_{j,p}^{(n-1)}}|_{x_k=1}\right]. \tag{37}$$

The complexity involved in the calculation of conditional mean implies that the computation of variance will also be difficult. Variance is evaluated in the Section 3.3.2.

### 3.3.2. The PIC Output Variance with APP Feedback

We now investigate the PIC output variance for the APP feedback case, using the model for residual feedback. We have,

$$\mathcal{E}\left[(\hat{x}_{k,a}^{(n)})^2\right] = \mathcal{E}\left[\left(x_k + \sum_{j\neq k} c_{jk}(x_j - \hat{x}_{j,a}^{(n-1)}) + z_k\right)^2 |_{x_k=1,\hat{x}_{j,e}^{(n-1)},\hat{x}_{j,a}^{(n-1)},\hat{x}_{j,p}^{(n-1)}\forall j}\right]. \tag{38}$$

For the sake of notational elegance, we remove the conditional information in the following.

$$\mathcal{E}\left[(\hat{x}_{k,a}^{(n)})^2\right] = \mathcal{E}\left[\left(x_k + \sum_{j\neq k} c_{jk}(x_j - \hat{x}_{j,e}^{(n-1)}) + \sum_{j\neq k} c_{jk}\hat{x}_{j,R}^{(n-1)} + z_k\right)^2\right]. \tag{39}$$

After some simplifications, i.e., $\mathcal{E}[z_k] = 0$, $x_k^2 = 1$ and $\mathcal{E}[z_k^2] = \sigma_n^2$, we get,

$$\begin{aligned}(\tilde{\sigma}_{k,a}^{(n)})^2 = 1 + \frac{1}{N}\sum_{j\neq k} c_{jk}\left(1-(\hat{x}_{j,e}^{(n)})^2\right) + \mathcal{E}\left[\left(\sum_{j\neq k} c_{jk}\hat{x}_{j,R}^{(n-1)}\right)^2\right] \\ -2\mathcal{E}\left[x_k\sum_{j\neq k} c_{jk}\hat{x}_{j,R}^{(n-1)}\right] \\ -2\mathcal{E}\left[\left(\sum_{j\neq k} c_{jk}(x_j - \hat{x}_{j,e}^{(n-1)})\right)\left(\sum_{j\neq k} c_{jk}\hat{x}_{j,R}^{(n-1)}\right)\right] - (\tilde{\mu}_{k,a}^{(n)})^2.\end{aligned} \tag{40}$$

Similarly to the PIC conditional mean for APP feedback, calculation of the expectations for $(\tilde{\sigma}_{k,a}^{(n)})^2$ is very difficult. Specifically, determining expectations of $\hat{x}_{j,R}^{(n)}$ conditioned on $x_k = 1$, given that $\hat{x}_{j,R}^{(n)}$ has terms in the denominator that are correlated with $x_k$, remains a challenge.

### 3.3.3. LLRs with APP Feedback

In Section 2.2 it was shown that when extrinsic feedback is utilised at the SISO, LLRs from previous iterations are simply a scaled noisy version of the current LLRs, hence nothing is to be gained using previous data. At this point we address the question "how does the analysis of Section 2.2 differ for the APP case?". Following the same line of reasoning shown in Section 2.2, we start with the mean and variance at the PIC output, which for the APP case are given in (36) and (40). We now make the LLR conversion for the APP case utilizing (10) and (11) and then can group similar terms and conclude:

$$\lambda_{1,k,a}^{(n)} = 2\frac{\tilde{\mu}_{k,a}^{(n)}}{(\tilde{\sigma}_{k,a}^{(n)})^2} \tag{41}$$

$$\omega_{k,a}^{(n)} = \frac{\sqrt{\tilde{\mu}_{k,a}^{(n)}}}{\tilde{\sigma}_{k,a}^{(n)}}\left(\sum_{j\neq k} c_{jk}(x_j - \hat{x}_{j,a}^{(n-1)}) + z_k\right). \tag{42}$$

It is noted that since $\tilde{\mu}_{k,a}^{(n)}$ is not unity, our terms in (41) and (42) differ from the extrinsic case. If the consistency property of Section 2.2 is maintained, the notion of APP feedback does not affect the expression for $\Lambda_{3,k,a}^{(n)}$, however we must substitute APP signal terms where appropriate. The expression for the $\varphi_{k,a}^{(n)}$ and the correlation factor $\rho_{k,a}^{(n)}$ are evaluated as;

$$\varphi_{k,a}^{(n)} = \frac{\lambda_{1,k,a}^{(n)}\omega_{k,a}^{(n)}\delta_{1,k}^{(n)} + \sum_{i=1}^{n-1}\prod_{j=i+1}^{n}\lambda_{1,k,a}^{(i)}\omega_{k,a}^{(i)}\delta_{1,k}^{(i)}\delta_{2,k}^{(j)}}{\lambda_{3,k,a}^{(n)}} \quad (43)$$

$$\rho_{k,a}^{(n)} = \mathcal{E}\left[\frac{\lambda_{1,k,a}^{(n-1)}\omega_{k,a}^{(n)}\omega_{k,a}^{(n-1)}\delta_{1,k}^{(n-1)} + \sum_{i=1}^{n-2}\prod_{j=i+1}^{n-1}\lambda_{1,k,a}^{(i)}\omega_{k,a}^{(n)}\omega_{k,a}^{(i)}\delta_{1,k}^{(i)}\delta_{2,k}^{(j)}}{\lambda_{3,k,a}^{(n-1)}}\right] \quad (44)$$

A Table 3 is given with system and maximum number of users. Table 3 gives a brief summary of the type of performance improvements that can be achieved via the application of MRC to a coded multiuser receiver. It is given with a comparison for a maximum number of users in case of PIC for various systems and probabilistic data association (PDA) system for MRC and truncated MRC (TMRC).

**Table 3.** Maximum Users for MRC Variants, $N$ = 8, $E_b/N_o$ = 5 dB, (5,7) code.

| Systems | Maximum Users | Systems | Maximum Users |
|---|---|---|---|
| PIC with no Combining | 9 | PDA with no Combining | 16 |
| PIC with Full MRC | 13 | PDA with Full MRC | 17 |
| PIC with TMRC | 15 | PDA with TMRC | 17 |

## 4. Results and Discussion

### *4.1. Behaviour of PIC Mean for APP Feedback*

Figure 5 is evaluated for the given parameters; $K/N = 20/10$, (5,7) encoding, $L = 1000$, $E_b/N_0 = 6$ dB. In Figure 5, we see the results for the sample mean $\tilde{\mu}_{k,a}^{(n)}$ at the PIC output plotted against iterations for a system with the (5,7) encoding scheme. For this data random interleaving was used and the data was averaged over 1000 experiments. Plotted on the same graph are the expectation of the residue multiplied by the transmitted data $\mathcal{E}\left[x_k \sum_{j\neq k} c_{kj}\hat{x}_{j,R}^{(n-1)}\right]$ and the sum of the sample mean and expectation of the *residue* multiplied by the transmitted data $\tilde{\mu}_{k,a}^{(n)} + \mathcal{E}\left[x_k \sum_{j\neq k} c_{kj}\hat{x}_{j,R}^{(n-1)}\right]$ the same conditions. The *residue* expectation was calculated using knowledge of the transmitted data. We note from the Figure 5 that when the sample mean and the *residue* are summed, the result is near unity; confirming that the *residue* is a key difference between the PIC output statistics for the APP-feedback and extrinsic-feedback cases. This result also confirms that the expectation in (36) is the key problem to solve when considering the PIC output mean for extrinsic feedback.

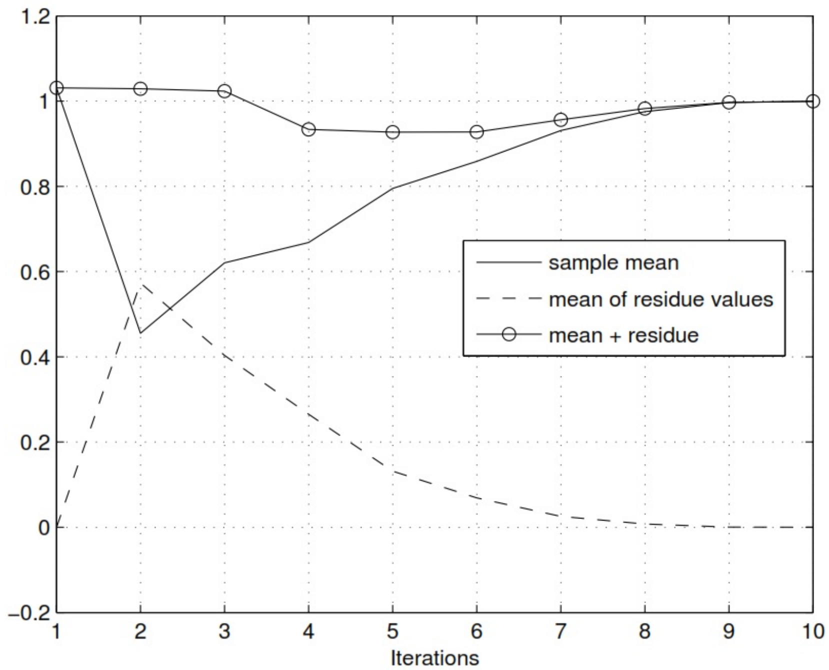


**Figure 5.** Mean and residue behaviour, $K/N = 20/10$, $L = 1000$, $E_b/N_0 = 6$ [dB].

We note from Figure 5 that the deviation of the sum of sample mean and residue from unity. It is also revealed that when calculating the mean based on extrinsic parameters for the case where APP data is passed in the system, that:

- This output mean drops below unity.
- This output mean value is the sum of the APP mean and the residue.

We therefore account for the deviation of the mean from unity by the fact that we are calculating an extrinsic mean in an APP system.

### 4.2. Behaviour of the Residue

Figure 6 is evaluated over a varying number of users for $K$, $N = 10$, $E_b/N_0 = 0$ [dB]. In Figure 6, it is noticed that the behaviour of the residue after two iterations for various system loads. The system under test had a processing gain of 10 dB, and the number of users was varied from 1 to 25. The SNR was set at 0 dB. Otherwise, the system parameters were the same as described in Section 4.1.

We see in Figure 6 that while the residue may be trivial at small loads, the residue builds to the substantial value of nearly 0.4 at a reasonably modest load of $K/N = 15/10$, and continues to grow as the load increases. Figure 6 demonstrates that the residue is significant for high system loads and therefore must be taken into account when determining PIC output statistics for systems with APP feedback.

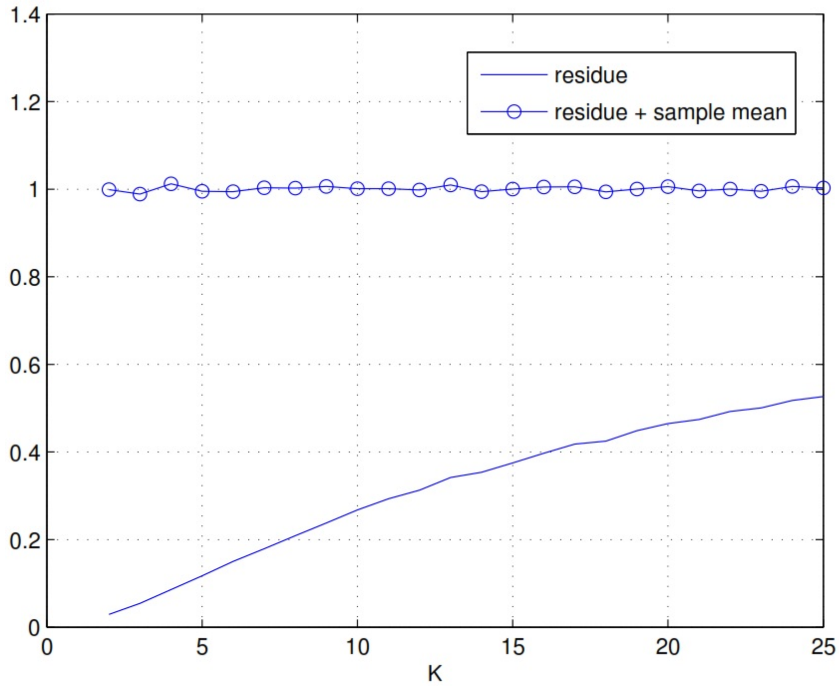


**Figure 6.** Residue Behaviour with $K$, $N = 10$, $E_b/N_0 = 0$ [dB].

Figure 7 is evaluated for a low SNRs, it is seen that the weighted delay filter (WDF) and the MRC filter shows the identical performance. It is also observed that WDF enters the waterfall region earlier with single user bound for a low value of SNR as compared to MRC filter. Further noted that WDF outperforms MRC giving a value of 0.8 dB.

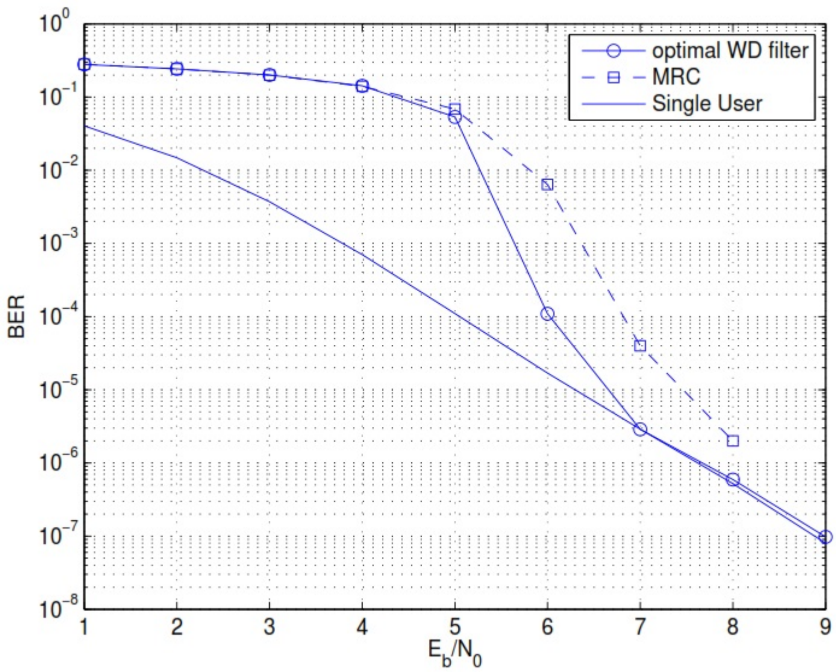


**Figure 7.** Weighted delay and MRC filter performance vs. SNR, $K/N$ = 20/10, $L$ = 1000, Iterations = 10.

Another simulation result is evaluated in Figure 8 considering the different number of users, i.e., 15 and 20. It was seen from the results that convergence for the unfiltered case is unlikely happen for 15 users, while for the filtered case, it is unlikely happen for 21 users.

The conclusion can be made that for a given values of SNR = 6 dB, and a processing gain of 10 dB, the system load increases to approximately 42% using the weighted delay filtering.

Figure 7 shows the simulation for the optimal filter's system capacity. The results show that WD performs better for larger SNR values. However, MRC and WD show the same performance for lower SNR values. Similarly, Figure 8 is plotted to show the contribution from different user points of view. It shows that we can see the system capacity achieved using the WD filtering for a fixed value of SNR, varying the iterations.

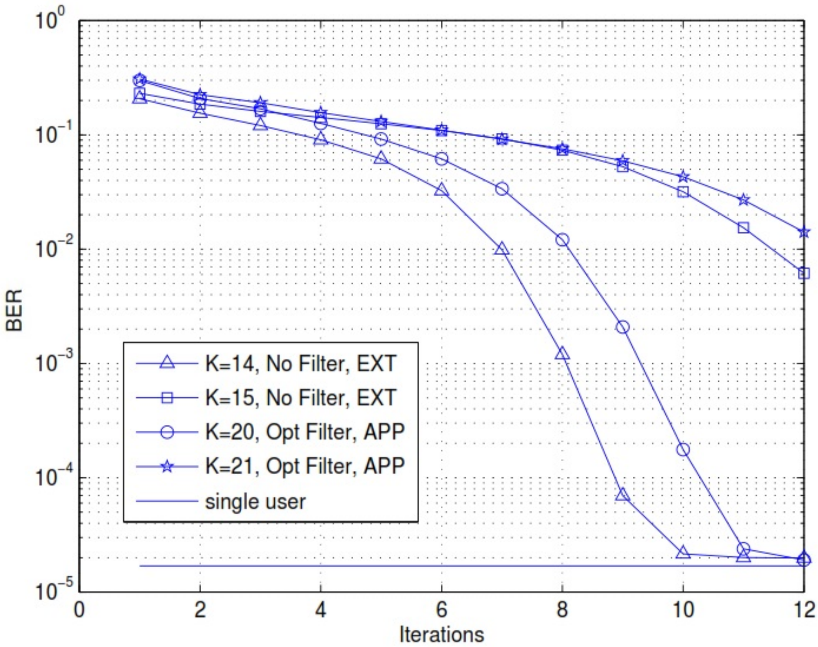


**Figure 8.** System Capacity, N = 10, $E_b/N_0$ = 6 dB, $L$ = 1000.

## 5. Conclusions

The iterative coded CDMA receivers with extrinsic feedback will not benefit from use of the weighted delay filter. Indeed, should a weighted delay filter be implemented, the filter will pick a scaling factor of $\delta_2 = 0$, and automatically ignore data from previous cycles. The reason for this behaviour is outlined in Section 2.2, where it is shown that when extrinsic feedback is used data from previous cycles is simply a scaled, noisy version of the data from the current cycle and hence no benefit is to be realised from its utilisation. Iterative coded CDMA receivers with APP feedback however will see a dramatic performance improvement via the use of a weighted delay filter, and will significantly outperform a traditional pure extrinsic feedback implementation. In an APP implementation, the optimal weighting factors are not 0 and 1. The key difference between the two scenarios is the residual feedback in the APP implementation. The existence of the residual feedback has a significant effect on the statistics at the output of the PIC, and as shown in Sections 3.2 and 3.3, and makes calculation of these statistics very challenging. The residue term is shown to be a key difference between the APP and extrinsic feedback systems with respect to the benefits to be had by weighted delay filtering. In Section 3.3 it is shown that the notion of "previous data being a scaled, noisy version of data from the current cycle" does not hold with APP feedback because of the residue information. Empirical results are given in Section 4.1 which verify that the residue is the key difference between the PIC statistics for APP and extrinsic data. It is shown in Section 4.2 that the residue is indeed substantial and hence, cannot be ignored in any rigorous analysis of an APP feedback system.

- The iterative receivers with suboptimal components may not always yield the best results.
- For a coded CDMA system with extrinsic feedback, no benefit will be realised by weighted delay filtering.

- Determining the PIC output statistics analytically in the case of APP feedback remains an open problem.
- The conclusion can be made that for a given values of SNR = 6 dB, and a processing gain of 10 dB, the system load increases to approximately 42% using the weighted delay filtering.
- Further noted that WDF outperforms MRC giving a value of 0.8 dB.

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