



Article In-Orbit Reliability Evaluation of Space TWTA Based on Copula Function and Bivariate Hybrid Stochastic Processes

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Abstract: Currently, it is still a challenge to study the degradation mechanisms of the space traveling wave tube amplifier (TWTA) with no failure and small sample tests. Given that the Copula functions are used to describe the correlation of multiple performance characteristics, this paper develops a bivariate hybrid stochastic degradation model to evaluate the in-orbit reliability of TWTA. Firstly, based on the impact analysis of the life of TWTA, helix current and anode voltage are selected as the performance degradation parameters. Secondly, stochastic processes with random effects based on the one-dimensional Wiener process and Gamma process are applied to describe the degradation of TWTA's helix current and anode voltage, respectively, and the corresponding marginal distribution function is obtained. Then, the Copula function is utilized to describe the correlation between two different performance parameters of TWTA. Meanwhile, this paper also proposed a two-step method to estimate the reliability level of TWTA based on its in-orbit telemetry data through a two-step method, which contains a Markov Chain Monte Carlo (MCMC) algorithm and a maximum likelihood estimation (MLE) algorithm. Besides, the Bayes-Bootstrap sampling method is also used to improve the evaluation accuracy to overcome the defect of an in-orbit small sample of TWTA. Finally, a TWTA degradation case with a set of telemetry data is carried out, and the results show that the method proposed in this paper is more applicable and more accurate than other methods.

Keywords: in-orbit reliability; space TWTA; Wiener process; Gamma process; random effects; Copula function

1. Introduction

Space TWTA is a key component of spacecraft transponders and spacecraft transmitters. It is mainly used in the payload of military communication satellites, navigation and positioning satellites, microwave remote sensing satellites, and so on, to realize the function of microwave power amplification [1]. As the core component of the satellite key subsystem, space TWTA is particularly important in aerospace applications, and its life directly affects the satellite's service life and in-orbit stable operation. Therefore, it is of great significance to study the in-orbit life of TWTA for the Prognostics and Health Management (PHM) of satellites.

In general, commonly used reliability prediction methods are divided into two main categories: failure mechanism-driven methods and data-driven methods. The failure mechanism-driven model is used to establish a degradation model through the relationship between the failure modes selected by the internal failure mechanism of the product and a large amount of historical operation data. However, the data-driven methods are used to predict the probability density function (PDF) and cumulative distribution function (CDF) of the product's life through its historical failure data and the existing observation



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). data. These methods do not rely on the operating principle of the product and the inherent failure mechanism, mainly including neural networks, support vector machines, state-space models, and stochastic processes. However, due to the high manufacturing cost and complex failure mechanisms of space TWTA, a single-cathode degradation model [2] cannot accurately reflect the degradation of TWTA, and it is also difficult to establish the physical failure model of the entire tube. Therefore, there are several difficulties in applying the prediction methods based on the failure mechanism models in the engineering application of the reliability evaluation of the TWTA. Performance degradation-based methods are one of the essential data-driven methods based on the fact that the hidden life information is reflected by one or multiple performance characteristics, and they have been recognized as an effective and important approach for the reliability evaluation of the high-reliability and long-life products. They also provide a reference to carry out the research on PHM of TWTA.

Many research results and theories have been obtained for reliability evaluation based on performance degradation methods, such as degenerate trajectory models, degenerate distribution models, and stochastic processes [3–8]. In comparison with the first two models, stochastic process-based methods are more in line with engineering practice because they can describe the uncertainties of performance degradation and the individual differences between different units. In engineering applications, most scholars mainly focus on one-dimensional stochastic processes. However, there are multiple degradation trajectories or components for a complex system. It may make the product or equipment reliability analysis fail to obtain relatively accurate results or even lead to rigorous conclusions [9], taking no consideration of their correlation with each other. Many experts and scholars have introduced the Copula function [10] to deal with relevant issues, for instance, the heavy machine tool [11], the lithium-ion battery [12], the rubidium atomic clock [13], and to study the multiple-parameter degradation modeling of complex systems and products because of the characteristics of its simple structure and accurate description of the correlation between failure modes and random variables, and have achieved fruitful research results in recent years. For example, Jiang et al. [14] used the Copula function to establish the joint PDF between multi-dimensional random variables for complex and multi-dimensional-related mechanical systems and thus proposed a system reliability analysis method. Liu et al. [15] proposed a life prediction method based on the Copula function under multiple degradations to predict the remaining life of the satellite momentum wheel and figured out a method to select the Copula function by the AIC criterion. Pan et al. [16] used the Wiener process to describe each degradation process separately, assuming that different parameters are correlated, used the Copula function to establish an accelerated degradation model, and applied the MCMC algorithm to estimate the unknown parameters in the model, then deduced the reliability function of the product under the normal stress. Zhang et al. [17] proposed a method of bearing the remaining life based on the bivariate Wiener process and analyzed the correlation between the two performance indices by the Copula function. Guangze et al. [18] proposed a multi-degradation modeling method based on a Wiener process and Copula function and took IGBT as an example to verify the model in engineering. However, there are only a few results of reliability correlation studies in the existing literature, and their research chiefly focuses on the field of reliability evaluation, assuming that the multi-parameter of the product obeys the same stochastic process, ignoring the inconsistent degradation laws of different performance parameters, which may lead to inaccurate assessments.

Up to now, the life prediction of TWTA has mainly been carried out based on the cathode's physics of failure. However, the working state of TWTA is affected by temperature, radiation, and other factors in-orbit. Some component performances may degrade slowly, leading to multiple failure modes of the product, and different failure modes may be related. Therefore, it is difficult to predict the life of the TWTA by using a one-dimensional degradation model which only has a deterministic feature. In this paper, we will use bivariate hybrid stochastic processes and the Copula function to perform the reliability

analysis of TWTA. The layout of the paper is organized as follows. Section 2 is devoted to describing the performance degradation of TWTA and identifies the characteristic parameter. In Section 3, a model of performance degradation based on one-dimensional stochastic processes with random effects is introduced. In Section 4, an introduction to the Copula function, comprehensive determination, and modeling with multi-degradation are discussed in detail. Section 5 introduces the parameter estimation of a two-step method. A practical TWTA degradation case is used to validate the method proposed in Section 6, and we present conclusions and future research in Section 7.

2. Performance Degradation Analysis of Space TWTA

The space TWTA is the most widely used vacuum electronic device consisting of a traveling wave tube (TWT) and an Electronic Power Conditioner (EPC). The TWT includes a slow-wave structure, an electron gun, RF input and output couples, a magnetic focusing system, and a collector [19], and its structure is shown in Figure 1. Engineering practices indicate that the main reasons for its failure are insufficient cathode emission ability and the deterioration of the vacuum degree. Through in-depth research on the failure mechanisms of the TWTA, helix current and anode voltage are selected as the in-orbit performance characteristic parameters, and the data are used to study the performance degradation of the TWTA.



Figure 1. Constructional structure of the TWT.

2.1. Analysis of Helix Current Degradation Characteristic

The helix current of TWTA is the current that is not collected by the collector but is intercepted by the helix. Its long-term stability is affected by the life of the cathode, the reliability of electron gun assembly, the change of heater current, the change characteristics of the internal vacuum, the gain stability of TWTA, the sensitivity of the whole TWTA to ambient temperature and other parameters, and so on. In addition, the adaptability of the EPC to the performance of the TWT also affects the subtle changes in the helix current, especially the stability of the output voltage of the TWTA and the adjustment ability influence of the anode voltage on the cathode emission current of the TWTA within the lifecycle. The helix current is a key comprehensive index that characterizes the functional properties of the TWTA and shows a non-monotonic change.

2.2. Analysis of Anode Voltage Degradation Characteristic

From above, the vacuum degree in the tube and the degradation degree of the cathode are the main factors affecting the life of the space TWTA. When the vacuum degree in the tube is good, the degradation degree of the cathode directly determines the life of the space

TWTA. From the output power $P_0 = I_k \times V_H \times \eta_e$, where I_k represents the cathode emission current, V_H is the helix voltage, and η_e means the electron efficiency, we can find that the decrease of the cathode emission current is the main reason for the decline of the output power of the TWTA with the increase in the working time. To protect the cathode emission current from falling too fast to affect the drop of the output power of the TWTA, it is usually used to improve the modulated anode voltage to increase the cathode emission current. Within the modulation range, the anode voltage increases monotonically with its rising rate and amplitude, mainly characterizing the degradation of the cathode performance of the TWTA.

3. One-Dimensional Degradation Models Based on Stochastic Processes

Based on the analysis of the performance degradation of the TWTA, the Wiener process and Gamma process are used to model the helix current and anode voltage, respectively.

3.1. Degradation Models Based on Univariate Wiener Process with Random Effects

The Wiener process [20] is generally suitable for modeling continuous and fluctuating degradation products. For a linear Wiener process X(t), X(t) has independent increments and is defined as the performance value of the product at a time t, which satisfies:

$$X(t) = X(0) + \lambda t + \sigma B(t)$$
(1)

Among them, X(0) represents the initial value of the product, generally X(0) = 0, and B(t) is the standard Brownian motion. λ and σ are unknown parameters, where λ is the drift parameter while σ is the diffusion parameter. Furthermore, the increments $\Delta X(t)$ follow a standard normal distribution, that is, $\Delta X(t) \sim N(\lambda \Delta t, \sigma^2 \Delta t)$.

Suppose the failure threshold of the product's parameter is l, and the life $T = \inf\{t | X(t) \ge l\}$ is defined as the first arrival time with X(t) degradation to the failure threshold l. According to the statistical characteristics of the Wiener process, the life T obeys the inverse Gaussian distribution, that is:

$$T \sim IG\left(\frac{l}{\lambda}, \frac{l^2}{\sigma^2}\right) \tag{2}$$

Therefore, the PDF and CDF of the life *T* are:

$$f(t) = \frac{l}{\sqrt{2\pi\sigma^2 t^3}} exp\left(-\frac{(l-\lambda t)^2}{2\sigma^2 t}\right)$$
(3)

$$F(t) = 1 - \Phi\left(\frac{l - \lambda t}{\sigma\sqrt{t}}\right) + exp\left(\frac{2\lambda l}{\sigma^2}\right) \left[1 - \Phi\left(\frac{l + \lambda t}{\sigma\sqrt{t}}\right)\right]$$
(4)

The reliability function corresponding to a one-dimensional degradation parameter which is described by the Wiener process can be expressed as:

$$R(t) = 1 - F(t) = \Phi\left(\frac{l - \lambda t}{\sigma\sqrt{t}}\right) - exp\left(\frac{2\lambda l}{\sigma^2}\right) \left[1 - \Phi\left(\frac{l + \lambda t}{\sigma\sqrt{t}}\right)\right]$$
(5)

Assume that there are individual differences between samples in the process of manufacturing and assembling, let the drift parameter, λ , be a random variable that describes random effects following a normal distribution [21], that is, $\lambda \sim N(\mu, \sigma_{\lambda}^2)$. Then, the distribution function of the Wiener process with random effects follows the normal distribution with the mean, μt , and variance, $(\sigma^2 t + \sigma_{\lambda}^2)t$.

The CDF of the product that takes into consideration random effects can be calculated by the following formula:

$$F_{\lambda}(t) = \Phi\left(-\frac{l-\mu t}{\sqrt{\sigma^2 t + \sigma_{\lambda}^2 t^2}}\right) + exp\left(\frac{2\mu l}{\sigma^2} + \frac{2\sigma_{\lambda}^2 l^2}{\sigma^4}\right) \Phi\left(-\frac{2\sigma_{\lambda}^2 l t + \sigma^2 (l+\mu t)}{\sigma^2 \sqrt{\sigma^2 t + \sigma_{\lambda}^2 t^2}}\right)$$
(6)

Thus, the reliability function considering random effects can be deduced as follows:

$$R_{\lambda}(t) = 1 - F_{\lambda}(t)$$

$$= \Phi\left(\frac{l - \mu t}{\sqrt{\sigma^2 t + \sigma_{\lambda}^2 t^2}}\right) - exp\left(\frac{2\mu l}{\sigma^2} + \frac{2\sigma_{\lambda}^2 l^2}{\sigma^4}\right) \Phi\left(-\frac{2\sigma_{\lambda}^2 l t + \sigma^2 (l + \mu t)}{\sigma^2 \sqrt{\sigma^2 t + \sigma_{\lambda}^2 t^2}}\right)$$
(7)

3.2. Degradation Models Based on Univariate Gamma Process with Random Effects

Different from the Wiener process, whose degradation path is not necessarily monotone, the Gamma process [22] not only imitates the monotonically increasing degradation process but also incorporates meaningful model parameters, random effects, and uncertainties. In this paper, we assume that the degradation path of a product's parameter satisfies the Gamma process, and the degradation data of this parameter are G(t) at a moment, t, with a shape parameter, α , and a scale parameter, β , which is a stochastic process satisfying:

- (1) G(0) = 0
- (2) $\{G(t), t \ge 0\}$ is stable and has independent increments
- (3) The increment at any time follows a Gamma distribution, namely, $\Delta G(t) = G(t + \Delta t) G(t) \sim Ga(\alpha \Delta t, \beta)$

where α and β are unknown parameters, and the shape parameter, α , describes the influence of the external environment and other factors on the product performance, which is used to describe the common attributes of all samples, while the scale parameter β describes the random effects, such as the influence of manufacturing techniques and raw materials, which is used to describe the individual differences between samples. According to the definition of the Gamma process, its PDF is:

$$f(g|\alpha,\beta) = \frac{(\beta)^{\alpha t}}{\Gamma(\alpha t)} g^{\alpha t-1} \cdot e^{-\beta g}, g > 0$$
(8)

where $\Gamma(\alpha) = \int_0^\infty g^{\alpha-1} e^{-g} dg$ is a Gamma distribution.

Considering the individual differences between the samples, random effects are introduced into the scale parameter β . To simplify the analysis, we usually assume that β follows a Gamma distribution [23], that is, $\beta \sim Gamma(\kappa, r)$, whose PDF is:

$$g(\beta) = \frac{r^{\kappa}}{\Gamma(\kappa)} \beta^{\kappa-1} \cdot e^{-r\beta}, \beta > 0$$
(9)

Then, Equation (9) can be deduced by applying the total probability formula of continuous random variables as follows:

$$f(g|\alpha) = \frac{\Gamma(\alpha t + \kappa)}{\Gamma(\alpha t)\Gamma(\kappa)} \cdot \frac{(r)^{\kappa} g^{\alpha t - 1}}{(r + g)^{\alpha t + \kappa}} = \frac{\Gamma(\frac{2\alpha t + 2\kappa}{2})}{\Gamma(\frac{2\alpha t}{2})\Gamma(\frac{2\kappa}{2})} \cdot \frac{g^{\frac{2\alpha t}{2} - 1}}{(r)^{\frac{2\alpha t}{2}}(1 + \frac{g}{r})^{\frac{2\alpha t + 2\kappa}{2}}}$$
(10)

For a fixed time, let $Z(t) = \frac{\kappa G(t)}{\alpha r t}$, then Equation (10) can be rewritten as:

$$f(\mathbf{z}|\boldsymbol{\alpha}) = \frac{\Gamma(\frac{2\alpha t + 2\kappa}{2})}{\Gamma(\frac{2\alpha t}{2})\Gamma(\frac{2\kappa}{2})} \left(\frac{\alpha t}{\kappa}\right)^{\frac{2\alpha t}{2}} \frac{z^{\frac{2\alpha t}{2} - 1}}{\left(1 + \frac{\alpha tz}{\kappa}\right)^{\frac{2\alpha t + 2\kappa}{2}}}$$
(11)

From Equation (11), it can be seen that the random variable Z(t) obeys an F-distribution with degrees of freedom $2\alpha t$ and 2κ , that is:

$$Z(t) \sim F_{2\alpha t, 2\kappa}(z) \tag{12}$$

According to the PDF $f(\mathbf{z}|\alpha)$ and the characteristic of the F-distribution, the distribution function of the degradation value G(t) can be derived from Equation (13):

$$F_{G(t)}(g) = P(G(t) \le g) = P\left(\frac{\kappa G(t)}{\alpha r t} \le \frac{\kappa g}{\alpha r t}\right) = P\left(Z(t) \le \frac{\kappa g}{\alpha r t}\right) = F_{2\alpha t, 2\kappa}\left(\frac{\kappa g}{\alpha r t}\right)$$
(13)

Suppose ω is the failure threshold of the product whose degradation path follows the Gamma process, the life, *T*, of the product is denoted by the time when the performance degradation *G*(t) exceeds its failure threshold for the first time, which is defined as:

$$T = \inf\{t | G(t) \ge \omega, t > 0\}$$
(14)

Considering that the Gamma process is a strictly monotonous stochastic process, the CDF of the product's life, *T*, is:

$$F_{\beta}(t) = P(T \le t) = P(G(t) \ge \omega) = 1 - F_{2\alpha t, 2\kappa} \left(\frac{\kappa \omega}{\alpha r t}\right)$$
(15)

Therefore, the reliability function of the product that takes into consideration random effects can be expressed as:

$$R_{\beta}(t) = 1 - F_{\beta}(t) = F_{2\alpha t, 2\kappa} \left(\frac{\kappa \omega}{\alpha r t}\right)$$
(16)

4. Modeling of Multi-Parameter Degradation

4.1. The Copula Functions

The Copula function is proposed by Sklar and is used to study the correlation between multiple variables. Sklar's theorem [24] pointed out that a joint distribution can be decomposed into n marginal distributions and a Copula function. This decomposition greatly simplifies the analysis of the correlation problem and the description of the degradation variable and model specification. Based on Sklar's theorem, the expression of the joint distribution function describing the correlation between variables is as follows:

$$H(x_1, x_2, \cdots, x_n) = C(u_1, u_2, \cdots, u_n; \theta)$$
(17)

where $H(x_1, x_2, \dots, x_n)$ is the joint distribution function of the variables x_1, x_2, \dots, x_n , $C(u_1, u_2, \dots, u_n; \theta)$ is the Copula function with the parameter θ , and u_1, u_2, \dots, u_n is the marginal distribution function corresponding to the variables x_1, x_2, \dots, x_n .

4.2. Types of Copula Function and Determination

There are many kinds of binary Copula functions, such as Frank, Gumbel, and Clayton. The three different widely used binary Copula functions are shown in Table 1, where μ and ν are the independent variables which are the marginal distribution functions of a product, such as $\mu = F_1(t)$ and $\nu = F_2(t)$, and θ values are the parameters describing the correlation between binary degradation performances.

For Frank and Clayton functions, the closer the θ value is to 0, the weaker the correlation between variables. On the other hand, for the Gumbel function, the closer the θ value is to 1, the stronger the correlation.

Copula Function	С(µ,ν;θ)	$c(\mu, u; oldsymbol{ heta})$		
Frank	$-rac{1}{ heta}lnigg(1+rac{(\mathrm{e}^{-\mu heta}-1)(\mathrm{e}^{- u heta}-1)}{\mathrm{e}^{- heta}-1}igg)$	$\frac{-\theta \mathrm{e}^{-(\mu+\nu)\theta}(\mathrm{e}^{-\theta}-1)}{\left((\mathrm{e}^{-\theta}-1)+(\mathrm{e}^{-\mu\theta}-1)(\mathrm{e}^{-\nu\theta}-1)\right)^2}$		
Gumbel	$e^{\left(-\left(\left(-ln\mu\right)^{\theta}+\left(-ln\mu\right)\theta\right)^{1/\theta}\right)}$	$\frac{C(\mu,\nu;\theta)(-lnC(\mu,\nu;\theta)+\theta-1)}{\mu\nu(-lnC(\mu,\nu;\theta))^{2-1/\theta}(-ln\mu)^{1-\theta}(-ln\nu)^{1-\theta}}$		
Clayton	$\left(\mu^{-\theta}+\nu^{-\theta}-1\right)^{-1/\theta}$	$\mu^{-\theta-1} \Big(\mu^{-\theta} + \nu^{-\theta} - 1 \Big)^{-1-1/\theta}$		

Table 1. The three different Copula functions.

Different Copula functions will lead to different results, and for making the prediction results more reasonable and accurate, it is necessary to determine the appropriate Copula function. As the Akaike Information Criterion (AIC) value is a quantitative method widely used to evaluate the goodness-of-fit, it can be applied to the determination of the Copula function. The AIC criterion is defined as:

$$AIC = -2lg(A) + 2m \tag{18}$$

where lg(A) is the log-likelihood function, and *m* is the number of estimated parameters of the model. If the AIC value is smaller, the selected model is more appropriate.

4.3. Reliability Modeling of Multiple Degradation Processes

Assume that the product has *n* multiple performance degradation indicators when any one of the degradation performances reaches the failure threshold specified, the product is considered to fail, and its failure time is *T*. At a time *t*, the performance degradation trajectories can be expressed as $X(t) = (X_1(t), X_2(t), \dots, X_n(t))$, and the corresponding failure threshold is $l = (l_1, l_2, \dots, l_n)$, then the product's life can be described as follows:

$$R(t) = P(T > t) = P(\min(T_1, T_2, \cdots, T_n) > t) = P(X_1(t) < l_1, X_2(t) < l_2, \cdots, X_n(t) < l_n)$$
(19)

When there are two performance degradation parameters, and taking the Frank function as an example, the reliability of the product can be shown as in Equation (20):

$$R(t) = P(T > t)$$

$$= P(\min(T_1, T_2, \dots, T_n) > t)$$

$$= 1 - P(T_1 \le t) - P(T_2 \le t) + P(T_1 \le t, T_2 \le t)$$

$$= R_1(t) + R_2(t) - 1 + C(F_1(t), F_2(t); \theta)$$

$$= R_1(t) + R_2(t) - 1 - \frac{1}{\theta} ln \left(1 + \frac{(e^{-\theta F_1(t)} - 1)(e^{-\theta F_2(t)} - 1)}{e^{-\theta} - 1} \right)$$
(20)

5. Parameter Estimation

Supposing there are *N* samples, and each sample is measured *M* times and $X_{k,i}(t_j)$ is the kth measured parameter of the ith sample of the product at a time t_j , where $i = 1, \dots, N, j = 1, \dots, M$, and k = 1, 2. Then the measurement data of the product's degradation parameters are $X_{k,i}(t_j) = \{(x_{1,1}(t_j), x_{2,1}(t_j)), \dots, (x_{1,N}(t_j), x_{2,N}(t_j))\}$ and $X_{k,i}(t_0) = 0$.

If the degradation increment of the ith sample of the first performance in the time interval $[t_{i-1}, t_i]$ follows the Wiener process, then its likelihood function can be expressed as:

$$L(\Delta X_1|\mu,\sigma_\lambda,\sigma) = \prod_{i=1}^N \prod_{j=1}^M \frac{1}{\sqrt{2\pi (\sigma^2 \Delta t_{ij} + \sigma_\lambda^2) \Delta t_{ij}}} exp\left\{-\frac{(\Delta x_{1ij} - \mu \Delta t_{ij})^2}{2(\sigma^2 \Delta t_{ij} + \sigma_\lambda^2) \Delta t_{ij}}\right\}$$
(21)

If the degradation increment of the ith sample of the second performance in the time interval $[t_{i-1}, t_i]$ obeys the Gamma process, its likelihood function is as follows:

$$L(\Delta X_2 | \alpha, \kappa, r) = \prod_{i=1}^N \prod_{j=1}^M \frac{\Gamma(\alpha \Delta t_{ij} + \kappa)}{\Gamma(\alpha \Delta t_{ij}) \Gamma(\kappa)} \cdot \frac{(r)^{\kappa} \Delta x_{2ij}^{\alpha \Delta t_{ij} - 1}}{(r + \Delta x_{2ij})^{\alpha \Delta t_{ij} + \kappa}}$$
(22)

According to the feature of the Copula function, its joint probability density function $f(\Delta X_1, \Delta X_2; \theta)$ can be expressed as:

$$f(\Delta X_1(t), \Delta X_2(t); \theta) = c(F(\Delta X_1(t)), F(\Delta X_2(t)); \theta) \cdot f_1(\Delta X_1(t)) \cdot f_2(\Delta X_2(t))$$
(23)

Then, the log-likelihood function of the joint distribution of the product's degradation can be obtained from Equation (23) as follows:

$$lnL(F_{\Delta X_{1}}, F_{\Delta X_{2}}; \theta) = \sum_{i=1}^{N} \sum_{j=1}^{M} [lnc(F(\Delta X_{1}(\Delta t)), F(\Delta X_{2}(\Delta t)); \theta) + lnf_{1}(\Delta X_{1}(\Delta t)) + lnf_{2}(\Delta X_{2}(\Delta t))]$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{M} \left[lnc\left(\Phi\left(\frac{\Delta x_{1}-\mu\Delta t_{ij}}{\sqrt{(\sigma^{2}\Delta t_{ij}+\sigma_{\lambda}^{2})\Delta t_{ij}}}\right), F_{2\alpha t, 2\kappa}\left(\frac{\kappa\Delta x_{2}}{\alpha r\Delta t_{ij}}\right); \theta\right) + ln\left(\frac{1}{\sqrt{2\pi(\sigma^{2}\Delta t_{ij}+\sigma_{\lambda}^{2})\Delta t_{ij}}}\exp\left\{-\frac{(\Delta x_{1ij}-\mu\Delta t_{ij})^{2}}{2(\sigma^{2}\Delta t_{ij}+\sigma_{\lambda}^{2})\Delta t_{ij}}\right\}\right) + ln\left(\frac{\Gamma(\alpha\Delta t_{ij}+\kappa)}{\Gamma(\alpha\Delta t_{ij})\Gamma(\kappa)} \cdot \frac{(r)^{\kappa}\Delta x_{2ij}}{(r+\Delta x_{2ij})^{\alpha\Delta t_{ij}+\kappa}}\right)\right]$$

$$(24)$$

Among them, $c(u_1, \dots, u_n, \dots, u_N) = \frac{\partial C(u_1, \dots, u_n, \dots, u_N)}{\partial u_1 \dots \partial u_n \dots \partial u_N}$, and $f(\cdot)$ is the PDF of the marginal distribution $F(\cdot)$.

From Equations (23)–(25), we can find that there are multiple unknown parameters in the likelihood function, and it is difficult to estimate parameters by the maximum likelihood estimation method only. Thus, the procedure of the two-step method is as shown in Figure 2.



Figure 2. Two-stage estimation method description.

Step 1: Parameter estimation of the marginal distribution

According to the edge distribution of the performance degradation data of the TWTA, the likelihood function of the edge distribution as shown in Equations (21) and (22) can be obtained, and the estimated value can be obtained by the MCMC method [25], which is used to estimate unknown parameters in the likelihood function of marginal distributions referred to in the Wiener process and the Gamma process, respectively. In addition, since the parameter estimation of the marginal distribution of the degradation model belongs to the parameter estimation in the case of small samples, the results obtained by the statistical method in which the mean, variance and shape parameters, and size parameters are unknown are generally low in reliability. For the overall characteristics that characterize the sample, such as the diffusion parameter and size parameter, the Bayes-Bootstrap method is used for resampling and re-estimation. The parameter estimation results are $\hat{\theta}_1 = \hat{\mu}$, $\hat{\sigma}_{\lambda}$, $\hat{\sigma}$ and $\hat{\theta}_2 = \hat{\alpha}$, $\hat{\kappa}$, \hat{r} .

Step 2: Parameter estimation of the Copula function model

For the binary degradation case, substitute the obtained parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ of the marginal distribution function into the Copula density function part, as shown in Equation (24), and then use the MLE method to obtain the parameter estimates, $\hat{\theta}$, of the Copula density function. Then, all unknown parameter estimates $\Theta = \{\hat{\mu}, \hat{\sigma}_{\lambda}, \hat{\sigma}, \hat{\alpha}, \hat{\kappa}, \hat{r}, \hat{\theta}\}$ have been received.

6. Case Application

According to the analysis in Section 2, the performance of space TWTA directly affects the stable operation and reliability of the system. As important telemetry parameters of TWTA, the helix current and anode voltage gradually deteriorate with the extension of inorbit time, whose relative failure thresholds are 2.5 mA and 400 V, respectively, representing the health status of TWTA, and provide effective information for the reliable life prediction of satellite subsystems. In the early phase of TWTA in-orbit, the product was still aging, and its performance was not very stable, and the performance parameters of the three TWTAs were recorded according to the remote monitoring system in this paper. All data were initialized and preprocessed. The parameter degradation trajectories after aging are shown in Figure 3, which shows the performance degradation trajectory of individual samples of TWTA, the changes of the average helix current and anode voltage of the in-orbit products, and the uncertainty brought by the linear model fitting.



Figure 3. The degradation trajectory of TWTA. (**a**) Helix current degradation path and (**b**) anode voltage degradation path.

To determine whether the degradation paths of the three TWTA's performance parameters follow the Wiener process and Gamma process separately, a goodness-of-fit was performed on the degradation data of the helix current and anode voltage. The Kolmogorov–Smirnov (K-S) test was used to test the normal distribution and Gamma distribution of goodness-of-fit for the degradation increments of the three samples with a 95% confidence level. The results of the K-S test for all samples were greater than 5% and show that it is capable of degradation modeling of the Wiener process and the Gamma process.

According to the model built in Section 3 and the MCMC parameter estimation method, sampling and estimating unknown parameters were carried out based on the Wiener process and the Gamma process, and all unknown parameters were assumed to have no prior distribution. The estimation results of unknown parameters are shown in Table 2, and the parameters posterior distribution and iteration traces of the helix current and anode voltage are shown in Figures 4 and 5.

Helix Current Anode Voltage σ_{λ}^2 σ^2 Parameter α μ κ r Evaluation 0.286 0.432 0.044 0.983 0.458 0.584



Table 2. The estimation results of unknown parameters.

Figure 4. The parameter posterior distribution of the helix current, (a) μ , (b) σ_{λ}^2 , (c) σ^2 .



Figure 5. The parameter posterior distribution of anode voltage (a) α , (b) κ , (c) r.

Due to the few in-orbit samples, the overall features, such as σ^2 and α , reflect-ing the performance degradation of the TWTA, showed more volatility in the itera-tions, and resulted in a larger estimation error than the other parameters. Therefore, for the above problems, the parameters reflecting the overall characteristics of the product were resampled and re-estimated using the Bayes-Bootstrap method. The re-sults after following this procedure are shown in Table 3.

Table 3. The estimation results of unknown parameters.

Methods	Parameter	Evaluation
МСМС	$\hat{\sigma^2}$ $\hat{\alpha}$	0.044 0.983
Bootstrap	$\hat{\sigma^2}$ $\hat{\alpha}$	0.04 1.04

The evaluation results of marginal distributions were substituted into Equation (24), and the correlation coefficient θ was obtained by MLE, which is shown in Table 4. Through the comprehensive determination of the Copula function, the AIC values of different Copula functions, including the Frank, Clayton, and Gumbel functions, are shown in

Table 4. The AIC value of Frank was the smallest, indicating that the goodness-of-fit was much better.

Гab	le 4	. AIC	values	of e	each (Copul	la i	function	•
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Copula Function	Frank	Gumbel	Clayton	
$\hat{ heta}$	3.267	1.65	2.394	
AIC	-3072.3	-3048.2	-3011.7	

Then, the reliability function curve of TWTA adopted the method in this paper can be obtained by Equation (20). For comparison, the reliability function curves of a singleparameter helix current, single-parameter anode voltage, and independent performance parameter case were also acquired. They are all shown in Figure 6.



Figure 6. Reliability function curves of different cases.

From Figure 6, we can find the following phenomena.

The reliability results of the four models are different, and the reliability of a singleparameter degradation model is higher than the reliability of the model proposed in this paper. With the increase of time, the difference among the models also increased. For a complex-mechanism product such as TWTA, different parameters have their own special contribution to the performance of the product, and it may overestimate the reliability of the TWTA if considering only the degradation of a single performance parameter, and the reliability of the product between the final prediction and the actual value discrepancy will increase with time if taking no account of the correlation of different degradation parameters. Combined with the reliability definition in Equation (20), it can be seen that there is a competitive relationship between the two performance characteristics. Reliability evaluations are more conservative than a single index and are more influenced by the performance index of the helix current that reaches the failure threshold first. In addition, there is a correlation between the helix current and anode voltage, as we described in Section 2.1. Consequently, it is unreasonable to assume complete independence between them. This is also illustrated by the differences between reliability evaluation results based on mutual independence and correlation, as shown in Figure 5.

Besides, the mean time to failure (MTTF) evaluated by the model proposed in this paper is 127.86 months, that is, 10.655 years. This is in line with the engineering estimation because there were still some key technologies of TWTA that had not been completely

broken through when they started the in-orbit application. Meanwhile, in engineering, it is generally agreed that the life of the TWTA is mainly determined by the cathode's life, while the helix current is a comprehensive index of the TWTA, and its variation is affected by many factors. It is difficult to describe the relationship between the helix current and the life of the TWTA by an equation and is worth further studying the failure mechanism of TWTA with the helix current.

7. Conclusions

In this paper, a reliability evaluation model for TWTA, which has two performance degradation characteristics based on the Copula function and bivariate hybrid stochastic processes, was developed to overcome the limitations and shortcomings of the existing methods. The degradation characteristics of the space TWTA were analyzed, and the degeneracy models with single degradation and bivariate degradation were established, respectively. The main innovations of this paper are its viewpoints of a space TWTA's performance degradation model based on the multi-degradation variables of the bivariate hybrid stochastic processes, taking into account random effects compared with the existing methods, and depicting the correlation with the different performance parameters by the Copula function. Besides, a two-step method was proposed to evaluate unknown parameters in the model, where the edge distributions of the helix current and anode voltage were modeled through the Wiener and Gamma processes, and then the Copula function was applied to calculate the joint distribution. The evaluation results obtained using this paper's method compared with the traditional methods that only consider a one-dimensional parameter or multiple independent parameters were presented. The mean time to failure (MTTF) evaluated by the model proposed in this paper was 127.86 months, or 10.655 years. This shows that it is necessary to consider the correlation between the helix current and anode voltage of TWTA. Therefore, the method proposed in this paper is more applicable and more accurate than other traditional methods when dealing with the reliability evaluation of multi-parameter degradation products.

In summary, this research provides a new insight for the reliability modeling and analysis of the space TWTA, and the proposed method can be further applied in complex systems with long life, high reliability, and measurable degradation data. However, this paper showed a correlation between the helix current and anode voltage as a preliminary study, and there is still a lot of work to be carried out because it is assumed that the initial degradation state of TWTA is consistent. In the future, we would like to continue carrying out the reliability analysis of TWTA, such as unfixed failure threshold, multi-source fusion, online prediction, etc.

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