



# Article Towards Watt-Level THz Sources for High-Resolution Spectroscopy Based on 5th-Harmonic Multiplication in Gyrotrons

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**Abstract:** We propose the concept of high-power THz radiation sources based on five-fold frequency multiplication in gyrotrons intended for plasma applications. The efficient excitation at the 5th cyclotron harmonic is due to the specific property of the eigenmodes of cylindrical waveguides, as a result of which, the conditions of simultaneous electrodynamic resonance at two selected TE modes are satisfied asymptotically with very high accuracy. Previously, we have verified this principle in experiments with a low-frequency kilowatt-level gyrotron in which, due to the low-density spectrum, the operating mode is excited with no competition from parasitic oscillations. The novel concept is a development of this idea applied to the systems with a denser spectrum, which is inevitable in higher frequency and power devices. Simulations within the averaged time-domain model demonstrate that, despite the mode competition, it is possible to excite Watt-level 1.25 THz 5th cyclotron harmonic in a recently developed sub-MW 0.25 THz gyrotron with TE<sub>19,8</sub> operating mode. The obtained results open a possibility for implementation of radiation sources with output power/frequency combination, practically inaccessible using other THz generation methods and highly sought for a number of applications, including high-resolution molecular spectroscopy.

**Keywords:** frequency multiplication; high-order cyclotron harmonics; gyrotrons; high-power THz radiation

## 1. Introduction

Multiple applications, including high-resolution spectroscopy, remote sensing, communications, medicine, and security [1–3], require continuous-wave (CW), compact and powerful radiation sources operating in the terahertz gap. Lack of powerful radiation sources is especially critical in the 0.5 to 3 THz band, which is "too low" for quantum electronic devices. Among vacuum electronics devices, which include backward wave oscillators, klystrons [3,4], vircators [5,6], etc., gyrotrons [7–14] are reported to successfully operate at 0.1 to 1 THz [10], and even beyond with high output power. This is due to the fact that gyrotrons are based on interaction of electrons rotating in homogeneous magnetic fields with fast waves excited in smooth cylindrical waveguides. Gyrotrons are usually energized by weakly relativistic electron beams; therefore, they are much more compact than, for example, free-electron lasers.

In gyrotrons, a helical electron beam excites  $TE_{m,q}$  modes of a cylindrical waveguide at the frequencies close to the cutoff ones defined as  $\omega_c = v_{m,q}/R$  ( $v_{m,q}$  is the *q*th root of the equation  $J'_m(x) = 0$ ,  $J_m(x)$ ,  $J'_m(x)$  are the *m*th order Bessel function and its first derivative, *m* and *q* are azimuthal and radial indexes of the operating mode). Thus, the condition of cyclotron resonance has the form  $\omega_c \approx s\omega_H$ , where  $\omega_H = eH/mc\gamma$  is the gyrofrequency,  $\gamma$  is the relativistic mass factor of electrons, *s* is the number of a radiated cyclotron harmonic (CH). Accordingly, operation of weakly relativistic ( $\gamma \sim 1$ ) gyrotrons at 1 THz with excitation of the 1st cyclotron harmonic (*s* = 1) requires a magnetic field of



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**Copyright:** © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). about 36 T achievable only in pulsed field magnets. State-of-the art steady-state magnets (cryomagnets [15]) produce the magnetic field of 10–15 T; thus, high harmonic operation (s > 1) appears to be the sole solution for producing terahertz radiation in CW regime.

It should be noted that there are two approaches for harmonic operation in gyrotrons, namely, selective excitation of a resonant mode at a given cyclotron harmonic and the effect of frequency multiplication. The first method makes it possible to provide a sufficiently high efficiency of the electron-wave interaction. However, in this case, the increase in frequency is limited by strong competition with modes at the fundamental cyclotron resonance. As a result, at frequencies above 100 GHz, selective excitation at the 2nd CH was achieved in conventional CW gyrotrons [11,13] with frequencies up to 0.5 THz, while so-called large-orbit gyrotrons (LOGs) provide CW operation at the 3rd harmonic [14] with frequencies up to 0.4 THz.

The effect of frequency multiplication [16–20] arises due to non-linear properties of the beam of rotating electrons; each of them represents a non-isochronous (non-linear) cyclotron oscillator. The non-isochronism of rotation is caused by the relativistic dependence of the gyrofrequency on the electron energy. As a result, the bunching of electrons in the field of a low-frequency (LF) wave leads to the emergence of high-frequency (HF) components of the electron current at every harmonic of this wave. Thus, LF and HF components radiate simultaneously from the interaction space of a gyrotron. In this case, no problem of mode competition exists as the HF mode is forcefully excited by the bunched electrons, while the total beam current is lower than the starting value.

However, efficient multiplication requires the electrodynamic system of the gyrotron, namely, a slightly irregular cylindrical waveguide, to be simultaneously resonant to two  $TE_{m,q}$  modes, one at the fundamental and one at the *s*-th CH. This requirement can be written as (see [16] for details):

$$v_{m_s,q_s} = s v_{m_1,q_1}, \quad m_s = s m_1.$$
 (1)

Unfortunately, conditions (1) cannot be exact in the non-equidistant spectrum of cylindrical waveguides. As a result, the efficiency of frequency multiplication  $K = P_s/P_1$  (where  $P_s$  and  $P_1$  are HF and LF radiation power, correspondingly) is extremely low and decreases with the harmonic number *s* as  $K \sim 10^{-2s}$  [20].

Nevertheless, as demonstrated in study [21], the condition (1) can be satisfied asymptotically for selected multiplication numbers s = 5, 9, 13... and selected operating modes, whose radial indices are related as:

$$q_s = sq_1 - 3(s-1)/4.$$
<sup>(2)</sup>

This relationship was established on the basis of the asymptotic representation of the derivatives of Bessel functions (Debye's asymptotic). In particular, for s = 5, modes TE<sub>*m,p*</sub> at the fundamental CH and TE<sub>5*m*,5*p*-3</sub> at the 5th CH will be resonant to the electron beam with high accuracy. This principle has been verified in experiments with a CW 45-GHz gyrotron driven by 25 keV/2 A beam at TE<sub>6,3</sub> mode, where ~100 mW radiation power at the 5th CH cyclotron harmonic (225 GHz) has been detected [21]. The obtained conversion ratio  $K \approx 1.5 \cdot 10^{-5}$  was an order of magnitude higher than for the case of the triple frequency multiplication estimated in [20].

In this paper, within the frame of numerical simulations, we demonstrate the possibility of using the developed approach to implement Watt-level THz radiation sources based on high-power gyrotrons intended for plasma applications. In these devices, the mode spectrum is dense and severe mode competition takes place unlike the situation presented in [16]. We show that under such conditions, efficient generation of high harmonic content is still possible.

### 2. Simulations of 5th-Harmonic Multiplication with Parameters of a 250-GHz High-Power Gyrotron

The possibility of reaching the THz range based on a five-fold frequency multiplication is obviously associated with the use of higher frequency gyrotrons than in the experiment [21]. Simultaneously, since the conversion coefficient remains rather low, it is necessary to use high-power fundamental-harmonic gyrotrons. Currently, the most powerful MW-level radiation is generated by gyrotrons intended for plasma applications, which typically operate up to 170 GHz [8]. At the same time, gyrotrons with operating frequencies of about 250 GHz are in demand for controlled thermonuclear fusion installations of the DEMO type. Note here that such gyrotrons typically exploit very high-order transverse modes. Thus, conditions of the double resonance (1) should be met with greater accuracy.

Further, we demonstrate in simulations that efficient 5th CH multiplication can be obtained in the DEMO-prototype gyrotron [9] which has been recently developed at IAP RAS (Figure 1a). The gyrotron provides the output power of 200 kW in CW operation regime at an accelerating voltage of 55 kV and an electron beam current of 12.5 A (see Figure 1b). The operating TE<sub>19,8</sub> mode is excited at the fundamental cyclotron resonance. In accordance with the selection rules (1) and (2), the effective excitation of the 5th cyclotron harmonic is expected at the mode TE<sub>95,37</sub>. The accuracy of condition (1) in this case is  $(\nu_{5m_1,q_5} - 5\nu_{m_1,q_1})/5\nu_{m_1,q_1} \approx 0.02\%$ .







For simulations, the time-domain multi-mode averaged model [20] was used. According to this model, the electric field of each mode in the interaction space (smoothly tapered cylindrical waveguide) is presented in the form  $\vec{E}_n = \text{Re}(A_n(z,t)\vec{E}_{\perp}(r)\exp(is_n\omega_H^0t - im_n\varphi))$ , where  $A_n(z,t)$  is the slowly varying amplitude of the *n*-th mode,  $\vec{E}_{\perp}(r)$  is its transverse structure,  $\varphi$  is the azimuthal angle. The electron-wave interaction is described by the following system with the parabolic equations standing for the evolution of mode amplitudes  $a_n$  and non-isochronous oscillator equations for the complex transverse momentum of electrons :

$$i\frac{\partial^{2}a_{n}}{\partial Z^{2}} + s_{n}\frac{\partial a_{n}}{\partial \tau} + (i\Delta_{n} + i\delta_{n}(Z) + \sigma_{n})a_{n} = i\frac{I_{n}}{4\pi^{2}}\int_{0}^{2\pi} \langle p^{s_{n}}\rangle_{\theta_{0}}e^{i(m_{n}-s_{n})\varphi}d\varphi,$$

$$\frac{\partial p}{\partial Z} + \frac{\overline{g}_{0}^{2}}{4}\frac{\partial p}{\partial \tau} + ip(|p|^{2}-1) = i\sum_{n}a_{n}(p^{*})^{s_{n}-1}e^{-i(m_{n}-s_{n})\varphi},$$
(3)

where  $a_n = \frac{eA_n I_{mn-s_n} \left( v_{mn,q_n} R_b / R_0 \right)}{mc\omega_H^0} \frac{s_n^{s_n}}{2^{s_n-1} s_n!} \frac{\beta_{\perp 0}^{s_n-4}}{\gamma_0}, Z = \frac{\beta_{\perp 0}^2}{2\beta_{||0}} \frac{\omega_H^0}{c} z, \tau = \frac{\beta_{\perp 0}^4}{8\beta_{\perp 0}^2} \omega_H^0 t, \beta_{\perp 0} = V_{\perp 0} / c$ and  $\beta_{||0} = V_{||0} / c$  are values of transverse and axial electron velocities at the entrance of the interaction space,  $g_0 = \beta_{\perp 0} / \beta_{||0}, \Delta_n = \frac{8\beta_{||0}^2 s_n^2}{\beta_{\perp 0}^4} \frac{s_n \omega_H^0 - \overline{\omega}_n^c}{\overline{\omega}_n^c}, \delta_n(Z) = \frac{8\beta_{||0}^2 s_n^2}{\beta_{\perp 0}^4} \frac{\overline{\omega}_n^c - \omega_n^c(Z)}{\overline{\omega}_n^c}$ are cyclotron and geometrical detunings.  $\omega_n^c(Z) = v_{mn,q_n} c / R(z)$  describes the axial dependence of the cut-off frequency, R(z) is the profile of the gyrotron cavity which includes the cutoff neck, the regular part with  $R(z) = R_0$ , and the collector widening,  $I_n = 64 \frac{eI_b}{m_e c^3} \frac{\overline{\beta}_{||0} \overline{\beta}_{\perp 0}^2}{\gamma_0} s_n^3 \left( \frac{s_n^{s_n}}{2^{s_n s_n}} \right)^2 \frac{f_{mn-sn}^2 \left( v_{mn,q_n} R_b / R_0 \right)}{\left( v_n^2 - m_n^2 \right) f_{mn}^2 \left( v_{mn,q_n} \right)}}$  is the field excitation factor for the beam with an injection radius of  $R_b$  and a current of  $I_b$ .  $\sigma_n = 4\beta_{||0}^2 \beta_{\perp 0}^{-4} Q_n^{-1} s_n^2$  is the parameter of Ohmic losses,  $Q_n$  is the Ohmic Q-factor. The radiation power in each mode can be found based on the following relation:  $P_n[kW] = 511.765 \cdot I[A] \cdot (\gamma_0 \beta_{\perp 0}^2 / G_n) \cdot \operatorname{Im}(a_n \cdot \partial a_n^* / \partial Z) \Big|_{Z=L}$ 

The motion equations in the system (3) have to be supplemented with standard boundary conditions corresponding to electrons uniformly distributed over the cyclotron rotation phases:  $p(Z = 0) = p_0 \exp(i\theta_0)$ ,  $\theta_0 \in [0, 2\pi)$ . For the mode amplitudes at the left (cathode) edge of the system Z = 0, we use zero boundary conditions corresponding to the cutoff neck. At the output cross-section Z = L located in the collector widening outside the region of interaction with electrons, we apply non-reflection boundary conditions:

$$a_n(\tau,L) + \frac{1}{\sqrt{i\pi s_n}} \int_0^\tau \frac{e^{-is_n^{-1}(i\Delta_n + i\delta_n(L) + \sigma_n)(\tau - \tau')}}{\sqrt{\tau - \tau'}} \frac{\partial a_n(\tau',L)}{\partial Z} d\tau' = 0$$
(4)

This condition means matching with the output waveguide, where all modes propagate far from the cutoff frequencies.

The results of simulations are presented in Figure 2. Figure 2a shows the operation zones for the operating  $TE_{19,8}$  mode, the most dangerous parasitic  $TE_{-17,9}$  mode at the 1st cyclotron harmonic and for the TE<sub>95,37</sub> mode at the 5th cyclotron harmonic. One can see that the zone optimal for excitation of the  $TE_{95,37}$  mode is partially overlapped by the zone of parasitic  $TE_{-17.9}$  mode generation. However, there is a region of magnetic fields in which effective excitation of the 5th CH is possible. Figure 2b demonstrates the establishment of the regime of stationary generation for the intensity of the magnetic field of 98.4 kOe. It should be noted that in this simulation, the level of initial azimuthal modulation, which is the same for all competing modes, is maintained in the beam during the entire time of the onset of oscillations. Nevertheless, the operating mode  $TE_{19.8}$  (blue curve) wins in the competition, despite the proximity of parasitic mode  $TE_{-17.9}$  (green curve). The red curve corresponds to the level of CW excitation at the 5th cyclotron harmonic. According to simulations, the power of radiation at the 5th CH (i.e., at the frequency of 1.25 THz) can reach 8W. Taking into account the spread in velocities of about 15–20%, which is typical for magnetron-injector guns of gyrotrons, the radiation power is reduced (orange curve in Figure 2a). However, according to the simulation, it remains at the level of several Watts  $(K \approx 1.5 \cdot 10^{-4}).$ 



**Figure 2.** (a) Results of simulations for the 5th harmonic multiplication. Output power vs. guiding magnetic field for competing  $TE_{19,8}$  and  $TE_{-17,9}$  modes at the 1st cyclotron harmonic and for  $TE_{95,37}$  mode at the 5th cyclotron harmonic. Orange symbols depict the dependence of output power for the spread of electron velocities of 15%. (b) Establishment of the stationary oscillations in the process of competition between 12 modes at the fundamental cyclotron resonance.

### 3. Conclusions

To conclude, we note that  $TE_{19,8}$  is not the only high-order mode that can be used for five-fold frequency multiplication in the free-running gyrotron operation regime. According to Figure 2a, the neighboring parasitic mode's excitation zone is shifted by 2–3% in terms of magnetic field. For lower operating modes, this shift is even larger. Correspondingly, the sub-MW-level gyrotrons operating at  $TE_{10,5}$  or  $TE_{12,5}$  can also be used for 5th CH excitation.

In order to increase the generated 5th CH power, one would have to use MW-level gyrotrons, which possess a much denser mode spectrum. In this case, excitation zones of different modes would overlap and the parasitic modes would prevent the excitation of the 5th CH at the optimal parameters. To avoid this, the generation region of the fundamental-harmonic operating mode can be expanded by locking the gyrotron with an external signal that suppresses parasitic oscillations (see details in [22]). According to preliminary estimations, frequency-locked MW-level gyrotrons can provide the radiation power at the level of 10–20 W in the sub-THz band. Such parameters of radiation can be of interest, for example, for the molecular spectroscopy [23], including observation of so-called forbidden transitions.

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