

Article

Mechanical Properties of the Functionally Graded Lining for a Deep Buried Subway Tunnel

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Abstract: With the rapid development of the subway rail transit, tunnels are buried at an increasing depth, raising the requirements of bearing capacity and waterproofness for linings. Functionally graded materials are introduced into the design of linings to save costs, and concrete with different elastic moduli is equipped at different positions to reduce the waste of materials, compared to the homogeneous lining. The significance of this study includes that the functionally graded lining for the buried subway tunnel is under the non-uniform confining pressure and the calculation model of internal force and deformation for the functionally graded lining is established. The elastic modulus of the lining is set to vary with the angle in the form of a power function, and the function parameters are analyzed on the basis of this model. The results show that the radial displacement of the lining axis decreases with the increase in a and b , but the deformation mode remains the same, and the reduction in deformation is smaller and smaller. With the increase in a and b , the distribution trend of the moment remains the same. The lateral pressure coefficient λ has a great impact on the safety of the structure, which exceeds the influence of the function parameters on the safety of the structure. The displacement of the lining axis and the section moment change linearly with the increase in λ . With the increase in λ , the shape of the lining changes significantly, which shows that the side with larger pressure deforms to the inside and the side with smaller pressure expands to the outside. When the maximum deformation occurs at 0° , the parameter a should be larger than b . When the maximum deformation occurs at 90° , the parameter b should be larger than a , so as to minimize the cost of materials and reduce the structural deformation. Finally, the numerical simulation is conducted to verify the theoretical results, showing that the calculation model of internal force and deformation is suitable for the cylinder with $t/R \leq 0.2$, and there is a certain gap between the theoretical calculation and numerical simulation, but the largest gap of the displacement is within 8%. Compared with Function I, Function II has some advantages in reducing the maximum deformation of the structure, but the advantages are relatively low. The analysis results have significant reference value for designers and relevant scholars.

Keywords: subway tunnel; functionally graded lining; deep buried; elastic modulus; mechanical properties



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1. Introduction

Underground spaces in Shanghai are congested with subways and vital urban facilities [1]. The utilization of deep underground spaces solves the problem regarding the shortage of shallow underground space [2–4]. Nevertheless, challenges facing deep buried tunnels are still considerable. For example, soil and water pressure in the stratum deeper than 50 m exceeds 1 MPa [5], which challenges the bearing capacity and waterproofness of deep buried tunnels [6]. In order to improve the supporting performance and waterproofness

of subway lining, two methods are usually applied: either to increase the thickness of the lining or to enhance the concrete strength [7]. However, both are extravagant in that concrete strength and thickness are the same in each position, which leads to a waste of concrete in corresponding positions.

In this paper, the functionally graded materials (FGMs) were introduced to the design of the lining of the subway tunnel, and concrete with different elastic moduli was arranged at different positions to reduce its cost on the premise of ensuring structural safety. The concept of FGMs was proposed in 1984 by materials scientists in the Sendai area as a means of preparing thermal barrier materials [8]. The composition of the functional graded materials changed continuously from one direction to another, resulting in a continuous change in the material properties (such as elastic modulus) [9]. The variations of composition and properties in conventional composite materials and FGMs are illustrated in Figure 1.

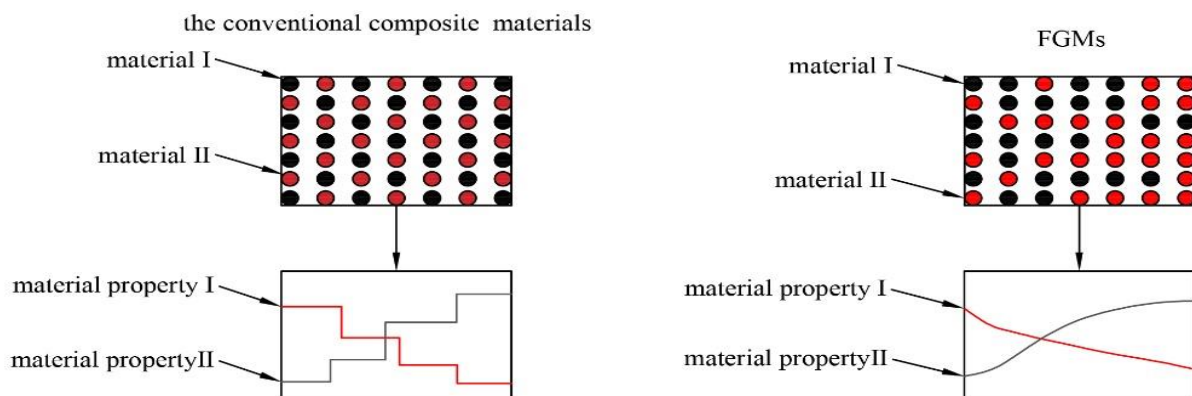


Figure 1. Variations of composition and properties in conventional composite materials and FGMs.

Extensive applications of FGMs have extended to many fields where the operating conditions are severe, including aerospace, chemical plants, nuclear energy reactors and so on [10]. Efforts were also made to study the performance of construction materials, such as metal and cement-based materials [11]. Liu et al. [12] developed the sustainable structure constructed with functionally graded concretes using fibers and recycled aggregates. Ahmadi et al. [13] studied the mechanical properties of the graded concrete specimens composed of recycled aggregates and steel wires recycled from waste tires. Dias et al. [14] established the concept of functionally graded fiber cement, and the use of statistical mixture designs was discussed to choose formulations and present ideas for the production of functionally graded fiber cement components. Shen et al. [15] employed a functionally graded material system to make fiber more efficient in a fiber reinforced cement composite with four layers, each with a different fiber volume ratio. Differently, the study introduced in this paper aims to develop a functionally graded cement-based lining for the subway tunnel. To achieve this, the elastic modulus function is adopted, which is a power law function with respect to angle α .

There have been many analytical results about functionally graded hollow cylinders subjected to mechanical stresses. Shi et al. [16] defined the elastic modulus as a linear function varying with the radius, and Poisson's ratio is set as a constant, then, the exact solutions of the hollow cylinder with continuously graded properties are obtained. Dai et al. [17] assumed the elastic modulus as a simple power law function varying through the wall thickness, and the exact solution for displacement and stress is determined when Poisson's ratio is assumed constant. Similarly, Batra et al. [18,19] analyzed the deformation of functionally graded cylinders composed of incompressible isotropic linear elastic materials, with the variations of the shear modulus in the radial direction given by a power law relation and constant Poisson's ratio. In the other case, the exact elasticity solution of a radially nonhomogeneous hollow cylinder was derived, whose elastic modulus varied in an exponential and power law function [20]. Beyond those above, the radial elastic

modulus was assumed as an arbitrary function [21]. In these works, material parameters such as elastic modulus or shear modulus were assumed in advance to be a function of the radius, and Poisson's ratio was a constant. The stresses and displacement distributions were calculated under the given elastic modulus $E(r)$ or the shear modulus $G(r)$. Differing from the mentioned works above, Zhang et al. [22] pre-assumed the desired stress distribution with the elastic modulus $E(r)$ being undermined, then the $E(r)$ was confirmed by back-calculations according to the stress distribution and loadings. Li et al. [23] studied the mechanical properties of the functionally graded concrete shaft lining with uniform confining pressure. However, all the above-mentioned works are based on axisymmetric loads, and the situation under non-axisymmetric loads has not been researched yet, such as linings of the subway tunnel. In this paper, the functionally graded lining of the subway tunnel was modelled as a functionally graded hollow cylinder under non-axisymmetric loads, and its mechanical properties are studied. The analytical results presented here should serve as benchmarks for verifying the numerical solutions of problems. The objective of this study is the mechanical properties of functionally graded lining for a deep buried subway tunnel with non-uniform confining pressure, which can offer a reference for the design of functionally graded lining.

2. Analytical Solutions for the Mechanical Properties of the Functionally Graded Lining

2.1. Basic Assumptions

The general agreement of these analytical models lies on the following basic assumptions: (1) the cross section for the lining is assumed to be circular, and the lining satisfies the plain strain condition [24]; (2) the material behavior of the lining is generally assumed to be elastic, because of the small deformation; (3) loads on the top, bottom and sides are distributed uniformly.

2.2. Governing Equations

For a homogeneous straight beam, the moment is defined to be positive when the structure tensile part is at the bottom, and the upward deformation is defined to be positive. Ignoring the axial deformation, the differential equation between the moment and the deformation is given by

$$\frac{d^2\omega}{dx^2} = \frac{M(x)}{EI} \quad (1)$$

For a homogeneous circular lining structure, as shown in Figure 2, the microelement can be analyzed as a straight beam, and Equation (1) should be satisfied between the moment and the radial deformation in the rectangular coordinate system.

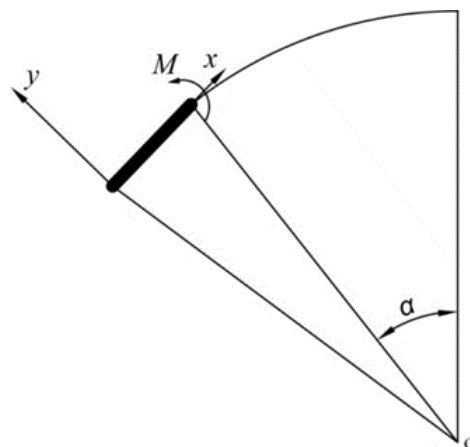


Figure 2. Microelement of the circular lining.

The polar coordinate system is established, whose origin is the center of the lining. The top of the structure is defined as 0° , and the anticlockwise direction is defined to be positive. Therefore, the differential equation of the functionally graded lining in the polar coordinate system is given by

$$\frac{d^2\omega}{d\alpha^2} = R_H^2 \frac{M(\alpha)}{E(\alpha)I} \quad (2)$$

where ω is the radial deformation of the lining and R_H is the calculated radius of the section.

2.3. Building Model

According to the symmetry of the lining and external loads, the quarter circle is considered. Therefore, the calculation model is as shown in Figure 3.

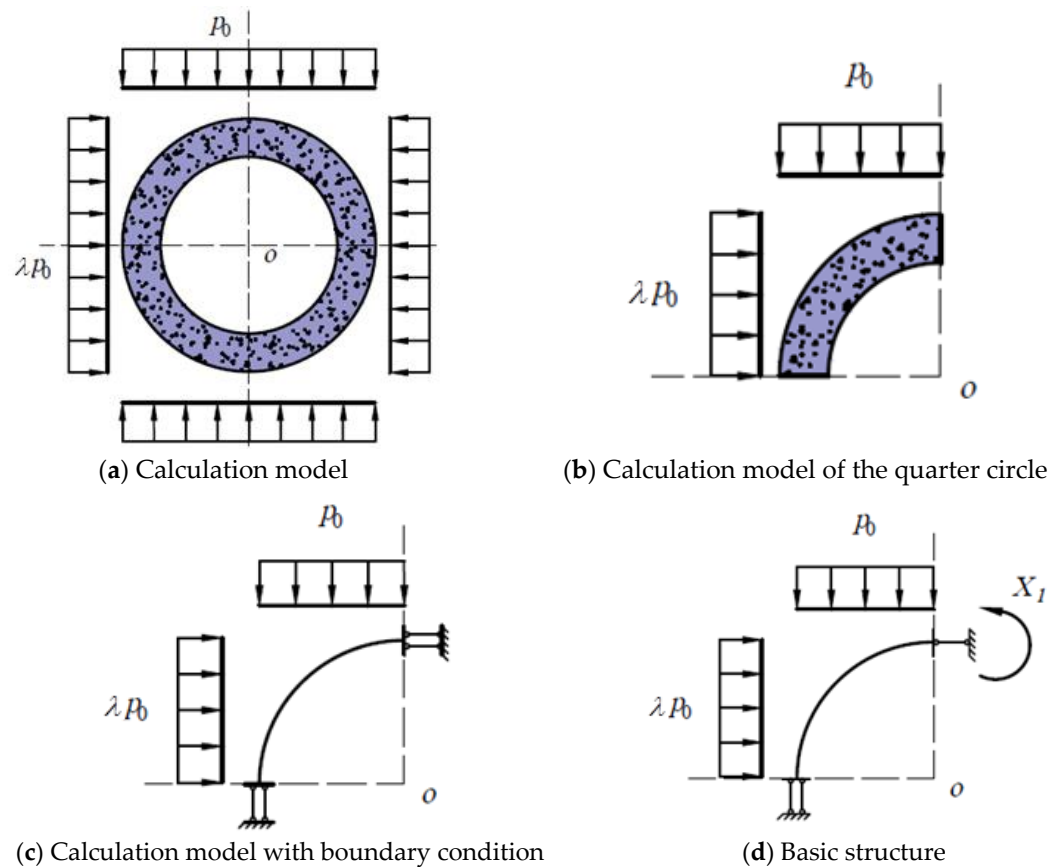


Figure 3. Calculation model.

At the interval $[0, \pi/2]$, the elastic modulus function versus α is given by

$$E(\alpha) = E_0[A\alpha^n + B] \quad (3)$$

where E_0 is the basic elastic modulus of lining.

At the interval $[\pi/2, \pi]$, the elastic modulus is symmetrical to that at the interval $[0, \pi/2]$.

For the sake of simplicity, let $E = aE_0$ at $\alpha = 0$ and $E = bE_0$ at $\alpha = \pi/2$, and the elastic modulus function transformed with parameters a and b is given by Equation (4), which is defined as Function I.

$$E(\alpha) = E_0 \left[\frac{b-a}{(\pi/2)^n} \alpha^n + a \right] \quad (4)$$

where a and b are the parameters of the elastic modulus.

2.4. Solutions of the Internal Force Coefficient of the Functionally Graded Lining

As shown in Figure 3d, the angular displacement is equal to zero at the position of $\alpha = 0^\circ$, and the force method equation is given by

$$\delta_{11}X_1 + \Delta_{1p} = 0 \quad (5)$$

where δ_{11} is the displacement at X_1 of $\bar{X}_1 = 1$; Δ_{1p} is the displacement at X_1 of external forces;

$$\delta_{11} = \int \frac{\bar{M}_1^2}{E(\alpha)I} ds = \int_0^{\pi/2} \frac{\bar{M}_1^2}{E(\alpha)I} R_H d\alpha = \int_0^{\pi/2} \frac{R_H}{E(\alpha)I} d\alpha = \frac{R_H}{E_0 I} m_1 \quad (6)$$

where

$$m_1 = \int_0^{\pi/2} \frac{1}{E(\alpha)/E_0} d\alpha$$

Moreover,

$$\Delta_{1p} = \int \frac{\bar{M}_1 M_p}{E(\alpha)I} ds = \int_0^{\pi/2} \frac{M_p}{E(\alpha)I} R_H d\alpha$$

where

$$M_p = \frac{1}{2}(\lambda - 1)p_0 R_H^2 \sin^2 \alpha$$

Therefore,

$$\Delta_{1p} = \int \frac{\bar{M}_1 M_p}{E(\alpha)I} ds = \int_0^{\pi/2} \frac{M_p}{E(\alpha)I} R_H d\alpha = \frac{1}{2}(\lambda - 1)p_0 R_H^3 \int_0^{\pi/2} \frac{\sin^2 \alpha}{E(\alpha)I} d\alpha = \frac{1}{2} \frac{(\lambda - 1)p_0 R_H^3}{E_0 I} m_2 \quad (7)$$

where $m_2 = \int_0^{\pi/2} \frac{\sin^2 \alpha}{E(\alpha)/E_0} d\alpha$; λ is the coefficient of lateral pressure.

So,

$$X_1 = \frac{-\Delta_{1p}}{\delta_{11}} = \frac{1}{2}(1 - \lambda)p_0 R_H^2 \frac{m_2}{m_1},$$

$$M_\alpha = \bar{M}_1 X_1 + M_p = \frac{1}{2}p_0 R_H^2 (1 - \lambda) \left(\frac{m_2}{m_1} - \sin^2 \alpha \right).$$

Further, the moment nondimensionalized is given by

$$\frac{M_\alpha}{p_0 R_H^2} = \frac{1}{2}(1 - \lambda) \left(\frac{m_2}{m_1} - \sin^2 \alpha \right). \quad (8)$$

The axial force can be obtained by calculating moment at the center, and the axial force nondimensionalized is given by

$$\frac{N}{p_0 R_H} = \sin^2 \alpha + \lambda \cos^2 \alpha. \quad (9)$$

Boundary conditions at $\alpha = 0$ and $\alpha = \pi/2$ can be obtained by the principle of virtual work.

$$\begin{aligned} \omega_0 &= - \left(\int_0^{\pi/2} \frac{\bar{M}_2 M_p}{E(\alpha)I} R_H d\alpha + \int_0^{\pi/2} \frac{\bar{M}_2 M_1}{E(\alpha)I} R_H d\alpha \right) \\ &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} \left(\int_0^{\pi/2} \frac{\sin^3 \alpha}{E(\alpha)/E_0} d\alpha - \frac{m_2}{m_1} \int_0^{\pi/2} \frac{\sin \alpha}{E(\alpha)/E_0} d\alpha \right) \cdot \\ &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} m_3 \end{aligned} \quad (10)$$

where $\bar{M}_2 = 1$ is exerted at $\alpha = 0^\circ$ vertically;

$$\begin{aligned}
 m_3 &= \int_0^{\pi/2} \frac{\sin^3 \alpha}{E(\alpha)/E_0} d\alpha - \frac{m_2}{m_1} \int_0^{\pi/2} \frac{\sin \alpha}{E(\alpha)/E_0} d\alpha. \\
 \omega_{\pi/2} &= -\left(\int_0^{\pi/2} \frac{\bar{M}_3 M_P}{E(\alpha)I} R_H d\alpha + \int_0^{\pi/2} \frac{\bar{M}_2 M_1}{E(\alpha)I} R_H d\alpha \right) \\
 &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} \left(\frac{m_2}{m_1} \int_0^{\pi/2} \frac{1 - \cos \alpha}{E(\alpha)/E_0} d\alpha - \int_0^{\pi/2} \frac{\sin^2 \alpha (1 - \cos \alpha)}{E(\alpha)/E_0} d\alpha \right) \quad (11) \\
 &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} m_4
 \end{aligned}$$

where $\bar{M}_3 = 1$ is exerted at $\alpha = 90^\circ$ horizontally to the right;

$$m_4 = \frac{m_2}{m_1} \int_0^{\pi/2} \frac{1 - \cos \alpha}{E(\alpha)/E_0} d\alpha - \int_0^{\pi/2} \frac{\sin^2 \alpha (1 - \cos \alpha)}{E(\alpha)/E_0} d\alpha.$$

From derivation, when external forces are determined, the internal force can be obtained, and the radial displacement can be obtained according to the governing equation and boundary conditions. The boundary conditions are concluded as follows.

$$\begin{aligned}
 \alpha = 0, \omega_0 &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} m_3 \\
 \alpha = \pi/2, \omega_0 &= \frac{(\lambda - 1)p_0 R_H^4}{2E_0 I} m_4 \quad (12)
 \end{aligned}$$

2.5. Single Factor Test on the Lining Model of the Functionally Graded Lining

The dimensionless parameter K is employed to reflect the displacement at the lining axis, and the dimensionless parameter P is employed to reflect the section moment. The definitions of K and P are given by

$$K = \omega / \frac{p_0 R_H^4}{E_0 I} \quad (13)$$

$$P = \frac{M_\alpha}{p_0 R_H^2} \quad (14)$$

The single factor tests are carried out to study the influence of the lateral pressure coefficient λ , the elastic modulus parameters a and b on K , and the typical parameters are given by $a = 1$, $b = 1$ and $\lambda = 0.5$.

In the single factor test, one of the factors is changed in turn, and the other factors are fixed according to the typical parameters. The variation range of each factor is summarized in Table 1.

Table 1. Factor levels of the single factor test.

Factors	Variation Range	Step
a	0.7~1.3	0.1
b	0.7~1.3	0.1
λ	0.5~1.5	0.2

Under the condition that the other parameters remain unchanged, the influence of a on the radial displacement and section moment in the range of $0 \sim 90^\circ$ at the axis of functionally graded lining is studied. In addition, the influence of a on the typical points of 0° , 15° , 30° , 45° , 60° , 75° and 90° at the axis of functionally graded lining is studied, and the results are shown in Figures 4 and 5.

As can be observed from Figure 4a, with the increase in a , the radial displacement at the lining axis decreases, but the deformation trend of the lining is consistent with the maximum displacement at 0° and 90° and the minimum displacement near 45° . In addition, from Figure 4b, with the increase in a , the slope of the radial displacement curve of each typical point decreases gradually. It shows that the increase in a can enhance the stiffness of the lining, but the reduction in deformation becomes smaller and smaller.

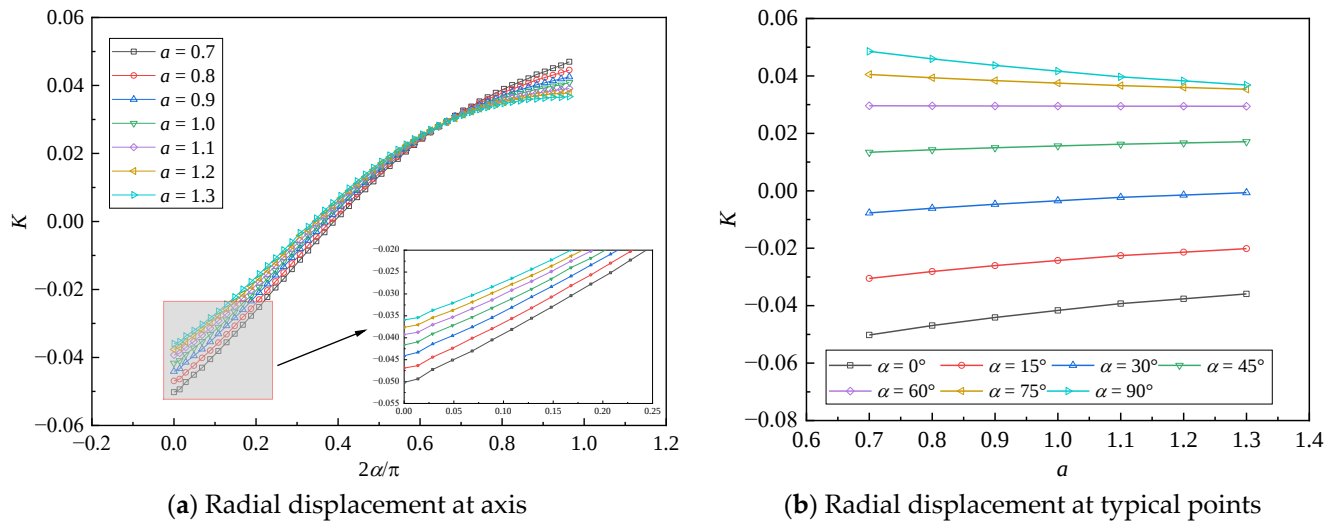


Figure 4. Radial displacement in the single factor test of parameter a .

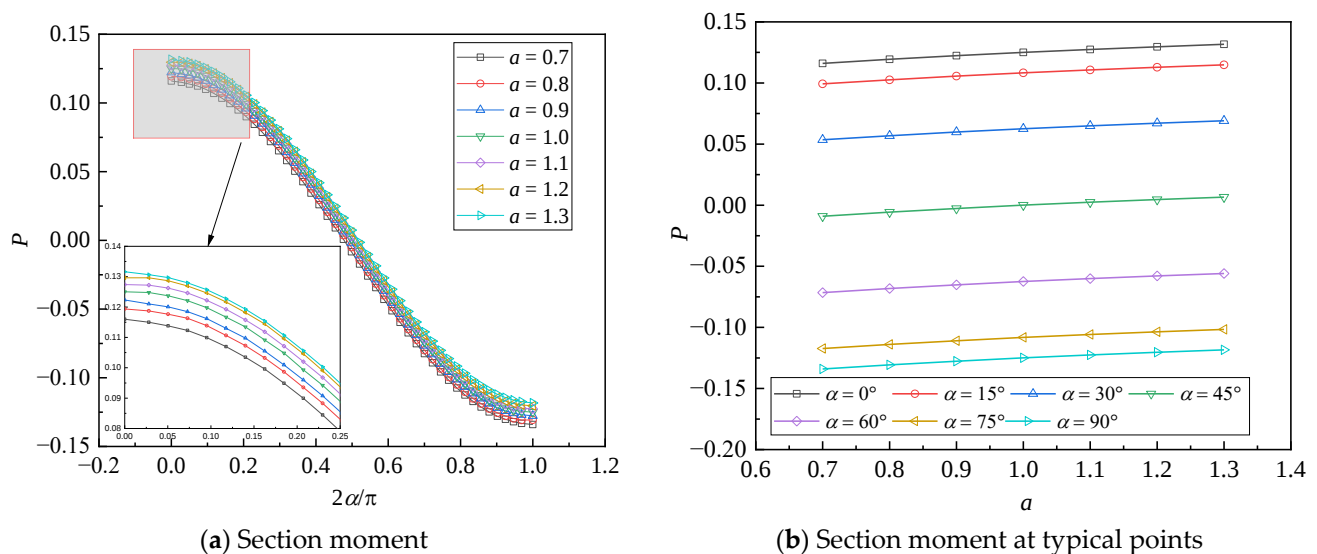


Figure 5. Section moment in the single factor test of parameter a .

From Figure 5a, with the increase in a , the distribution trend of section moment is consistent with the maximum moment at 0° and 90° and the minimum moment near 45° . In addition, from Figure 5b, with the increase in a , the negative section moment at typical points decreases linearly, and the positive section moment increases linearly. The maximum section moment transfers from the section of 90° to the section of 0° , and the maximum section moment decreases.

It can be observed from the distribution of the displacement and internal force that the optimal mode of the elastic modulus is to configure the larger elastic modulus at 0° and 90° to improve the rigidity, and the smaller elastic modulus at 45° to achieve the full utilization

of the material. The optimized elastic modulus function is given by Equation (15), which is defined as Function II.

$$E(\alpha) = E_0 \left[\frac{a-b}{(\pi/4)^n} \left| \alpha - \frac{\pi}{4} \right|^n + b \right] \quad (15)$$

Under the condition that the other parameters remain unchanged, the influence of b on the radial displacement and section moment in the range of $0^\circ \sim 90^\circ$ at the axis of functionally graded lining is studied. In addition, the influence of the typical points of $0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ and 90° on the axis of functionally graded lining is studied, and the results are shown in Figures 6 and 7.

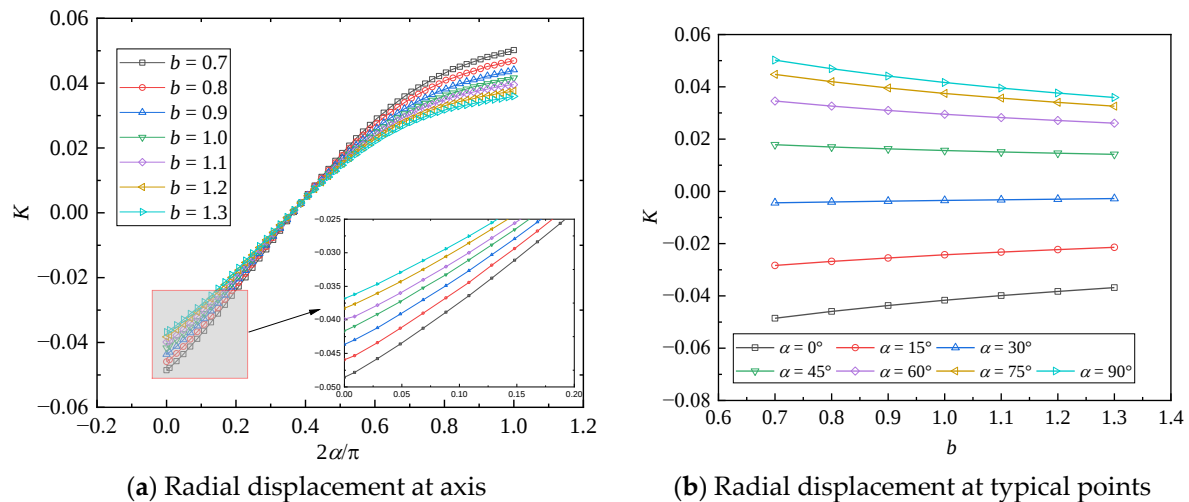


Figure 6. Radial displacement in the single factor test of parameter b .

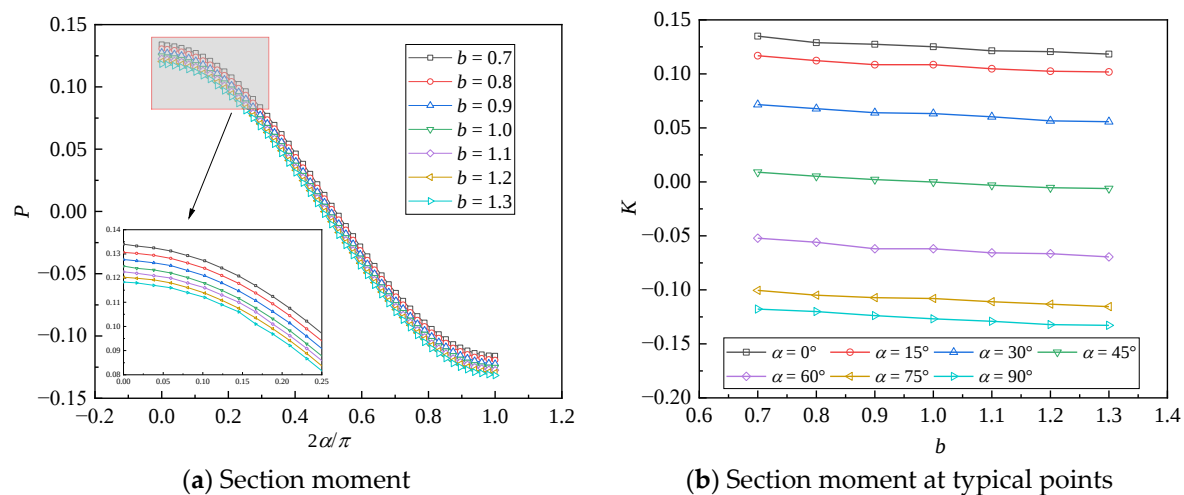


Figure 7. Section moment in the single factor test of parameter b .

As can be observed from Figure 6a, with the increase in b , the radial displacement at the lining axis decreases, but the deformation trend of the lining is consistent with the maximum displacement at 0° and 90° and the minimum displacement near 45° . In addition, from Figure 6b, with the increase in b , the slope of the radial displacement curve of each typical point decreases gradually. It shows that the increase in a can enhance the stiffness of the lining, but the reduction in deformation becomes smaller and smaller. Comparing Figure 6 with Figure 4, it can be found that the effect of changing a or b alone on the deformation characteristics of functionally gradient lining is the same when other parameters remain unchanged.

From Figure 7a, with the increase in b , the distribution trend of section moment is consistent with the maximum moment at 0° and 90° and the minimum moment near 45° . In addition, from Figure 7b, with the increase in b , the negative section moment at typical points increases linearly, and the positive section moment decreases linearly. The maximum section moment transfers from the section of 0° to the section of 90° , and the maximum section moment decreases. Comparing Figure 7b with Figure 5b, it can be found that with the increase in the section stiffness, the section moment also increases.

Under the condition that the other parameters remain unchanged, the influence of λ on the radial displacement and section moment in the range of 0° to 90° on the axis of functionally graded lining is studied. In addition, the influence of the typical points of 0° , 15° , 30° , 45° , 60° , 75° and 90° at the axis of functionally graded lining is studied, and the results are shown in Figures 8 and 9.

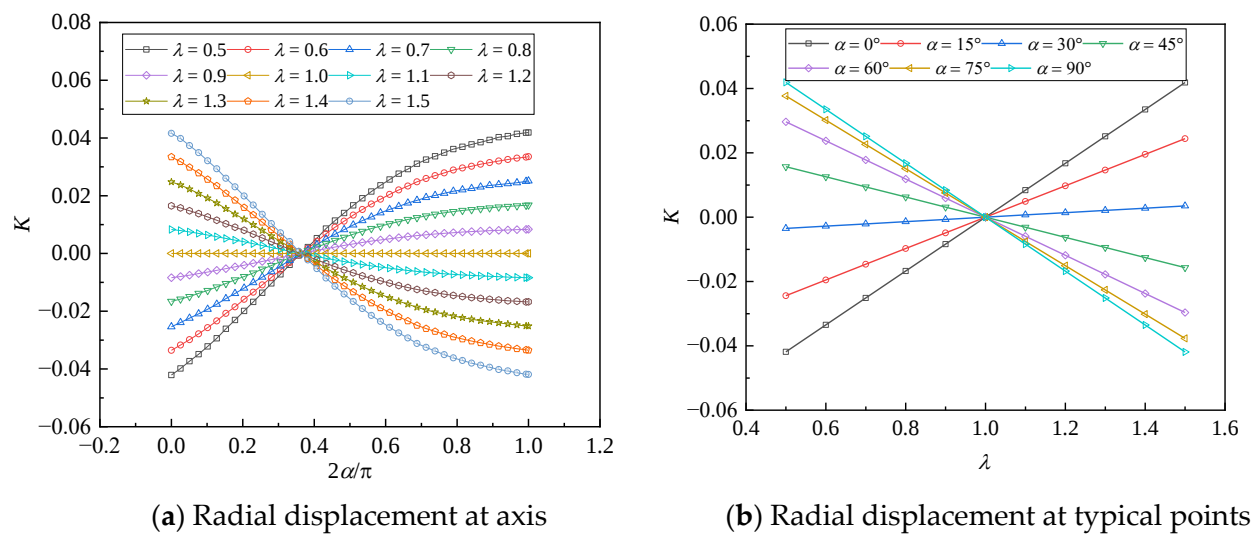


Figure 8. Radial displacement in the single factor test of parameter λ .

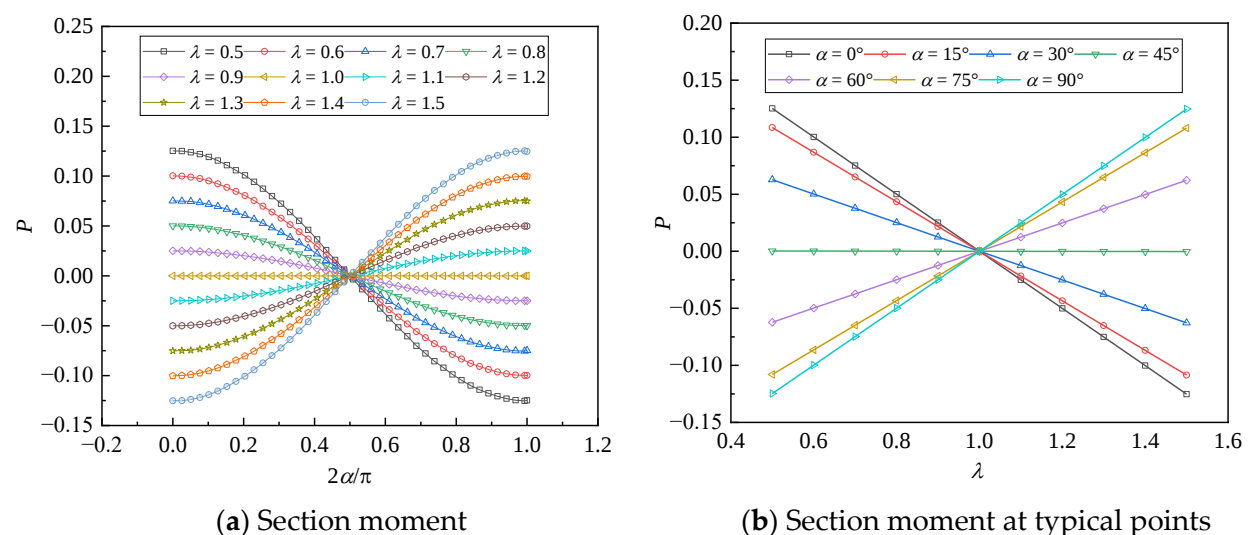


Figure 9. Section moment in the single factor test of parameter λ .

From Figures 8a and 9a, the radial displacement of the lining axis and the section moment are 0 when $\lambda = 1$, which means that when the upper load is equal to the lateral load, the lining is in the safest state. When $\lambda = 0.5 \sim 1.5$, the curves of the radial displacement and the section moment are symmetric with the curve of $\lambda = 1$.

From Figures 8b and 9b, it can be observed that the displacement of the lining axis and section moment changes linearly, and the deformation mode of the lining changes significantly with the increase in λ . It turns into the deformation with large lateral pressure to the inside, and the side with small pressure to the outside, which conforms to the actual deformation characteristics.

2.6. Orthogonal Test of Lining Model of Functionally Graded Lining

According to the above single factor test, factors a , b and λ are selected for the orthogonal test to compare the significance. Because the axial radial displacement and section moment curve of the functional gradient lining are symmetrical about the curve of $\lambda = 1$, the part of $\lambda < 1$ is taken. Three levels are taken for the three factors. The values of levels and the orthogonal test scheme are summarized in Tables 2 and 3.

Table 2. Level values of every factor in orthogonal test.

Factors	a	b	λ
Levels	0.9	0.9	0.5
	1.0	1.0	0.6
	1.1	1.1	0.7

Table 3. Orthogonal test scheme.

Number	Parameters			
	a	b	λ	Empty Column
A1	0.9	0.9	0.5	1
A2	0.9	1	0.6	2
A3	0.9	1.1	0.7	3
A4	1	0.9	0.6	3
A5	1	1	0.7	1
A6	1	1.1	0.5	2
A7	1.1	0.9	0.7	2
A8	1.1	1	0.5	3
A9	1.1	1.1	0.6	1

2.6.1. Deformation Mode Analysis

1. Calculation results of radial displacement

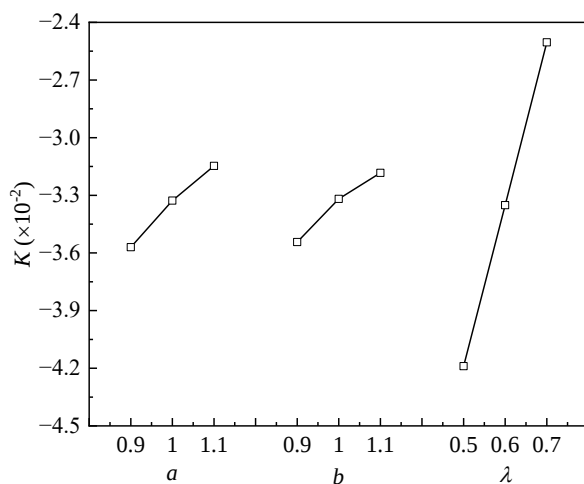
In the orthogonal test, the radial displacement at the lining axis is used to represent the lining deformation mode, and the calculation results are summarized in Table 4.

Table 4. Radial displacement of functionally graded lining.

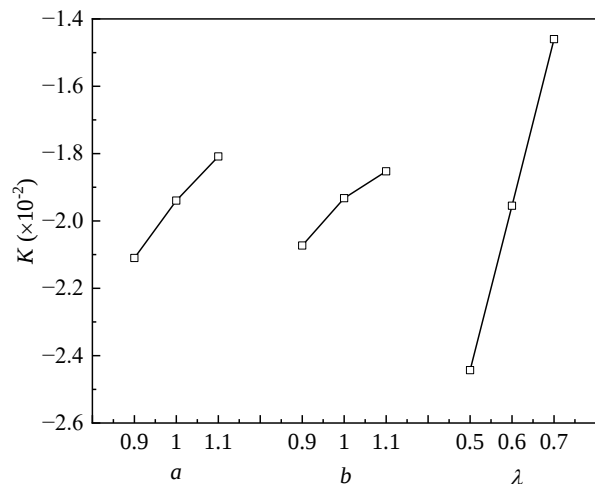
Number	a	b	λ	Dimensionless Parameter of the Displacement $K (\times 10^{-2})$						
				0°	15°	30°	45°	60°	75°	90°
A1	0.9	0.9	0.5	−4.653	−2.752	−0.505	1.569	3.122	4.082	4.658
A2	0.9	1	0.6	−3.528	−2.085	−0.374	1.120	2.364	3.070	3.494
A3	0.9	1.1	0.7	−2.529	−1.493	−0.262	0.867	1.694	2.187	2.482
A4	1	0.9	0.6	−3.494	−2.038	−0.300	1.300	2.477	3.166	3.528
A5	1	1	0.7	−2.501	−1.458	−0.208	0.938	1.772	2.251	2.501
A6	1	1.1	0.5	−3.988	−2.323	−0.322	1.508	2.824	3.567	3.953
A7	1.1	0.9	0.7	−2.482	−1.429	−0.159	1.008	1.852	2.321	2.529
A8	1.1	1	0.5	−3.927	−2.254	−0.229	1.624	2.947	3.661	3.966
A9	1.1	1.1	0.6	−3.032	−1.742	−0.177	1.250	2.261	2.799	3.032

2. Intuitive analysis of influence factors

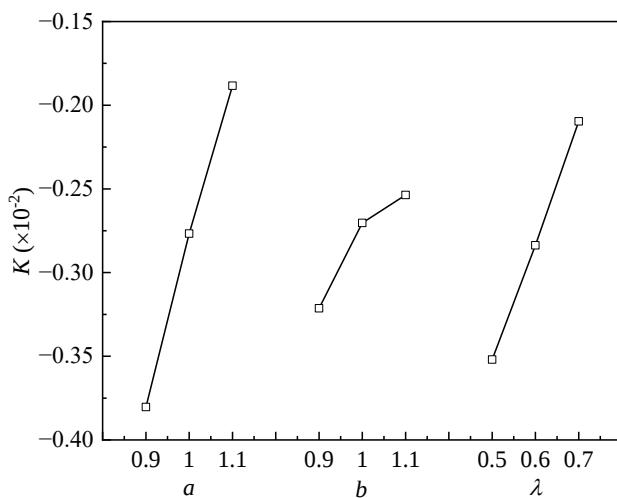
From Figure 10, the increase in the elastic modulus parameter a or b reduces the deformation, but the influence of a and b on different positions is different. When $\alpha < 45^\circ$, the displacement range caused by the change in parameter a is slightly larger than that caused by the change in parameter b , so changing a has a great influence on the deformation. When $\alpha > 45^\circ$, the displacement range caused by the change in parameter b is slightly larger than that caused by the change in parameter a , so changing b has a great influence on the deformation. Therefore, when the maximum deformation is at $\alpha = 0^\circ$, a can be increased to reduce the maximum deformation. When the maximum deformation is at 90° , b can be increased to reduce the maximum deformation. In addition, the radial displacement is most affected by the lateral pressure coefficient λ , and decreases linearly with the increase in λ , which shows that the lateral pressure coefficient has a great influence on the safety of structures, which exceeds the influence of the structural parameters. At 30° , the influence of a on the radial displacement exceeds the other two parameters. At 45° , the increase in a increases the radial displacement, but within $30\sim 45^\circ$, the radial displacement is also small, which belongs to the junction of positive and negative deformation, so the abnormality brought by this range is not considered.



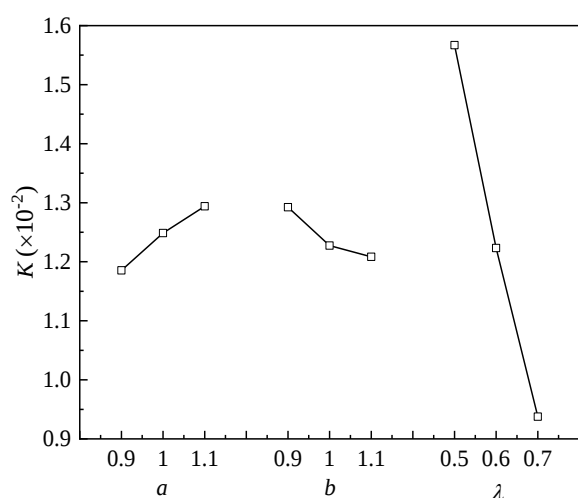
(a) Intuitive analysis of the radial displacement at 0°



(b) Intuitive analysis of the radial displacement at 15°

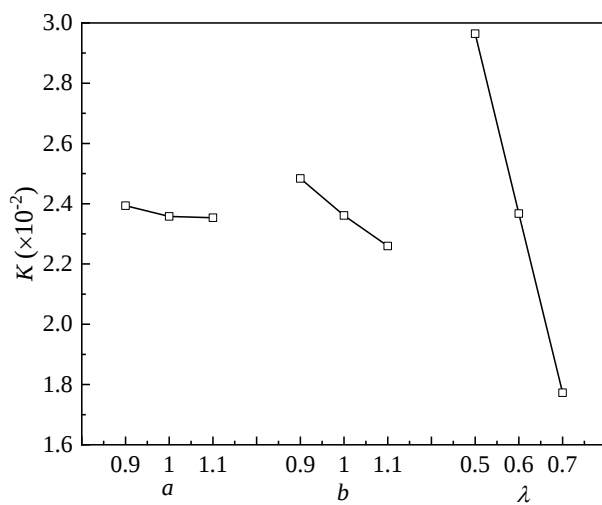


(c) Intuitive analysis of the radial displacement at 30°

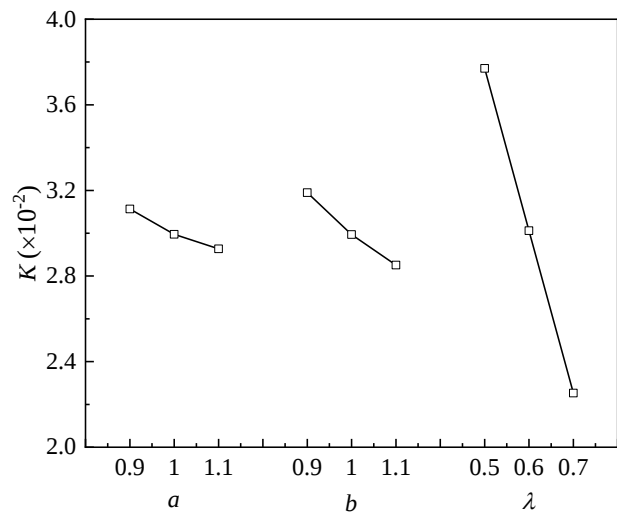


(d) Intuitive analysis of the radial displacement at 45°

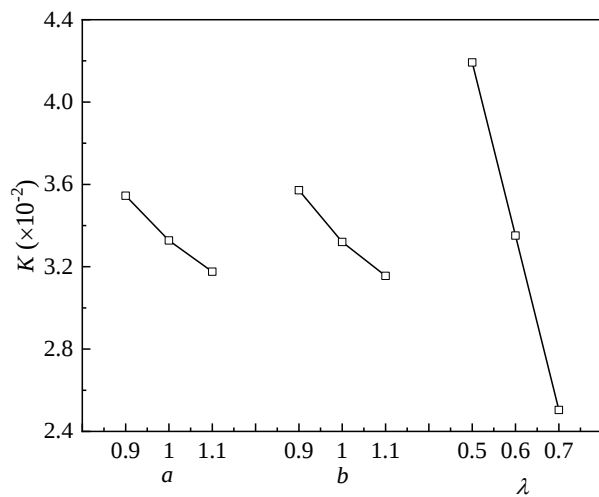
Figure 10. Cont.



(e) Intuitive analysis of the radial displacement at 60°



(f) Intuitive analysis of the radial displacement at 75°



(g) Intuitive analysis of the radial displacement at 90°

Figure 10. Intuitive analysis of influence factors of the radial displacement. Intuitive analysis of influence factors of the radial displacement.

3. Variance analysis of influence factors

The variance analysis of influence factors of radial displacement is summarized in Table 5.

Table 5. Variance analysis of influence factors of radial displacement.

Position	Parameters	Dof	Square of Deviance	F	Confidence			Significance
					99%	95%	90%	
0	a	2	0.270	26.061	99	19	9	**
	b	2	0.198	19.123	99	19	9	**
	λ	2	4.261	410.793	99	19	9	***
	Error	2	0.010					
15	a	2	0.137	28.193	99	19	9	**
	b	2	0.075	15.339	99	19	9	*
	λ	2	1.449	297.711	99	19	9	***
	Error	2	0.005					

Table 5. Cont.

Position	Parameters	Dof	Square of Deviance	F	Confidence			Significance
					99%	95%	90%	
30	<i>a</i>	2	0.055	32.980	99	19	9	***
	<i>b</i>	2	0.007	4.438	99	19	9	*
	λ	2	0.030	18.095	99	19	9	***
	Error	2	0.002					
45	<i>a</i>	2	0.018	4.040	99	19	9	-
	<i>b</i>	2	0.012	2.631	99	19	9	-
	λ	2	0.596	134.648	99	19	9	***
	Error	2	0.004					
60	<i>a</i>	2	0.003	1.258	99	19	9	-
	<i>b</i>	2	0.075	32.857	99	19	9	**
	λ	2	2.130	927.119	99	19	9	***
	Error	2	0.002					
75	<i>a</i>	2	0.053	10.111	99	19	9	*
	<i>b</i>	2	0.173	32.974	99	19	9	**
	λ	2	3.452	656.317	99	19	9	***
	Error	2	0.005					
90	<i>a</i>	2	0.206	20.093	99	19	9	**
	<i>b</i>	2	0.263	25.636	99	19	9	**
	λ	2	4.276	416.241	99	19	9	***
	Error	2	0.010					

“***” means very significant, “**” means significant, “*” means there is some influence but not significant, “-” means no influence.

According to Table 5, when $\alpha < 45^\circ$, parameters *a* and λ have a very significant influence on the radial displacement of the functional gradient lining, and the significance of λ is greater than that of parameter *a*. With the increase in α , the significance of parameter *a* on the displacement increases gradually, and that of parameter *b* on the displacement decreases gradually, but the significance of parameter *a* is always greater than that of parameter *b*. When $\alpha > 45^\circ$, parameters *b* and λ have a very significant influence on the radial displacement of the functional gradient lining, and the significance of λ is greater than that of parameter *b*. With the increase in α , the significance of parameter *a* on the displacement increases gradually, and that of parameter *b* on the displacement decreases gradually, but the significance of parameter *b* is always greater than that of parameter *a*. At $\alpha = 45^\circ$, parameters *a* and *b* have no effect on the radial displacement, but λ has a significant effect on the radial displacement.

In conclusion, for the functionally graded lining with linear elastic modulus distribution, in the part of $\alpha < 45^\circ$, the change in parameter *a* has a greater impact on the lining deformation. In the part of $\alpha > 45^\circ$, the change in parameter *b* has a greater impact on the lining deformation. Therefore, when the maximum deformation occurs at 0° , the parameter $a > b$. When the maximum deformation occurs at the position of 90° , the parameter $a < b$ can save material costs and reduce the structural deformation.

2.6.2. Section Moment Analysis

1. Calculation results of section moment

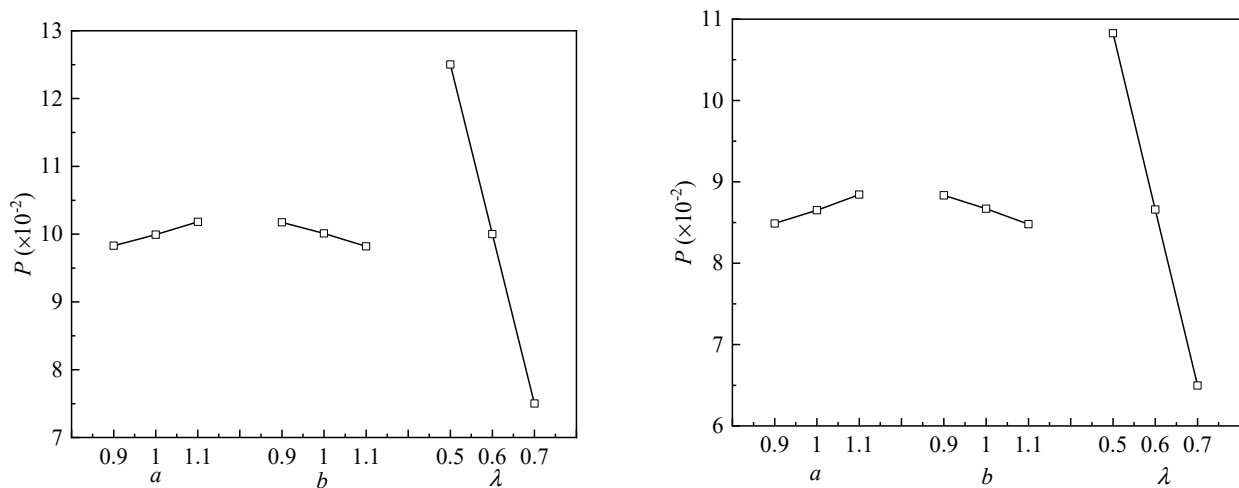
The calculation results of the section moment of functional gradient lining in the orthogonal test are summarized in Table 6.

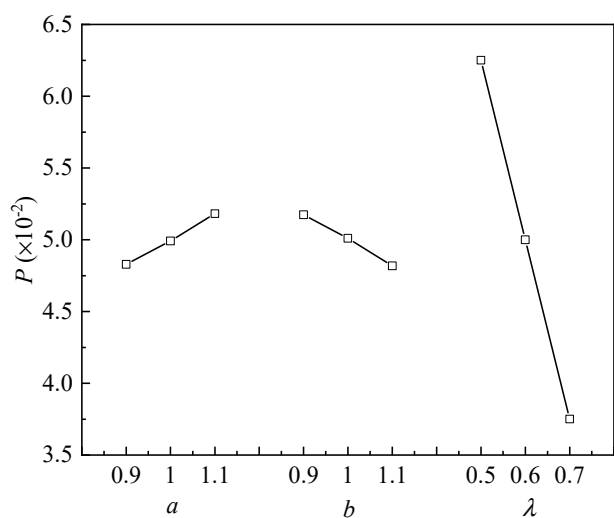
Table 6. Section moment of functionally graded lining.

Number	a	b	λ	Dimensionless Parameters of Section Moment $P (\times 10^{-2})$						
				0°	15°	30°	45°	60°	75°	90°
A1	0.9	0.9	0.5	12.503	10.828	6.253	0.0028	−6.247	−10.823	−12.497
A2	0.9	1	0.6	9.787	8.447	4.787	−0.213	−5.213	−8.874	−10.213
A3	0.9	1.1	0.7	7.195	6.191	3.445	−0.305	−4.055	−6.800	−7.805
A4	1	0.9	0.6	10.213	8.874	5.213	0.213	−4.787	−8.447	−9.787
A5	1	1	0.7	7.502	6.497	3.752	0.0015	−3.748	−6.494	−7.498
A6	1	1.1	0.5	12.259	10.584	6.009	−0.241	−6.491	−11.067	−12.741
A7	1.1	0.9	0.7	7.805	6.800	4.055	0.305	−3.445	−6.191	−7.195
A8	1.1	1	0.5	12.741	11.067	6.491	0.241	−6.009	−10.584	−12.259
A9	1.1	1.1	0.6	10.000	8.660	5.000	0	−5.000	−8.660	−10.000

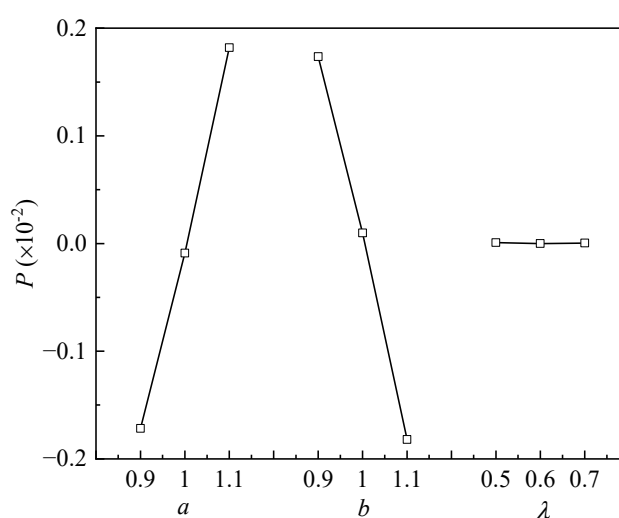
2. Intuitive analysis of influence factors

It can be observed from Figure 11 that λ has the greatest influence on the section moment. When $\alpha < 45^\circ$, with the increase in parameter a , the section moment increases. With the increase in parameter b , the section moment decreases, and the increase in the section moment caused by the increase in a is almost the same as the decrease in the section moment caused by the increase in b . When $\alpha > 45^\circ$, with the increase in parameter a , the section moment decreases, and with the increase in parameter b , the section moment increases. The decrease in the section moment caused by the increase in a is almost the same as the increase in the section moment caused by the increase in b . When $\alpha = 45^\circ$, the section moment is small, and the influence of parameters a and b on the section moment is greater than that of λ . Because this is the junction of positive and negative bending moment, and the section moment is small, the abnormality generated here is not considered. Therefore, it can be found that when the elastic modulus of the lining increases, the section moment also increases, which conforms to the idea of “flexible yielding” in structural design.

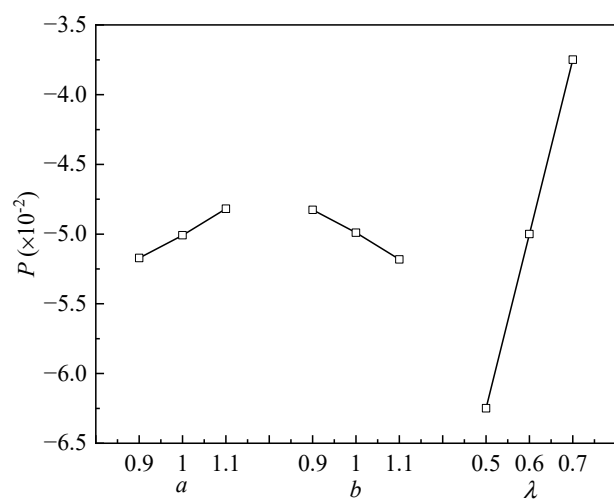
(a) Intuitive analysis of the section moment at 0° (b) Intuitive analysis of the section moment at 15° **Figure 11.** Cont.



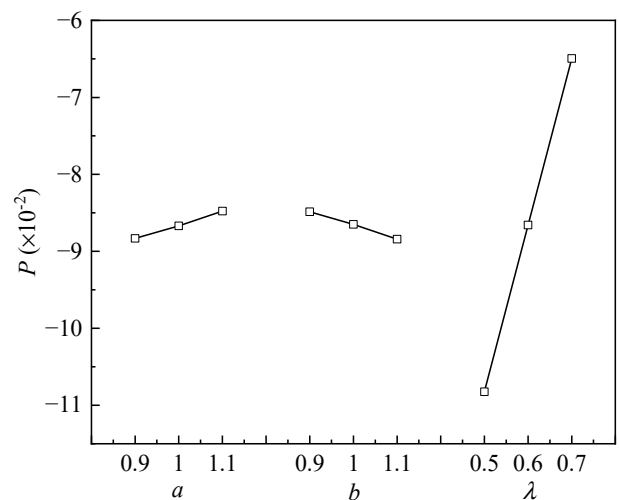
(c) Intuitive analysis of the section moment at 30°



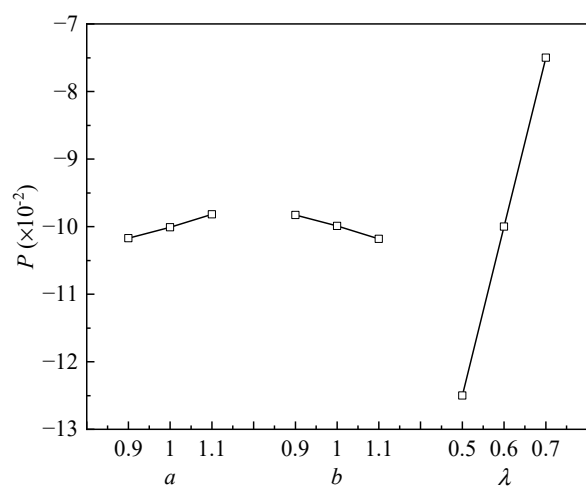
(d) Intuitive analysis of the section moment at 45°



(e) Intuitive analysis of the section moment at 60°



(f) Intuitive analysis of the section moment at 75°



(g) Intuitive analysis of the section moment at 90°

Figure 11. Intuitive analysis of influence factors of the section moment.

3. Variance analysis of influence factors

The variance analysis of influence factors of the section moment is summarized in Table 7.

Table 7. Variance analysis of influence factors of section moment.

Position	Parameters	Dof	Square of Deviance	F	Confidence			Significance
					99%	95%	90%	
0	<i>a</i>	2	0.188	12.698	99	19	9	*
	<i>b</i>	2	0.190	12.843	99	19	9	*
	λ	2	37.505	2533.057	99	19	9	***
	Error	2	0.015					
15	<i>a</i>	2	0.188	12.448	99	19	9	*
	<i>b</i>	2	0.190	12.590	99	19	9	*
	λ	2	28.128	1862.404	99	19	9	***
	Error	2	0.015					
30	<i>a</i>	2	0.188	12.698	99	19	9	*
	<i>b</i>	2	0.190	12.843	99	19	9	*
	λ	2	9.378	633.349	99	19	9	***
	Error	2	0.015					
45	<i>a</i>	2	0.188	12.704	99	19	9	*
	<i>b</i>	2	0.190	12.838	99	19	9	*
	λ	2	0.000	0.000	99	19	9	-
	Error	2	0.015					
60	<i>a</i>	2	0.188	12.698	99	19	9	*
	<i>b</i>	2	0.190	12.843	99	19	9	*
	λ	2	9.373	633.011	99	19	9	***
	Error	2	0.015					
75	<i>a</i>	2	0.188	12.471	99	19	9	*
	<i>b</i>	2	0.190	12.565	99	19	9	*
	λ	2	28.119	1861.611	99	19	9	***
	Error	2	0.015					
90	<i>a</i>	2	0.188	12.698	99	19	9	*
	<i>b</i>	2	0.190	12.843	99	19	9	**
	λ	2	37.495	2532.381	99	19	9	***
	Error	2	0.015					

“***” means very significant, “**” means significant, “*” means there is some influence but not significant, “-” means no influence.

According to the variance analysis in Table 7, the influence of λ on the section moment is very significant, and the influence on the positions of $\alpha = 0^\circ$ and $\alpha = 90^\circ$ is the most significant. The influence of elastic modulus parameters *a* and *b* on the section moment is significant, and the significant degree is similar. The section of 45° is abnormal in the variance analysis, where λ has no effect on the section moment, and the influence of the elastic modulus parameters on the section moment is similar to that of other parts. Because of the small section moment, this abnormality is not considered.

In conclusion, for the functionally graded lining with linear elastic modulus distribution, the influence of the non-uniformity of external load on the section moment is the largest, far more than the influence of elastic modulus parameters.

2.7. Verification of the Numerical Simulation

In the numerical calculation, the lining of subway tunnels has a certain thickness, but in the theoretical calculation, the lining is simplified as a curved rod structure. Therefore, under the condition of the different thickness diameter ratio, the displacement calculation results of the inner wall, outer wall and axis of the lining in the numerical calculation are

compared with the theoretical calculation results, so as to determine the corresponding position of the radius R_H in the theoretical calculation. The calculation results are shown in Figure 12.

From Figure 12, it can be observed that the radial displacement at the inner edge of the cylinder is the largest, followed by the axis, and the outer edge is the smallest, indicating that the inner edge of the cylinder structure should be used as the basis for failure under the action of external force. When $t/R \leq 0.2$, the theoretical calculation results are close to the radial displacement of the inner edge of the cylinder in the numerical simulation results. With the increase in the thickness diameter ratio, the numerical simulation results of the radial displacement of the inner edge at 0° and 90° gradually increase. When $t/R = 0.2$, the numerical simulation results of the radial displacement of the inner edge at 0° and $\alpha = 90^\circ$ are closest to the theoretical calculation results. However, when $t/R > 0.2$, the numerical simulation results of the radial displacement of the inner edge at 0° and 90° exceed the theoretical calculation results, and the gap between them gradually becomes larger. It shows that when $t/R \leq 0.2$, the theoretical calculation results can be used to calculate the radial displacement of the inner edge, and the calculation results are conservative. However, when $t/R > 0.2$, if the theoretical model continues to be applied to calculate the radial displacement at the most unfavorable position, the results will be smaller, which is not conducive to the safety of the structure. In conclusion, the theoretical model of internal force and deformation is suitable for the $t/R \leq 0.2$ cylinder structure.

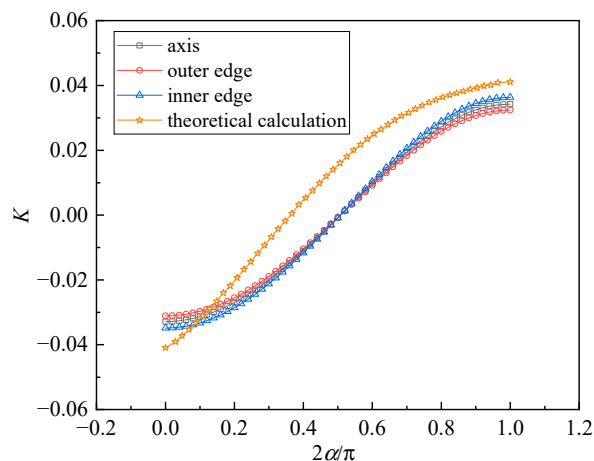
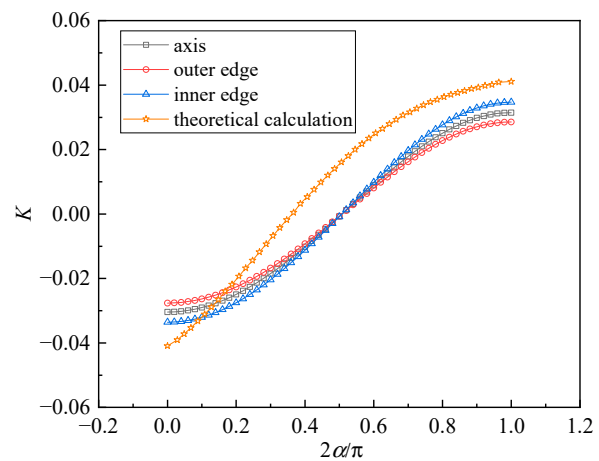
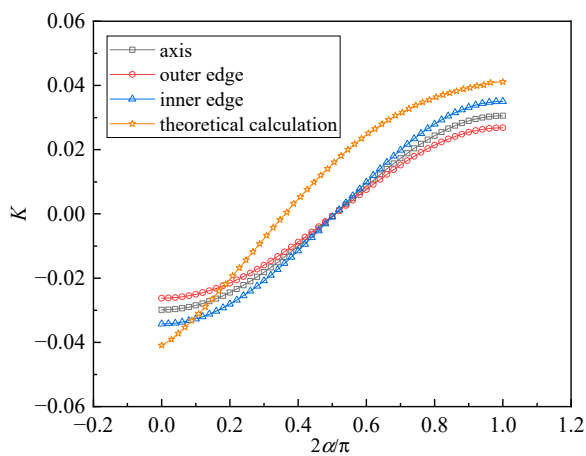
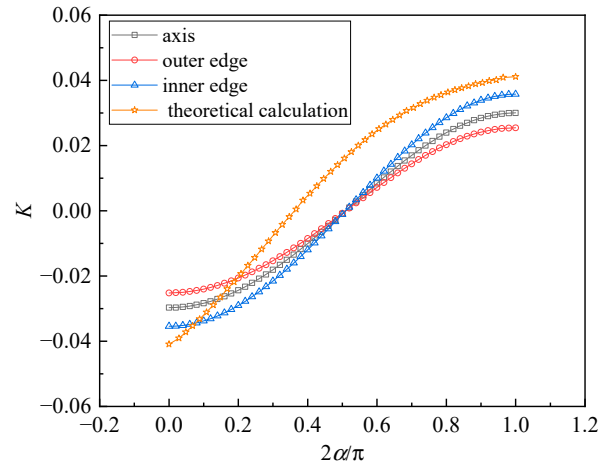
(a) $t/R = 0.03$ (b) $t/R = 0.05$ (c) $t/R = 0.07$ (d) $t/R = 0.09$

Figure 12. Cont.

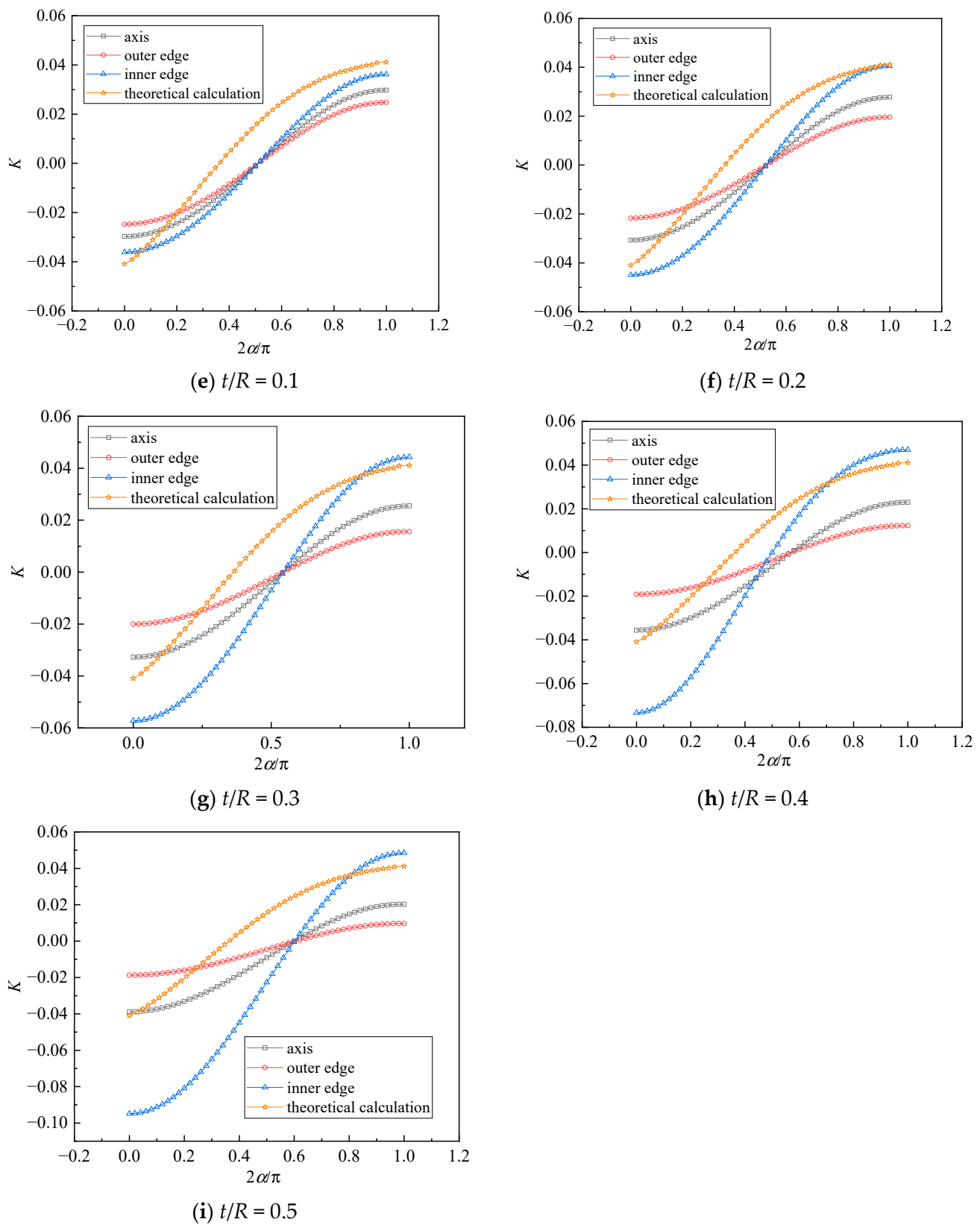


Figure 12. Comparison of numerical simulation results and theoretical calculation results of radial deformation of homogeneous cylinder at different positions under different thickness diameter ratio.

According to the comparison of the radial displacement between the numerical simulation and the theoretical calculation under the different thickness diameter ratios, the

function gradient cylinder under the condition of $t/R = 0.2$ is simulated. The adopted elastic modulus functions are Function I and Function II. In order to reflect the economic characteristics of the functionally graded structures (FGS), two groups of elastic modulus parameters $a = 1, b = 0.9$ and $a = 0.9, b = 1$ are used. The vertical load is 1 MPa, $\lambda = 0.5$ and the fundamental elastic modulus $E_0 = 36$ GPa.

Because ABAQUS 6.14 cannot realize the continuous change in the elastic modulus along the circular direction, the cylinder structure is adopted to give the elastic modulus along the circular direction in sections. When there are enough segments, it can be considered that the cylinder with segments given the elastic modulus is equivalent to the functional gradient cylinder in mechanical properties. Next, the number of segments is explored. The change trend of Mises stress and deformation coefficient $|K|$ with the number of segments at typical points is shown in Figure 13.

From Figure 13, it can be observed that with the increase in the number of segments, the Mises stress and radial displacement coefficient $|K|$ at the inner edge of the cylinder gradually converge to a certain value, so when the number of segments is large enough, the method of elastic modulus given by segments can be used to simulate the functional gradient lining. It can be also observed that when the number of segments is 16, the stress and deformation begin to converge. When the number of segments is 32, the change degree of Mises stress and the radial deformation coefficient of each typical point is less than 0.05% and 0.02%, respectively. Therefore, in order to ensure the accuracy of calculation and reduce the calculation, in the next simulation of the mechanical properties of the FGM cylinder, the 1/4-cylinder structure is divided into 16 sections to simulate the FGM cylinder.

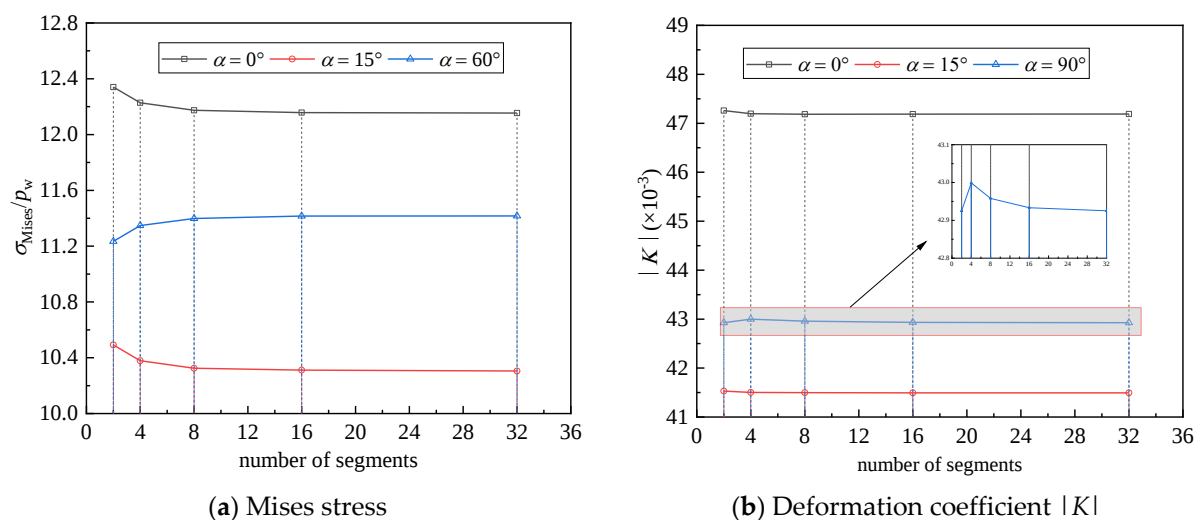


Figure 13. The change trend of cylinder mechanical properties with the number of segments.

Based on the above demonstration of the thickness diameter ratio and the number of segments of the functional gradient cylinder in the numerical simulation, the mechanical properties of functional gradient linings with $t/R = 0.2$ and the number of segments of 16 are analyzed, and the deformation coefficient K of the functional gradient lining under the condition of the elastic modulus Function I is compared with the theoretical calculation result, and the comparison result is shown in Figure 14. The radial displacement of FGM lining with different elastic modulus parameters in the theoretical calculation and numerical simulation is shown in Figure 15. Compared with the radial displacement of FGM linings with two elastic modulus functions, the result is shown in Figure 16.

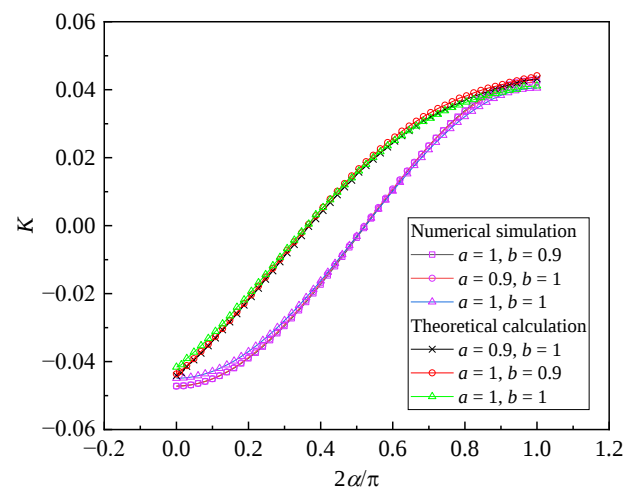


Figure 14. Comparison between the theoretical and numerical results of radial displacement of functionally graded lining under the condition of elastic modulus Function I.

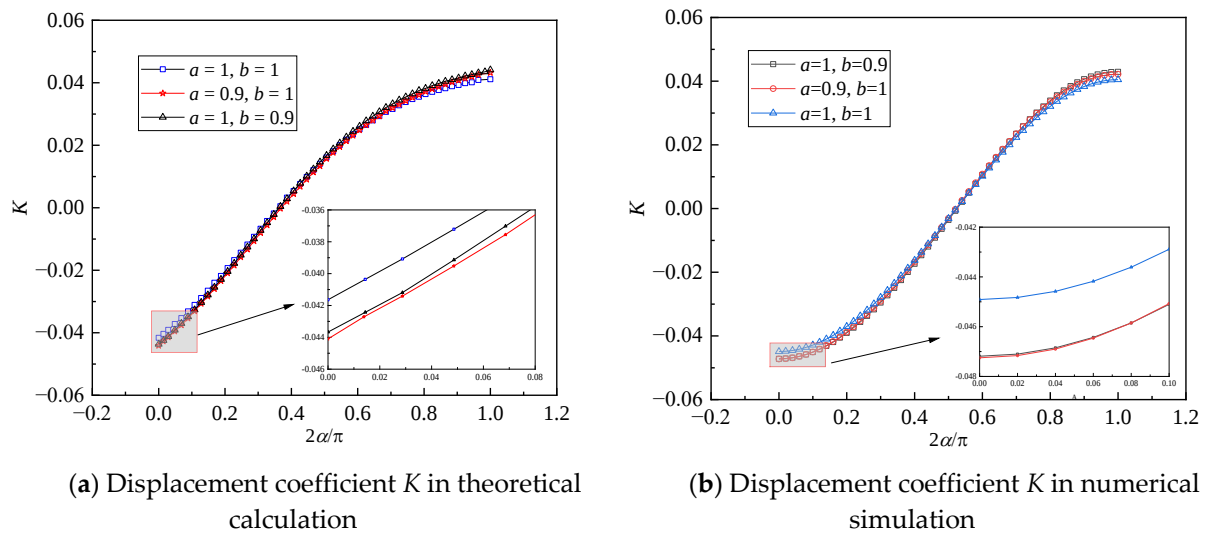


Figure 15. Comparison of radial displacement calculation results of functionally graded lining with different elastic modulus parameters under the condition of elastic modulus Function I.

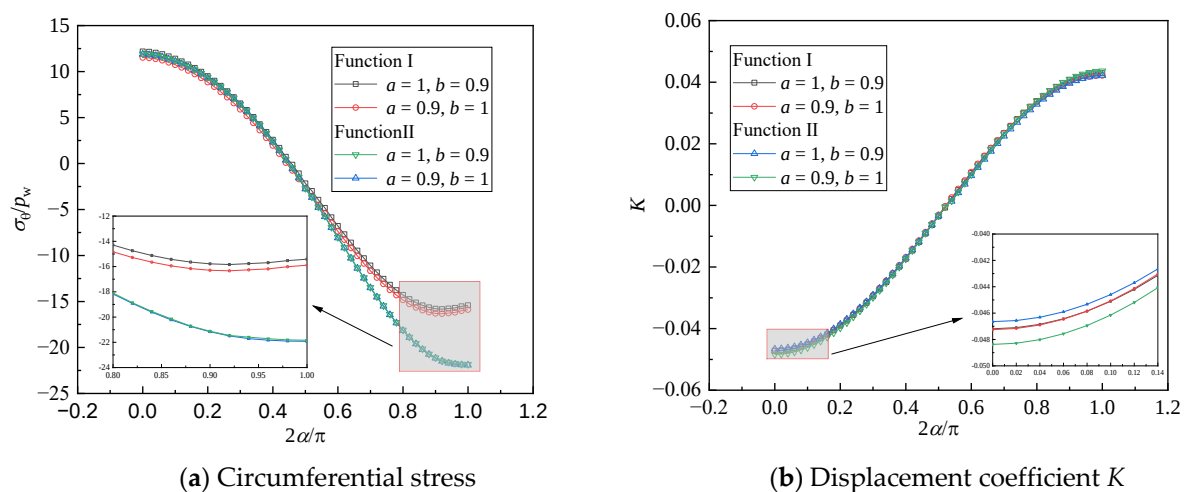


Figure 16. Comparison of mechanical properties of functionally graded lining under two elastic modulus functions.

From Figure 14, the radial displacement curves of FGM lining almost coincide under the conditions of $a = 1, b = 0.9$ and $a = 0.9, b = 1$ in the results of both the theoretical analysis and the numerical simulation. There is a certain gap in the results between the theoretical analysis and the numerical simulation, but the theoretical analysis and the numerical simulation results are close at the two places of 0° and 90° with large displacement, where the gap between the two calculation results is within 8%, so the theoretical calculation results can be used as the basis for the deformation of the lining.

From Figure 15, under the condition of the linear distribution of the elastic modulus, the radial deformation of lining is slightly different with different elastic modulus parameters in both the theoretical calculation and the numerical simulation. From Figure 15a, the maximum radial displacement is at 0° . The maximum radial displacement of homogeneous lining is the smallest and the functional gradient lining with elastic modulus parameter $a = 1, b = 0.9$ is the second, and the functional gradient lining with elastic modulus parameter $a = 0.9, b = 1$ is the largest. In addition, the difference between the maximum deformation of functionally graded lining with $a = 1, b = 0.9$ and that of homogeneous lining is no more than 5%. From Figure 15b, similarly, the maximum radial displacement is at 0° in the numerical simulation. The maximum radial displacement of homogeneous lining is the smallest and the functional gradient lining with elastic modulus parameter $a = 1, b = 0.9$ is the second, and the functional gradient lining with elastic modulus parameter $a = 0.9, b = 1$ is the largest. In addition, the difference between the maximum deformation of functionally graded lining with $a = 1, b = 0.9$ and that of homogeneous lining is no more than 5%. In conclusion, although there are some differences in the results between the theoretical analysis and the numerical simulation, the laws shown by them are consistent. When $\lambda = 0.5$, the maximum deformation is at 0° , and $a > b$ is conducive to reduce the maximum deformation.

From Figure 16, the deformation of the functionally graded lining corresponding to the two elastic modulus functions is similar, but the circumferential stress is quite different. From Figure 16a, the circumferential stress of functional gradient lining corresponding to elastic modulus Function II is larger near 90° and is about 30% larger than that corresponding to Function I. From Figure 16b, the radial displacement of the four functional gradient linings reaches the maximum value at 0° , and the deformation difference is about 2%. The radial displacement of the functional gradient lining corresponding to Function II with $a = 1, b = 0.9$ is the minimum, and that of the functional gradient lining corresponding to Function I with $a = 1, b = 0.9$ is the second. It can be observed that Function II has some advantages over Function I in reducing the maximum deformation of the structure, but the advantages are relatively low and the overall stiffness of the functional gradient structure represented by Function II is also larger. Therefore, when Function I can meet the deformation requirements, it is unnecessary to use Function II to reduce the deformation further.

3. Conclusions

This study aims to develop a new approach to design the functionally graded lining of subway tunnels. With two parameters of the elastic modulus function and lateral pressure coefficient, the mechanical properties of the functionally graded lining were studied, proving that the new lining shows great characteristics of economy and security.

In the study, the mechanical properties of the functionally graded lining were researched and by analyzing theoretically and numerically, some conclusions are drawn as follows:

- (1) The single factor test shows that the radial displacement of the lining axis decreases with the increase in a and b , but the deformation mode remains the same, and the reduction in deformation is smaller and smaller. In addition, with the increase in a and b , the distribution trend of moment remains the same. With the increase in a , the positive section moment increases linearly and the negative bending moment decreases linearly. With the increase in b , the negative moment increases linearly and the positive moment decreases linearly, which embodies the idea of “flexible yielding”. The displacement of the lining axis and the section moment change linearly with the

increase in λ . With the increase in λ , the shape of the lining changes significantly, which shows that the side with large lateral pressure deforms to the inside, and the side with small lateral pressure expands to the outside.

- (2) The orthogonal test shows that when the maximum deformation occurs at 0° , the parameter a should be larger than b . When the maximum deformation occurs at 90° , the parameter b should be larger than a , so as to save material costs on the premise of ensuring safety. In addition, the lateral pressure coefficient has a great impact on the safety of the structure, which exceeds the influence of structural parameters on the safety of the structure.
- (3) The numerical simulation shows that the calculation model of internal force and deformation is suitable for the cylinder with $t/R \leq 0.2$. There is a certain gap between the theoretical analysis and numerical simulation, but the gap between the theoretical analysis and numerical simulation results is within 8% at 0° and 90° with large displacement. In addition, the conclusion of the theoretical analysis is verified. There is little difference in deformation between the two kinds of functional graded linings, but there is a big difference in circumferential stress. It can be observed that compared with Function I, Function II has some advantages in reducing the maximum deformation of the structure, but the advantages are relatively low.

In further study, the solution model of internal force and the displacement of functionally graded lining under specific elastic function should be conducted based on the theory of elasticity.

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