

Article

Weighted Multi-Band Turbo-Coded FSK for Reliable Underwater Communications

Hyun-Woo Jeong  and Ji-Won Jung * 

Department of Radio Communication Engineering, Korea Maritime and Ocean University, Busan 49112, Korea; gusdn0930@g.kmou.ac.kr

* Correspondence: jwjung@kmou.ac.kr; Tel.: +82-51-410-4424

Abstract: Multi-band communication technology allocates the same data to different frequency bands, improving both performance and propagation efficiency. However, since the performance loss in a particular band affects the entire band, the multi-band structure may have worse performance than the single-band one. To avoid performance degradation, this paper proposed estimated coded bit error rates (EC-BER) algorithm. EC-BER algorithm is a method of analyzing the reliability of the received data based on the performance difference between demodulated and decoded data. It analyzes the performance of each band and allocates lower weighting values to inferior bands. Furthermore, an iterative turbo-coded equalization algorithm, which iteratively exchanges probabilistic information between the equalizer and turbo decoder, is applied. By employing a direct sequence spread spectrum (DSSS) with frequency shift keying (FSK) and multi-band techniques for reliable underwater communications, a lake trial was conducted. Experimental results show that the performance is improved as the number of spreading factors and turbo iterations increase. Especially the addition of the EC-BER algorithm dramatically improved the reliability of the system, resulting in minimal to zero errors.

Keywords: underwater communications; direct sequence spread spectrum; multi-band; estimated coded BER; turbo equalizer



Citation: Jeong, H.-W.; Jung, J.-W. Weighted Multi-Band Turbo-Coded FSK for Reliable Underwater Communications. *Appl. Sci.* **2022**, *12*, 4883. <https://doi.org/10.3390/app12104883>

Academic Editors: Alexander Sutin and Jacek Tomków

Received: 28 February 2022

Accepted: 9 May 2022

Published: 11 May 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

For reliable communications, covertness in military security has recently received much attention in the field of air and underwater environments [1,2]. Nowadays, Unmanned Air Vehicles (UAVs) are playing an important role in global society due to their ability to carry a payload [3]. In respect to underwater acoustic communications, a covert underwater communication system designed for the purpose of transmission signals is not intercepted by other unintended receivers. Typically, covert communication systems use a spread spectrum technique called the direct sequence spread spectrum (DSSS) [4]. DSSS involves increasing the system's symbol rate by a factor of chips. The larger bandwidth helps mitigate performance degradation due to the narrowband interference. The problem of the DSSS technique in an underwater acoustic channel involves extreme inter-symbol interferences (ISI) caused by multi-path, propagation loss, and Doppler effects [5,6]. These are the main factors that degrade performance in underwater channel with a low SNR. Therefore, multi-band technology with forward error correction (FEC) are essential for reliable underwater communications. For the purpose of achieving high reliability, a turbo-coded frequency shift keying (FSK) method was employed in the paper [7]. Multi-band technology, where a larger carrier spacing for the same data helps to improve the robustness of frequency variation was also used [8]. To apply the multi-band turbo-coded FSK technology, the available bandwidth needs to be divided into specific bands, and the turbo-coded FSK symbols allocate the different specific bands. It overcomes frequency-selective fading due to a large delay spread and sparse channel impulse responses and

noise. However, since the performance loss in a particular band affects the entire band, the multi-band structure may have worse performance than the single-band one. This problem can be solved by using a receiving end that analyzes the receiver power of each band and allocates the lower weights to inferior bands. Therefore, it is very important to find which band is inferior one. There are two conventional methods of deciding inferior band [9,10]. The first is the Effective Signal to Noise Ratio (ESNR) estimation. To estimate a receiving SNR, this method uses a pilot signal or preamble data, which are already known between the receiving and transmitting ends. The relation between the estimated SNR, and the coded error rate of a channel decoding algorithm is utilized to set a weighting value that satisfies the Quasi-Error-Free (QEF) condition of the channel-coding algorithm. The second is the Post-Equalization SNR (PES) estimation. The PES method measures an error element, which is the difference in output between an equalizer and a received signal, at the equalizer that applies Least Mean Square (LMS) or Recursive Least Square (RLS) types to the receiving end [11,12]. Due to various multi-paths, the measured values using the ESNR and the PES methods in receiving end of each band are not much different between bands. We cannot distinguish which band is inferior by using these conventional methods [13]. Therefore, this paper proposed an estimated coded bit error rates (EC-BER) algorithm, which analyzes the performance of each band and allocates lower weighting values to inferior bands. Since the demodulation error rate and the decoded error rate have a close relationship, the demodulation error rate must be low to satisfy the QEF condition after channel decoding. By using this point, the EC-BER algorithm is a method for measuring the reliability of the received data by checking how many differences there are between the demodulation and the decoded data. Because EC-BER curve falls sharply when the decoded BER is perfectly corrected, we decide optimal weighting values in the range of the lowest error rate of the EC-BER. Finally, the turbo-coded equalization algorithm, which improves the performance by receiver-based iteration, is also used [14]. For the analysis, an underwater experiment was conducted in a lake by using spread 4-FSK, turbo codes with one-third coding rates, and four bands.

Through the lake trial tests, the performance was improved as the number of spreading factors and turbo iterations increased. Furthermore, the multi-band performance improved when the proposed EC-BER weighting algorithm was applied.

This paper is arranged as follows: Section 2 presents system model of the multi-band FSK with a DSSS is described in detail. In Section 3, the proposed EC-BER algorithm is described, and showed how an optimal weighting value is assigned by simulation results. Section 4 analyzes the performance of the proposed algorithm through lake experiments. Concluding remarks are given at the end of Section 5.

2. System Model

Figure 1 shows the structure of the weighted multi-band FSK transceiver model with a DSSS. The turbo encoder has a rate of one-third. Information bits, K , are passed through the encoder. After encoding, N coded bits are generated, and they are input to the interleaving block. The interleaving shuffles the input sequence to improve the performance of the encoder against noisy burst errors [15]. The interleaving output is then packetized with preamble. The packet has $N_p (= N + n)$ bits, consisting of N bits of turbo-coded data, $C = \{c_0, c_1, c_2, \dots, c_{N-1}\}$, and n bits of preamble data, $P = \{p_0, p_1, p_2, \dots, p_{n-1}\}$. Preamble data are added to the encoded data to detect the start of the packet.

where $r_p(t)$ denotes the received preamble symbol, P denotes the reference preamble symbol. When the preamble and received sequence exactly align with each other, the correlation function, $C(t)$, reaches the maximum value, and the start of the packet is located. The Rx sub-band processing block is a non-coherent FSK demodulator. The non-coherent receiver typically employs energy detection for each subcarrier. It separates the frequency band by a bandpass filter and detects the highest energy by an envelope detector for different frequencies. After Rx sub-band processing, due to reflections from multiple obstacles in the underwater environment, a RAKE receiver is designed to counter the effects of multi-path fading for spread spectrum communications. The RAKE receiver consists of multiple correlators, in which the receive signal is multiplied by time-shifted versions of a locally generated spreading code sequence. The delays in each received

signal are compensated and are fed to the combiner, integrator, and comparator, which combines them suitably with different appropriate time delays. In particular, the RAKE receiver architecture allows for an optimal combining of energy received over paths with different [18]. The RAKE processing output includes the residual Doppler shift, which may be expressed as Equation (4):

$$z_{k,i} = \sum_{i \in C} \tilde{c}_i \exp(jf_{d,k}), \quad 1 < i < N, \quad 1 < k < N_b \quad (4)$$

where $z_{k,i}$ denotes the i -th RAKE output at the k -th band. $\tilde{c}_i$ denotes the i -th de-spreading bit, and $f_{d,k}$ denotes the residual Doppler shift at the k -th band. After RAKE processing, a decision feedback equalizer (DFE) removes multi-path interference from each band [19]. The output of the DFE at time t for k -th band, $z_k(t)$, is given by:

$$z_k(t) = \sum_{i=0}^{m_c-1} c_i[t] z_{k,t-i} - \sum_{j=0}^{m_b-1} b_j[t] z_k[t-j] \quad (5)$$

where $c_i[t]$ ($i = 0, 1, \dots, m_c - 1$) are the forward equalizer taps at time t , $b_j[t]$ ($j = 0, 1, \dots, m_b - 1$) are the feedback taps at time t . The LMS updates the algorithm for the feedforward and feedback filter taps [19].

In the multi-band configuration, the output signal of each band is combined, and they are input into the decoder. In combining process, the multi-band configuration may have worse performance than a single-band one. This is because the performance degradation in a particular band affects the output from all bands [20]. This problem can be solved by the proposed EC-BER algorithm, which analyzes the error rates of each band, sets threshold values, and allocates lower weighting values to inferior bands. EC-BER algorithm is a method of analyzing the reliability of the received data based on the performance difference between demodulated and decoded data. Here, the input symbol of the decoder can be expressed by Equation (6):

$$L_e^D = \sum_{k=1}^{N_b} W_k z_k(t) \quad (6)$$

After threshold detection, a weighting value W_k for the k -th band is assigned. The extrinsic value L_c^D that calculates the post-probability is the error-correction term. The re-interleaving of the computed value as $L_c^D - L_e^D$ is input to DFE. Then, L_c^I is updated to compensate for the errors. As the number of iterations in turbo equalization increases, the updated information may approach the original signal to be transmitted, thereby improving the BER performance [14].

3. EC-BER Algorithm

Underwater channels have various performance degradation factors, such as multiple paths and Doppler shifts. Particularly, it is possible to estimate the channel information by preamble data, which are known on both the transmitter and receiver [10]. With the estimated channel information, coded data are compensated. Because the underwater communication channel is a time-varying channel, as it does fluctuate with time, we cannot guarantee that the estimated channel information in the preamble data field is the same as that of the coded data field. This makes the data field unable to be completely compensated. As shown in Equation (4), a small amount of the remaining Doppler shift in the coded data field can still have a large influence on system performance [21]. Since each band has a different Doppler shift, the performance degradation in a particular band affects the output from all bands. To overcome the performance loss of the multi-band configuration, we propose an EC-BER algorithm in order to allocate lower weight to inferior band. The EC-BER algorithm is a method that can estimate the reliability of received data by checking how many differences there are between the demodulated and decoded symbols.

The detailed structure of the EC-BER block of Figure 1 is shown in Figure 2. The EC-BER block counts how many differences there are between the re-encoded and hard-decisioned demodulated symbols. We denote the hard decision of $z_k(t)$ as

$\bar{Z}_k = \{\bar{z}_{k,1}, \bar{z}_{k,2}, \bar{z}_{k,3}, \dots, \bar{z}_{k,N}\}$ and the re-encoded bit as $\hat{Z}_k = \{\hat{z}_{k,1}, \hat{z}_{k,2}, \hat{z}_{k,3}, \dots, \hat{z}_{k,N}\}$. The error counting processing is shown in Equations (7) and (8):

$$e_{k,i} = \begin{cases} 1 & \text{if } \bar{z}_{k,i} \neq \hat{z}_{k,i} \quad 1 < i < N \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

$$N_{k,e} = \sum_{i=1}^N e_{k,i} \quad (8)$$

where $e_{k,i}$ is the index of error for the i -th bit at the k -th band, and $N_{k,e}$ is the total number of errors for the k -th band. Thus, $N_{k,e}$ has a close relationship with the decoding performance. If $N_{k,e}$ is low, the EC-BER would be also small because the errors input to the decoder are small.

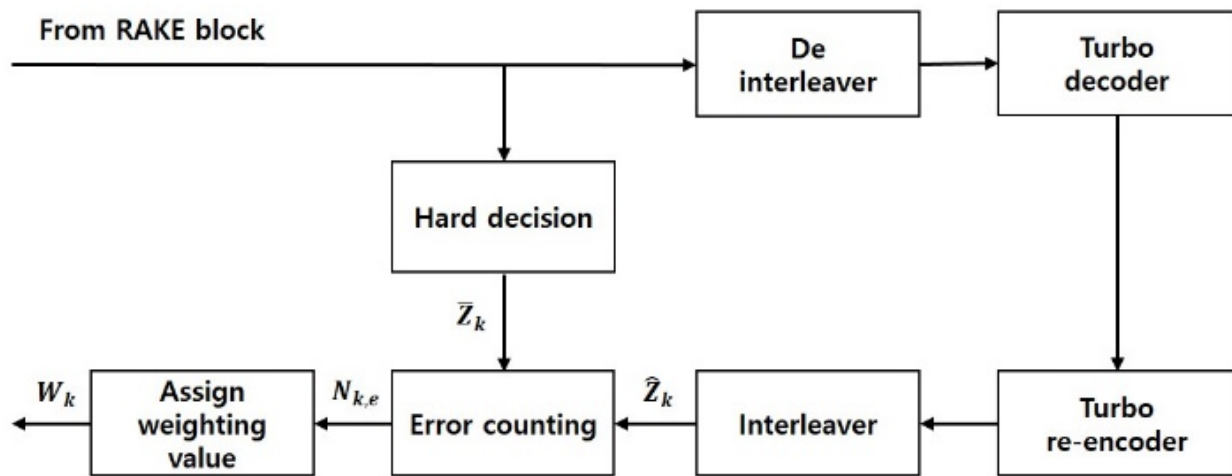


Figure 2. Block diagram of EC-BER block.

Figure 3 shows the relation between the EC-BER and decoded BER in a single-band through the simulation. It shows that the EC-BER falls sharply when the decoded BER is perfectly corrected. Therefore, the EC-BER has a close relationship with the performance of each band.

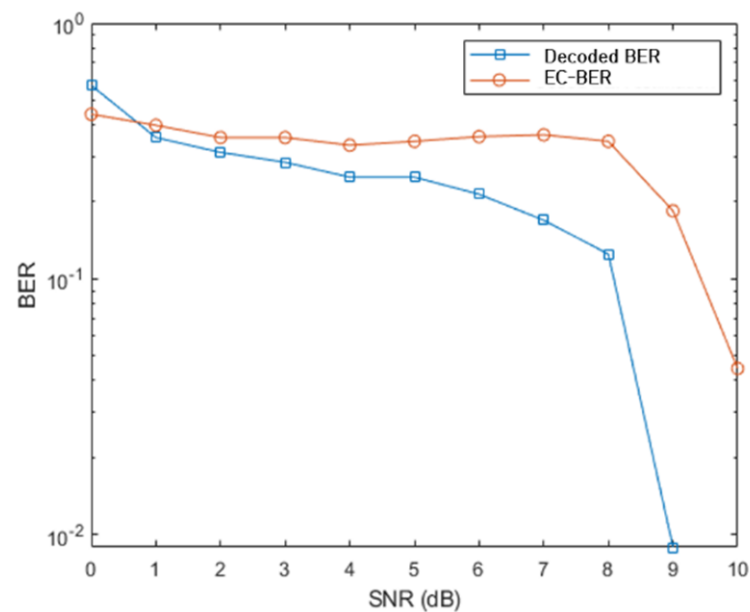


Figure 3. Relation of decoded BER and EC-BER.

For the multi-band configuration to avoid performance loss induced from an inferior band, a simulation was conducted to decide the optimal weighting value for each band according to the EC-BER. Employing turbo-coded data with rate of one-third ($N = 336$ bits) and 4-FSK, to avoid complexity, we used only two bands of f_1 and f_2 in the simulation. As shown on Figure 4, fixing the error rate of the f_1 band to less than 10%, we changed the error rate of the f_2 band from 10% to 50%. This is because a maximum demodulation error rate of 10% may be perfectly decoded in the turbo code with a rate of one-third. The weighting value of the f_1 band was fixed at 1, we tried the weighting value of the f_2 band was changed from 0 to 1 in intervals of 0.1. Figure 4 shows the EC-BER according to weighting values for two bands.

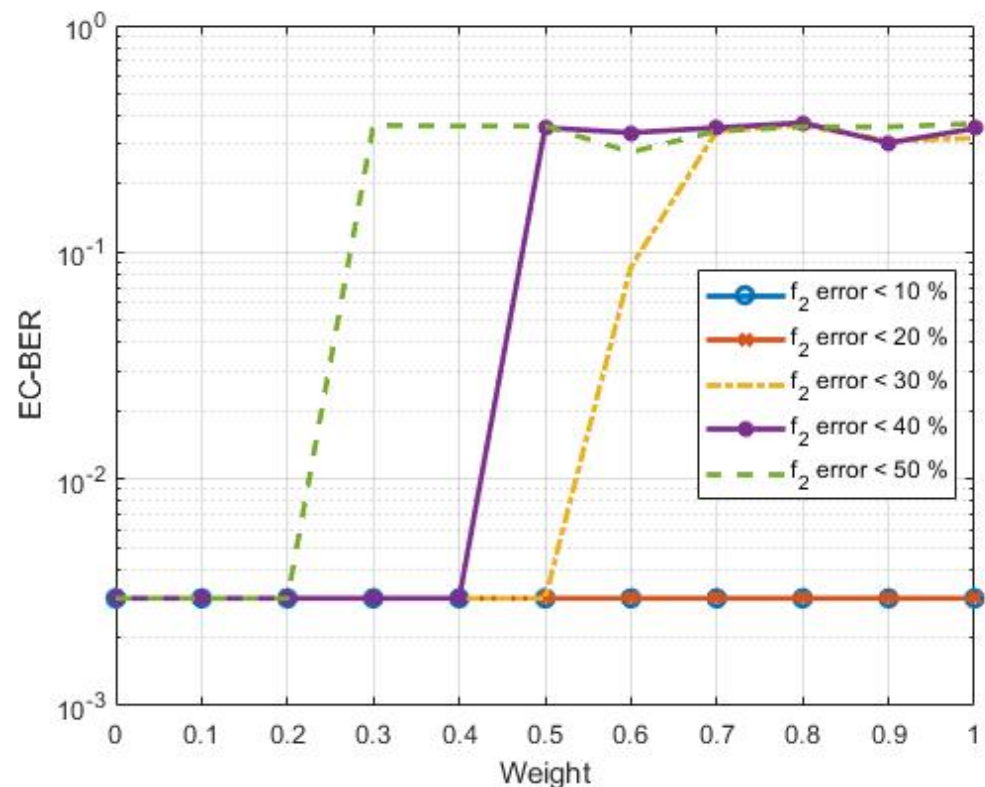


Figure 4. EC-BER for various weighting values.

For example, less than EC-BER of 20%, the error was perfectly corrected regardless of the weighting value. Less than EC-BER of 30 %, the error is perfectly corrected when the weighting value is less than 0.6. Similarly, EC-BER weighting values less than 40% and 50% were allocated as less than 0.4 and 0.2, respectively. The results mean that the higher the EC-BER is, the lower weight assigned. We confirmed that a higher EC-BER of lower weight is assigned. Based on Figure 4, we optimized the weighting values as shown in Table 1. To prove the efficiency of the weighting values as shown in Table 1, a simulation was conducted with the parameters of a four-band FSK signal with turbo codes.

Table 1. Weighting value according to EC-BER.

EC-BER (%)	Weighting Value
<20	1.0
<30	0.6
<40	0.4
<50	0.2
others	0

In the simulation result of Figure 5a, to investigate how well the weighting algorithm operates, the specific f_2 band is only set to high SNR and the EC-BER of the f_2 band decreases rapidly, meaning the decoded error is perfectly corrected. Figure 5b shows the performance gain when weighting is applied or not. When weighting is applied based on Table 1, errors were corrected perfectly on an SNR of 3 [dB]; in the other case, errors were corrected on an SNR of 6 [dB]. We knew 3 [dB] of performance gain was obtained by applying the weighting algorithm.

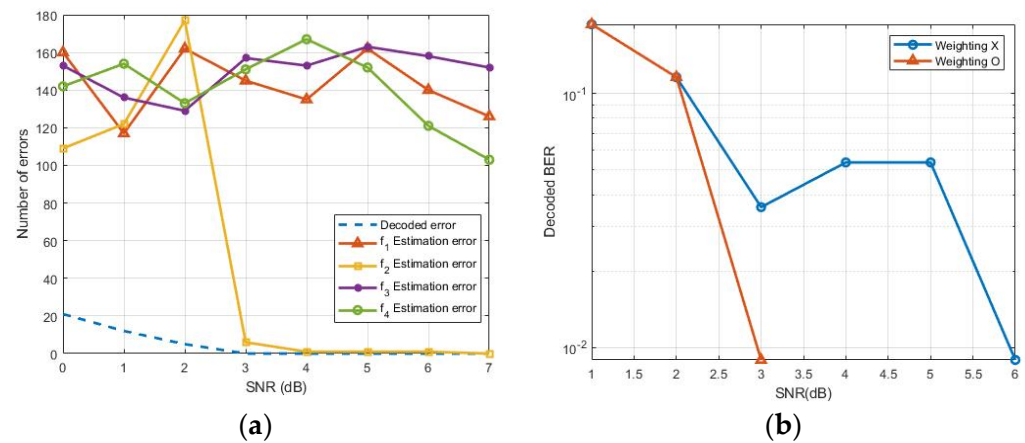


Figure 5. BER performance of weighted multi-band: (a) number of errors for multi-band and (b) performance comparison between weighting and non-weighting.

Finally, we confirmed the proposed method is effective to measure the performance of each band and improved the performance of the multi-band transmission method in underwater channel environments. With respect to the computational complexity compared to the state-of-the-art, such as ESNR and PES described in [13], in contrast to ESNR and PES which need several complex operations (N squaring and divide operations), the EC-BER algorithm only needs a shift register operation in the turbo re-encoding processing and to compare logic. Furthermore, it can predict the reliability of the coded data field exactly in time-varying underwater channels. However, in deciding the optimal weighting values using EC-BER, the values listed in Table 1 are a little different depending on amount of multi-path and Doppler spread.

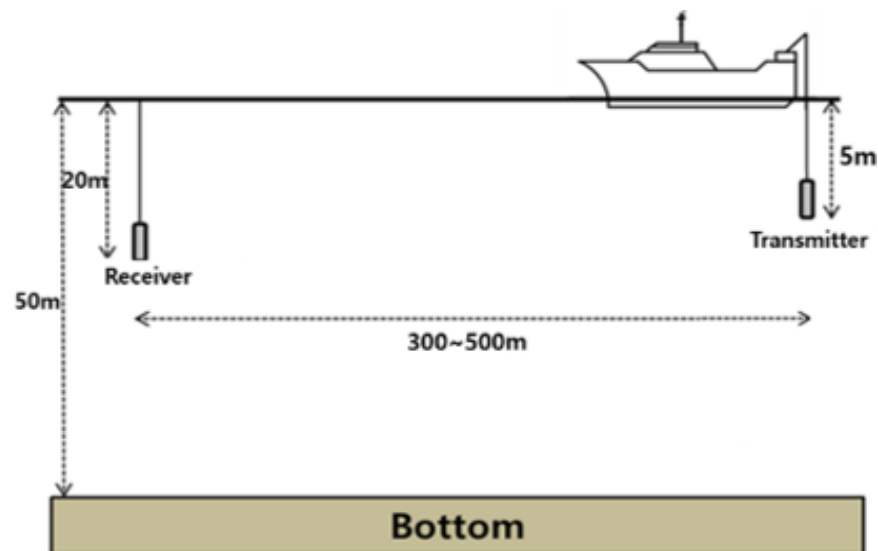
4. Experiment Result

A lake trial was run in Munkyeong city, Korea, in March 2021. The goal was to demonstrate the performance of EC-BER algorithms in controlled underwater acoustic environmental conditions. Table 2 lists parameters of the underwater test as described in Section 2. Five trials of the same packet were iteratively tested, and four bands with 4-FSK modulation and turbo codes with a rate of one-third was tested by changing several chips with 8 and 32. The center frequencies of 4 bands were 14 kHz, 18 kHz, 22 kHz, and 26 kHz. The sampling frequency was 192 kHz, and the data rate was 20 bps. The iteration of the turbo equalizer was set to five times.

A cross-section of the deployment in lake is shown in Figure 6. The transceiver hydrophone was deployed at 20 m and 5 m in a maximum water depth of 50 m. We collected data at the receiver anchored to the lakebed. Then, we transmitted data by moving the boat in the range of 300 m to 500 m. To transmit the communication waveforms, wave files were generated and saved in a database. The files were played using the transmit laptop sound file, amplified to the appropriate level.

Table 2. Experimental parameters.

Parameters	Value
FEC	Turbo code with rate of 1/3 ($K = 112$, $N = 336$)
Preamble bit (n)	255
Number of chips (N_c)	8, 32
Number of multi-bands (N_b)	4
Modulation	4-ary FSK
Equalizer	LMS-DFE
Number of turbo equalization iteration	5
Center frequencies of multi-band	$f_1 = 14$ kHz $f_2 = 18$ kHz $f_3 = 22$ kHz $f_4 = 26$ kHz
Sampling frequency	192 kHz
Data rate	20 bps
Distance	300~500 m
Depth	Water depth: 50 m Transmitter: 5 m Receiver: 20 m

**Figure 6.** Deployment of the lake trial.

A chirp sequence, called linear frequency modulation (LFM), is transmitted to characterize the channel impulse response (CIR) in the presence of Doppler. It shows the channel is affected by multi-path propagation caused by reflections from the surface and the bottom. After processing, the CIR was obtained using an LFM, as shown in Figure 7a. The multi-path delay spread is measured as a function of time and is approximately 0.3 s. It can be seen that there are multi-path echoes following the end of transmission. The frequency shift is also measured as a function of time, shown in Figure 7b. The frequency shift can be caused by the relative motion from one platform to the other, and it can either increase or decrease the frequency of the acoustic signal. Doppler spread occurs when multiple signals are received with different frequency shifts. The effect of Doppler spread is about 4 Hz. Specific objectives of the lake trial are to confirm the weighted multi-band underwater acoustic communication configuration.

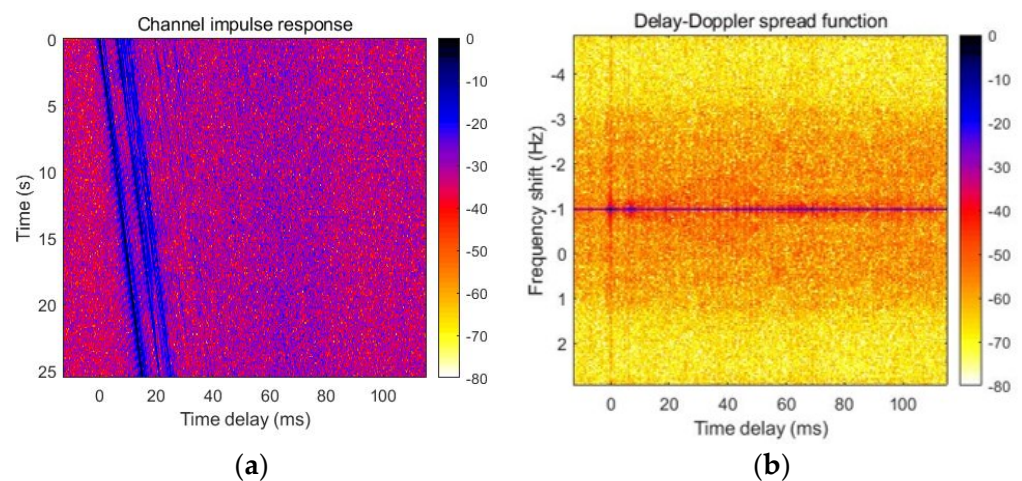


Figure 7. Channel characteristic: (a) Channel impulse response and (b) Doppler spread.

Fixed on $N_b = 4$, Tables 3 and 4 show lake trial results for different numbers of chips by applying a turbo equalization algorithm. We knew that two or three packets were succeeded among five packets. The errors of the succeeded packets are close to zero as the number of iterations increases.

Table 3. Performance analysis according to the number of Turbo iterations. ($N_c = 8$, $N_b = 4$).

Turbo Iteration Packet Number	1	3	5
1	$10^{-0.60}$	$10^{-2.05}$	0
2	$10^{-0.51}$	$10^{-0.52}$	$10^{-0.54}$
3	$10^{-0.97}$	0	0
4	$10^{-0.37}$	$10^{-0.37}$	$10^{-0.37}$
5	$10^{-0.37}$	$10^{-0.37}$	$10^{-0.37}$

Table 4. Performance analysis according to the number of Turbo iterations. ($N_c = 32$, $N_b = 4$).

Turbo Iteration Packet Number	1	3	5
1	$10^{-0.94}$	$10^{-1.09}$	0
2	$10^{-0.94}$	0	0
3	$10^{-0.43}$	$10^{-0.46}$	$10^{-0.56}$
4	0	0	0
5	$10^{-0.69}$	$10^{-0.77}$	$10^{-0.82}$

As shown in Tables 3 and 4, some packets still failed to decode. In the case of $N_c = 32$, the third and fifth packets still failed, even though the number of turbo equalization iterations was 5. We applied optimized weighting values of EC-BER to the error occurrence packets. After measuring the EC-BER, we assigned different weighting values to each band based on Table 1. As a result, we successfully decoded the third and fifth packets, as shown in Table 5. Regardless of applying the weighting algorithm to $N_c = 8$, errors were not corrected. This means that the more spreading there is, the better the performance is. The addition of the EC-BER algorithm drastically improved the reliability of the system resulting in minimal to zero errors.

Table 5. Performance analysis according to weighting. ($N_c = 32$, $N_b = 4$).

Packet Number	Band of Frequency	Estimation Error Rate (%)	Weighting Value	Decoded BER According to Number of Turbo Iterations		
				1	3	5
3	f_1	45	0.2	$10^{-1.01}$	$10^{-1.27}$	0
	f_2	49	0.2			
	f_3	6	1.0			
	f_4	53	0.2			
5	f_1	42	0.4	$10^{-2.05}$	0	0
	f_2	38	0.4			
	f_3	7	1.0			
	f_4	6	1.0			

5. Conclusions

Since underwater acoustic channels have severe ISI caused by multi-path, propagation loss, and Doppler effects, multi-band techniques with spreading are essential in a low SNR environment for reliable underwater communications. In multi-band configuration, the performance loss occurs because specific inferior band affects to entire band. It is very important to decide which band is the inferior one. In order to decide the inferior band, we proposed the EC-BER algorithm which estimates the reliability of the received data by checking the how many differences between re-encoded and demodulated symbols. The EC-BER has a close relationship with the performance of each band. Therefore, we can improve the performance as allocate lower weight to inferior band. The benefits of the proposed EC-BER algorithm are low computational complexity and exact performance prediction of coded data field for time-varying channel compared to state-of-the art such as ESNR and PES.

Through the simulations, we optimized weighting values using the proposed EC-BER, and we confirmed performance gain was obtained compared to conventional one. To prove the effectiveness of the EC-BER algorithm, a lake trial was conducted by employing four bands, a spread 4-FSK modulation, and turbo codes with rate of one-third. The experimental results show that the performance was improved by increasing the number of chips and iterations of the turbo equalization. However, some packets still failed to decode even though the turbo equalization reached five times. We applied optimized weighting values of the EC-BER to the failed packets. As a result, failed packets are successfully decoded, and we confirmed that the EC-BER algorithm drastically improved the reliability of the system, resulting in minimal to zero errors. Through the lake experimental results, the proposed algorithm is a useful technology for reliable underwater channels. For future studies, based on the proposed algorithm, we will apply it to the exact Doppler estimation. Finally, it will be applied to long-range underwater communication that will further increase the distance between the transceivers.

Author Contributions: Conceptualization, H.-W.J.; methodology, J.-W.J.; software, H.-W.J.; validation J.-W.J.; formal analysis, J.-W.J.; data curation, H.-W.J.; writing—original draft preparation, H.-W.J.; writing—review and editing, J.-W.J.; supervision, J.-W.J.; project administration, J.-W.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Agency for Defense Development (contract No. UD200010DD). The authors acknowledge with thanks the Agency for Defense Development for its technical and financial support.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare that there is no conflict of interest in this paper.

References

1. Yang, T.C.; Yang, W.B. Low probability of detection underwater acoustic communication using direct-sequence spread spectrum. *J. Acoust. Soc. Am.* **2008**, *124*, 3632–3647. [[CrossRef](#)] [[PubMed](#)]
2. Sozer, E.M.; Proakis, J.G.; Stojanovic, R.; Rice, J.A.; Benson, A.; Hatch, M. Direct Sequence Spread Spectrum Based Modem for Under Water Acoustic Communication and Channel Measurements. In Proceedings of the Oceans’99 MTS/IEEE, Seattle, WA, USA, 13–16 September 1999.
3. Citroni, R.; Di Paolo, F.; Livreri, P. A Novel Energy Harvester for Powering Small UAVs: Performance Analysis, Model Validation and Flight Results. *Sensors* **2019**, *19*, 1771. [[CrossRef](#)] [[PubMed](#)]
4. Wang, Y.; Zeng, Z.; Li, Y.; Zhang, J.; Jin, S. Multicarrier spread spectrum communication scheme for cruising sensor network in confined underwater space. *Int. J. Distrib. Sens. Netw.* **2014**, *10*, 165749. [[CrossRef](#)]
5. Zhang, X.; Cui, J.H.; Das, S.; Gerla, M.; Chitre, M. Underwater wireless communications and networks: Theory and application: Part 1. *IEEE Commun. Mag.* **2015**, *53*, 40–41.
6. Milica, S.; James, P. Underwater acoustic communication channels: Propagation models and statistical characterization. *IEEE Commun. Mag.* **2009**, *47*, 84–89.
7. Jeong, H.W.; Shin, J.E.; Jung, J.W. Multiband FSK with Direct Sequence Spread Spectrum for Underwater Acoustic Communications. In Proceedings of the 2021 Twelfth International Conference on Ubiquitous and Future Networks, Barcelona, Spain, 5–8 July 2021.
8. van Walree, P.A.; Leus, G. Robust Underwater Telemetry with Adaptive Turbo Multiband Equalization. *IEEE J. Ocean.* **2009**, *34*, 645–655. [[CrossRef](#)]
9. Wan, L.; Zhou, H.; Xu, X.; Huang, Y.; Zhou, S.; Shi, Z.; Cui, J.H. Adaptive Modulation and Coding for Underwater Acoustic OFDM. *IEEE J. Ocean. Eng.* **2015**, *40*, 327–336. [[CrossRef](#)]
10. Baek, C.U.; Jung, J.W.; Do, D.W. Study on the structure of an efficient receiver for covert underwater communication using direct sequence spread spectrum. *Appl. Sci.* **2018**, *8*, 58. [[CrossRef](#)]
11. Ayvaz, E.N.; Meryem, M.; Ali, Ö. A fast start-up modified LMS algorithm. In Proceedings of the 26th Signal Processing and Communications Applications Conference (SIU), Izmir, Turkey, 2–5 May 2018; pp. 1–4.
12. Djigan, V.I.; Shakhtarin, B.I.; Likhoedenko, K.P.; Zhurakovskiy, V.N. Multichannel stabilized fast RLS algorithm for nonstationary signal processing. In Proceedings of the East-West Design & Test Symposium (EWDTS), Novi Sad, Serbia, 29 September–2 October 2017; pp. 1–7.
13. Jung, J.W.; Kim, I.S. A study on threshold detection algorithm for adaptive transmission in underwater acoustic communication. *J. Acoust. Soc. Korea* **2020**, *39*, 585–591.
14. Zheng, Y.R.; Wu, J.; Xiao, C. Turbo equalization for single-carrier underwater acoustic communications. *IEEE Commun. Mag.* **2015**, *53*, 79–87. [[CrossRef](#)]
15. Deaett, M.A.; Audi, P.P. Interleaver performance for FSK transmission on the acoustic fading channel. In Proceedings of the Symposium on Autonomous Underwater Vehicle Technology, Washington, DC, USA, 5–6 June 1990; pp. 313–317.
16. Stefano, B. *Synchronization of Digital Telecommunications Networks*; Wiley: Hoboken, NJ, USA, 2002; pp. 210–221.
17. Li, Y.; Zhang, X.; Benson, B.; Kastner, R. Hardware Implementation of Symbol Synchronization for Underwater FSK. In Proceedings of the 2010 IEEE International Conference on Sensor Networks, Ubiquitous, and Trustworthy Computing, Newport Beach, CA, USA, 7–9 June 2010.
18. Blackmon, F.; Sozer, E.; Stojanovic, M.; Proakis, J. Performance comparison of RAKE and hypothesis feedback direct sequence spread spectrum technique for underwater communication applications. In Proceedings of the MTS/IEEE Oceans Conference, Biloxi, MI, USA, 29–31 October 2002; Volume 1, pp. 594–603.
19. Salz, J. Optimum mean-square decision feedback equalization. *Bell Syst. Tech. J.* **1973**, *52*, 1341–1373. [[CrossRef](#)]
20. Lee, H.S.; Baek, C.U.; Jung, J.W. A Weighted Turbo Equalized Multi-Band Underwater Wireless Acoustic Communications. *Appl. Sci.* **2018**, *8*, 1711. [[CrossRef](#)]
21. Shin, J.E.; Jeong, H.W.; Jung, J.W. Doppler Compensation Algorithm Using Coded Estimation BER in Underwater Acoustic Communication. In Proceedings of the OCEANS 2021, San Diego, CA, USA, 20–23 September 2021.