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Article

Numerical Simulation of Rockfill Materials Based on Fractal Theory

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Abstract: With the use of the particle flow code in two dimensions, a fractal model is established with the number of particles of different particle fractions used as the statistics to study the fractal characteristics of particle size distribution. Numerically simulated specimens obtained by four scale methods are subjected to the relative density test and the biaxial compression test to explore the influences of fractal dimension *D* on the macroscopic and mesomechanical properties of specimens, as well as to study the relationship between fractal dimension *D* and different mechanical performance indexes. Results show that the particle size distribution of each of the four groups after scale exhibits fractal characteristics, with the fractal dimension *D* ranging from 1.27 to 2.03. The number of fine particles in the specimen increases with the fractal dimension *D*, the particle aggregates become more compact, the macroscopic mechanical properties of the specimens are improved, and a linear relationship exists between the fractal dimension *D* and different mechanical performance indexes. A large fractal dimension *D* corresponds to a great mesoparticle coordination number.

Keywords: numerical simulation; rockfill materials; fractal model; mesomechanical properties

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1. Introduction

Rockfill materials have been used as a main filling material for earth-rockfill dams, especially with the construction of high earth-rock dams in recent years. Thus, their functional characteristics need to be acquired accurately. In practical engineering, the particle size of rockfill materials is 400–600 mm and even reaches more than 1000 mm. Given the limitations of laboratory test instruments, rockfill materials that exceed the permissible maximum particle size needs to be scaled. To determine the mechanical properties of rockfill materials on site, analog simulation is adopted in laboratory tests to fabricate test specimens that are consistent with the prototype rockfill materials in terms of the internal structure, thereby determining the engineering features of the prototype rockfill materials. Thus far, many scholars at home and abroad have conducted extensive research on factors that influence the size effect of rockfill materials from the aspects of scale method [1–4], specimen size [5–7], particle shapes [8–10], and scale ratio [11–13]. Their research methods are still dominated by conventional triaxial tests [2–4,6–8], and laboratory tests are limited by test conditions and costs, as well as difficulties in analyzing changes in the internal mesoscopic mechanisms of rockfill materials. Numerical simulation can be used to compensate for the shortcomings of laboratory tests, allowing for the real-time monitoring of the law of evolution of rockfill materials in the internal crack, energy and mesostructured [14,15].

Since its introduction [16], fractal theory has made some valuable achievements in the aspects of rock fracture [17], soil particle morphology [18,19], and rockfill materials breakage [20–22]. Fractal geometry theory was drawn into the gradation of rock-fill materials. It is found that the truncation error is one of the main factors affecting the density scale effect of rockfill materials [23].

Wu et al. [20] studied the correspondence between the relative density (RD) of rockfill materials and the fractal dimension based on fractal theory and explained the four common scale methods. Turcotte et al. [24] found that, with particles being crushed increasingly, the initially distributed particles show similar fractal distributions. Bouzeboudja and Melbouci [25] selected two methods for calculating the fractal dimension and explored the relations among the fractal dimension and the size, shape, and properties of particles. Hou et al. [26] found that the fractal dimension increases with the confining pressure in the test specimen, and the fractal dimension depends on the particle size distribution. Muto et al. [27] observed crushed particles of a rock fracture zone under an optical microscope and found that the fractal dimension that was obtained on site was apparently higher than those measured in laboratory tests but close to the theoretical fractal dimension. The above findings are mainly concentrated in the aspects of fractal dimension and particle breakage, scale methods, and mechanical properties, and the research techniques are mostly based on Tyler's fractal model for mass distribution [19]. At present, most scholars have studied the mechanical properties of rockfill materials using the quality fractal model proposed by Tyler, but the fractal model is not reported with the number of particles between different particle groups as the statistical number in the newspaper.

In this paper, with the use of the particle flow code in two dimensions and on the basis of fractal theory, a fractal model is established with the number of particles of different particle fractions as the statistics to explore the fractal characteristics of the particle size distribution. Using the fractal model, the original grading curve of rockfill materials is scaled in four conventional scales. Taking the fractal dimension of the scaled sample as the index reflecting its structural parameters, the relative density test and biaxial compression test under different fractal dimensions are carried out, and the physical and mechanical properties of rockfill materials under different fractal dimensions are studied. The research results provide a new method for studying the mechanical properties rockfill materials.

2. Grading Scale Methods

The commonly used scale methods for the grading of rockfill materials include screening, similar gradation, equivalent substitution, and mixing. Screening refers to the removal of particles that exceed the permissible maximum particle size. Thus, the content of fine particles smaller than 5 mm is relatively increased with the mechanical properties of granular materials. Screening is suitable only when few particles exceed the permissible maximum particle size. Self-similarity gradation remains the coefficient of curvature of gradation, and the nonuniformity coefficient remains unchanged with the increasingly larger content of fine particles and the smaller content of coarse particles, thus changing the mechanical properties. Equivalent substitution ensures that the content of coarse particles and that of fine particles are unchanged even with changes in the grading, nonuniformity, and curvature coefficients of the coarse particles. In mixing, the particle size is reduced by self-similarity gradation at a certain scale, and the equivalent substitution is used. Whatever scale method is chosen, the relationship of the coarse and fine particles in the filling process changes markedly, and the mechanical properties of the rockfill materials change significantly before and after scaling. Therefore, how a reasonable scale method should be chosen awaits further study.

To investigate the influences of the fractal dimension D on the mechanical properties of rockfill materials, this paper chooses the grading curve designed in [28] and the grading curves obtained from the four scale methods; the maximum particle size after scale is 60 mm. The scale methods are equivalent substitution, screening, self-similarity gradation, and mixing, which are denoted by DL, TC, XS, and HH, respectively. The grading curve designed for rockfill materials and the gradation characteristic curve after scaling are shown in Figure 1.

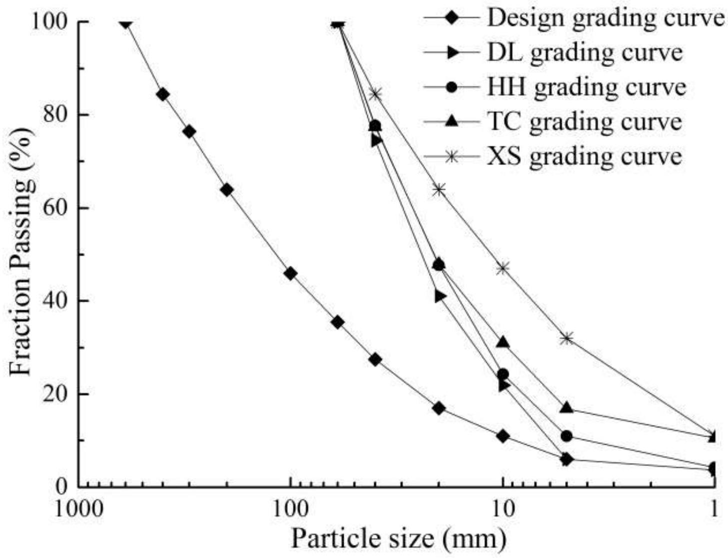


Figure 1. Grading curves.

3. Fractal Model

The particle shape and size distribution of rockfill materials, as well as the clearance between particles and surface behaviors, have certain influences on compactness. These features are random in a certain range and are difficult to describe in the conventional language of mathematics. Therefore, the fractal dimension D is taken as a macro-meso mechanical index to study rockfill materials, with the following basic definition based on fractal theory:

$$N \propto \left(\frac{1}{r}\right)^{D} \tag{1}$$

where r represents the graphic measure, N represents the measured value from the corresponding r graph, and D represents the fractal dimension of the graph.

Given that the numerically simulated specimen makes obtaining the number of particles of different particle fractions easier, the particle aggregate inside the numerically simulated specimen is considered a measure, and the composition of particles is a structural pattern without characteristic length. In this paper, a fractal model that is different from that presented by Turcotte [21] is established, with the ratio of the upper limit d_i for the particle size of the particle fraction to the maximum particle size d_{max} of the specimen as the measure, and the ratio of the number of particles of the particle fraction to the total number of particles of the specimen as the measured value. Then, the particle size distribution function reflected by the number of particles is

$$\frac{N_i}{N_{sum}} \propto \left(\frac{d_i}{d_{max}}\right)^{-D} \tag{2}$$

where N_i represents the number of particles with a particle size within the range of $[d_{i-1}, d_i]$, N_{sum} represents the total number of particles of the specimen, d_i represents the upper limit for the particle size of the particle fraction, and d_{max} represents the maximum particle size in the specimens.

With the introduction of a proportional constant C, Equation (2) can be expressed by

$$\frac{N_i}{N_{sum}} = C\left(\frac{d_i}{d_{max}}\right)^{-D} \tag{3}$$

From Equation (3), the number N_i of particles in the particle fraction is related to the proportional constant C, the upper limit d_i for the particle size of the particle fraction, the maximum particle size d_{max} inside the specimen, and the fractal dimension D of the specimen. This equation gives the basic mathematical model for the particle size distribution of the numerically simulated specimen. The numerically simulated specimen during particle formation has no particle breakage, so $N_i \propto (d_i)^{-D}$. With the introduction of another proportional constant λ, Equation (3) can be expressed by

$$N_i = \lambda (d_i)^{-D} \tag{4}$$

Both sides of Equation (4) are operated logarithmically. Then

$$\lg(N_i) = -D\lg(d_i) + \lg\lambda \tag{5}$$

Equation (5) presents the method of solving the fractal dimension. The double logarithmic chart of the number of particles in the particle fraction N_i and the upper limit for the particle size of the particle fraction d_i are plotted. The regression curve is fitted by using the least squares method, and the inverse of the slope of the line is also the fractal dimension D. The number of particles in the numerically simulated specimen is shown in Table 1. Linear fitting $\lg(d_i)$ and $\lg(N_i)$ is shown in Figure 2.

Table 1. The number of particles for numerical samples and test results.

Scale Method	Particle Size (mm)					Fractal Dimension	Correlation Coefficient
	60~40	40~20	20~10	10~5	5		
DL	20	72	164	544	461	1.27	0.839
HH	18	65	201	458	850	1.49	0.935
TC	18	64	146	487	1314	1.64	0.978
XS	13	45	148	521	2500	2.03	0.992

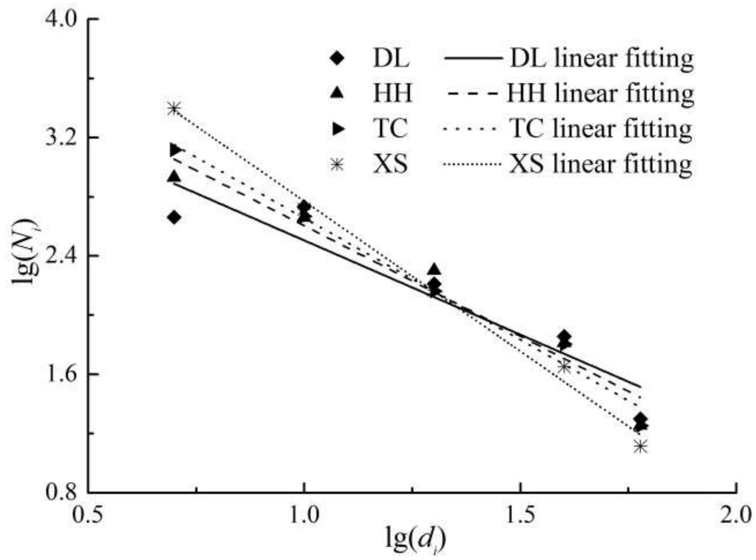


Figure 2. Linear fitting between $\lg(d_i)$ and $\lg(N_i)$.

The fitting results in Table 1 and Figure 2, except for the relatively poor linear correlation (correlation coefficient $R^2 = 0.838$) that corresponds to the numerically simulated specimen from the equivalent substitution, indicate that the numerically simulated specimens from the other three scale methods show better linear fitness, with a correlation coefficient within the range of 0.935–0.992. These findings indicate that the fractal dimension D can basically reflect the changing trend of particle size distribution. Only the equivalent substitution can cause a greater disturbance to the particle size distribution.

4. Analysis of the Results of the Numerical Test

4.1. Compactness Test

RD is an important indicator for controlling the shear strength of granular material [3]. Thus, the numerical test on RD is performed to analyze the influences of the fractal dimension D on the compactness of rockfill materials. The maximum void ratio e_{max} and minimum void ratio e_{min} of the numerically simulated specimen must be determined first. In this paper, the maximum void ratio is determined by using a method presented by Deluzarche et al. [13], that is, frictional particle aggregate under the action of dead load reaches the loosest state defined by the natural accumulation state. However, the minimum void ratio is obtained by compressing the rigid boundary of frictionless particle aggregate in all directions. The specimen size is φ 300 mm × 600 mm, and the maximum particle size d_{max} is 60 mm. The relationship between dense degree index and the fractal dimension D is shown in Figure 3.

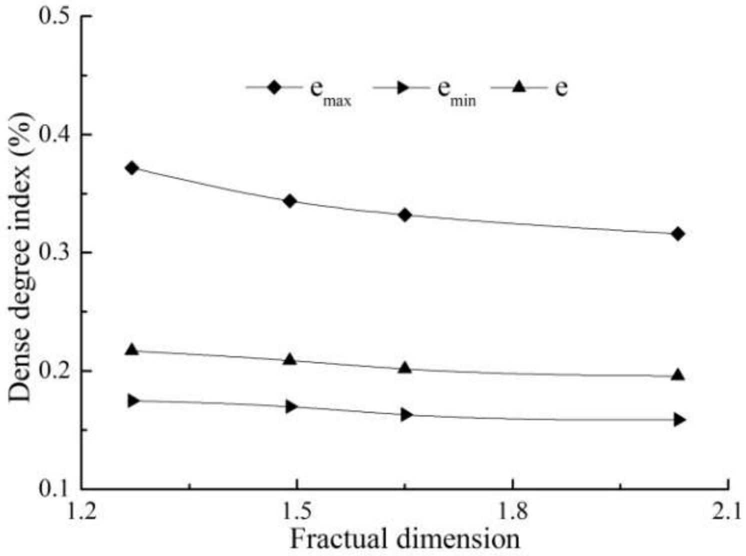


Figure 3. Relationship between dense degree index and fractal dimension.

Figure 3 shows that the fractal dimension D increases from 1.27 to 2.03, and the maximum void ratio e_{max} and the minimum void ratio e_{min} of the numerically simulated specimen, as well as the void ratio e that corresponds to the RD of 0.89, decrease. With the increasing fractal dimension D, the scale method changes, and the number of fine particles contained in the specimen increases gradually with the number of contacts between the inner particles. However, the number of contacts is closely related to the compactness. A high content of fine particles corresponds to great compactness and low void ratio. For smooth round particles without breakage, a small particle size corresponds to great compactness and a large fractal dimension D.

4.2. Biaxial Compression Test

To eliminate the influences of different compactness on the macroscopic strength and deformation characteristics of the specimen, the RD (0.89) is taken to prepare the numerically simulated specimens that are made by the four different scale methods, and the specimen size is φ 300 mm × 600 mm. In this paper, the linear stiffness model without considering particle breakage is adopted, and the calibration of meso parameters refers to references [29,30]. The mesonumerical parameters are set as follows: the modulus of elasticity is 2.0 GPa; the ratio of the normal contact stiffness of the particle to the tangential contact stiffness is 1.5; the friction coefficient between the particles is 0.48; the friction factor between the particle and the rubber film and that between the particle and the compression plate is 0.001; and the density is 2.45 g/cm^3.

Figure 4 shows the deviatoric stress and volumetric strain with axial strain under different fractal dimensions *D*. At the initial stage of loading, the deviatoric stress increases rapidly with the axial strain, thereby indicating that the rockfill materials has not yet been completely compacted, and as the axial strain advances, the deviatoric stress increases slowly until the peak deviatoric stress is reached. At that stage, the specimen is wholly shrunk, and the volumetric strain of different specimens is greatly affected by the fractal dimension *D*. Without considering particle breakage, after reaching the peak deviatoric stress, the deviatoric stress-axial strain curve exhibits softening, and the entire specimen produces a dilatancy effect, a great fractal dimension *D* corresponds to a more obvious dilatancy effect. The more fine-grained soil in rockfill materials, the more obvious the shear dilatancy. The numerically simulated deviatoric stress and volumetric strain with axial strain curve fluctuates violently, because particles with a particle size of 5 mm are substituted equivoluminally for particles with a particle size smaller than 5 mm. Without the filling of smaller particles, the sliding and rolling of large particles become relatively easy. Thus, the particle aggregate changes the dilatancy significantly.

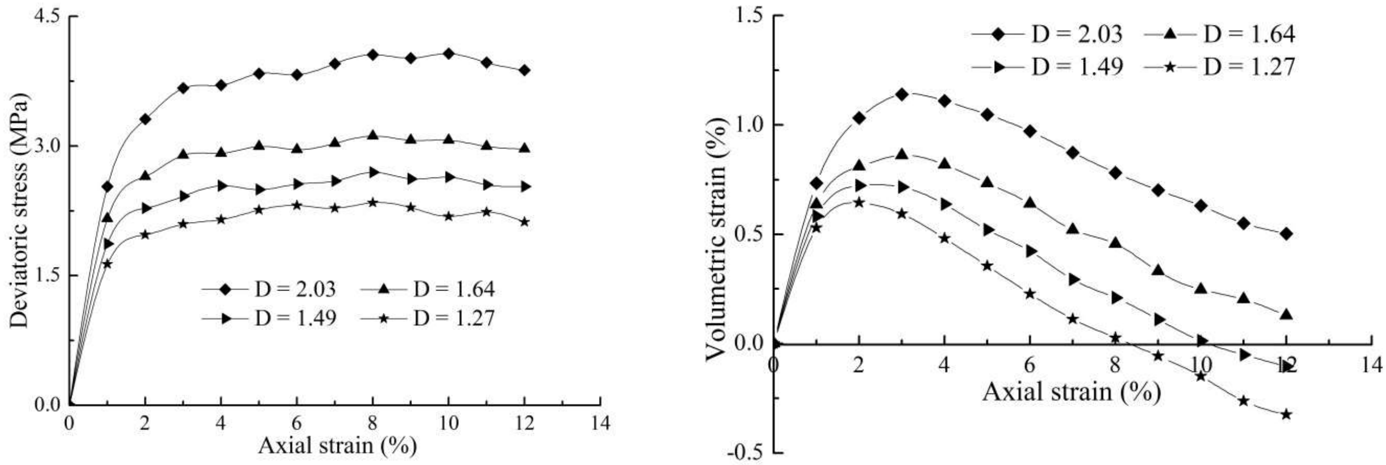
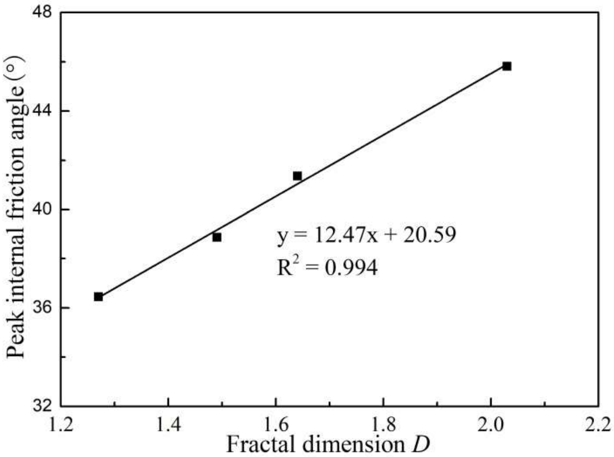
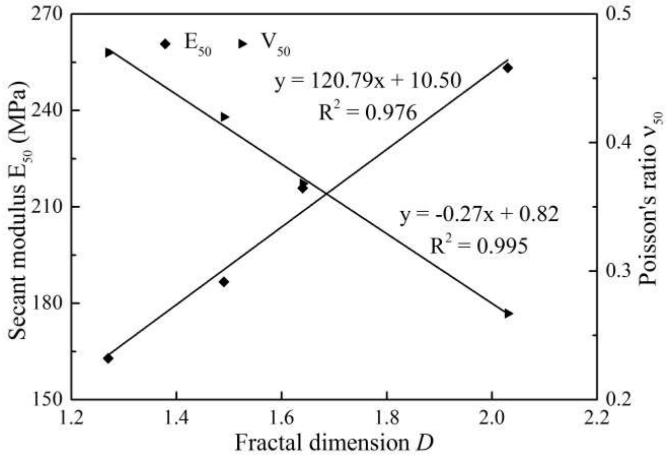


Figure 4. Relation curves of deviatoric stress and volumetric strain with axial strain under different fractal dimensions.

Figure 5 shows that the fractal dimension *D* greatly affects the mechanical properties of the numerically simulated specimen. The peak internal friction angle of the specimen and the secant modulus rise with the fractal dimension *D*, because an increase in the number of fine particles in the specimen increases the friction between particles and intensifies the sliding and rolling of particles, with the peak deviatoric stress increasing. An increase in the content of fine particles improves the filling of particles, thereby forming a more stable and dense structure with minor axial deformation under the initial stress, but the secant modulus of the specimen increases. The Poisson's ratio of the specimen decreases with the fractal dimension *D*, thereby indicating that a large number of fine particles in the specimen corresponds to evident aggregate interlock behavior and thus weak lateral deformation of the specimens. The peak internal friction angle of the specimen, the secant modulus, and the Poisson ratio show better linear fitting with the fractal dimension *D*.



(**a**)



(**b**)

Figure 5. Relationships between index of strength and deformation characteristics with fractal dimension *D*. (**a**) strength characteristics, (**b**) deformation characteristics.

4.3. Micromechanical Response

The numerical simulation allows the real-time monitoring of the evolution of rockfill materials in the mesostructure [14,15]. The coordination number *M* refers to the average number of contacts in the specimen that reflects the contact characteristics of rockfill materials from a meso point of view A great coordination number corresponds to more compact filling of coarse particles and thus higher structure stability. The evolutionary curve of the coordination number is shown in Figure 6. Figure 6 shows that in the initial stage of loading, the finer particles are gradually filled between the coarse particles, the contact points between the particles are increasing, the corresponding coordination number is also increasing, and the sample is in a compressed state; After the coordination number reaches the maximum value, with the continuous increase of load, relative sliding occurs between some coarse particles, the number of contact points between particles decreases, the corresponding coordination number decreases, and the sample is in the state of shear expansion; The coordination number changes corresponding to the four fractal dimensions *D* are similar. The larger the fractal dimension *D* is, the larger the coordination number is, but it does not coincide, indicating that the influence of fractal dimension *D* on numerical simulation calculation is objective.

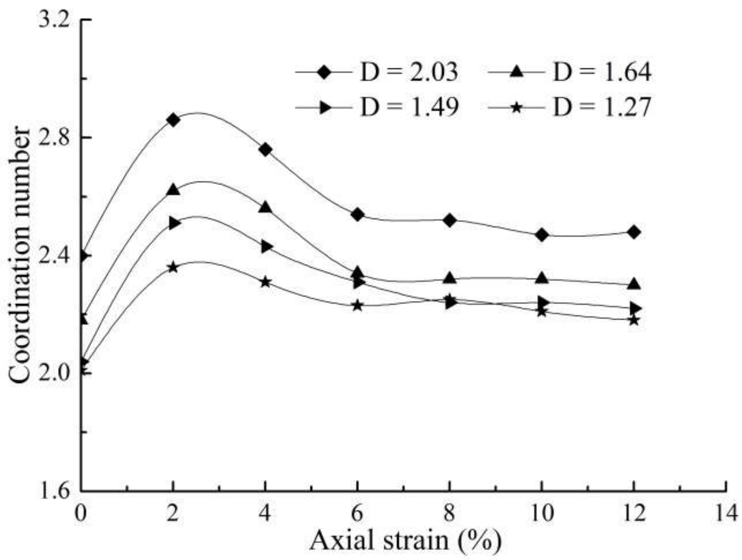


Figure 6. Evolutionary curves of the coordination number.

5. Conclusions

Based on the fractal theory, a fractal model is established with the number of particles between different particle groups as the statistical number. The relationship between the fractal dimension D and different mechanical performance indexes is discussed. Several conclusions can be drawn as follows:

(1) With the use of the particle flow code in two dimensions and on the basis of fractal theory, a fractal model was established with the number of particles used as the statistics. Compared with the specimens obtained by equivalent substitution, which considerably disturbed the particle size distribution, the numerically simulated specimens obtained by the other three scale methods exhibited better fractal characteristics.

(2) A great fractal dimension D corresponds to more obvious deviatoric stress curve softening and volumetric strain curve dilatancy and thus a larger peak deviatoric stress. The peak internal friction angle of the specimen φ_{p}, the secant modulus E_{50}, and the Poisson's ratio v_{50} show better linear fitting with fractal dimension D.

(3) The fractal dimension D often changes the mechanical properties of the specimen with certain macro-responses, that is, with the increasing fractal dimension D, the compactness indicators e_{max}, e_{min}, and e decrease little by little, the strength indicator φ_{p} increases gradually, the deformation indicator v_{50} diminishes, and E_{50} rises. A meso-response is that the fractal dimension D increases with the coordination number.

(4) Using the particle aggregate model, which is used to calculate the rockfill materials in accordance with the fractal theory, the relationship between fractal dimension, peak strength and confining pressure of rockfill materials is established. The limit value of particle breakage is given by using fractal dimension.

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