



Article Multi-Objective Aerodynamic Design Optimisation Method of Fuel Cell Centrifugal Impeller Using Modified NSGA-II Algorithm

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Abstract: This paper presents a modified NSGA-II algorithm based on the spatial density (SD) operator, combined with computer graphics-based surface parameterisation methods and computational fluid dynamics (CFD) simulations. This was done to optimise the multi-objective aerodynamic design of a centrifugal impeller for a 100-kW vehicle-mounted fuel cell and improve the multi-conditions aerodynamic performance of the centrifugal impeller of the vehicle-mounted fuel cell (FC). The optimisation objectives are to maximise the isentropic efficiency of the rated and common operating conditions. The optimisation results showed that the efficiency of rated working conditions had an increase of 1.29%, mass flow increase of 8.8%, pressure ratio increase of 0.74% and comprehensive margin increase of 6.2%. The efficiency of common working conditions had an increase of 1.2%, mass flow increase of 9.1%, pressure ratio increase of 0.24% and comprehensive margin increase of 10%. The optimisation effect is obvious under the premise of satisfying the constraints, which proves the optimisation method's engineering effectiveness and provides technical support and methodological research for the multi-objective aerodynamic design optimisation of centrifugal impellers for vehicle-mounted FCs.

Keywords: multi-objective aerodynamic optimisation; centrifugal impeller; DNSGA; spatial density

1. Introduction

New energy vehicles have been vigorously developed in recent years due to the problems of environmental pollution and oil shortages. Among them, fuel cell vehicles (FCVs) have a long-range, fast energy replenishment and high efficiency, which have more potential and are more attractive than lithium battery vehicles and traditional internal combustion engine vehicles. However, the high cost also limits their development. The fuel cell system (FCS) mainly consists of an electric stack, air supply system, hydrogen supply system, and water and thermal management systems. First, the electric motor drives a centrifugal impeller to pressurise the air. Then, the pressurised and humidified air is sent to the hydrogen fuel reactor for a chemical reaction. The final output of electricity provides energy for the electric motor and the whole vehicle. The compressor is the core component of the air supply system, accounting for 16.89% of the overall vehicle cost, second only to the fuel stack in the fuel cell (FC). Improving the aerodynamic performance and efficiency of the compressor can effectively save the overall cost.

Common types of air compressors are screw, scroll, roots, and centrifugal. A large mass flow is easy to achieve for the screw, roots and centrifugal compressors, and high pressure can be easily realized for the screw and centrifugal compressors [1]. The scroll, screw, and slide compressors are all in a type of positive displacement, which can boost the pressure through the shrinkage of inner chambers. In contrast, centrifugal compressors take in the gas and elevate the kinetic energy through a rotating impeller at high speed to boost the pressure of air. Centrifugal compressors have high efficiency, a compact structure



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and a high-pressure ratio, as well as favourable noise, vibration and harshness (NVH) performance [2]. Li et al. [3] highlighted centrifugal compressors as a good choice for FCSs due to their relatively low weight and high pressure. Therefore, centrifugal compressors are considered to be more suitable for FCs. As the most important compressor component, the performance of the high-speed centrifugal impeller has a vital impact on the overall efficiency of an air compressor and fuel cell system which then affects the cost of the whole vehicle [4].

Although high-speed centrifugal impellers have been used extensively in internal combustion engines (ICE) and have similarities to FC centrifugal impellers, concerning size, weight, NVH behaviour and cost, centrifugal impellers for internal combustion engines are unfortunately not directly usable in FCS for several reasons.

- 1. The working environment is different. The FC reactor requires oil-free air, while the centrifugal impeller used in ICE is driven by oil-lubricated bearings, which can cause reactor contamination.
- 2. The rated rotational speed is different. The ICE can make the centrifugal impeller reach more than 200,000 revolutions per minute (rpm) using high-temperature exhaust gas. Meanwhile, the exhaust gas of the FC cannot drive the centrifugal impeller to the required speed. The centrifugal impeller can only be driven by oil-free electric bearings, and its maximum speed can only reach 140,000 rpm. Further, the rated speed of 80,000 rpm–110,000 rpm is considered reasonable.
- 3. It has a different working condition range. The specific speed of the centrifugal impeller in the FC is usually smaller than that of the ICE [5]. Further, their operating conditions are different. The operating range of the FC centrifugal impeller is narrow and usually runs along the surge boundary, while the operating range of the ICE centrifugal impeller is larger. If the ICE centrifugal impeller is applied to the FC system, there is a high risk of surge or blockage. In summary, FC centrifugal impellers need to be specifically designed and optimised for their own use.

Since the total pressure ratio of the FC air compressor can reach 0.2–0.4 Mpa, the driving power consumption can reach 25% of the output power of FCS [1,6]. Further, it cannot recover enough energy from the low-temperature exhaust gas to drive the high-speed motor like the ICE. Therefore, improving the aerodynamic performance of the centrifugal impeller can effectively reduce the system power consumption. According to the current study's results [7], the improvement of impeller efficiency can improve the material utilisation and comprehensive performance in FC to achieve further cost reduction. This work can be achieved through the design optimisation of the aerodynamic performance of the centrifugal impeller.

In FC gas supply systems, the higher the air pressure, the better the cost reduction. Studies have shown that the platinum content can be reduced from 0.36 g to 0.2 g/kW when the inlet pressure is increased from 1.5 to 3.0 atm. Second, the higher the air pressure, the higher the system's output power [8]. Higher air pressure is beneficial to increase the energy density of the stack, reducing the size and weight of the FC system and saving the manufacturing cost. Third, higher air pressure is beneficial to the water balance [4]. However, too high a stack inlet pressure consumes additional parasitic power, which reduces the efficiency of the whole machine. Therefore, when optimising the design of the aerodynamic performance of the FC centrifugal impeller, an appropriate increase in the pressure ratio is beneficial for the overall cost reduction. As well, the mass flow has a positive correlation with the FCS. The output power of the FCS will increase with the increase of the mass flow, which is beneficial to the efficiency of the FCS.

The traditional centrifugal impeller design method relies on the iterative design from 1D to 3D by experts. Since the 1990s, optimisation design methods have been introduced into the field of impeller aerodynamic design. They have been widely studied for their advantages of short design cycles, lack of dependence on expert experience, and breakthroughs from traditional design limitations [9,10]. Li et al. [9] presented a 3D multiobjective aerodynamic optimisation method by the multi-objective differential evolutionary algorithm, 3D blade parameterisation method, self-adaptive integration, and RANS. The optimisation results show that the isentropic efficiency and the total pressure ratio were increased by 3.06% and 1.26%, respectively. Ekradi et al. [10] used a genetic algorithm (GA), a 3D blade parameterisation method, a CFD solver and an artificial neural network, presenting a method for 3D optimisation of a transonic centrifugal impeller. The isentropic efficiency increased by 0.97%, the mass flow increased by 0.65%, and the total pressure ratio increased by 0.74%.

The optimisation algorithm is one of the core aspects of the centrifugal impeller aerodynamic design optimisation method, which determines the optimisation-seeking capability and convergence speed in the established design space. Optimisation algorithms for turbomachinery are generally classified into gradient methods, agent model methods (e.g., neural networks) and meta-heuristic algorithms (e.g., genetic algorithms). The gradient method is more efficient in the case of continuous and single-peaked variable space, but it can often only obtain the optimal local solution of the problem, not suitable for the optimisation search of complex multi-peaked problems. The agent model method can better reduce the computational cost, but it is very difficult to establish an agent model with sufficient accuracy since the dimensionality of decision variables increases. At the same time, the method is more difficult to deal with multi-objective multi-constraint problems. Although the meta-heuristic algorithms is computationally expensive, it has been widely used in impeller mechanical design optimisation due to it can obtain global optimisation solutions and has strong robustness [11]. It should be emphasised that the meta-heuristic algorithm is particularly suitable for the optimisation search of multi-objective multi-constraint problems due to its inherent concurrency property.

In recent years, meta-heuristic algorithms have been widely used to solve multiobjective optimization problems, and effectively applied to engineering practice, which are classified into two dominant classes: swarm intelligence and evolutionary. Swarm intelligence algorithms imitate the intelligence of swarms, flocks and herds of creatures while evolutionary algorithms imitate the mechanisms of evolution in nature. Mirjalili et al. [12] proposed a novel optimization algorithm named multi-objective salp swarm algorithm (MSSA) for solving optimization problems multiple objectives by imitate swarm behaviour of salps in oceans. MSSA had been tested on mathematical optimization functions and the results show that it is able to refine the initial random solutions effectively and convergence and is successfully applied to the multi-objective optimization problem of maximizing the efficiency and minimizing the cavitation of the propeller to achieve the optimal blade shape. Cao et al. [13] proposed a bi-swarm particle swarm optimizer with novel neighbourhood topology strategy (BPSO-NT) to solve the optimization of intermodal transportation planning in the field of logistics. Ten well-known benchmark functions are utilized to test the performance of BPSO-NT and the results show that the proposed BPSO-NT is superior to others PSO variants algorithm. In addition, BPSO-NT has achieved good results in solving practical engineering problems. Nadeem Shaukat et al. [14] presented an intelligent technique using modified genetic algorithms, combined with SuperMC code, to find the optimum loading pattern for the nuclear fuel, which is favourable to maximize effective multiplication factor and neutron flux densities inside the reactor core under design limits. In 2002, Deb et al. [15] proposed a modified version of NSGA, the well-known NSGA-II algorithm, which has a superior comprehensive performance compared with other evolutionary algorithms and is widely used in optimizing the centrifugal impellers. Duan et al. [16] used NSGA-II to optimise the conflict between pressure head and hydraulic efficiency. The tests proved that the optimised mini-pump increased the pressure head by about 24% and the hydraulic efficiency by 4.75% compared with the original design prototype, and the flow field analysis showed that the optimised model could alleviate the impeller wake and improve the pressure distribution and flow direction. Chen et al. [17] used NAGS-II to optimise the design of the automotive torque converter impeller. The results show that the zero-order polynomial of the Gaussian model is the best prediction

for the stall torque ratio. They proposed an effective solution for the design of the hydraulic torque converter impeller geometry.

However, the NSGA-II algorithm is excessively focused on convergence, leading to controversy over its performance in the population diversity. Wang et al. [18] pointed out that the defect of NSGA-II is the selecting principle of crowding distance which is based on the cubic distance to the adjacent points. For instance, in the ZDT problems, with continuous evolution, more and more points would cluster to a small range. The diversity of individuals is lost which leads to a local optimum. Huang et al. [19] indicated that the crowding distance operator in NSGA-II can guarantee the diversity along the non-dominated front, but it may lose the horizontal diversity when wiping off excessive individuals. Due to the wide range of FC operating conditions and the higher requirements for aerodynamic performance under different operating conditions, the multi-objective problems using the classical NSGA-II algorithm easily fall into the local optimal solution.

In order to improve the population diversity of solution sets for multiple operating conditions performance of FC centrifugal impeller, a modified NSGA-II optimisation algorithm (i.e., DNSGA) is proposed, which has a novel classification method for non-dominated ranking and crowding distance operator (i.e., SD operator with weights).

2. DNSGA Algorithm

2.1. Algorithm Framework

There are two aspects of shortcomings for the NSGA-II algorithm. On the one hand, it is easy to fall into local optimal solutions and into premature convergence due to its focus on convergence. On the other hand, when some individuals have large differences in decision variables while their objective function values are approximately equal, the adoption of NSGA-II for the crowdedness ranking may remove potentially excellent individuals, which is unfavourable to the diversity of the population. The DNSGA algorithm is proposed to remedy the shortcomings, which essentially increases the optional range of individual decision space by improving the non-dominated ranking and spatial density ranking. This is conducive to the inheritance of excellent individuals to the next generation, thus improving the diversity of individuals in the population.

The next generation of DNSGA is selected in three main steps. First, the parents and offspring are divided into three groups (M1, M2 and M3) according to the non-dominance result. The number of individuals in M1 does not exceed 50% of the population size. The number of individuals in M2 is not smaller than the population size. The remaining individuals are divided into M3. Second, M3 is deleted. M1 and M2 are combined into group Q. Finally, the individuals with the lowest spatial density in M2 are sequentially deleted according to the SD until the number of individuals in Q is equal to the population size.

The main workflow of the algorithm is shown in Figure 1, and the specific steps are as follows:

- (1) Randomly generate an initial population P_0 to start the evolution;
- (2) Perform the binary tournament selection, simulated binary crossover and polynomial variation on P_0 to generate new offspring P_g^* ;
- (3) Evaluate the fitness function so that multi-objective values are obtained for each individual;
- (4) Sort all individuals in P_g and P_g^* into $F_1, F_2, F_3 \dots F_n$ based on non-dominance.
- (5) Divide $F_1, F_2, F_3 \ldots F_n$ into the three groups of M1, M2 and M3;
- (6) Generate the next population P_{g+1} according to the spatial density operator ordering;
- (7) Return to step 2 until the end of the maximum number of iterations.



Figure 1. DNSGA algorithm workflow chart.

2.2. Spatial Density Ordering

The spatial density expressions for the two individuals of $Y_1 = (x_1^1, x_1^2, ..., x_1^n)$ and $Y_2 = (x_2^1, x_2^2, ..., x_2^n)$ with n-dimensional decision variables is expressed as Formula (1).

$$d(Y_1, Y_2) = \sum_{i=1}^n \left| x_1^i - x_2^i \right| \, (1 \le i \le n) \tag{1}$$

Empirically, the influence of the decision variables of different dimensions on the aerodynamic performance of the centrifugal impeller varies widely. Therefore, different weights are assigned to different decision variables. The mathematical expression of spatial density with weights is Equation (2).

$$d_w(Y_1, Y_2) = \sum_{i=1}^n w_i \left| x_1^i - x_2^i \right| \left(1 \le i \le n, w_i \ge 0 \right)$$
⁽²⁾

where w_i is the weight of the *i*th dimension variable.

In this paper, a spatial density operator ranking with weights is used to remove the redundant individuals in *M*2, and the procedure is as follows:

- (1) *M1* and *M2* are combined into group *Q*.
- (2) The two individuals with the lowest spatial density in *Q* are found, and at least one belongs to *M*2.
- (3) If one individual belongs to *M*1 and the other belongs to *M*2. The individual belonging to *M*2 is directly removed from *Q*. If both individuals belong to *M*2, the one who has a lower spatial density with other individuals in *Q* is deleted.
- (4) Return (2) and (3) in a loop until the total number of individuals in Q reaches the population size.

2.3. Algorithm Validation

Five classical multi-objective algorithms, including VEGA, SPEA2, EMOEA, NSGAII and DNSGA, are run 30 times independently under two classical test functions (Table 1) from the ZDT problems proposed by Ziztler et al. [20] to test the performance of the presented DNSGA.

Name	Dimension	Variable Range	Objective Functions (Minimised)	Population Size	Max Generations
ZDT2	30	[0, 1]	$F = (f_1(x), f_2(x))$ $f_1(x) = x_1$ $f_2(x) = g(x) \left[1 - (x_1/g(x))^2 \right]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n-1)$	40	200
ZDT3	30	[0, 1]	$F = (f_1(x), f_2(x))$ $f_1(x) = x_1$ $f_2(x) = g(x) \left[1 - \sqrt{x_1/g(x)} - (x_1/g(x)) \sin(10\pi x_1) \right]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n-1)$	40	200

Table 1. Test function parameter configuration.

The hypervolume ratio (HVR) and generational distance (GD) are recorded, and statistical results are shown in Table 2 and the box plots are shown in Figures 2 and 3. On the one hand, the HVR metric measures the volume covered by the obtained PF for problems where all objectives are to be minimised. A larger value of the metric indicates better diversity. On the other hand, the GD metric indicates how far the tested PF is from the real PF. A low GD value reflects good convergence performance.

 Table 2. Statistical results of different classical multi-objective evolutionary algorithms.

Algorithm	Max	Min	Mean	Standard Deviation
		ZDT2-H	VR	
VEGA	0.3828	0.2293	0.311390000	0.043664290
SPEA2	0.6513	0.4933	0.611976667	0.048765225
EMOEA	0.6263	0.4835	0.512056667	0.024596811
NSGAII	0.6436	0.4859	0.615383333	0.038940014
DNSGA	0.8502	0.8033	0.838260000	0.012841796
		ZDT3-H	VR	
VEGA	0.5610	0.3502	0.457723333	0.051953901
SPEA2	0.7978	0.7716	0.790000000	0.006022347
EMOEA	0.7957	0.7715	0.784923333	0.006618493
NSGAII	0.7988	0.7883	0.793970000	0.002069485
DNSGA	0.8473	0.8323	0.842230000	0.003388623
		ZDT2-G	D	
VEGA	0.3498	0.1197	0.206133333	0.055084906
SPEA2	0.0644	0.0243	0.032513333	0.007023804
EMOEA	0.0991	0.0066	0.046606667	0.019133303
NSGAII	0.0678	0.0192	0.033546667	0.008564412
DNSGA	0.0493	0.0283	0.043323333	0.004708976
		ZDT3-G	D	
VEGA	0.1526	0.0572	0.104370000	0.024940384
SPEA2	0.0066	0.0008	0.001750000	0.001029158
EMOEA	0.0053	0.0008	0.001863333	0.001091630
NSGAII	0.0022	0.0008	0.001293333	0.000248909
DNSGA	0.0125	0.0044	0.006093333	0.001383940



Figure 2. Performance comparison of different multi-objective evolutionary algorithms on the test function ZDT2.



Figure 3. Performance comparison of different multi-objective evolutionary algorithms on the test function ZDT3.

As shown in Table 2, Figures 2 and 3, for test functions the ZDT2 and the ZDT3, the mean values of the HVR metrics for DNSGA are significantly higher than that of other classical multi-objective algorithms, and the standard deviations of the HVR metrics for DNSGA are also relatively lower than other multi-objective algorithms, indicating that the population diversity performance of DNSGA is superior to others evolution algorithms.

For the ZDT2, the mean value and standard deviation of the GD metrics for DNSGA do not differ much from the other algorithms. However, for the ZDT3, the mean value of the GD metrics for DNSGA lower than VEGA and higher than other algorithms, and the standard deviation of the GD metrics for DNSGA do not differ much from the other algorithms.

To solve the defects of NAGAII algorithm, the proposed DNSGA algorithm trades off the population diversity and the convergence. By the comparison of HVR and GD metric, although the convergence of DNSGA is slightly inferior to that of other classical multiobjective optimization algorithms, DNSGA is superior to other algorithms in population diversity. For the multi-objective aerodynamic optimization problem of complex operating conditions for FC, we put more emphasis on the performance of population diversity.

3. Multi-Objective Aerodynamic Design Optimisation of Centrifugal Impeller

Based on the DNSGA algorithm, a new multi-objective optimisation method for the centrifugal impeller is constructed by combining the MDOF method and the CFD technique. This optimisation method is used to carry out multi-objective optimisation for the centrifugal air compressor of the vehicle-mounted FC to obtain a global optimisation scheme at different rotational speeds.

3.1. Numerical Methods and Validation

3.1.1. Numerical Method

CFD techniques have been widely used to study the internal flow of centrifugal impellers [21–24]. In this paper, the steady-state flow field of a centrifugal impeller is obtained by solving the three-dimensional (3D) steady-state Reynolds averaged Navier-Stokes equation using NUMECA's Fine Turbo solver. The turbulence model adopts the S-A model, which is a simple one-equation model that can capture turbulence in the end-wall constrained case. Meanwhile, the S-A model is insensitive to the numerical errors caused by

mesh roughness and has been widely used in impeller machinery. The fourth-order explicit Runge–Kutta model is used for the time marching. A finite volume central difference scheme with second-order and fourth-order artificial viscosity terms is used to control the pseudo numerical oscillation in spatial discretisation. The convergence is accelerated by using multigrid, local time step, and implicit residual. The blade surface grid adopts HOH topology, the blade tip and hub gap grid adopt 4OH topology, and the thickness of the first grid layer near the wall is 1×10^{-6} m, $Y^+ \leq 1$.

3.1.2. Mesh Generation and Independence Verification

The grid independence of a single channel of the impeller at a rated speed is verified to ensure the grid quality of flow field calculation. The total grid number of the main blade and splitter blade is divided into 0.3 million, 0.66 million, 1.03 million and 1.5 million, respectively. The results are shown in Figure 4. When the grid number reaches 1.03 million, the grid independence requirement can be satisfied. Therefore, the subsequent optimisation process selects 1.03 million as the grid template for calculation. The boundary conditions for the numerical calculation are set as follows: the total compressor inlet temperature is 293 K, the total pressure is 101,325 Pa, the inlet direction is axial, and the outlet is the average static pressure. The convergence point before the first divergence point is the near-surge point by gradually increasing the backpressure and advancing the calculation from the blockage point to the near-surge point. The blade surface and end wall are set as no-slip boundary conditions.



Figure 4. Grid independence verification.

3.1.3. Numerical Method Validation

The validity of the numerical simulation was verified using the Krain impeller as the test model [25,26], which was verified by our team [5,27]. The numerical simulation results matched well with the Krain experimental data concerning the trend of the performance curves, indicating that the numerical method adopted in this paper can provide an accurate qualitative analysis of the aerodynamic performance changes before and after optimisation.

3.2. Optimisation Object

In this paper, the performance of FCVs single-stage high-speed centrifugal impeller with a rated output of 100 kW is optimised, and its CAD model is shown in Figure 5, and the aerodynamic and geometric parameters are shown in Table 3.



Figure 5. CAD of the original impeller.

Table 3. Aerodynamic and geometric parameters of the centrifugal compressor.

Parameter	Value
Fluid medium	Air
Rated output power of FCS (kW)	100
Rated speed (rpm)	100,000
Design point Mass flow (g/s 100,000 rpm)	118.32
Design point Total pressure ratio (100,000 rpm)	2.7
Design point Isentropic efficiency (100,000 rpm)	83.5%
Normal condition speed (rpm)	70,000
Design point Mass flow $(g/s 70,000 \text{ rpm})$	77.36
Design point Total pressure ratio (70,000 rpm)	1.7
Design point Isentropic efficiency (70,000 rpm)	85.5%
Non-dimensional speed coefficient	0.577
Blade numbers	16 (8 + 8)
Blade thickness (mm)	1
Tip clearance (mm)	2.5
Radial exit angle (°)	85

3.3. Optimisation Objectives and Constraints

Centrifugal impeller optimised design conditions are usually selected at rated speed for geometric parameter improvement. Further, non-design conditions are often selected at other mass flow at the same speed, which can meet the requirements for industrial compressors working in a single environment. However, the working speed range of the vehicle-mounted centrifugal impeller is wide, and the performance of the rated speed can be improved by optimising the rated speed only. However, the performance of the common operating conditions cannot be guaranteed. The isentropic efficiency of the aerodynamic performance of the centrifugal impeller of FC at the rated operating point and common operating point is optimised by multi-objective optimisation in this paper to improve them at the same time. Take 100,000 rpm, mass flow of 118.32 g/s and pressure ratio of 2.7 as the rated working condition. The selection of the common condition point is based on the U.S. Department of Energy (DOE) technical requirements for the same class of automotive FC power systems. A total of 70% of the rated speed of the centrifugal impeller, 77.36 g/s mass flow and 1.7 total pressure ratio are set as the common operating point. The original performance curve under two working conditions is shown in Figure 6.



Figure 6. Aerodynamic performance curve at different conditions.

According to the above description, maximising the isentropic efficiency at two operating points is the optimisation objective, with the optimised mass flow not decreasing and the optimised total pressure ratio in-between 1 and 1.01 times the original total pressure ratio as the constraint. The mathematical expression is as follows:

 $\max Eff_{d 70k}$

 $\max Eff_{d \ 100k}$

Objective function:

Constraint function:

$$Mass_{o\ 70k} \ge Mass_{d\ ori\ 70k}$$
 (5)

$$Mass_{o_100k} \ge Mass_{d_ori_100k} \tag{6}$$

$$0 \le \frac{T p r_{o_{-70k}} - T p r_{d_{-}ori_{-70k}}}{T p r_{d_{-}ori_{-70k}}} \le 0.01$$
(7)

$$0 \le \frac{Tpr_{o_100k} - Tpr_{d_ori_100k}}{Tpr_{d_ori_100k}} \le 0.01$$
(8)

$$L \le x_i \le x_i^U$$
 (9)

 $Eff_{d_{-70k}}$ is the isentropic efficiency at the design point of 70,000 rpm, $Eff_{d_{-100k}}$ is the isentropic efficiency at the design point of 100,000 rpm, and $Mass_{d_{-0ri_{-70k}}}$ and $Mass_{o_{-70k}}$ are the mass flow at the original impeller design point and the optimised impeller design point at 70,000 rpm, respectively. $Mass_{d_{-0ri_{-100k}}}$ and $Mass_{o_{-100k}}$ are the mass flow at the original impeller design point at 100,000 rpm, respectively. $Tpr_{d_{-0ri_{-70k}}}$ and $Tpr_{o_{-70k}}$ are the total pressure ratios at the original impeller design point and the optimised impeller design point at 70,000 rpm, respectively. $Tpr_{d_{-0ri_{-100k}}}$ are the total pressure ratios at the original impeller design point and the optimised impeller design point at 70,000 rpm, respectively. $Tpr_{d_{-0ri_{-100k}}}$ and $Tpr_{o_{-100k}}$ are the total pressure ratios at the original impeller design point at 100,000 rpm, respectively. x_i is the optimisation variable, b x_i^{U} is the upper bound of the optimisation variable, and x_i^{L} is the lower bound of the optimisation variable.

 x_i^l

3.4. Parameterisation Method

The development of impeller surface parameterisation methods has gone through from half-blade full parameterisation [28,29] to full-blade surface parameterisation [30], and then to the latest MDOF [27,31]. These methods are based on computer graphics by changing the blade suction surface and pressure surface control points to generate new blades directly. MDOF surface parametrisation has obvious advantages over other parametrisation methods in the process of blade surface generation, with the advantages of a low number of optimisation variables, smooth blade, flexible reshaping, and guaranteed

(3)

(4)

blade strength. In this paper, the MDOF parameterisation method is used to parameterise the surfaces of the main blade and the splitter blade separately using two 6 × 3 order Bezier surfaces. As shown in Figure 7, each surface has seven control vertexes in the ξ direction (0, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0) and four points in the η direction (0, 0.4, 0.7, 1.0). It is necessary to keep ξ_1 and ξ_2 fixed to ensure the first-order continuity of the leading edge. At the same time, the variation of the purple control vertex of the suction surface is consistent with the variation of the green control vertex of the corresponding pressure surface during the reshaping process to ensure that the blade thickness does not get thinner. Therefore, the values of the normal coordinates of the green control vertexes on the pressure surface are the optimisation variables, and the total number is only (5 × 4) × 2 = 40. By trial and error, the variable ranges for control vertexes are set as shown in Table 4.



Figure 7. Distribution of the parameterised control vertexes.

(a) Main blade					
Main Blade/mm	ξ,3	ξ4	ξ,5	ξ,6	ξ7
η1	[-2.0, 0.5]	[-2.0, 0.5]	[-2.0, 0.5]	[-1.0, 0.5]	[-0.5, 0.5]
η2	[-2.0, 2.0]	[-2.0, 2.0]	[-2.0, 2.0]	[-1.0, 1.0]	[-0.5, 0.5]
η3	[-2.0, 2.0]	[-2.0, 2.0]	[-2.0, 2.0]	[-1.0, 1.0]	[-0.5, 0.5]
η4	[-0.3, 2.0]	[-0.3, 2.0]	[-0.3, 2.0]	[-0.3, 1.0]	[-0.5, 0.5]
		(b) Splitter bl	ade		
Splitter Blade/mm	ξ3	ξ4	ξ5	ξ6	ξ7
η1	[-2.0, 0.5]	[-2.0, 0.5]	[-2.0, 0.5]	[-1.0, 0.5]	[-0.5, 0.5]
η2	[-2.0, 2.0]	[-2.0, 2.0]	[-2.0, 2.0]	[-1.0, 1.0]	[-0.5, 0.5]
η3	[-2.0, 2.0]	[-2.0, 2.0]	[-2.0, 2.0]	[-1.0, 1.0]	[-0.5, 0.5]
η4	[-0.3, 2.0]	[-0.3, 2.0]	[-0.3, 2.0]	[-0.3, 1.0]	[-0.5, 0.5]

 Table 4. Range of variation of each control vertexes.

3.5. Optimisation Process and Algorithm Parameter Setting

The detailed steps of the optimisation process for this experiment are shown in Figure 8, and the optimisation parameters are set as shown in Table 5.



Figure 8. Optimisation process.

Table 5. Parameter settings of DNSGA.

Option	Value	
Population size	40	
Number of generations	30	
Rate of crossover	0.98	
Rate of mutation	0.01	

In Step 1, the population individuals were initialised, and the fitness values of the parent individuals were obtained using the MDOF parameterisation method and CFD numerical calculations. The more detailed processes of the CFD implemented into the optimisation model are as follows: Firstly, the mesh template is obtained through the mesh division and independence verification of the original impeller; Secondly, the new impeller geometry is generated by using MDOF parameterization and optimization algorithm, and then the geometric mesh of the new impeller can be obtained through the original mesh template division. Finally, the NUMECA-FINE module is called to perform aerodynamic calculations on the new mesh to achieve the aerodynamic performance of the centrifugal impeller, including flow rate, total pressure ratio and efficiency. In addition, the mass flow and total pressure ratio should satisfy the constraints, and the efficiency is searched for the maximum value in the optimization process.

In Step 2, the offspring were generated based on selection, crossover, and variation. The fitness values of the calculated offspring individuals were obtained using the MDOF parameterisation method and CFD numerical calculations.

In Step 3, the parents and offspring were merged, and the next generation of parent individuals was selected by non-dominated sorting and spatial density operator sorting. The parameters of the weight coefficients of the spatial operator decision variables are set as shown in Table 6. The setting value of the weighting coefficient is based on engineering experience. The main blade of the centrifugal impeller, the middle of the blade, the

trailing edge, the root and the tip of the blade have a greater impact on the aerodynamic performance, so these positions are set to a larger weighting coefficient.

Main Blade/Splitter Blade	ξ3	ξ4	ξ5	ξ,6	ξ7
η1	1.4/1.2	1.6/1.4	1.4/1.2	1.6/1.4	1.6/1.4
η2	1.2/1.0	1.4/1.2	1.2/1.0	1.4/1.2	1.4/1.2
η3	1.2/1.0	1.4/1.2	1.2/1.0	1.4/1.2	1.4/1.2
$\eta 4$	1.4/1.2	1.6/1.4	1.4/1.2	1.6/1.4	1.6/1.4

Table 6. Value of weight of each control point.

In Step 4, the process returned to Step 2 and continued until the number of iterations was satisfied.

3.6. Optimisation Results and Analysis

3.6.1. Comparison of Blade Geometry

Based on engineering experience and aerodynamic requirements for centrifugal impellers, the best individual is selected from the PF solution (i.e., the optimal geometry blade). Figure 9 shows the geometric deformation of the blade before and after optimisation. Figure 10 shows the deformation contour of the blade surface before and after optimisation. Figure 11 shows the comparison of the hub, middle and tip geometry before and after optimisation.



Figure 9. Comparison of impeller blade geometry before and after optimisation.



Figure 10. Comparison of Bezier surface deformation contour before and after optimisation.



Figure 11. Comparison of the hub, middle and tip geometry before and after optimisation.

For the main blade along the span-wise direction, the deformations at the tip and hub are larger, while the middle section is essentially unchanged. Along the chord length direction, the change is smaller at the leading edge. The tip and middle section at the middle position bend toward the suction surface, and the tip bends toward the pressure surface. At the trailing edge, the tip bends toward the suction surface, the middle section basically remains unchanged, and the hub bends toward the pressure surface.

For the splitter blade along the span-wise direction, the deformations at the tip and hub are larger, while the middle section position changes less. Along the chord length direction, the leading edge is basically unchanged. The middle position of the tip bends to the suction surface, the middle section is basically unchanged, and the hub bends to the pressure surface. The tip of the trailing edge bends to the suction surface, the middle section is unchanged, and the hub bends to the pressure surface.

In general, the lower and middle regions of the flow path between the pressure surface of the main blade and the suction surface of the splitter blade become wider, and the top becomes narrower after optimisation. Meanwhile, the lower and middle regions of the flow path between the suction surface of the main blade and the pressure surface of the splitter blade become narrower, and the top becomes wider. The change of blade geometry and flow channel area will inevitably cause aerodynamic performance and flow field structure changes.

3.6.2. Comparison of Multi-Conditions Aerodynamic Performance before and after Optimisation

Tables 7 and 8 show the comparison of the aerodynamic performance before and after the optimisation at the design point of rated condition and the design point of a common condition, respectively. Further, Figure 12 shows the comparison of the performance curves before and after the optimisation of the rated condition and common condition. After optimisation, the efficiency at the design point of the rated operating conditions increased by 1.29%, the mass flow increased by 8.8%, the total pressure ratio increased by 0.74%, and the comprehensive margin increased by 6.2%. The efficiency of the common operating conditions improved by 1.2%, the mass flow improved by 9.1%, the pressure ratio improved by 0.24%, and the comprehensive margin improved by 10%. The centrifugal impeller aerodynamic performance has been significantly improved, and the performance under different working conditions can be improved simultaneously.

Table 7. Comparison of the performance at the design point before and after optimisation (rated operating condition).

100,000 rpm	Mass Flow (g/s)	Total Pressure Ratio	Isentropic Efficiency	Comprehensive Surge Margin ¹
Original	118.32	2.70	83.54%	19.4%
Optimised	128.76	2.72	84.83%	25.6%
Relative change	+8.8%	0.74%	+1.29%	+6.2%

¹ Comprehensive surge margin = $\left(\frac{Ps \times Md}{Pd \times Ms} - 1\right) \times 100\%$, *Ps*: total pressure at the surge point; *Pd*: total pressure at design point; *Ms*: mass flow at the surge point; *Md*: mass flow at the design point.

Table 8. Comparison of the performance at the design point before and after optimisation (common operating condition).

70,000 rpm	Mass Flow (g/s)	Total Pressure Ratio	Isentropic Efficiency	Comprehensive Surge Margin ²
Original	77.36	1.7	85.53%	18.9%
Optimised	84.43	1.704	86.73%	28.9%
Relative change	+9.1%	0.24%	+1.2%	+10%

² Comprehensive surge margin = $\left(\frac{Ps \times Md}{Pd \times Ms} - 1\right) \times 100\%$, *Ps*: total pressure at the surge point; *Pd*: total pressure at design point; *Ms*: mass flow at the surge point; *Md*: mass flow at the design point.



Figure 12. Aerodynamic performance curve at different conditions before and after optimisation.

3.6.3. Flow Field Analysis

Figure 13 shows the entropy contour of the S3 section before and after the optimisation of the rated operating condition. After optimisation, the high entropy areas in channels I and II, A, B, C and D are reduced, and the losses are significantly reduced. The reason for the decrease of inlet loss is that the shock wave at the blade inlet is weakened due to the better matching of the inlet attack angle. Figure 14 shows the entropy contour of the S3 section before and after the optimisation of the common operating condition. The entropy value of A, B, C, D, E and F is obviously reduced after optimisation, given that the incidence angle and blade geometry are better matched, making the acceleration section of the blade inlet shorter and acceleration slower. This is beneficial to reduce the loss. As can be seen from Figures 10–12, the top of the flow channel between the pressure surface of the main blade and the suction surface of the splitter blade becomes narrower. Further, the back-bending angle is increased after optimisation, which reduces the pressure expansion angle and reduces the return loss. This is the main reason for the reduction of the top



clearance and loss at the outlet of the rated and common operating conditions and the increase of efficiency. This is the main reason for the reduction of tip and outlet losses.

Figure 13. Entropy contour of the S3 cross-section before and after optimisation (rated operating condition).



Figure 14. Entropy contour of the S3 cross-section before and after optimisation (common operating condition).

Figure 15 shows the isentropic efficiency changes along the span-wise at the downstream outlet for different operating conditions before and after optimisation. It can be seen that the most significant efficiency improvement is roughly in the area of 80% in the span-wise direction. As such, the flow field structure at the 80% height (H) of the impeller was chosen as the focus of the analysis.

Figure 16 shows the static pressure distribution on the blade surface at the 80% height of the impeller before and after the optimisation of the rated operating condition. Figure 17 shows the relative Mach number contour on the B2B surface at the 80% height of the impeller before and after the optimisation of the rated operating condition. After optimisation, the positive angle of the attack at the inlet is reduced, the matching of airflow is improved, the area and maximum value of the high relative Mach number of the front edge A and B of the main blade suction are reduced, and the shock loss at the inlet is reduced. At the same time, the loss of the interaction between the shockwave and the boundary layer is reduced, which makes the flow separation in the latter half of the suction surface move backwards. Further, the separation area is reduced, and the separation loss is

reduced. The decrease of the positive angle of attack also increases the flow area, which increases the mass flow after optimisation. The static surface pressure of the suction surface of the main blade and splitter blade is significantly reduced, increasing the relative Mach number in the C, D and E regions. Meanwhile, the reduction of the area of the low-speed region reduces the reverse pressure gradient and improves the flow performance of the downstream outlet F. Further, it can be seen from the comparison diagram of the static pressure distribution that the blade load increases after optimisation, which is conducive to the increase of work.



Figure 15. Span-wise isentropic efficiency.



Figure 16. Comparison of the static pressure distribution on the blade surface at 80% height before and after optimisation (Rated operating condition).



Figure 17. Comparison of the entropy contour of B2B surface at 80% height before and after optimisation (rated operating condition).

Figure 18 shows the static pressure distribution on the blade surface at the 80% height of the impeller before and after optimisation of the common operating condition. Figure 19 shows the relative Mach number contour on the B2B surface at the 80% height of the impeller before and after optimisation of the common operating condition. After optimisation, the matching of inlet flow angle is improved, the acceleration section is shortened, the area and maximum value of the high relative Mach number of suction leading-edge A and B are reduced, and the shock loss and separation loss at the inlet is reduced. Moreover, the decrease of the positive angle of attack also increases the flow area, which increases the mass flow after optimisation. The static pressure on the suction surface of the main blade and splitter blade is significantly reduced, increasing the relative Mach number in the areas of C, D and E, effectively reducing the low-speed area of the channel, reducing the reverse pressure gradient, delaying the separation position, and improving the aerodynamic performance of the downstream outlet F.



Figure 18. Comparison of the static pressure distribution on the blade surface at 80% height before and after optimisation (common operating condition).



Figure 19. Comparison of the entropy contour of B2B surface at 80% height before and after optimisation (common operating condition).

4. Conclusions

In this paper, the multi-condition aerodynamic optimisation of the centrifugal impeller is studied, and the optimisation algorithm DNSGA is proposed and verified on the test function. An automatic optimisation method is studied and designed, combining the DNSGA with MDOF surface parameterisation and CFD flow-field analysis. The method is successfully applied to the aerodynamic optimisation of the FC centrifugal impeller. The main conclusions are as follows:

- Compared with other classical multi-objective evolutionary algorithms, DNSGA performs well in population diversity and convergence, especially concerning greater improvement in diversity. This optimisation algorithm improves the traditional method of NSGA-II for selecting the value of the objective function by improving the division of non-dominated ranking and spatial density ranking on decision variables. This provides a new scientific method for the application and research of the multiobjective evolutionary algorithm.
- 2. The optimised geometric deformation of the blade increases the back-bend angle of the blade, the positive angle of attack at the inlet, and the relative Mach number in the blade channel. These changes reduce the diffuser angle and reverse the pressure gradient, reducing the separation loss and improving the flow at the downstream outlet. Under multi-working conditions, the aerodynamic performance curve moved upward as a whole, and the flow condition was significantly improved. The isentropic efficiency under the rated conditions increased by 1.2%, the flow increased by 9.1%, the pressure ratio increased by 0.24%, and the comprehensive margin increased by 10%. The isentropic efficiency under common working conditions increased by 1.29%, the flow increased by 8.8%, the pressure ratio increased by 0.74%, and the comprehensive margin increased by 6.2%. Moreover, it is verified that the aerodynamic optimisation of the FC impeller can improve the efficiency of rated and common working conditions simultaneously. The centrifugal impeller design optimisation system presented in this paper provides technical support and a new method for scientific research to study the aerodynamic optimisation of centrifugal impellers for vehicle-mounted FCs.
- 3. The parametric method used in this paper is based on the Bezier surface blade optimization method. The limitation of this method is that the change of one variable control point on the Bezier surface will cause the change of the entire surface and cannot achieve the local optimization of the blade geometry, so as not to get the best

blade geometry. In order to overcome this drawback, we will introduce a more flexible B-spline surface parameterization method or Free-Form deformation method, which can do to control the local surface variation and thus may obtain a blade geometry with better aerodynamic performance.

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Nomenclature

Abbreviations	

SD	spatial density
FC	fuel cell
FCS	fuel cell system
FCVs	fuel cell vehicles
CFD	computational fluid dynamics
NVH	noise, vibration and harshness
3D	three-dimensional
ICE	internal combustion engines
MDOF	multi-degree-of-freedom
GA	genetic algorithm
DNSGA	spatial density NSGA-II
PF	Pareto frontier
HVR	The hypervolume ratio
GD	generational distance
Ori	original
Opt	optimized
L.E.	leading edge
T.E.	trailing edge
DOE	Department of Energy
Η	height
Symbol	
W	output power of the FC stack
Q_m	mass flow
V	average voltage of the cell
λ	stoichiometric ratio of the air
P_0	initial population
P_g^*	new offspring
$d(Y_1, Y_2)$	spatial density operator
$d_w(Y_1, Y_2)$	spatial density operator with weights
w_i	weight of the <i>i</i> th dimension variable
rpm	revolutions Per Minute
$Eff_{d_{-70k}}$	isentropic efficiency at the design point of 70,000 rpm
Eff_{d_100k}	isentropic efficiency at the design point of 100,000 rpm

Mass _{d_ori_70k}	mass flow at the original impeller design point at 70,000 rpm
$Mass_{o_{70k}}$	mass flow at the optimised impeller design point at 70,000 rpm
Mass _{d ori 100k}	mass flow at the original impeller design point at 100,000 rpm
$Mass_{o_{100k}}$	mass flow at the optimised impeller design point at 100,000 rpm
Tpr _{d ori 70k}	total pressure ratios at the original impeller design point at 70,000 rpm
$T pr_{o 70k}$	total pressure ratios at the optimised impeller design point at 70,000 rpm
Tpr _{d ori 100k}	total pressure ratios at the original impeller design point at 100,000 rpm
$T pr_{o_{100k}}$	total pressure ratios at the optimised impeller design point at 100,000 rpm
x_i	optimisation variable
x_i^U	upper bound of the optimisation variable
x_i^{L}	lower bound of the optimisation variable
ξ	unitized bezier plane horizontal coordinates
η	unitized bezier plane vertical coordinates
P_s	Total pressure at the surge point
Pd	Total pressure at the design point
Ms	Mass flow at the surge point
Md	Mass flow at the design point

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