

Article

Experimental Validation of Formula for Calculation Thermal Diffusivity in Superlattices Performed Using a Combination of Two Frequency-Domain Methods: Photothermal Infrared Radiometry and Thermoreflectance

Michał Pawlak ^{1,*}, Timo Kruck ², Nikolai Spitzer ², Dariusz Dziczek ¹, Arne Ludwig ²  and Andreas D. Wieck ²

¹ Institute of Physics, Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, Grudziadzka 5, 87-100 Torun, Poland; dziczek@umk.pl

² Chair of Applied Solid-State Physics, Experimental Physics VI, Ruhr-University Bochum, Universitaetsstrasse 150, D-44780 Bochum, Germany; timo.kruck@ruhr-uni-bochum.de (T.K.); nikolai.spitzer@rub.de (N.S.); arne.ludwig@rub.de (A.L.); Andreas.Wieck@ruhr-uni-bochum.de (A.D.W.)

* Correspondence: mpawlak@fizyka.umk.pl; Tel.: +48-56-611-32-50

Abstract: In this paper, we validate two theoretical formula used to characterize thermal transport of superlattices at different temperatures. These formulas are used to measure cross-plane thermal conductivity and thermal boundary resistance, when it is not possible to obtain heat capacity or thermal diffusivity and in-plane thermal conductivity. We find that the most common formula for calculating thermal diffusivity and heat capacity (and density) can be used in a temperature range of $-50\text{ }^{\circ}\text{C}$ to $50\text{ }^{\circ}\text{C}$. This confirms that the heat capacity in the very thin silicon membranes is the same as in bulk silicon, as was preliminary investigated using an elastic continuum model. Based on the obtained thermal parameters, we can fully characterize the sample using a new procedure for characterization of the in-plane and cross-plane thermal transport properties of thin-layer and superlattice semiconductor samples.

Keywords: superlattice; thin films; thermal transport; thermal wave methods



Citation: Pawlak, M.; Kruck, T.; Spitzer, N.; Dziczek, D.; Ludwig, A.; Wieck, A.D. Experimental Validation of Formula for Calculation Thermal Diffusivity in Superlattices Performed Using a Combination of Two Frequency-Domain Methods: Photothermal Infrared Radiometry and Thermoreflectance. *Appl. Sci.* **2021**, *11*, 6125. <https://doi.org/10.3390/app11136125>

Academic Editor: Valentina Belova

Received: 21 May 2021

Accepted: 26 June 2021

Published: 30 June 2021

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1. Introduction

Quantitative description of heat dissipation in multi-layered nanostructures and thin layers is a challenging task. One of the most popular methods is time-domain thermoreflectance (TDTR) [1–14]. For superlattices, the strategy for investigating their thermal properties using the TDTR method relies on measuring thermal conductivity. The other cross-plane and in-plane thermal parameters (thermal diffusivity and thermal boundary resistance) are calculated based on this parameter. However, in order to measure the thermal conductivity in the TDTR, method, researchers need to use the heat capacity of the bulk material. A preliminary theoretical investigation, used an elastic continuum model, suggested that the heat capacity in the very thin silicon membranes is the same as in bulk silicon [15]. For superlattices, this relation is more complex. However, frequency-domain measurements are simple and less expensive and can measure all thermal transport properties. The benefit of using frequency domain methods over time-resolved methods is their unique profilometry capability. This relies on the observation that one can relate the origin of a signal at a given frequency to the depth or penetration length of the thermal wave (i.e., thermal diffusion length). This allows the thermal conductivity as a function of depth to be probed by varying the frequency. Another major advantage of frequency domain methods is that they are simpler to implement experimentally and do not require the use of expensive femtosecond lasers. There are many frequency-domain methods, such as photoacoustic [16,17], photothermal beam reflection [18,19], thermoreflectance [20,21] photothermal infrared radiometry [22–30], and modulated free carrier absorption [31,32].

Recently, we reported on cross-plane thermal transport properties of superlattices and thin films by using a frequency-domain PTR (photothermal infrared radiometry) method [24]. We demonstrated that it is possible to estimate values of the cross-plane thermal parameters for one-dimensional heat flow. In this paper, we study thin-layer and superlattice samples using both frequency domain methods, PTR and thermoreflectance (FDTR), at different temperatures. The purpose of this study is to demonstrate that both methods are complementary and that all in-plane and cross-plane heat transport properties can be fully measured using both methods. All parameters are then used for the experimental validation of formulas calculating thermal transport properties in superlattice in different temperature.

2. Materials and Methods

2.1. Frequency Domain Photothermal Infrared Radiometry Set Up

The PTR experimental set-up is presented in Figure 1 [24]. The DPSS laser ($\lambda = 532$ nm, 800 mW CW power, model CNI laser MGL-FN-532-1W) is intensity-modulated by an acoustic-optical modulator (model Crystal Technology, Inc. AOM 3080-120) in the frequency range between 100 Hz and 10 MHz using a 30 MHz waveform generator (Agilent 33522A). The beam spot ($1/e^2$) has a diameter of 1.8 mm. Two Au-coated off-axial parabolic mirrors are used to collect and collimate the emitted IR radiation from the sample, focusing it on a photoconductive mercury cadmium telluride (MCT) detector (Kolmartech KMPV 11-1-J1/DC/Ge). The detector has peak sensitivity at a wavelength of $9.6 \mu\text{m}$ and an active area of 1 mm^2 (with sensitivity between $2 \mu\text{m}$ and $12 \mu\text{m}$). It is cooled with liquid nitrogen and has an antireflection-coated Ge window. Finally, the signal from the detector is processed by a lock-in amplifier (LIA, SR865 from Stanford Research, able to work in the frequency range between DC and 2 MHz and SR844 in the frequency range of 2 MHz and 10 MHz). Additionally, the system is prepared for temperature-dependent measurements using a Linkam System (LTS420) able to work between -200°C and 420°C .

The disadvantage of the PTR method is that the signal strongly decreases with the temperature with law $\Delta E \sim \sigma T^3 \Delta T$ (where σ is Stefan-Boltzmann constant) as shown in Figure 2. For comparison, the results from frequency-domain thermoreflectance (FD-TR) measurements are also presented.

Although the PTR signal can be increased by decreasing the beam size, we should bear in mind that there is a trade-off between one-dimensional heat propagation in the sample and the beam spot size. Thus, there is an optimum spot-size that maximizes the PTR signal while keeping the heat-transport one-dimensional. From Figure 2 one can see that at low temperature (below $-100^\circ\text{C}/173 \text{ K}$), the PTR signal is very small and measurements cannot be taken in frequency-scans up to MHz. On the other hand, thermoreflectance can be measured down to 50 K [33]. The thermoreflectance signal can be described by the formula [33]:

$$S_{\text{TR}} = \frac{\Delta R}{R} = \left(\frac{1}{R} \frac{\partial R}{\partial T} \right) \Delta T = C_{\text{TR}} \Delta T \quad (1)$$

where ΔT is the thermal wave oscillations (which depends on the temperature via thermal transport parameters) and C_{TR} is the thermoreflectance coefficient. In paper [16], authors show that signals above room temperature increase linearly with temperature. In our paper we show that this trend is also observed for lower temperatures for a layered sample. The deviation from linear behavior was already observed by other authors [34].

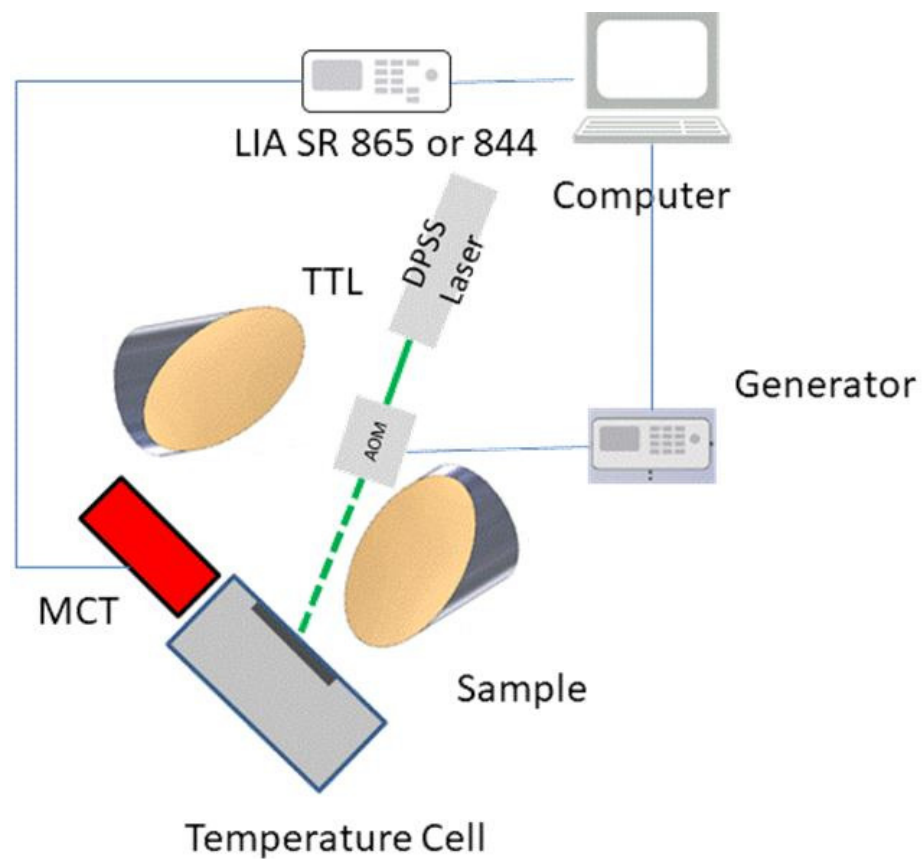


Figure 1. Frequency-domain photothermal infrared radiometry set-up with temperature cell. The beam size was 1.8 mm.

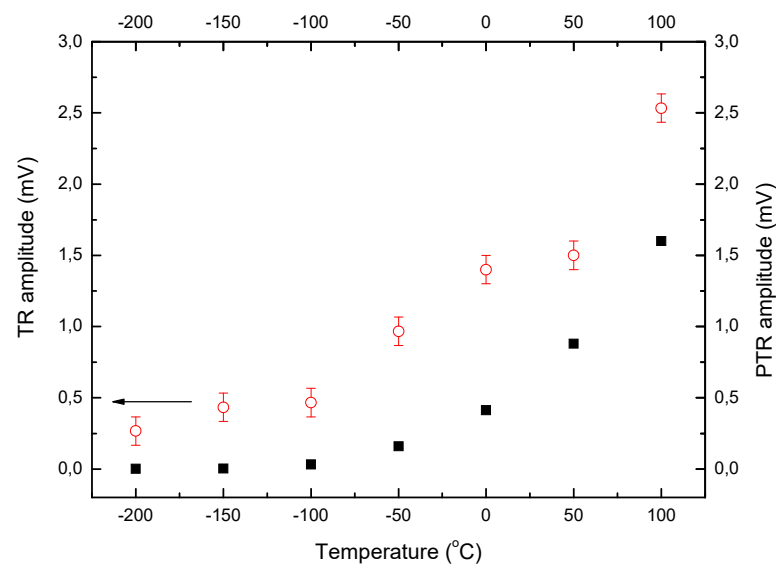


Figure 2. Temperature-dependent of the PTR and TR amplitudes at 1 kHz for a GaAs wafer covered by 50 layer of Ti and 2 μm AlGaAs thin layer with 100 nm of gold layer, respectively. The data are not normalized. The signals were depicted for 1 kHz. Note that PTR amplitude is very stable with error bars within experimental points.

2.2. Frequency Domain Thermoreflectance Set Up

The FD-TR setup is presented in Figure 3.

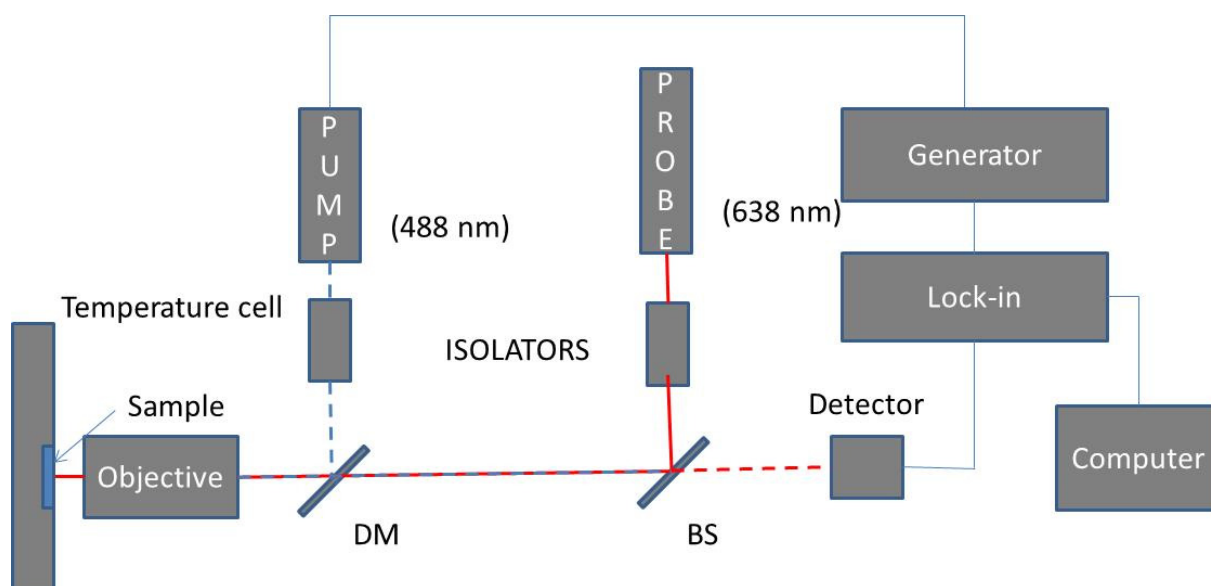


Figure 3. FD-TR setup.

The 100 mW modulated pump laser (488 nm Cobolt model 06-MLD) was intensity modulated using the sinusoidal signal from the 30 MHz waveform generator (Agilent 33522A). While the 2 mW probe beam (638 nm Cobolt model 06-MLD) was used to monitor the changes of the sample reflectivity due to changes of the temperature induced by the pump beam. The beam spots ($1/e^2$) have been measured (Beam Profiler Slit 2 μm S-BR2-Si) to be 3 μm in diameter. Laser beams were focused on the sample by an objective Edmund Optics, 100X model 46-401. The signal was processed by a lock-in amplifier (LIA, SR865 from Stanford Research able to work in the frequency range between DC and 2 MHz). The amplitude A and phase (φ) were acquired, but only the phase was analyzed by a computer for each modulation frequency. The temperature-dependent measurements were performed using a Linkam System able to work between $-200\text{ }^\circ\text{C}$ and $420\text{ }^\circ\text{C}$. The reference phase was measured by placing the detector instead of the sample. The reference phase was then extracted from the measured phase of the thermoreflectance signal.

2.3. Materials

Sample #1 consisted of a GaAs substrate wafer that was used as a reference. Samples #2 consisted of $\text{Al}_{0.33}\text{Ga}_{0.67}\text{As}$ thin film alloys grown on GaAs substrate wafers while samples #3 and #4 consisted of AlAs/GaAs superlattices grown on GaAs substrate wafers. The GaAs substrate wafers had a 3" (76 mm) diameter with a (001)-orientation and were nominally undoped. The $\text{Al}_x\text{Ga}(1-x)\text{As}$ semiconductor alloys and superlattice samples were grown via molecular beam epitaxy (MBE). The GaAs substrate temperature in the MBE chamber during growth was $600\text{ }^\circ\text{C}$ (measured by a pyrometer) and the As-flux (beam equivalent pressure BEP) was 9.6×10^{-6} Torr in the MBE machine. The growth of all wafers was preceded by careful oxide desorption at $600\text{ }^\circ\text{C}$ and the growth of a GaAs buffer layer. The metal layers were deposited by thermal evaporation in a separate deposition chamber with current-heated evaporation boats. Sample #2 had a 200 nm GaAs buffer layer with a nominally 2000 nm thick $\text{Al}_{0.33}\text{Ga}_{0.67}\text{As}$ layer. Sample #3 and #4 had a 150 nm GaAs buffer layer and a superlattice consisting of 10 repetitions of nominally 26 nm AlAs and 26 nm GaAs (with the superlattice ending with a GaAs layer). Samples #2 and #4 were covered by a 100 nm thin Au-layer, while sample #1 and #3 are covered by a Ti

layer with a thickness of 50 nm. The deposition processes on all samples were done in a vacuum with pressure less than 10^{-6} mbar.

Figure 4 presents Scanning Electron Microscope image (SEM) of superlattice sample.

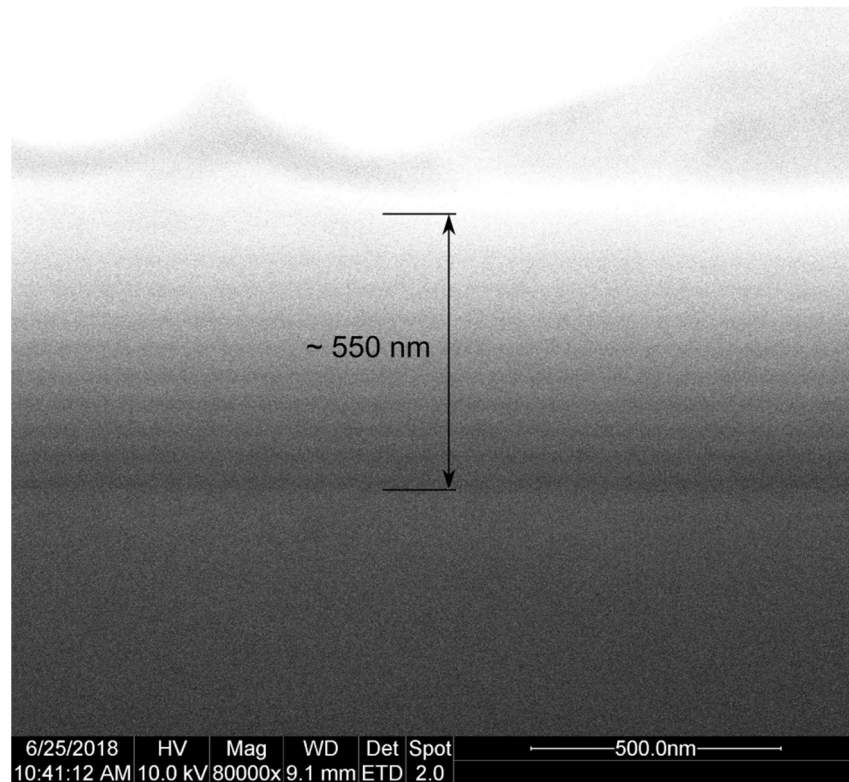


Figure 4. SEM image of superlattice sample (samples #3 and #4).

The difference in film thickness measurements using SEM and reflectometry is due to slight differences in film thickness across the sample surface. The samples measured using reflectometry were used for the measurements. The samples properties are presented in Table 1.

Table 1. Summary of the sample properties.

	Thicknesses Estimated from Reflectivity Measurement [24]	Description	Metallic Layer on the Top
Sample #1	-	Undoped GaAs used as substrate;	Ti 50 nm
Sample #2	1980 nm	Thin $\text{Al}_{0.33}\text{Ga}_{0.67}\text{As}$ alloy	Au 100 nm
Sample #3	520 nm	AlAs/GaAs superlattice with period thickness of 52 nm	Ti 50 nm
Sample #4	520 nm	AlAs/GaAs superlattice with period thickness of 52 nm	Au 100 nm

3. Theoretical Models

As shown in Figure 5 we used two geometries: cross and in-plane. In-plane geometry is achieved by significantly reducing the width of the excitation beam.

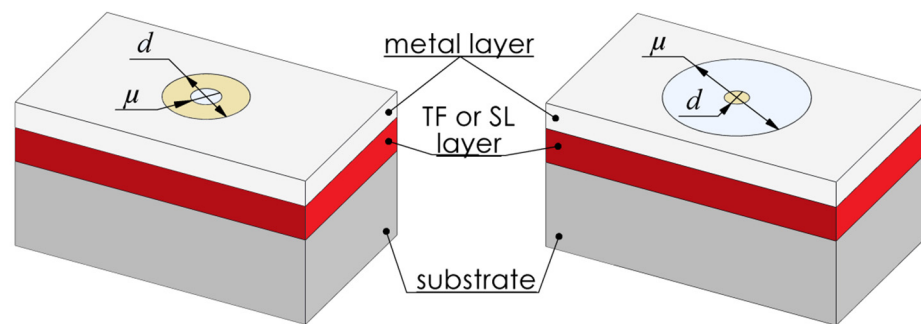


Figure 5. Cross-plane (left) and in-plane (right) geometry in frequency-domain methods.

Although Figure 5 suggests that the lower the frequency the better the measurement of in-plane thermal conductivity, it still depends on the thermal parameters (the better the parameter determination sensitivity, range shifts toward higher frequencies, see Figure 6). The three-dimensional signal can be written based on the models already developed and published literature in [23]:

$$\Theta(\omega, 0) = -\frac{Q}{2\pi} \int_0^\infty \frac{D(\lambda)}{C(\lambda)} e^{-\frac{(\lambda w_0)^2}{8}} \lambda d\lambda \quad (2)$$

where Q is the laser power and w_0 is the laser spot $1/e^2$. The coefficient C and D can be calculated using:

$$\begin{bmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{bmatrix} = A_3 M_{32} A_2 M_{21} A_1 \quad (3)$$

Table 2. Constant parameters used for calculation.

Parameter	Three Layer
$k_1 = k_{Ti}$ (W/mK) at -50 °C	22.9
$k_1 = k_{Ti}$ (W/mK) at 0 °C	21.5
$k_1 = k_{Ti}$ (W/mK) at 50 °C	21.0
$\alpha_1 = \alpha_{Ti}$ (m^2/s) for all temperatures	8×10^{-6}
$k_1 = k_{Au}$ (W/mK) at -50 °C	321
$k_1 = k_{Au}$ (W/mK) at 0 °C	318
$k_1 = k_{Au}$ (W/mK) at 50 °C	315
$\alpha_1 = \alpha_{Au}$ (m^2/s) for all temperatures	127×10^{-6}
$R_{Ti/GaAs}$ (R_{12})	2.0×10^{-8} ($m^2 KW^{-1}$) from reference measurement
$k_3 = k_{GaAs}$ (W/mK) at -50 °C	70
$\alpha_3 = \alpha_{GaAs}$ (m^2/s) at -50 °C	4.2×10^{-5}
$k_3 = k_{GaAs}$ (W/mK) at 0 °C	60
$\alpha_3 = \alpha_{GaAs}$ (m^2/s) at 0 °C	3.2×10^{-5}
$k_3 = k_{GaAs}$ (W/mK) at 50 °C	30
$\alpha_3 = \alpha_{GaAs}$ (m^2/s) at 50 °C	1.7×10^{-5}

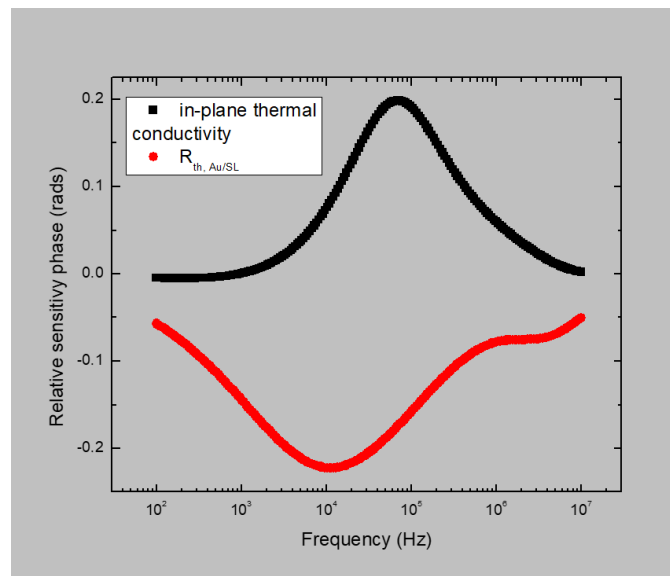


Figure 6. Phase relative sensitivities of model described by Equation (1) to the parameters ($R_{\text{AlGaAs/GaAs}} = 1.0 \times 10^{-9} \text{ m}^2 \text{ K/W}$, $k_{\text{AlGaAs}} = 12 \text{ W/mK}$, $\alpha_{\text{AlGaAs}} = 6 \times 10^{-6} \text{ m}^2/\text{s}$, $L = 500 \text{ nm}$) and with other parameter values taken from Table 2.

Each layer in the model is described by matrices A_3 , A_2 , A_1 specified by:

$$A_n = \begin{bmatrix} \cosh(q_n d_n) & \frac{-1}{k_{zn}} \sinh(q_n d_n) \\ -k_{zn} q_n \sinh(q_n d_n) & \cosh(q_n d_n) \end{bmatrix} \quad (4)$$

Here d_n is the layer thickness and k_{zn} the cross-plane thermal conductivity of the n th layer, and:

$$q^2 = \frac{k_m \lambda^2 + \rho_n c_n i \omega}{k_{zn}} \quad (5)$$

where ρ_n is the density of the n th layer, c_n is the specific heat capacity of the n th, and k_m and k_{zn} are the in-plane and cross-plane thermal conductivities of the n th layer, respectively. For calculations, we used the thermal diffusivity, defined as:

$$\alpha_{zn} = \frac{k_{zn}}{\rho_n c_n} \quad (6)$$

The resistances of the thermal interfaces are described by matrices:

$$M_{n,n-1} = \begin{bmatrix} 1 & -R_{n,n-1} \\ 0 & 1 \end{bmatrix} \quad (7)$$

where $R_{n,n-1}$ is the thermal interface resistance between the n th and the $(n-1)$ th layer.

In turns, one-dimensional PTR signal can be written as [24]:

$$\theta(\omega, 0) = \left(\frac{(1-R)I_0}{2k_1 \sigma_1} \right) \left(\frac{1+y_1}{1-y_1} \right) \quad (8)$$

Here, y_1 is a dimensionless quantity defined and $\sigma_i = \sqrt{i\omega/\alpha_i}$ as:

$$y_1 = e^{-2L_1 \sigma_1} \left(\frac{U_1^+}{U_1^-} \right) \quad (9)$$

where $L_1 = x_1$ is the length of first layer and $U_1^\pm$ is another dimensionless quantity defined as:

$$U_1^\pm = (1 + y_2) - R_{12}k_2\sigma_2(y_2 - 1) \pm \left(\frac{k_2\sigma_2}{k_1\sigma_1}\right)(y_2 - 1) \quad (10)$$

The multi-layer solution takes the form of a recursive sequence. For the n th layer, the dimensionless quantity y_n is defined as:

$$y_n = e^{-2L_n\sigma_n} \left(\frac{U_n^+}{U_n^-} \right) \quad (11)$$

where $L_n = x_n - x_{n-1}$ and $U_n^\pm$ is a dimensionless quantity defined as:

$$U_n^\pm = (1 + y_{n+1}) - R_{n,n+1}k_{n+1}\sigma_{n+1}(y_{n+1} - 1) \pm \left(\frac{k_{n+1}\sigma_{n+1}}{k_n\sigma_n}\right)(y_{n+1} - 1) \quad (12)$$

4. Results

It is worth emphasizing that the in-plane thermal conductivity can be measured by either changing the spot-size of the laser on the sample or spatially separating the position of the pump and probe spots [21,35]. Moreover, using the TD-TR method it is not possible to measure the in-plane thermal conductivity by changing its spot size, but one can perform measurements using separation of beams or by the elliptical-beam method [36]. It was found that for time-domain methods the recommended method is to spatially separate the position of the pump and probe spots [35]. In this paper we demonstrate that, using the frequency-domain method, it is possible to measure in-plane thermal conductivity. To test that the thermoreflectance is able to measure in-plane thermal conductivity, we performed the measurement of isotropic AlGaAs thin-layer sample and calculated the sensitivity. Figure 6 shows the relation between the relative sensitivity vectors of these parameters calculated using Equation (1). These relative sensitivity vectors are calculated as $S_{\text{Phase},p} = \partial(\text{Phase})/\partial \ln(p)$, where $S_{\text{Phase},p}$ is phase sensitivities to parameter p of the model, respectively [26].

After theoretically demonstrating that it is possible to measure in-plane thermal conductivity, we used the 2 μm thick AlGaAs sample to experimental validation of the method. 2 μm thick AlGaAs is thermally isotropic. Figure 7 shows the measured TR phase of sample #2.

In order to decrease the number of unknown parameters, we use the cross-plane thermal conductivity, diffusivity and thermal interface boundary of the Al_{0.33}Ga_{0.67}As thin film from the literature ($k = 12 \text{ W/mK}$, $\alpha = 6.5 \times 10^{-6} \text{ m}^2$, $R_{\text{AlGaAs/GaAs}} = 1 \times 10^{-9} \text{ (m}^2 \text{ KW}^{-1})$) [29,37]. The fitting results using Matlab function `lsqcurvefit` using Equation (1) yield the value of in-plane thermal conductivity to be $16 \pm 4 \text{ W/mK}$. This demonstrates that the sample is thermally isotropic. It is worth to notice that the thermal boundary resistance between gold and the layer was found to be one order of magnitude higher than in the interface Ti-GaAs ($R_{\text{Au/AlGaAs}} = (2.0 \pm 0.5) \times 10^{-7} \text{ (m}^2 \text{ KW}^{-1})$), as expected [36].

Figure 8 presents the PTR results for temperatures -50 , 0 and 50°C measured for sample #3.

The fitting procedure was identical as in our previous paper [24]. We used Matlab function `lsqcurvefit` for numerical calculations. First, we estimate R_{th} between Ti layer GaAs (sample #1). We found out that with changing temperature between -50°C and 50°C thermal interface boundary remains constant within the estimated error. Secondly, we use these values and data for sample #1 in estimating cross-thermal transport properties of sample #3. For the latter calculation, we used the parameters presented in Table 2.

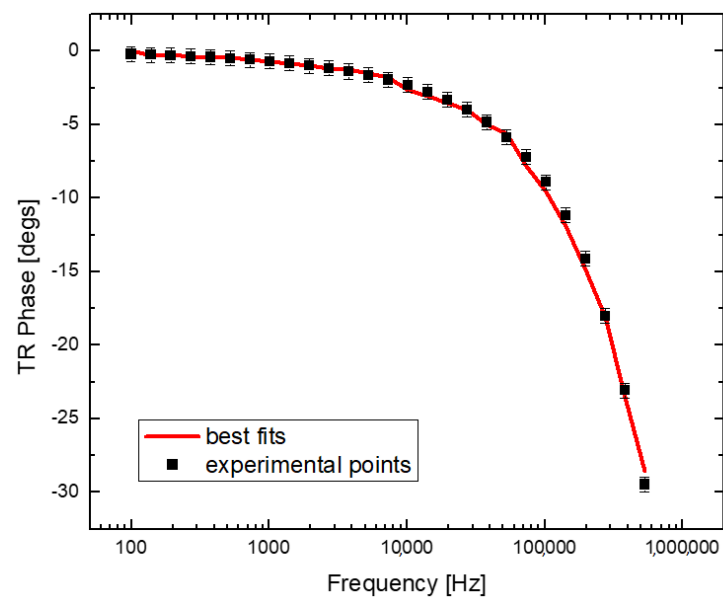


Figure 7. Measured TR phase of 2 μm AlGaAs thin layer covered by 100 nm of gold layer between 100 Hz and 500 kHz. The best fits are obtained using Equation (1).

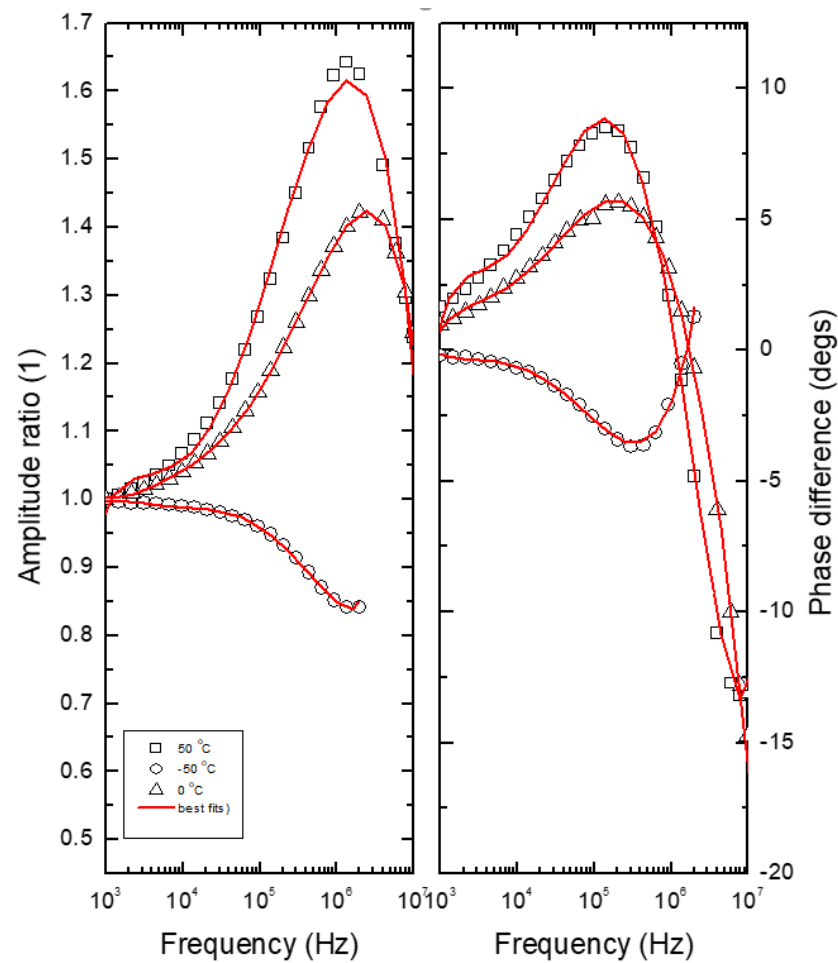


Figure 8. PTR experimental results (points) for sample#3 in three different temperatures: $-50\text{ }^{\circ}\text{C}$, $0\text{ }^{\circ}\text{C}$ and $50\text{ }^{\circ}\text{C}$. Additionally, the best fittings are presented as solid lines using Equation (8).

Note that we assume that the cross-plane thermal conductivity of the gold layer and the GaAs substrate are the same their in-plane thermal conductivity at the different temperatures. The fitting results are presented in Table 3.

Table 3. Cross-plane thermal transport parameters of the AlAs/GaAs superlattice, obtained from fits to the PTR data in Figure 5 for different temperatures.

Temperature (°C)	$k_{\text{GaAs/AlAs}}$ (W/mK)	$\alpha_{\text{GaAs/AlAs}}$ (m ² /s)	$R_{\text{th},23}$ (m ² KW ^{−1})
−50	25 ± 2	$(2.0 \pm 0.6) \times 10^{-5}$	$(2 \pm 0.2) \times 10^{-8}$
0	16 ± 3	$(1.5 \pm 0.5) \times 10^{-5}$	$(1 \pm 0.5) \times 10^{-8}$
50	8 ± 3	$(4.0 \pm 2.5) \times 10^{-6}$	$(1.5 \pm 0.9) \times 10^{-8}$
RT [15]	19 ± 5	$(1.0 \pm 0.2) \times 10^{-5}$	$(3.0 \pm 0.8) \times 10^{-8}$

The obtained results are in good agreement with the previous measurement, and some discrepancy may be due to the fact that the signal was not taken at the same location. As can be seen, all thermal transport properties decrease with increasing temperature, as expected. Only the thermal boundary resistance appears to be constant within the estimated errors. At this point, it is worth noting that the maximum observed in Figure 5 not only changes with the layer thickness (as in Figure 5 in [24]), but also with the change of the thermal transport properties. This maximum shifts to a lower frequency with increasing temperature (because the heat transport properties are deteriorated at higher temperatures).

Using data presented in Table 3, the measurement of the in-plane thermal conductivity of sample #4 and the R_{th} between Au layer and SL layer are calculated. The obtained results are presented in Figure 9 and Table 4.

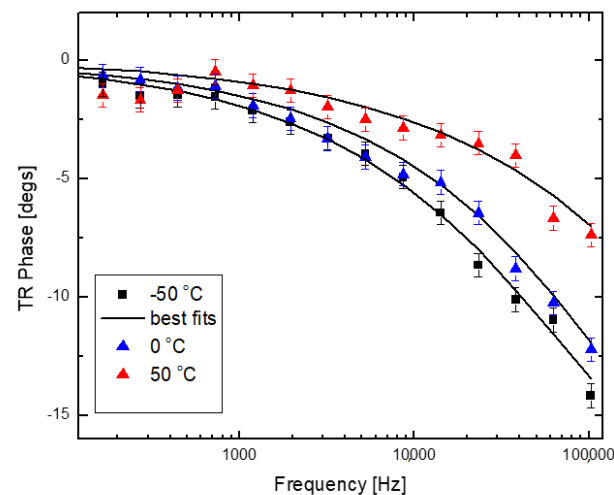


Figure 9. TR measurement of sample #4 for different temperatures. Experimental points are circles, while fits are lines. The red (50 °C), black (−50 °C) and blue (0 °C). The best fits are obtained using Equation (1).

Table 4. In-plane thermal transport parameters of AlAs/GaAs superlattice obtained from fits to the PTR data in Figure 5 for different temperatures.

Temperature (°C)	$k_{\text{GaAs/AlAs}}$ (W/mK)	$R_{\text{th},12}$ (m ² KW ^{−1})
−50	45 ± 5	$(0.9 \pm 0.2) \times 10^{-7}$
0	40 ± 5	$(1 \pm 0.9) \times 10^{-7}$
50	20 ± 5	$(1.5 \pm 0.9) \times 10^{-7}$

Note that the experimental results are consistent with the sensitivity analysis. The sensitivity to frequency about 1 kHz is 0, while the experimental phase to frequency of about 1 kHz is also close to 0 degs.

As can be seen, the in-plane thermal conductivity values are two times larger than the cross-plane thermal conductivity, as expected. The value is also consistent with those reported by authors for room temperature [36]. On the other hand, the thermal boundary resistance between gold layer and SL is within the error the same as for AlGaAs sample (sample #2). It is worth mentioning that, as with the R_{th} of Ti/SL layer, the thermal boundary resistance with changing temperature is constant with the estimation errors. It is also worth noting the experimental results for the same SL sample. The only difference is the metal covering the layer, but its influence is very small. Such a significant difference results from the difference in heat propagation for the one-dimensional (PTR) and three-dimensional (TR) systems.

Finally, Table 5 presents the summary in studying thermal transport properties of thin layers and superlattice using both methods frequency-domain photothermal radiometry (PTR) and frequency-domain thermoreflectance.

Table 5. The summary of frequency-domain photothermal radiometry (PTR) and frequency-domain thermoreflectance.

Feature	Frequency Domain PTR		Frequency Domain TR	
	Advantages	Disadvantage	Advantages	Disadvantage
Measurement of cross-plane thermal transport properties	Possible			Difficult but possible
Measurement of in-plane thermal transport properties		Difficult but possible	Possible	
Complexity of optical system	Relatively easy			Relatively difficult
Temperature-measurements at low temperatures	Difficult		Possible	
Temperature measurements at high temperatures	Possible		Possible	

As one can see from Table 5, both methods complement each other rather than contend for being the most appropriate method for thermal transport parameters measurements.

5. Discussion

If we define an equivalent volumetric heat capacity ρc_{eff} as a weighted average of the AlAs and GaAs heat capacities, the equivalent thermal diffusivity is

$$\alpha_{eff} = \frac{k_{measured}}{(\rho c)_{eff}} (\rho c)_{eff} \equiv (l_{GaAs}/l) \rho_{GaAs} c_{GaAs} + (l_{AlAs}/l) \rho_{AlAs} c_{AlAs} \quad (13)$$

where l is the superlattice period thickness.

Figure 10 presents a relation between measured and calculated values of thermal diffusivity.

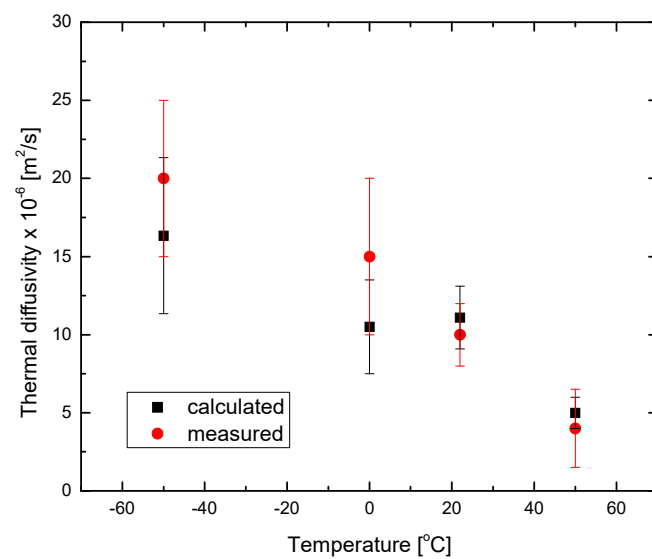


Figure 10. Relation between measured and calculated values of thermal diffusivity as function of temperature.

As can be seen in Figure 10, the measured and calculated thermal diffusivities are in good agreement. In superlattices, the in-plane thermal conductivity is two times bigger than cross-plane. Figure 11 demonstrated this relation in AlAs/GaAs superlattice sample. The thermal anisotropy of the superlattice layer confirms the expectation that the thermal conductivity is (about two times) higher in the plane than perpendicular to it which is consistent with phonon reflections at the GaAs – AlAs interfaces.

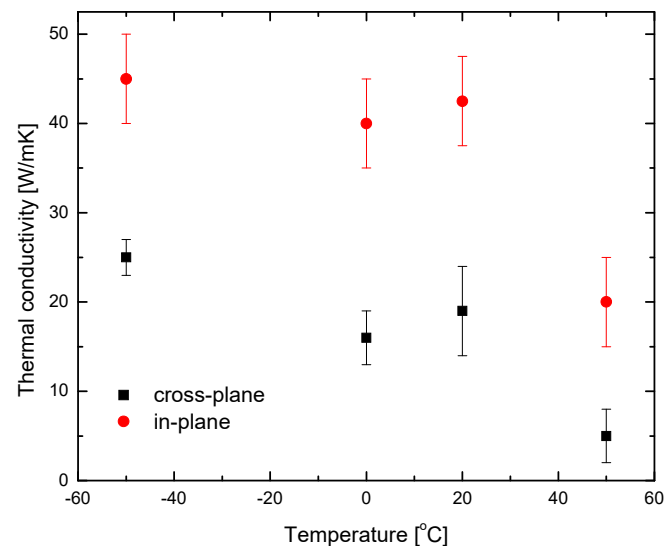


Figure 11. Relation between in-plane and cross-plane thermal conductivity measured for AlAs/GaAs superlattice sample as function of temperature.

As can be observed in the figure, the tested relationship is valid for temperatures below 50 °C. In this range, the in-plane thermal conductivity is twice that of the cross-plane. For 50 °C, a slight discrepancy between the tested relationship and the obtained values can be observed, but the results are within the limits of measurement uncertainty.

Authors should discuss the results and how they can be interpreted from the perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

6. Conclusions

In this paper, we present the combination of two methods, FD-PTR and FD-TR, which can completely characterize the thermal properties of thin films as well as a superlattices, and experimentally verified formulas used in other studies on the thermal properties of superlattices. The results obtained are consistent with those obtained using other methods as well our previous study [21]. We found that formulas for the calculation of thermal diffusivity can be used in all studied temperature ranges. The results obtained will serve other authors to gain more confidence in the assumptions they use to calculate the specific heat of nanolayers based on their bulk material properties, perhaps in a wider temperature range.

Author Contributions: Conceptualization, M.P.; methodology, M.P., D.D.; software, M.P.; validation, M.P.; formal analysis, M.P.; investigation, M.P., N.S.; resources, A.L., T.K., A.D.W.; writing—original draft preparation, M.P.; writing—review and editing, A.L., T.K., A.D.W.; visualization, M.P.; supervision, M.P., A.D.W.; project administration, M.P.; funding acquisition, M.P., A.D.W. All authors have read and agreed to the published version of the manuscript.

Funding: This work was funded by the National Science Centre Poland grant based on Decision no. 2015/19/D/ST3/02388. A.L. and A.D.W. acknowledge the support of the DFH/UFA CDFA-05-06.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Acknowledgments: Authors would like to thank Jose Miranda-Ordonez for the help in developing the model and Nathan Jukam for discussions. We thank Nadine Viteritti for sample preparations.

Conflicts of Interest: The authors declare no conflict of interest.

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