



Article DC-DC Zeta Power Converter: Ramp Compensation Control Design and Stability Analysis

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Abstract: The design of robust and reliable power converters is fundamental in the incorporation of novel power systems. In this paper, we perform a detailed theoretical analysis of a synchronous ZETA converter controlled via peak-current with ramp compensation. The controller is designed to guarantee a stable Period 1 orbit with low steady state error at different values of input and reference voltages. The stability of the desired Period 1 orbit of the converter is studied in terms of the Floquet multipliers of the solution. We show that the control strategy is stable over a wide range of parameters, and it only loses stability: (i) when extreme values of the duty cycle are required; and (ii) when input and reference voltages are comparable but small. We also show by means of bifurcation diagrams and Lyapunov exponents that the Period 1 orbit loses stability through a period doubling mechanism and transits to chaos when the duty cycle saturates. We finally present numerical experiments to show that the ramp compensation control is robust to a large set of perturbations.

Keywords: DC-DC synchronous ZETA converter; floquet multipliers; lyapunov exponents; ramp compensation control; nonlinear phenomena

1. Introduction

DC/DC Power converters are nowadays ubiquitous in technological applications. The ability to transform an input voltage to a desired output value is fundamental in electronic devices, energy harvesting systems, and power storage, just to mention a few. Converters that are able to operate in both step-up and step-down modes are of special interest in renewable power systems, where fluctuations in the environment may produce varying levels of input voltages, which, in some cases might be lower than the output requirements. Examples of these are the Ćuk, SEPIC, and ZETA converters [1]. Out of these circuits, the ZETA converter has the advantage of a non-inverted polarity in the output voltage [2]. The relatively simple design of the ZETA converter formed by two inductors, two capacitors, a controlled switch, and a diode, makes it quite appealing for a vast range of applications: DC motors [3,4], LED technologies [5,6], hybrid vehicles [7,8], and power factor correction [9–13], among others.

A special design of the ZETA converter that has gained attention in the last years is the synchronous ZETA converter, in which the diode is replaced by a secondary switch synchronized with the controlled one [14–17]. This type of topology increases the efficiency of the converter, as it eliminates the voltage drop in the diode, a desired feature especially in step-down tasks. This versatility of the ZETA converter calls for the design of reliable and robust control methods which have been developed during the last decade and include PI/PID control [18–21], adaptive control [22,23], and full state feed-back control [24,25].



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Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). The common feature in most of the controllers analyzed so far in the literature for the ZETA converter is that they are based on the state-space averaging technique (SSA) (one remarkable exception can be found in [26]).

In SSA, the nonlinear dynamics of the converter inherited from the switching behavior is replaced by an averaged linear model which is easy to analyze and allows the application of widely known linear-control theory techniques [27]. However, average linear models only avoid slow-scale phenomena [28], and the nonlinearity induced by the switch is known to produce fast-scale instabilities in power converters, such as quasi-periodicity and chaos [29], which decrease the efficiency and saturate the controllers. Therefore, to capture these non-trivial dynamics in switched systems, one needs to employ Filippov's formalism to model the behavior of the ZETA converter [30]. Within this more realistic framework, the desired operation in a power converter is a periodic orbit swirling nearby the operation point as the switch turns ON and OFF according to a control law. The problem is then to asses the stability of such solutions in terms of suitable dynamical indicators such as Floquet multipliers and Lyapunov exponents.

On the one hand, Floquet multipliers are used to describe the stability of periodic orbits and are therefore useful in the analysis of power converters [31]. With the aim of calculating the Floquet multipliers, it is necessary to compute the fundamental matrix associated to an equivalent discrete map [32]. The analytical procedure implies the use of a saltation matrix [33–35], in order to infer some implicit time derivatives related to the switching times. This tool has been used in several power converters such as boost converter [36], buck converter [32,33,37], boost-flyback [34,35], Cuk [38], and buck-boost [39], among others. On the other hand, Lyapunov exponents are used to characterize the stability of generic solutions (not only periodic), and they are especially useful to characterize chaotic behavior. Lyapunov exponents have also been used to describe the dynamics of power converters (see, e.g., [40–42]), but not for the ZETA converter. For this reason, Lyapunov exponents and Floquet multipliers are complementary indicators. While Lyapunov exponents are unable to capture the transitions from a periodic dynamics to another (for example, period doubling transitions), Floquet multipliers capture precisely these changes in the periodic solutions. Meanwhile, Floquet multipliers can only capture the stability of periodic orbits and do not provide any information of other types of dynamics such as quasi-periodic motion and chaotic behavior.

With these ideas in mind, in this paper, we propose to control and theoretically analyze the behavior of the synchronous ZETA (sZETA) converter. The control includes a proportional-integral control action (a PI controller) to avoid steady state error, handle the transient dynamics, and add robustness to the system [18–21]. Hence, the values of this controller are tuned to satisfy these two requirements. Next, a sawtooth signal of period T is subtracted from the PI output and compared with the current flowing through one of the coils of the converter. This is a strategy that has been widely used in power converters and is known as peak-current control with ramp compensation [2]. Two remarkable features of this control are: (i) its relatively easy implementation; and (ii) a fixed switching frequency. Hence, one of the main parameters in the peak-current control with ramp compensation is the slope of the ramp. Several techniques to design the ramp compensation exist, which include fixed slopes [43,44], adaptive slopes [45-47], and polynomial shaped ramps [48,49]. In this paper, we propose to tune the slope of the ramp in such a way that it guarantees the stability for varying values of input and output voltages, using Floquet multipliers. Once the ramp slope has been set, we complete the stability analysis of the controlled system using Lyapunov exponents and bifurcation diagrams.

As far as we know, such complete analysis of the sZETA converter has not been performed to date; with this aim, the paper is organized as follows. In Section 2, we introduce the model of the sZETA converter together with the proposed control. In Section 3, we describe the stability analysis tools that we used, namely calculation of Floquet multipliers and Lyapunov exponents. Next, we devote Section 4 to tuning the value of the ramp slope that guarantees stable operation in several combinations of input and output voltages and presenting a numerical study of the stability regions of the desired solutions, using the mathematical tools developed in Section 3. In this section, we also describe the way in which such solutions lose stability and we demonstrate the robustness of the proposed control to a wide range of disturbances. Finally, in Section 5, we conclude the paper with some discussion and remarks.

2. Synchronous ZETA Converter

2.1. Mathematical Model

The sZETA converter is schematized in Figure 1. In its simplest form, it is composed by an input voltage source V_{in} and five linear elements, namely two capacitors with capacitance C_1 and C_2 , two coils characterized by the inductance L_1 and L_2 , and the load resistance R. Additionally, two synchronous switches complete the design: whenever the control signal u = 1, the main switch is set to the ON position and the complementary switch is automatically set to the OFF position ($\bar{u} = 0$), and vice versa. The state of the sZETA converter is completely described by the currents flowing through the coils (i_1 and i_2) and the voltages across the capacitors v_1 and v_2 . Notice that, since we are interested in a theoretical approach to the stability of the converter, all elements are assumed to be ideal. The output voltage of the converter is given by v_2 . Furthermore, with the aim of defining a PI control in Section 2.2, we extend the state vector to include the integral of the error between a desired output V_{ref} and the output voltage, namely $e = V_{ref} - v_2$. Altogether, we can fully describe the sZETA converter via the state vector:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} := \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \\ \int edt \end{bmatrix}$$
(1)

which evolves according to the following equations:

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{f}_{ON}(\mathbf{x}) & \text{if } u = 1\\ \mathbf{f}_{OFF}(\mathbf{x}) & \text{if } u = 0 \end{cases}$$
(2)

where

$$\mathbf{f}_{ON} = \begin{bmatrix} V_{in}/L_1 \\ v_1/L_2 - v_2/L_2 + V_{in}/L_2 \\ -i_2/C_1 \\ i_2/C_2 - v_2/(RC_2) \\ V_{ref} - v_2 \end{bmatrix} \quad \mathbf{f}_{OFF} = \begin{bmatrix} -v_1/L_1 \\ -v_2/L_2 \\ i_1/C_1 \\ i_2/C_2 - v_2/(RC_2) \\ V_{ref} - v_2 \end{bmatrix}$$
(3)

are the vector fields that govern the dynamics in each one of the topologies defined by the position of the main switch. The gain of the sZETA converter depends on the value of the duty cycle d, i.e., the fraction of time during a PWM cycle that the switch remains in the ON position. It can be shown that the gain of the controller is given by [18]

$$\mathcal{G} = \frac{d}{1-d} \tag{4}$$

which means that the sZETA can work as a step-down (d < 0.5) or step-up converter (d > 0.5).



Figure 1. Schematic representation of the synchronous ZETA converter controlled via ramp compensation.

2.2. Control via Ramp Compensation

Now, we add a feedback loop to the system which allows us to obtain the desired output value V_{ref} . To do so, we implement a ramp compensation which is schematized in the feedback loop of Figure 1. The ramp compensation I_{ramp} is formed by a ramp periodic signal of period T that is subtracted at the output of a PI controller. The PI control operates over the output voltage and its aim is to reject disturbances while eliminating the steady state error and controlling the transient state. Notice that the PI controller corresponds to an external control loop of the voltage, as:

$$I_{ramp} = k_V (V_{ref} - v_2) + k_{int} x_5 - \frac{A}{T} \mod (t, T),$$
(5)

where k_V and k_{int} are the gains of the PI control and A is the ramp slope. At the beginning of the cycle, the main switch is set to the ON position (u = 1). The main switch is then set to the OFF position (u = 0) when I_{ramp} is equal to the current in the first coil i_1 (internal loop), or equivalently when:

$$i_1 = I_{ramp}.$$
 (6)

After this, the switch remains OFF until the beginning of the next cycle, completing the design of the controller. The switching condition (6) can be rewritten in the form $h_r(\mathbf{x}, t) = 0$ with

$$h_r(\mathbf{x},t) = k_V(V_{ref} - v_2) + k_{int}x_5 - \frac{A}{T} \mod (t,T) - i_1.$$
(7)

Figure 2 shows an example of the ramp controlled sZETA converter. In Figure 2A, we display the evolution of I_{ramp} and i_1 , during two cycles in the steady state. It is possible to see that, when both functions are equal, the control signal changes from u = 1 to u = 0 (Figure 2B). Figure 2C shows the evolution of the voltages v_1 and v_2 and depicts how, after a short transient, the output voltage (red line) asymptotically converges to the reference value V_{ref} (dashed black line). Throughout the paper, we make use of the values of the parameters shown in Table 1 unless otherwise stated.

A few words must be said about the values of the gains chosen. On the one hand, k_V was chosen to be relatively small to avoid oscillatory behavior during transient state. Moreover, k_{int} has a rather large value to guarantee fast convergence to the steady state and small steady error. Finally, the most critical parameter is the ramp compensation, i.e., the slope A, which is tuned in such a way to guarantee stable operation in a set of points chosen in the range of parameters that we use in this paper. To do so, we make use of the stability criteria that we describe in the following section.



Table 1. Parameter values used throughout the paper.

Figure 2. (**A**) Dynamics of the ramp compensation I_{ramp} and current i_1 showing the transitions of the switch; (**B**) Position of the switch as a function time; (**C**) Evolution of the voltages v_1 and v_2 with the reference voltage in dashed lines. Inset: Steady state evolution of the voltages across the capacitors showing a good agreement of v_2 with V_{ref} . To solve this system, a fourth-order Runge–Kutta method with variable step method was used with an event detection algorithm.

3. Stability Analysis

3.1. Floquet Multipliers

As shown in Figure 2, the ideal operation of the sZETA converter consists of a periodic orbit with period T, which commutes the position of the switch between 1 and 0 and assures that the orbit oscillates around the desired operation point. We now turn our attention to describe the stability of this periodic orbit, by making use of Floquet theory.

Let us assume a periodic solution $\mathbf{x}_p(t) = \mathbf{x}_p(t+T)$. According to Floquet theory, the stability of an infinitesimally perturbation around the reference orbit $\delta \mathbf{x}_p$ evolves from one period to the other as:

$$\delta \mathbf{x}_p(t+T) = M \delta \mathbf{x}_p(t). \tag{8}$$

Here, *M* is the monodromy matrix and is equal to the state transition matrix $\Phi(\mathbf{x}(t_0), t_0, t)$ evaluated at $t = t_0 + T$, namely:

$$M = \Phi(\mathbf{x}(t_0), t_0, t_0 + T) \tag{9}$$

The eigenvalues of the monodromy matrix μ_i are the so-called Floquet multipliers and indicate the stability of the periodic orbit as follows:

- If $|\mu_i| < 1$ for all *i*, then the orbit is stable.
- If at least one $|\mu_i| > 1$, the orbit is unstable.

The definition of the monodromy matrix in Equation (9) is valid for smooth systems. However, consider that the sZETA converter switches its dynamics from topology u = 1 to topology u = 0 whenever condition (7) is met, which requires special treatment for the calculation of the monodromy matrix. Let us call \mathbf{x}^* the state of the system where $h_r(\mathbf{x}^*, t^*) = 0$, and the corresponding switching time is t^* . It is easy to see that t^* can be expressed as $t^* = t_0 + dT$ if we set t_0 at the beginning of the ramp cycle. Without loss of generality, we can make $t_0 = 0$, therefore $\mathbf{x}^* = \mathbf{x}(dT)$. Then, for a non-smooth system that switches its dynamics from a vector field \mathbf{f}_{ON} to \mathbf{f}_{OFF} , the monodromy matrix can be constructed as follows (see [31] for a detailed explanation):

$$M = \Phi_{OFF}(\mathbf{x}(dT), dT, T) \cdot S \cdot \Phi_{ON}(\mathbf{x}(0), 0, dT),$$
(10)

where Φ_{ON} (Φ_{OFF}) are the state transition matrices associated with \mathbf{f}_{ON} (\mathbf{f}_{OFF}). Thanks to the linear nature of each individual vector field, the state transition matrices can be straightforwardly calculated as:

$$\Phi_{ON} = e^{A_{ON}Td} \tag{11}$$

$$\Phi_{OFF} = e^{A_{OFF}T(1-d)} \tag{12}$$

with

$$A_{ON} = \frac{\partial \mathbf{f}_{ON}}{\partial \mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/L_2 & -1/L_2 & 0 \\ 0 & -1/C_1 & 0 & 0 & 0 \\ 0 & 1/C_2 & 0 & -1/(RC_2) & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$
(13)
$$\begin{bmatrix} 0 & 0 & -1/L_1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$A_{OFF} = \frac{\partial \mathbf{f}_{OFF}}{\partial \mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & -1/L_2 & 0\\ 1/C_1 & 0 & 0 & 0 & 0\\ 0 & 1/C_2 & 0 & -1/(RC_2) & 0\\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$
(14)

The matrix S is the so-called saltation matrix, which accounts for the non-smoothness at the switching time t^* and has the form:

$$S = I + \frac{(\mathbf{f}_{OFF}(\mathbf{x}^*) - \mathbf{f}_{ON}(\mathbf{x}^*)) \cdot \mathbf{n}^T}{\mathbf{n}^T \cdot \mathbf{f}_{ON}(\mathbf{x}^*) + \frac{\partial h_r(\mathbf{x},t)}{\partial t}},$$
(15)

where $\mathbf{n} = \nabla h_r(\mathbf{x}, t)$ is the normal vector to the switching surface, which in our case can be written as

$$\mathbf{n} = [-1, 0, 0, k_V, k_{int}]^T,$$
(16)

and, similarly,

$$\frac{\partial h_r(\mathbf{x},t)}{\partial t} = -\frac{A}{T}.$$
(17)

With this, we can summarize the steps to establish the stability of a Period 1 orbit in the following algorithm:

1. Find the state at the beginning of the periodic orbit $\mathbf{x}(0)$ and the duty cycle *d* that fulfill the following conditions

$$\mathbf{x}(0) = \mathbf{x}(T)$$

$$h(\mathbf{x}(dT), dT) = 0$$
 (18)

- 2. With the solution d and $\mathbf{x}(0)$, calculate the state transition matrices (11) and (12) and the saltation matrix (15), to obtain the monodromy matrix (10).
- 3. Obtain the Floquet multipliers as the eigenvalues of *M*.

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3.2. Lyapunov Exponents

Lyapunov exponents provide an additional analysis tool to study the orbits in power converters [40]. Recall that Floquet multipliers are restricted to the study of periodic orbits. However, it can be the case that a power converter shows more complex phenomena that go beyond a Period 1 solution, such as quasi-periodicity and chaos. To quantify this, one has to make use of Lyapunov exponents. For a system whose evolution is governed by $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, given an infinitesimally small perturbation $\delta \mathbf{x}(0)$ around a reference orbit (notice that here we do not restrict the perturbation to a given periodic solution as in Section 3.1), the maximal Lyapunov exponent λ_{max} is defined as:

$$\lambda_{max} = \lim_{t \to \infty} \frac{1}{t} \log \frac{||\delta \mathbf{x}(t)||}{||\delta \mathbf{x}(0)||}.$$
(19)

The maximal Lyapunov exponent quantifies the rate of convergence ($\lambda_{max} < 0$) or divergence ($\lambda_{max} > 0$) of the reference orbit and the infinitesimally perturbed one. Hence, λ_{max} describes the nature of a given orbit as follows:

- If $\lambda_{max} < 0$, the attractor is a stable fixed point.
- If $\lambda_{max} = 0$, the attractor is a stable periodic (or quasi-periodic) orbit.
- If $\lambda_{max} > 0$, the attractor is chaotic.

In Equation (19), $\delta \mathbf{x}(t)$ is the evolution in time of the infinitesimal perturbation (also called tangent space) and its dynamics is governed by the variational equation

$$\delta \dot{\mathbf{x}} = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \delta \mathbf{x}$$
(20)

As in Section 3.1, the tangent evolution needs to be specially treated at the times where the system changes from a vector field to another. To solve this, we make use of the Mueller algorithm to solve the tangent evolution for systems with discontinuities [50]. We first describe the general procedure.

Let \mathbf{x}^- be the state vector right before a discontinuous event, which occurs when an indicator function $h(\mathbf{x}) = 0$. In addition, let \mathbf{x}^+ be the state right after the discontinuous event which may undergo a discontinuous change given by a so-called impact function $\mathbf{g}(\mathbf{x}^-)$ that maps the state of the system right before/right after the discontinuous event, namely $\mathbf{x}^+ = \mathbf{g}(\mathbf{x}^-)$. Similarly, \mathbf{f}^- and \mathbf{f}^+ are the vector fields that govern the dynamics before and after the discontinuity, respectively. With these considerations, the correction of the tangent space at the discontinuity is:

$$\delta \mathbf{x}^{+} = G(\mathbf{x}^{-})\delta \mathbf{x}^{-} + [G(\mathbf{x}^{-})\mathbf{f}^{-}(\mathbf{x}^{-}) - \mathbf{f}^{+}(\mathbf{x}^{+})]\delta t.$$
(21)

In Equation (21), δt is defined as:

$$\delta t = \frac{-\mathbf{H}(\mathbf{x}^{-})\delta\mathbf{x}^{-}}{\mathbf{H}(\mathbf{x}^{-})\mathbf{f}^{-}(\mathbf{x}^{-})}.$$
(22)

and

$$G(\mathbf{x}) = \frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}}; \qquad \mathbf{H}(\mathbf{x}) = \frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}.$$
 (23)

In order to apply the Mueller algorithm to the sZETA converter controlled via ramp compensation, it is convenient to express the dynamics of the system in an autonomous way including the evolution of the ramp. To do this, we extend the original system (3) to account for the ramp dynamics, making $\tilde{\mathbf{x}} := [i_1, i_2, v_1, v_2, \int e dt, x_6]^T$ where $x_6 = -\frac{A}{T}t$, hence:

$$\tilde{\mathbf{f}}_{ON} = \begin{bmatrix} V_{in}/L_1 \\ v_1/L_2 - v_2/L_2 + V_{in}/L_2 \\ -i_2/C_1 \\ i_2/C_2 - v_2/(RC_2) \\ V_{ref} - v_2 \\ -A/T \end{bmatrix} \quad \tilde{\mathbf{f}}_{OFF} = \begin{bmatrix} -v_1/L_1 \\ -v_2/L_2 \\ i_1/C_1 \\ i_2/C_2 - v_2/(RC_2) \\ V_{ref} - v_2 \\ -A/T \end{bmatrix}$$
(24)

In this framework, two discontinuous events occur and therefore two different corrections to the tangent evolution must be done:

• The first one corresponds to the transition of the switch from ON to OFF position, which is given by the indicator function

$$\tilde{h}_r(\tilde{\mathbf{x}}) = k_V (V_{ref} - v_2) + k_{int} x_5 + x_6 - i_1$$
(25)

Here, the fields $\mathbf{f}^- = \mathbf{\tilde{f}}_{ON}$, $\mathbf{f}^+ = \mathbf{\tilde{f}}_{OFF}$, and the states do not change their values abruptly, hence the impact function $\mathbf{\tilde{g}}_r(\mathbf{\tilde{x}})$ is trivially

$$\tilde{\mathbf{g}}_r(\tilde{\mathbf{x}}) = [i_1, i_2, v_1, v_2, x_5, x_6].$$
 (26)

With these ingredients, the different terms to be applied in Equations (21) and (22) are:

$$\mathbf{H}_{r} = [-k_{I}, 0, 0, -k_{V}, k_{int}, 1]$$
(27)

and

$$G_r = I \tag{28}$$

• The second discontinuous event occurs when the ramp resets to 0 at the end of every cycle, or equivalently when $x_6 = -A$ which corresponds to the indicator function

$$\tilde{h}_c(\tilde{\mathbf{x}}) = x_6 + A \tag{29}$$

Special care must be taken at this point. If the duty cycle is not saturated, then at each *T*-cycle $\mathbf{f}^- = \tilde{\mathbf{f}}_{OFF}$ and $\mathbf{f}^+ = \tilde{\mathbf{f}}_{ON}$. However, it might happen that the cycle is saturated and the switch remained in the ON position during the whole period, in which case $\mathbf{f}^- = \mathbf{f}^+ = \tilde{\mathbf{f}}_{ON}$. In both cases, x_6 changes its value abruptly to a constant value $x_6 = 0$, hence the impact function is:

$$\tilde{\mathbf{g}}_{c}(\tilde{\mathbf{x}}) = [i_{1}, i_{2}, v_{1}, v_{2}, x_{5}, 0],$$
(30)

which leads to

$$\mathbf{H}_{c} = [0, 0, 0, 0, 0, 1] \tag{31}$$

and

$$G_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
(32)

With these corrections, the λ_{max} calculation can be summarized in the following steps:

1. Evolve the extended system:

$$\begin{bmatrix} \mathbf{\dot{\tilde{x}}} \\ \delta \mathbf{\dot{\tilde{x}}} \end{bmatrix} = \begin{cases} \begin{bmatrix} \mathbf{\tilde{f}}_{ON}(\mathbf{\tilde{x}}) \\ \frac{\partial \tilde{f}_{ON}(\mathbf{\tilde{x}})}{\partial \mathbf{\tilde{x}}} \delta \mathbf{\tilde{x}} \end{bmatrix} & \text{if} \quad u = 1 \\ \\ \begin{bmatrix} \mathbf{\tilde{f}}_{OFF}(\mathbf{\tilde{x}}) \\ \frac{\partial \tilde{f}_{OFF}(\mathbf{\tilde{x}})}{\partial \mathbf{\tilde{x}}} \end{bmatrix} & \text{if} \quad u = 0 \end{cases}$$
(33)

in the interval $[t_0, t_0 + \tau]$ with randomly chosen initial conditions for $\delta \tilde{\mathbf{x}}(t_0)$, such that $||\delta \tilde{\mathbf{x}}(t_0)|| = 1$.

- 2. During the evolution, identify the times of each of the possible discontinuous events given by (25) or (29) and apply the correction (21) in accordance with the nature of the discontinuous event.
- 3. When $t = t_0 + \tau$, calculate and store

$$\lambda_i = \frac{1}{\tau} \log \frac{||\delta \mathbf{\tilde{x}}(t)||}{||\delta \mathbf{\tilde{x}}(t_0)||}$$

- 4. Set $t_0 = t_0 + \tau$ and renormalize $||\delta \tilde{\mathbf{x}}(t_0)|| = 1$.
- 5. Repeat Steps 1–4 for a sufficiently large number of steps *N*.
- 6. Calculate the maximal Lyapunov exponent as:

$$\lambda_{max} = \frac{1}{N} \sum_{i=1}^{N} \lambda_i,$$

4. Results

4.1. Ramp Compensation Design

We first sought to choose a suitable value of the ramp compensation slope A, as it is one of the most critical parameters in the control design. To do so, we calculated the stability of the Period 1 orbit via Floquet multipliers in the biparametric space (A, V_{in}) with $R = 100 \Omega$. This situation is depicted in Figure 3 for different values of V_{ref} . An interesting observation is that, for small values of V_{ref} , the system tends to lose stability when two conditions are met: (i) $V_{in} \approx V_{ref}$; and (ii) A is rather large, as seen in the dark red area in Figure 3A, which corresponds to $|\mu_{max}| > 1$. Interestingly, at increasing values of V_{ref} , the controller does not lose stability anymore at large A, and the conditions for stability loss are: (i) $V_{in} < V_{ref}$ (equivalently when the sZETA converter works as a boost); and (ii) Ais small, which produces the red triangular areas in Figure 3B–D. Since one of the main attributes of the sZETA converter is the ability to work both as a step-down and step-up converter, we chose a value of A which guarantees stability for most of the cases depicted in Figure 3. This value corresponds to A = 10, which is shown with the magenta line in all panels. In the next section, a deep stability analysis considering continuous variation of not only V_{in} and V_{ref} but also R is presented.





Figure 3. Norm of the maximal Floquet multiplier as a function of the input voltage V_{in} and the ramp slope A at a fixed value of V_{ref} , indicated on the title of each panel. (A) $V_{ref} = 5$ V; (B) $V_{ref} = 15$ V; (C) $V_{ref} = 25$ V; (D) $V_{ref} = 35$ V. In all panels, the magenta dashed line denotes the selected value of A = 10 that guarantees stable operation for most of the parameters selected.

4.2. Stability Regions

Now that we have chosen a suitable value of A, we aim to characterize the stability regions in the ramp compensation controlled sZETA converter at variations of different parameters and to characterize the mechanisms by which the stability is lost. To do this, we calculate the stability of the Period 1 solution taking R, V_{in} , and V_{ref} as bifurcation parameters (recall that the nominal parameters are $R = 100 \Omega$, $V_{ref} = 15$ V, and $V_{in} = 10$ V). The results are depicted in Figure 4. It is possible to see in Figure 4A that the load has little effect on the stability of the controller, as testified by $|\mu|_{max} < 1$ in the whole range of resistances $R \in [1 \Omega, 1000 \Omega]$ that we considered. It is also not surprising that the value of the duty cycle remains unchanged over the whole range and very close to the expected gain \mathcal{G} defined in Equation (4), as the gain only depends on the values V_{in} and V_{ref} . Next, we focus on the evolution of $|\mu|_{max}$ as a function of V_{ref} in Figure 4B. Here, we can observe that the orbit is stable in an ample range of V_{ref} , and it only loses stability at either very small references (V_{ref} < 1 V) or very large ones (V_{ref} > 42.9 V). This of course corresponds to very low or very high values of the duty cycle where power converters do not perform adequately. As above, the duty cycle solution obtained by solving (4) is in good agreement with the expected gain. Finally, we found a similar behavior choosing V_{in} as the bifurcation parameter (see Figure 4C). Namely, the system loses stability only at very large values of V_{in} corresponding to extreme values of the duty cycle. This suggests that the peak-current with ramp compensation is a quite stable strategy, except for cases in which the gain is either very small of very large.



Figure 4. Norm of the maximal Floquet multiplier as a function of different bifurcation parameters: (**A**) R; (**B**) V_{ref} ; (**C**) V_{in} . In all panels, the inset shows the solution of the duty cycle which solves the self-consistent Equation (18) (symbols) and the corresponding steady state expected solution (4) (red).

To test this hypothesis, we proceeded to calculate $|\mu|_{max}$ in the two-dimensional parameter space (V_{ref} , V_{in}) and the value of the duty cycle consistent with the Period 1 solution in Equation (18). The results are shown in Figure 5, where it can be seen that, indeed, the regions where the Period 1 orbit loses stability are those when the ratio V_{in}/V_{ref} is either too large or too small (bright red regions in Figure 5A). These are indeed parameter regions where the duty cycle has very large or very low values (see Figure 5B), hence supporting the initial hypothesis. A rather interesting exception appears for cases in which $V_{in} \approx V_{ref} < 5$ V. This is not surprising, because, as shown in Figure 3A, for the chosen value of the ramp slope A, there is a region of small and comparable values of V_{ref} and V_{in} where the controller is unstable. This result indicates that the proposed controller handles much better medium to high values of voltages, at least for the parameters chosen here.



Figure 5. (**A**) Heat map of the maximal Floquet multiplier in the parameter space (V_{ref}, V_{in}) ; (**B**) Heat map of the duty cycle solution (18) as in (**A**).

To better understand the mechanism of stability loss, we proceeded to calculate the bifurcation diagram of the duty cycle in the interval $V_{ref} \in [42 \text{ V}, 50 \text{ V}]$ at fixed $V_{in} = 10$ V. As depicted in Figure 4B, we expect a stability change at $V_{ref} \approx 42.9 \text{ V}$. The bifurcation diagram of the duty cycle shows the transition to a Period 2 orbit at that same value, as shown with the red vertical line in Figure 6A. Such orbit remains stable, as testified by $\lambda_{max} = 0$ (see Figure 6B), up to the value $V_{ref} \approx 43.54$ V, where the duty cycle starts saturating and producing seemingly chaotic values of *d*. The chaotic nature of the solutions after this critical value of V_{ref} is indeed revealed by $\lambda_{max} > 0$ after this point. From the definition of λ_{max} , it is possible to deduce that two nearby trajectories will be completely uncorrelated after a few milliseconds. The transition to chaos due to saturated *d* has been described previously as the route to chaos via skipped cycles in the literature [51]. Despite the fact that the chaotic regime is definitely undesired in any real application, it should be noticed that, nonetheless, the values of the error remain rather small (<2%) in the whole studied interval (see inset).



Figure 6. (A) Bifurcation diagram of the duty cycle; (B) maximal Lyapunov exponent near the disappearance of the Period 1 orbit in the interval $V_{ref} \in [42 \text{ V}, 50 \text{ V}]$. Inset: Bifurcation diagram of the error. For each point in this diagram, a transient evolution of 2000 cycles is allowed before calculating the tangent evolution during 3000 cycles. Renormalization of the tangent vector is performed every 10 cycles.

4.3. Disturbance Rejection

We now proceed to evaluate the performance of the ramp controlled sZETA converter under different types of perturbations. First, we focus on disturbances at the load *R*. This simulation is depicted in Figure 7. In this figure, *R* is set to the nominal value of 100 Ω , where one can see that the converter rapidly converges to the reference voltage in around 8 ms, starting from $\mathbf{x}(0) = \mathbf{0}$ (see Figure 7A), while the error rapidly decreases below 1% (Figure 7B). At t = 15 ms, the load is abruptly changed to $R = 200 \Omega$, where it again rapidly responds to the perturbation by transiently changing the duty cycle, until it relaxes again to the expected value $d = (V_{ref}/V_{in})/(1 + V_{ref}/V_{in})$, which for the chosen values of the reference and input voltage corresponds to d = 0.6 (see Figure 7C). Finally, another perturbation is presented to the system at t = 30 ms, where *R* is again changed to 50 Ω . It can be seen here that the controller handles the perturbation in a satisfactory manner. In fact, the last two disturbances are practically negligible. This means that, once the system reaches the steady state, disturbances in the load are unimportant for the system's performance.



Figure 7. Time trace of the ramp controlled sZETA converter under perturbations of the load: (**A**) voltage trace; (**B**) error percentage; and (**C**) duty cycle. In all panels, $R = 100 \Omega$ for t < 15 ms, $R = 200 \Omega$ for 15 ms $\leq t < 30$ ms, and $R = 50 \Omega$ for $t \geq 30$ ms.

Similarly, we performed a perturbation analysis in a more interesting set-up, i.e., by introducing disturbances in V_{ref} in such a way that the converter switches its behavior from step-up to step-down regime. This situation is depicted in Figure 8. From the time trace of the output voltage, we can see that the system reaches the three desired values of the reference despite the rather large deviations from one value to the other. Notice that, in the second transition, V_{ref} changes from $V_{ref,2} = 5$ V to $V_{ref,3} = 20$ V, which corresponds to a percent change of 300%. Even in this extreme case, the system reaches steady operation in approximately 5 ms, as seen by the small error values in Figure 8B a few milliseconds after the presentation of the disturbance. As expected, the transitions to different reference voltages are accompanied by variations in the steady state value of the duty cycle, which meet the expected value of the gain \mathcal{G} (Figure 8C).



Figure 8. Time trace of the ramp controlled sZETA converter under perturbations of the reference voltage: (**A**) voltage trace; (**B**) error percentage; (**C**) duty cycle. In all panels, $V_{ref,1} = 12$ V for t < 15 ms, $V_{ref,2} = 5$ V for 15 ms $\leq t < 30$ ms, and $V_{ref,3} = 20$ V for $t \geq 30$ ms

We performed the same perturbation analysis with V_{in} as a perturbation parameter. Following the same procedure as for V_{ref} , V_{in} was varied in such a way that the sZETA converter alternates between step-up and step-down behavior. In particular, it transits from step-up to step-down at t = 15 ms and returns to a step-up regime at t = 30 ms, as shown in Figure 9. As reported with the disturbances in V_{ref} , the converter rapidly reaches steady evolution in all the considered cases, despite the rather large variations of V_{in} . It is worth noticing that variations in V_{in} produce transient oscillations of smaller amplitude when compared with variations of V_{ref} , which produces lower deviations of the error (see Figure 9B) that is kept below 10%. This is also reflected in a much faster evolution of the duty cycle towards the asymptotic value, as seen in Figure 9C. This phenomenon can be explained because changing the input voltage requires only the adjustment of the already desired value. On the contrary, when V_{ref} changes, it is necessary for all the states, including the voltages of the capacitors, to adjust their values.



Figure 9. Time trace of the ramp controlled sZETA converter under perturbations of the input: (**A**) voltage trace; (**B**) error percentage; (**C**) duty cycle. In all panels, $V_{in} = 10$ V for t < 15 ms, $V_{in} = 18$ V for 15 ms $\leq t < 30$ ms, and $V_{in} = 12$ V for $t \geq 30$ ms.

Finally, with the aim of further showing the robustness of the controller, we carried out a mixed perturbation test in which all three parameters were varied simultaneously. This scenario is shown in Figure 10, and the results support the initial findings, namely that the controller is robust to even large variations from the nominal values of the parameters, including transitions from step-up (for t < 30 ms) to step-down regimes (t > 30 ms). In this figure, it is possible to notice an additional feature of this converter that is not clearly observable in Figure 7, namely that the last transition, which corresponds to the largest value of R, has a much faster transient response. Indeed, the transient time elapsed for the reference voltage $V_{ref,2} = 18$ V to $V_{ref,3} = 10$ V is significantly smaller than the initial transient taken to reach $V_{ref,1} = 10$ V (≈ 5 ms vs. ≈ 10 ms, respectively). This is despite the fact that the initial errors are comparable (see Figure 10B,C). This result highlights the importance of understanding the particular application when tuning the parameters of the controller, especially if fast transient response is a requirement.



Figure 10. Time trace of the ramp controlled sZETA converter under mixed perturbations of the input: (**A**) voltage trace; (**B**); error percentage; (**C**) duty cycle. In all panels, the following transitions were defined: for t < 15 ms, $(R, V_{in}, V_{ref,1}) = (100 \Omega, 10 V, 15 V)$; for 15 ms $\leq t < 30$ ms, $(R, V_{in}, V_{ref,2}) = (150 \Omega, 8 V, 18 V)$; and, for $t \geq 30$ ms, $(R, V_{in}, V_{ref,3}) = (200 \Omega, 15 V, 10 V)$.

5. Discussion and Conclusions

In this study, we theoretically analyzed the stability of the sZETA converter controlled via ramp compensation using non-smooth systems tools. We found that the proposed control technique is stable over a large parameter range including operation in step-up and step-down regimes. We demonstrated that the system loses the stability of the Period 1 orbit at extremely low values of the duty cycle, which are in general unrealistic in most applications. Nevertheless, we found that the sZETA converter also loses stability for high—but still realistic—values of the gain (i.e., d > 0.8) where a period doubling occurs and eventually the system transits to chaos by means of a skipped-cycle mechanism. In this sense, it is known that the sZETA converter is unsuited for high gains applications due to decreased efficiency of the converter [52]. To deal with this, some improvements have been proposed for the sZETA converter to deal with extreme conditions [52,53] using switched capacitors, but this requires the addition of several non-linear elements to the circuit, and the cheapest solution is still to choose a more suited topology for the application in hand (for a complete review on new step-up topologies, the reader is referred to [54]).

It should be noticed that we used two different (although very much related) tools in the analysis of our system, namely Floquet multipliers and Lyapunov exponents. The former characterizes the stability of a particular target periodic orbit, which is often the requirement in the design of power converters, and the latter describes the nature of the attractor in which the system evolves (fixed point, periodic, quasi-periodic, or chaotic). Unfortunately, it is uncommon in the literature to see the evolution of these two indicators, and some authors prefer one tool over the other. We showed that, using both indicators, one can have a much deeper understanding of the system and the way in which a certain dynamics loses stability. Both Floquet multipliers and Lyapunov exponents, however, are based on linear stability, therefore they only characterize the attractors in a neighborhood. This means that we can only guarantee local stability of the solutions, and more refined formalisms are required to capture the global convergence, especially in switched systems [55,56]. Despite this drawback, the disturbance rejection analysis showed that indeed the stable solutions attract a large perturbation. This feature is particularly important in applications where disturbances are important factors to consider, for instance in renewable energies applications, where the unsteady nature of the renewable sources may produce variations in the input power. As a matter of fact, the ZETA converter has been proven to be suitable on small scale (<350 W) and medium scale (<10,000 W) applications [57], where the converter supports a high efficiency over 90% [58]. Such is the case of the single-switch ZETA converter [59], using basic cells to step up the input voltage and get high gain and efficiency.

To the authors' knowledge, this is the first time that a complete dynamical description using non-linear systems tools was performed for the ZETA converter, with the only exception being the work of Ellappan and Anbukumar [60], which was made in a different configuration of the converter. Despite this, we propose as a future direction of research applying our non-smooth methodology to the analysis of other previously reported controllers for the ZETA converter which used the SSA approximation, with the aim of revealing possible nonlinearities that are not explained by the averaging technique. This also opens a wide range of possibilities that allow us to apply non-smooth systems tools and analyses other types of controllers that have been shown to perform adequately in power converters [34,35,61], including MPPT controller for photovoltaic applications [62], as well as the design of slope compensation to avoid overcompensation on light loads [43]. Finally, given that the proposed control was shown to be stable in a wide range of parameter variation, we propose to experimentally validate the results shown here in a future work.

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