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Adaptive Sensor Array Error Calibration Based Impact Localization on Composite Structure

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Received: 4 May 2020; Accepted: 10 June 2020; Published: 11 June 2020

Abstract: Gains and phases delay induced by sensor position error would significantly degrade the performance of high-resolution two-dimensional multiple signal classification (2D-MUSIC) algorithm, which resulting in low positioning estimation accuracy and poor imaging. In this study, adaptive piezoelectric sensor array calibration based method is proposed for impact localization on composite structure. First, observed signal vector from the sensor array is represented by error calibration matrix with unknown gains and phases, and then it used to construct the cost function including sensor array parameters. Second, a 2D-MUSIC algorithm based on linear attenuation calibration is applied for estimating the initial estimate of impact location. Finally, substituting the initial estimate, the cost function is minimized by adaptive iterative to calculate the sensor array error parameters and the exact location of the impact source. Both finite element method (FEM) simulation and experimental results on carbon-fiber composite panel demonstrate the validity and effectiveness of the proposed method.

Keywords: array error calibration; 2D-MUSIC algorithm; composite structure; impact location

1. Introduction

Structural health monitoring is an emerging technology with signal processing technology to monitor the status of composite structures in real time or whenever necessary [1–3]. Defect would be induced during service, such as from impacts by a foreign object. Despite the advantages of composite structure, they are susceptible to barely visible impact damage [4,5]. Therefore, impact localization is an important part of structural health monitoring system to ensure the safety of composite structure.

Array signal processing for source localization is one of the major interests in many applications. Over several decades, this topic was extensively studied and many algorithms have been proposed, such as spatial filter based method [6] and phased array based method [7]. Among these existing localization methods, the multiple signal classification (MUSIC) algorithm is easy implementation with arbitrary array configurations [8]. Yang et al. [9] proposed a MUSIC method based on array signal processing for impact source localization in a plate. He et al. [10] developed an imaging method based on the time-reversal operator and MUSIC algorithm for damage imaging in a metallic plate. Yuan et al. [11] presented a near-field impact monitoring method based on the two-dimensional multiple signal classification method for the composite structure. Zhong et al. [12,13] proposed a plum-blossom sensor array-based 2D-MUSIC method to realize omnidirectional impact localization of composite structures. Zuo et al. [14] presented a model-based 2D-MUSIC damage identification algorithm for plate-like structures. However, those MUSIC based methods in the literature mentioned above require precise knowledge of the signals received from a standard source located at any direction.

The performance of those methods depend strongly on the accuracy of the observed array signal vector from sensor array. In fact, array signal vectors have various error forms including array element position error, gain-phase error caused by structural propagation path deviation, etc. Therefore, imprecise sensor array signal vectors would significantly degrade the performance of the MUSIC algorithm, and there is still the problem of ensuring array steering vector calibration for practical engineering applications. To reduce the localization error caused by sensor phase errors, Bao et al. [15] presented an anisotropy compensated MUSIC algorithm. This method experimentally verifies that the correction of the sensor phase error makes the MUSIC algorithm more accurate and reliable for the damage localization on composite structures. However, the process of measuring the sensor phase error by experiments may be time consuming and expensive. A practical approach to alleviating the problems introduced by imprecise array calibration is to use the received signals to adjust or fine-tune the array calibration. In radar fields, self-calibrating antenna arrays have been developed and tested by Steinberg [16] and others. However, self-calibration sensor array for composite structural health monitoring seen to have received little attention.

In this study, adaptive sensor array error calibration based method is proposed for impact localization on composite structure. First, observed signal vector from the sensor array is represented by error calibration matrix with unknown gains and phases, and then it used to construct the cost function including sensor array parameters. Second, 2D-MUSIC algorithm based on linear attenuation calibration in ref. [11] is applied for estimating the initial estimate of impact location. Finally, substituting the initial estimate, the cost function is minimized by adaptive iterative to calculate the sensor array error parameters and the exact location of the impact source.

This study is organized as follows. Section 2 introduces the signal modal of imprecise sensor array and Section 3 simulates a FEA model which was created in ABAQUS. In Section 4, impact localization experiments are conducted on carbon-fiber composite panel to verify the proposed method.

2. Signal Modal of Imprecise Sensor Array

Assuming that a uniform linear sensor array consisting of $(2 \times M + 1)$ piezoelectric (PZT) sensors is arranged on the structure. The distance between two sensors is d. Considering the impact induced elastic waves $\mathbf{s}(t)$ with a certain frequency component of ω_0 arriving at the sensor array, and θ denotes the wave propagating direction caused by impact with respect to the coordinate y axis. The observed data output from qth PZT under the near-field situation $\mathbf{x}(t)$ can be presented as

$$\mathbf{x}_q(t) = \frac{r}{r_q} s(t) e^{-j\omega_0 \tau_q} + \mathbf{n}_q(t),\ q = -M, \cdots, 0, \cdots, M \tag{1}$$

where r and r_q denotes the distance between impact source and PZT 0 and PZT i, respectively, $\mathbf{n}_q(t)$ is the output corresponding to the background noise, τ_q is the arriving time difference among different sensor elements in the sensor array, which can be expressed as

$$\tau_q = \frac{(-d \sin\theta)}{c}(q-1) + (-\frac{d^2}{cr}\cos^2\theta)(q-1)^2 + O(\frac{d^2}{r^2}) \tag{2}$$

where c is the elastic wave average velocity. Denotes the array steering vector of precisely sensor array as $\mathbf{A}(r,\theta)$, and can be presented as

$$\mathbf{A}(r,\theta) = [a_{-M}(r,\theta), \cdots, a_q(r,\theta), \cdots a_M(r,\theta)] \tag{3}$$

where

$$a_q(r,\theta) = \frac{r}{r_q}\exp(-j\omega_0\tau_q)$$

In fact, array signal vectors have various error forms including array element position error, gain-phase error caused by structural propagation path deviation, etc. For an imprecise PZT sensor array seen in Figure 1, the observed data output from qth PZT should be rewritten as

$$\mathbf{x}(t) = \alpha_q \mathbf{s}(t - \psi_q) + \mathbf{n}_q(t) \tag{4}$$

where α_q and ψ_q is the qth sensor-related gain and delay due to element position error of the imprecise array.

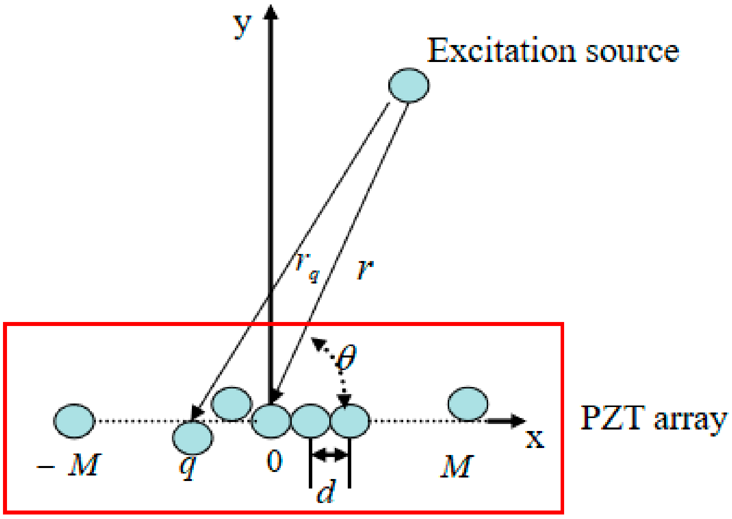


Figure 1. Array element position error schematic diagram.

Defined Γ_q as the gain and delay the qth sensor, and

$$\Gamma_q = \alpha_q \exp(-jw_0\psi_q), q = -M, \cdots, M \tag{5}$$

Moreover, array sensor position error matrix could be obtained as

$$\mathbf{\Gamma}(r, \theta) = diag\{\Gamma_{-M}, \cdots, \Gamma_M\} \tag{6}$$

By introducing the element position error matrix, the array steering vector for the near-field impact signal could be represented as

$$\tilde{\mathbf{A}}(r, \theta) = \mathbf{\Gamma}(r, \theta)\mathbf{A}(r, \theta) \tag{7}$$

For the whole sensor array, the observed input signal vector can be represented as

$$\mathbf{x}(t) = \tilde{\mathbf{A}}(r, \theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{8}$$

where

$$\mathbf{x(t)} = [\mathbf{x_{-M}(t)}, \cdots, \mathbf{x_0(t)}, \cdots, \mathbf{x_M(t)}]^{\mathbf{T}}$$
$$\tilde{\mathbf{A}}(r, \theta) = [\widetilde{a}_{-M}(r, \theta), \cdots, \widetilde{a}_0(r, \theta), \cdots, \widetilde{a}_M(r, \theta)]^{\mathrm{T}}$$
$$\mathbf{n}(t) = [\mathbf{n}_{-M}(t), \cdots, \mathbf{n}_0(t), \cdots, \mathbf{n}_M(t)]^{\mathrm{T}}$$

Adaptive Piezoelectric Sensor Array Error Calibration Based 2D-MUSIC

The basic idea of the MUSIC algorithm is obtaining the signal subspace and noise subspace through eigenvalue decomposition of the signal covariance matrix, and then estimating the signal parameter using the orthogonality of two spaces.

The covariance matrix of the observed signal vector from the imprecise sensor array is

$$\mathbf{R}_x = E[\mathbf{x}\mathbf{x}^H] = \mathbf{\Gamma A R_s A^H \Gamma}^H + \delta^2\mathbf{I} \quad (9)$$

where,$\mathbf{x}$ the narrow band signals extracted from the sensor outputs using the Gabor wavelet transform [17]. $\mathbf{R}_s$ is the covariance matrix of signal and δ^2 is noise power.

The eigenvalue decomposition of $\mathbf{R}_x$ is:

$$\mathbf{R}_x = \mathbf{U}_\mathrm{S}\sum\nolimits_\mathrm{S}\mathbf{U}_\mathrm{S}{}^\mathrm{H} + \mathbf{U}_\mathrm{N}\sum\nolimits_\mathrm{N}\mathbf{U}_\mathrm{N}{}^\mathrm{H} \quad (10)$$

Here, $\boldsymbol{U}_\mathrm{S}$ denotes the signal subspace corresponding to the largest eigenvalue $\sum_\mathrm{S}$.$\boldsymbol{U}_\mathrm{N}$ denotes the noise subspace corresponding to those small eigenvalues $\sum_\mathrm{N}$.

To describe the orthogonal properties, the spatial spectrum of imprecise sensor array can be calculated by

$$P_{2D-MUSIC} = \mathbf{A}(r,\theta)^H\mathbf{\Gamma}^H U_N U_N^H \mathbf{\Gamma A}(r,\theta) = \|U_N^H \mathbf{\Gamma}\tilde{\mathbf{A}}(r,\theta)\|^2 \quad (11)$$

Construct a cost function J as

$$J = \boldsymbol{\gamma}^H \|\tilde{\mathbf{A}}(r,\theta)^H U_N U_N^H \tilde{\mathbf{A}}(r,\theta)\| \boldsymbol{\gamma} \quad (12)$$

where $\boldsymbol{\gamma} = [\Gamma_{-M}, \cdots, \Gamma_M]^T$. Then, reasonable estimates of (r,θ) and $\mathbf{\Gamma}(r,\theta)$ could be obtained by minimizing the cost function, that is

$$\{\mathbf{\Gamma}(r,\theta), r, \theta\} = \mathrm{argmin}(\boldsymbol{\gamma}^H \|\tilde{\mathbf{A}}(r,\theta)^H U_N U_N^H \tilde{\mathbf{A}}(r,\theta)\| \boldsymbol{\gamma}) \quad (13)$$

Minimize (13) with respect to $\boldsymbol{\gamma}$ under the constraint $\boldsymbol{\gamma}^H\boldsymbol{\omega} = 1$ where $\boldsymbol{\omega} = [1, 0, \cdots, 0]^T$. The result of this minimization problem is well known and is given by

$$\boldsymbol{\gamma}^{(k+1)} = WW_k{}^{-1}\boldsymbol{\omega} / (\boldsymbol{\omega}^T WW_k{}^{-1}\boldsymbol{\omega}) \quad (14)$$

where $WW_k = diag\left\{\widetilde{a}(r_k,\theta_k)^H \mathbf{U}_N \mathbf{U}_N^H \widetilde{a}(r_k,\theta_k)\right\}$. Given a preset threshold, the algorithm performs the iterations until J converges, and the process seen as Figure 2.

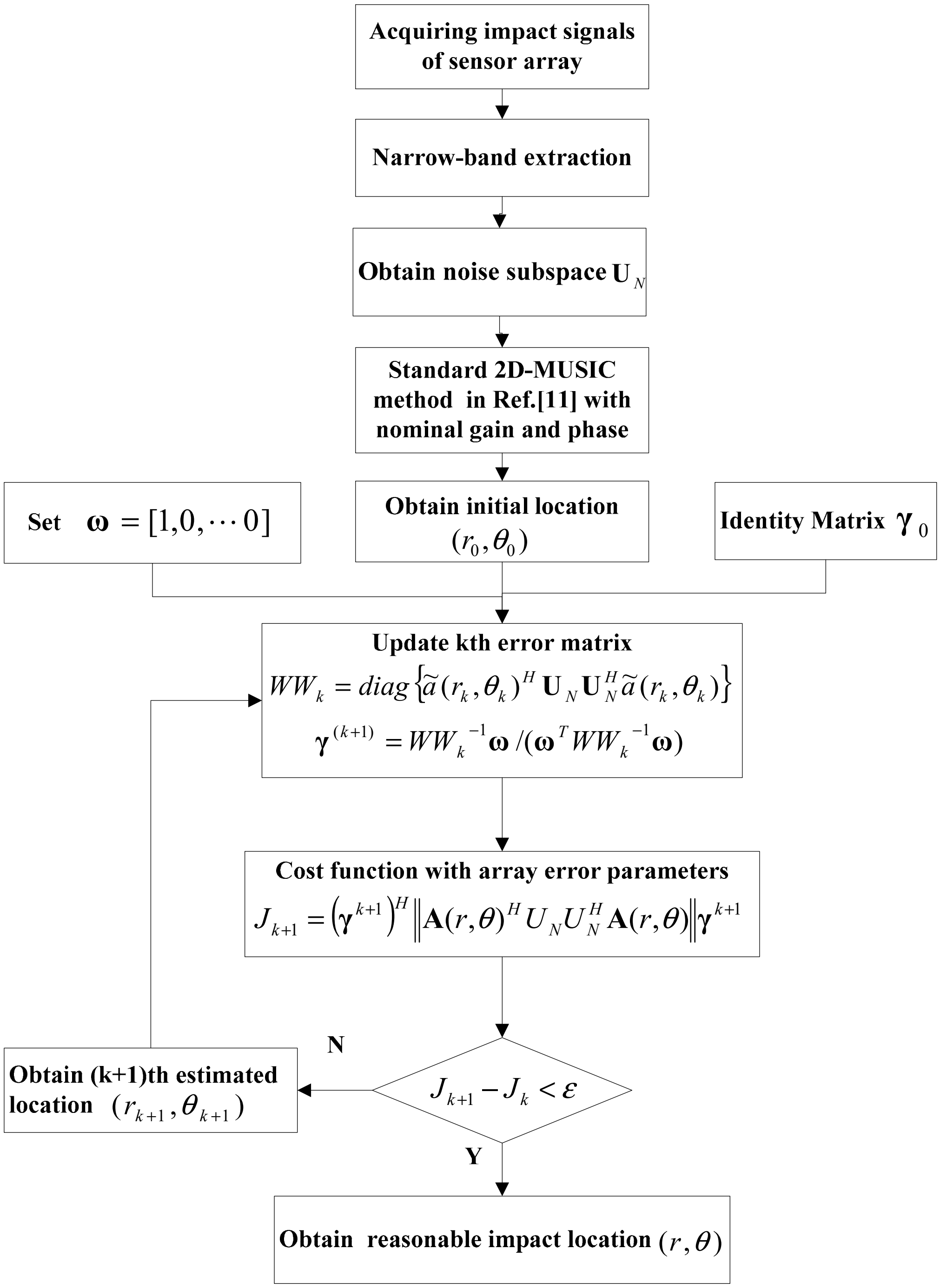


Figure 2. Process of adaptive piezoelectric sensor array error calibration based on the 2D-MUSIC method.

3. Finite Element Simulation

In order to understand the influence of the array element position parameters on 2D-MUSIC algorithm, a FEA plate model was created in ABAQUS. The model is a 600 mm × 600 mm × 2 mm aluminum plate with four sides fixed. C3D8I three-dimensional solid elements were used in the simulation. Load a concentrated dynamic load at the center of the plate to simulate the excitation signal source with a center frequency of 50 kHz, as shown in Figure 3. A set of seven nodes with a space of 10 mm from left to right is selected for simulating sensors arranged. The coordinates (x, y) of sensor array are shown in Table 1.

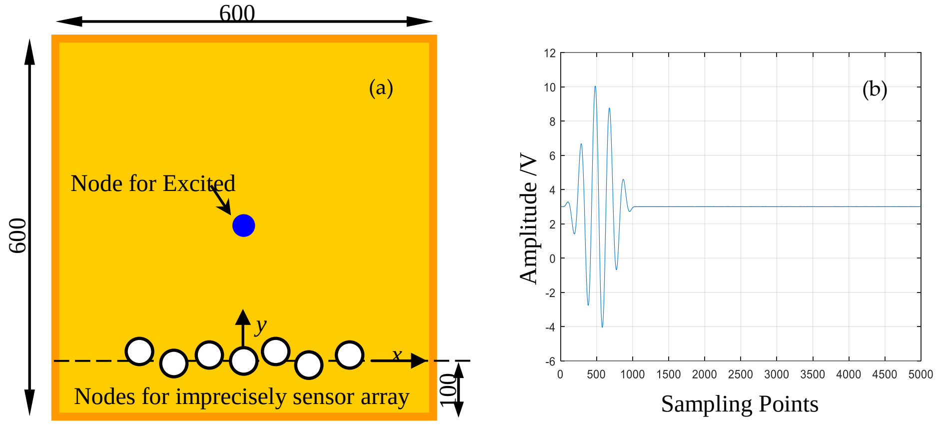
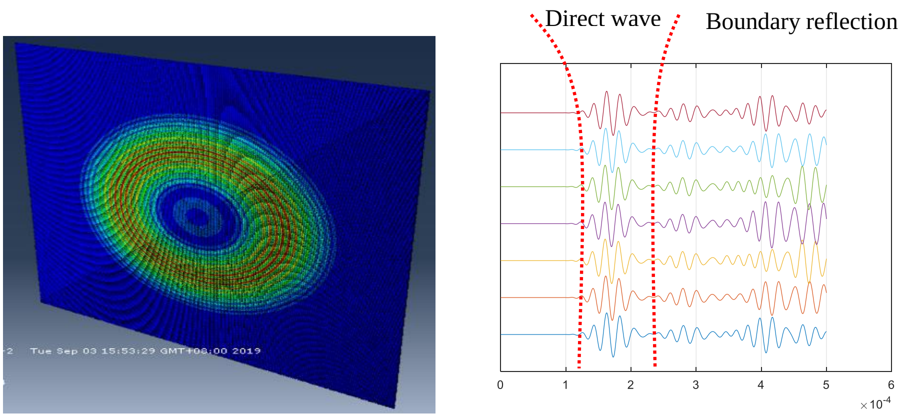


Figure 3. (**a**) Finite element method (FEM) model with error position of sensor array, (**b**) excitation signal.

Table 1. The coordinates (x, y) of FEM nodes selected for sensor array.

Sensor Array	Sensor #1	Sensor #2	Sensor #3	Sensor #4	Sensor #5	Sensor #6	Sensor #7
Precisely	(−30, 100)	(−20, 100)	(−10, 100)	(0, 100)	(10, 100)	(20, 100)	(30, 100)
Imprecisely	(−30, 104)	(−22, 98)	(−10, 100)	(2, 100)	(10, 100)	(18, 100)	(30, 100)

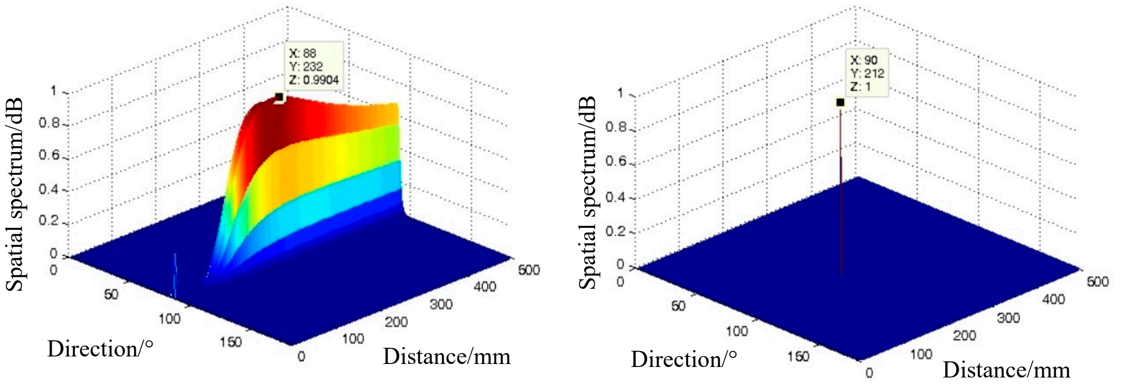
The position of the excitation signal source is loaded at the Cartesian coordinates (0, 200) and its polar coordinates is (90°, 200 mm). Lamb wave propagation simulation and response signal of the imprecise sensors array seen in Figure 4. According to the process of adaptive piezoelectric sensor array error calibration based 2D-MUSIC method described in previous section, the sensor array signals simulated by finite element simulation are analyzed.



(**a**) (**b**)

Figure 4. (**a**) Lamb wave propagation simulation and (**b**) response signal of the imprecise sensors array.

Figure 5 shows the comparison of the spatial spectrum estimated by standard 2D-MUSIC and 2D-MUSIC after error calibration, respectively. After considering the position error of sensors array elements, 2D-MUSIC algorithm has a better localization result. The estimation error in distance and direction is 1.2 cm and 0°, respectively. The FEM simulation results show that the proposed adaptive sensor array error calibration based 2D-MUSIC method is effective for signal source localization. In addition, almost all of the side lobes disappear after error calibration the adaptive correction seen from Figure 5, that is the beam directivity performance is enhanced.



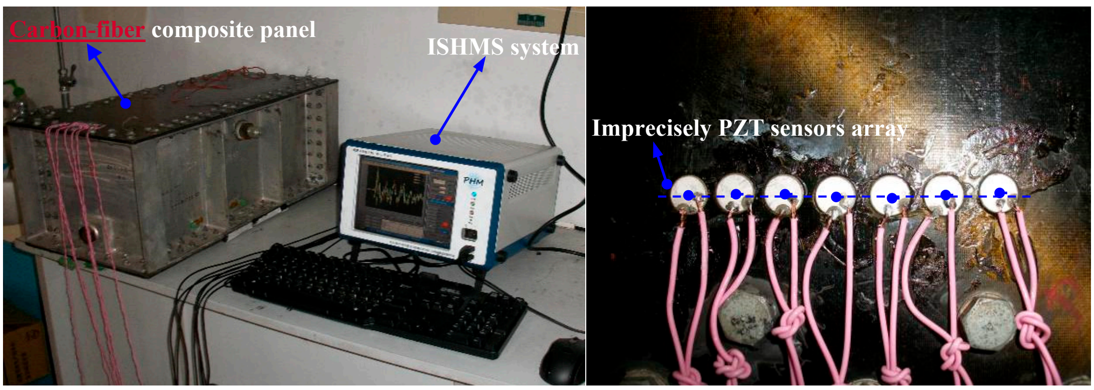
(a) **(b)**

Figure 5. Spatial spectrum estimated by (**a**) standard 2D-MUSIC and (**b**) 2D-MUSIC after error calibration

4. Evaluation on Carbon-Fiber Composite Panel

4.1. Experimental Setup

A 610 mm × 310 mm × 310 mm carbon-fiber composite panel, which was a part of a real aircraft oil tank structure, was selected for evaluation, shown in Figure 6. The composite panels were fixed by two rows of rivets with the width of 45 mm to the rid at four sides. The integrated structural health monitoring scanning system (ISHMS) was adopted here as the monitoring equipment, which was used to control the excitation and sensing of the PZT sensor array. Figure 6a shows the experimental set up. Uniform linear sensor arrays were used in the experiment including 7 PZT sensors bonded on the structure. The diameter of the PZT sensor was 8 mm, and the thickness was 0.48 mm. These sensors were arranged with a space of 10 mm, which were labeled as PZT1, PZT2 . . . PZT7, respectively. However, the sensor array always had a certain array element position error; this was defined as an imprecise PZT sensor array, seen in Figure 6b.



(**a**) (**b**)

Figure 6. (**a**) Experimental setup and (**b**) imprecise PZT sensors array bonded on the aircraft oil tank structure.

4.2. Impact Localization Results

In this section, four different impact cases are evaluated which list in Table 2. To compare the proposed method, classical 1D-MUSIC algorithm [8], adaptive error calibration based 1D-MUSIC, and standard 2D-MUSIC [11] are applied for locating the four impact cases.

Table 2. Polar coordinates of actual impacts applied.

Parameters	Case #1	Case #2	Case #3	Case #4
r (mm)	131	190	76	189
θ (°)	108	76	121	102

Figure 7 shows the respond array signals of case #1, and its narrow band signals extracted with a center frequency of 20 kHz. From figure, the direct waves between two dashed lines were intercepted as observed signal vectors and entered into the 1D-MUSIC and 2D-MUSIC model. The spatial spectrum of case #1 at the point (131 mm, 108°) are shown in Figure 8, obtained by classical 1D-MUSIC algorithm, 1D-MUSIC algorithm after error calibration, standard 2D-MUSIC method and the proposed method. The peak of Figure 8a,b represents the direction of impact (1D-MUSIC localization); the highest pixel point of Figure 8c,d represents the distance and direction of impact (2D-MUSIC localization). From figures, both the beam-formings of 1D-MUSIC and 2D-MUSIC algorithm are getting better after sensor array error calibration. The other three spatial spectra obtained are shown in Figures 9–11.

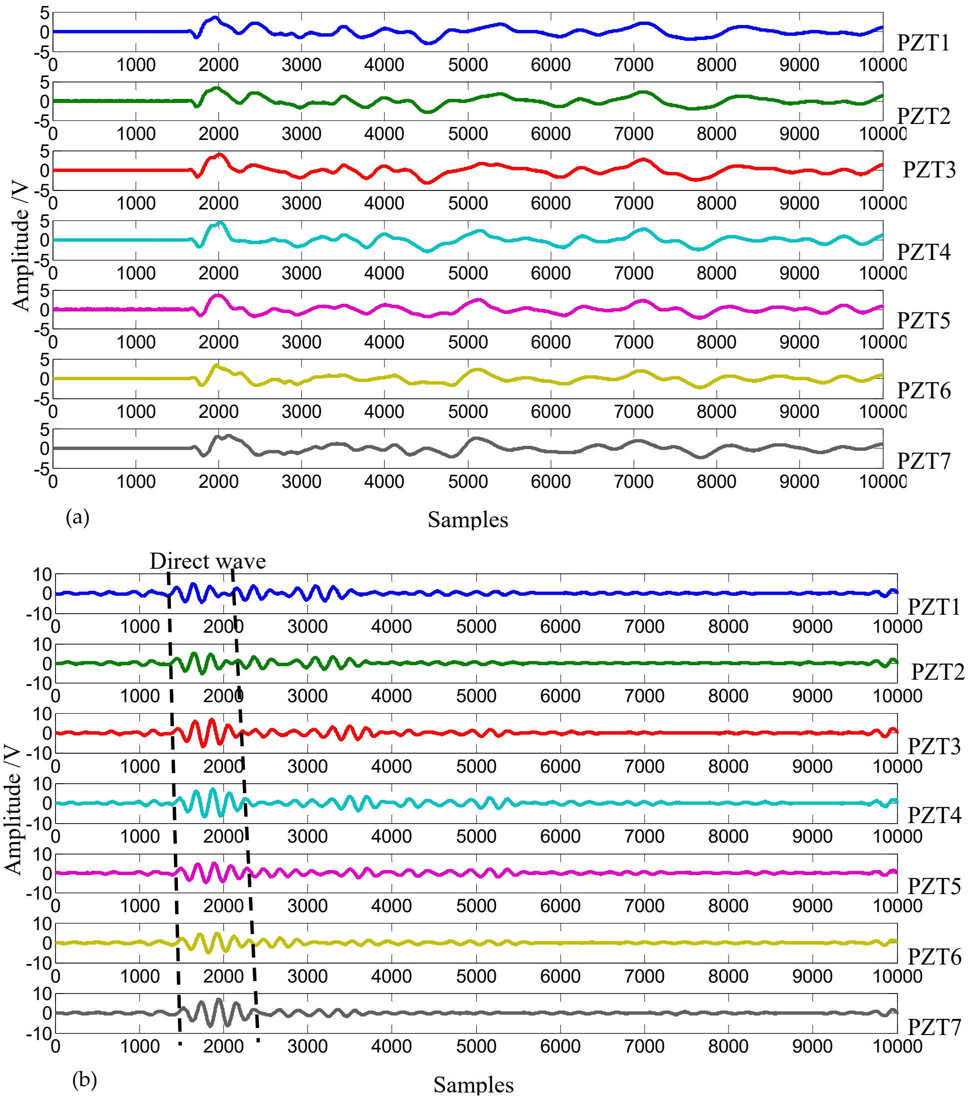


Figure 7. (**a**) Respond array signals of case#1 and (**b**) its narrow band signals extracted.

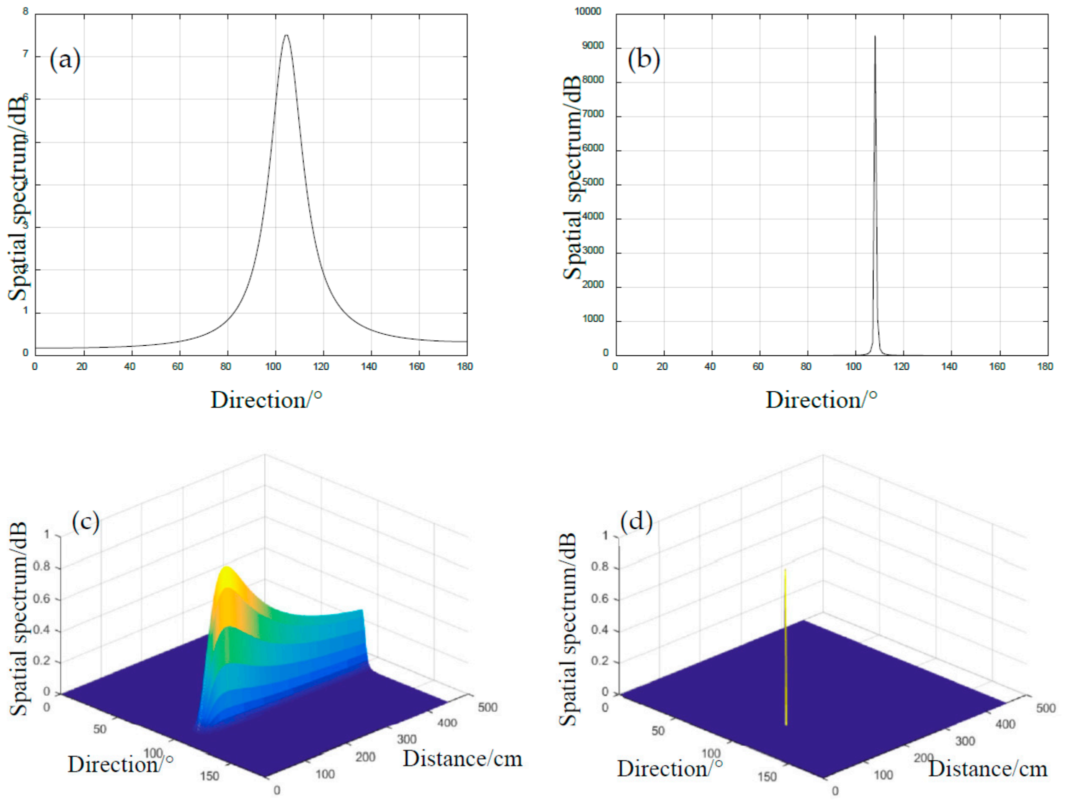


Figure 8. Spatial spectrum of case #1 estimated by (**a**) classical 1D-MUSIC algorithm, (**b**) error calibration based 1D-MUSIC algorithm, (**c**) standard 2D-MUSIC method and (**d**) the proposed method.

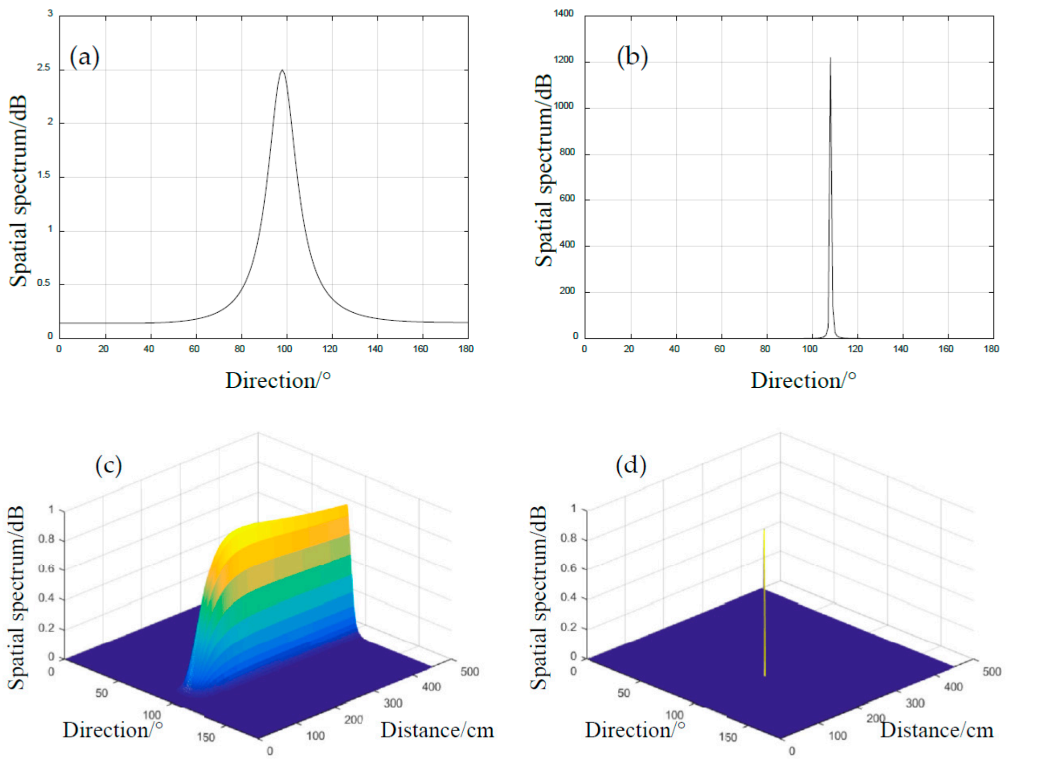


Figure 9. Spatial spectrum of case #2 at (189 mm, 102°) estimated by (**a**) classical 1D-MUSIC algorithm, (**b**) error calibration based 1D-MUSIC algorithm, (**c**) standard 2D-MUSIC method and (**d**) the proposed method.

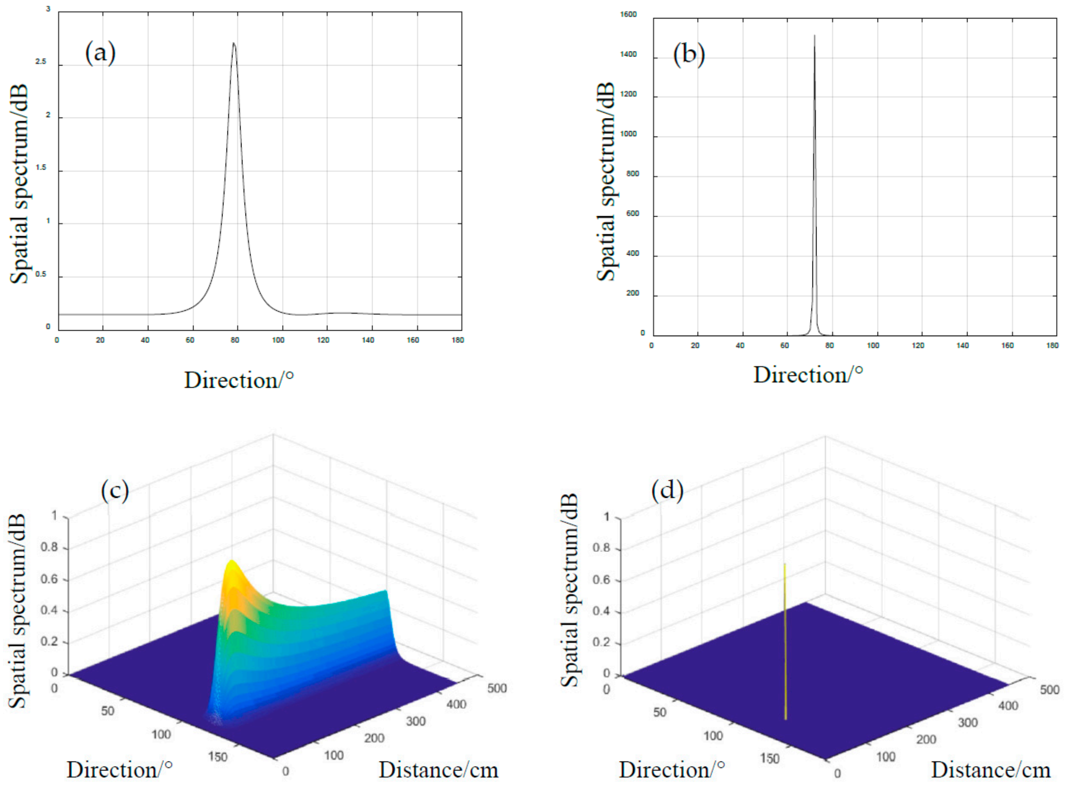


Figure 10. Spatial spectrum of case #2 at (76 mm, 121°) estimated by (**a**) classical 1D-MUSIC algorithm, (**b**) error calibration based 1D-MUSIC algorithm, (**c**) standard 2D-MUSIC method and (**d**) the proposed method.

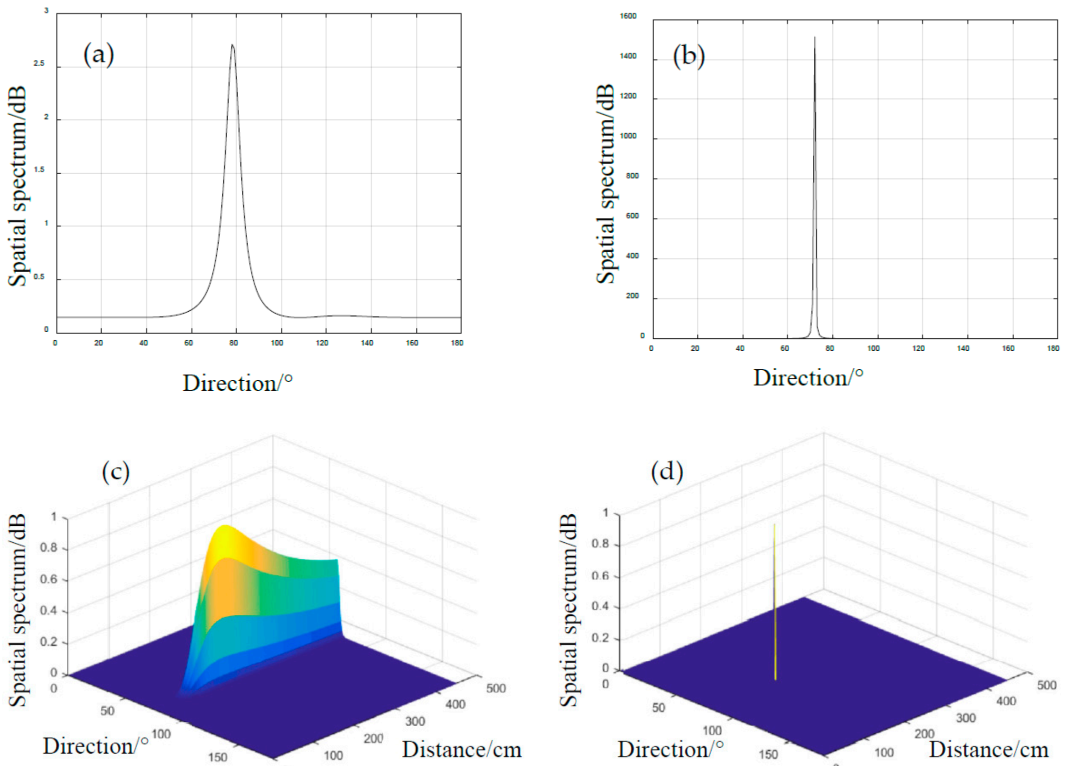


Figure 11. Spatial spectrum of case #2 at (190 mm, 76°) estimated by (**a**) classical 1D-MUSIC algorithm, (**b**) error calibration based 1D-MUSIC algorithm, (**c**) standard 2D-MUSIC method and (**d**) the proposed method.

The results of four impact cases and their errors estimated by classical 1D-MUSIC, standard 2D-MUSIC and the proposed method are summarized in Table 3. Compared with the actual impact points, the maximum error of direction estimated by classical 1D-MUSIC is 9° and all of the direction estimated errors decrease after error calibration. Table 3 also gives a comparison between the standard 2D-MUSIC in ref. [11] and the proposed method. The maximum direction and distance estimation error of standard 2D-MUSIC method is 4° and 2.2 cm, respectively. The maximum direction and distance estimation error of the proposed method is 2° and 2.2 cm, respectively. The results show that the proposed adaptive sensor array error calibration based 2D-MUSIC method is effective for impact localization on a real composite structure, which have more accurate localization results than standard 2D-MUSIC when sensors array is imprecise. In addition, the beam-formings both of 1D-MUSIC and 2D-MUSIC algorithm are getting better after sensor array error calibration.

Table 3. Summary of estimated results and errors.

Impact Case		Classical 1D-MUSIC		Standard 2D-MUSIC		Proposed Method	
r/mm	$\theta/°$	$\widehat{\theta}_c/E_c^\theta$	$\widehat{\theta}_{ac}/E_{ac}^\theta$	$\widehat{r}_f/E_s^r$	$\widehat{\theta}_f/E_s^\theta$	$\widehat{r}_f/E_p^r$	$\widehat{\theta}_f/E_p^\theta$
131	108	105/3	108/0	114/17	104/4	114/17	106/2
189	102	98/4	105/3	176/13	100/2	176/13	102/0
76	121	112/9	116/5	72/4	117/4	72/4	119/2
190	76	74/2	76/0	167/22	75/1	167/22	76/0

Note: $\widehat{\theta}_c/E_c^\theta$ and $\widehat{\theta}_{ac}/E_{ac}^\theta$ are estimated direction and error of classical and error calibration based 1D-MUSIC, $\widehat{r}_f/E_s^r$ and $\widehat{\theta}_f/E_s^\theta$ are estimated direction and error of standard 2D-MUSIC, $\widehat{r}_f/E_p^r$ and $\widehat{\theta}_f/E_p^\theta$ are estimated direction and error of the proposed method.

5. Conclusions

This article presents an adaptive piezoelectric sensor array calibration based method for composite structure impact localization. First, observed signal vector from the sensor array is represented by error calibration matrix with unknown gains and phases, and then it used to construct the cost function including sensor array parameters. Second, 2D-MUSIC algorithm based on linear attenuation calibration in ref. [11] is applied for estimating the initial estimate of impact location. Finally, substituting the initial estimate, the cost function is minimized by adaptive iterative to calculate the sensor array error parameters and the exact location of the impact source.

Both FEM simulations and experiment results on carbon-fiber composite panel with the imprecise PZT sensor array have demonstrated the validity and effectiveness of the proposed method. Compared with classical 1D-MUSIC and standard 2D-MUSIC method, the proposed method has more accurate localization results. The maximum direction and distance estimation error of the proposed method is 2° and 2.2 cm, respectively. It shows that the proposed adaptive sensor array error calibration based 2D-MUSIC method is effective for impact localization on a real composite structure. In addition, the beam-formings both of 1D-MUSIC and 2D-MUSIC algorithm are getting better after sensor array error calibration.

Further research is still worth doing to address systematically the influence of environmental factors to sensor array and to establish the signal model within environmental influence to improve the accuracy of impact localization.

Author Contributions: Y.Z. and J.X.; methodology, Y.Z. and L.R.; software, L.R.; validation, L.R., Y.Z., J.X. and Z.W.; formal analysis, L.R.; investigation, L.R.; data curation, L.R.; writing—original draft preparation, L.R.; Y.Z. and J.X.; writing—review and editing, Y.Z. and J.X.; supervision, Y.Z. and J.X.; project administration. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China [No. U1909217, 51905242], the Zhejiang Special Support Program for High-level Personnel Recruitment of China [No. 2018R52034] and the Qing Lan Project of Jiangsu Province of China.

Conflicts of Interest: The authors declare no conflict of interest.

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