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The Costs of Ambiguity in Strategic Contexts

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Abstract: With this research, we contribute to the study of ambiguity by analyzing how it can be handled in a rational, objective manner across the main strategic decision-making contexts that entrepreneurs and organizations face. Differentiating from most previous managerial and entrepreneurial studies, we conduct the analysis from a mathematical rather than an experimental approach, doing so by considering variants of a robust, yet simple, decision problem. Significantly, the analysis offers a simple model and approach to consider as benchmarks when assessing the impact of an ambiguity level of information against cases where more precise information is available. We identify the many costs of ambiguity, including direct absolute and relative maximum harms, as well as indirect possible penalties. We discuss the strategic entrepreneurial and managerial implications.

Keywords: ambiguity; uncertainty; risk; value of information; decision-making; optimization

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1. Introduction

Ambiguity poses a challenge for strategic and entrepreneurial decision-making[1] (e.g., Einhorn and Hogarth 1986; Petkova et al. 2014; Srivastava 2015). At its worst, it makes the identification of better, let alone optimal, choices impossible. That is why it has been the subject of several streams of research literature in management. Three streams have dominated. The first considers the behavioral aspects, focusing on ambiguity aversion (e.g., Ellsberg 1961). The second applies models of preference functions and subjective probability beliefs in order to collapse otherwise irreducible uncertainty into forms where expectations can be calculated precisely and still align with observed biases (e.g., Klibanoff et al. 2005). The third focuses on reducing ambiguity through managerial actions into more familiar probability problems (e.g., Cerreia-Vioglio et al. 2013). A focus on managing ambiguity 'as given' is missing in the research. Specifically, there is a gap in the literature concerning *the understanding of the costs of irreducible ambiguity across strategic decision-making contexts*. This paper is our attempt to address that gap, using a theoretical approach, and illustrating our points by drawing on a simple but robust model. The research question we explore then is, *what are the possible costs of holding only the ambiguity level of information in the main strategic decision-making contexts?*

We contribute to the study of ambiguity with an answer; one based on an analysis perspective that differs from and complements those dominating the literature (i.e., it is a conceptual paper that does not require reference to lab experiments, field studies or any other real-world data, and similar to other such analyses in micro-economics, can contribute by providing insights based on deduction alone). First, we analyze ambiguity 'as given'—as irreducible uncertainty—rather than as something that is transformed into the equivalent of optimizable probability. Second, we take an objective and neutral stance rather than attempting to account for ambiguity aversion in the calculus of any utility of the decision-maker; we look to maximize value in a given decision problem un-adjusted for any decision-maker subjective attitudes towards risk or ambiguity[2]. Third, we consider one base decision problem across the full range of main strategic contexts[3]. The significance of our analysis is that it describes a clear benchmark model and analysis approach for reference and comparison against cases where more precise information is available. That model is useful in providing, in simple terms, the relative costs of ambiguity versus more precise levels of information, so that decisions on how much to spend to improve informational precision can be made on a clear basis. Our analysis delineates when ambiguity is costly, and in what ways such costs may be measured. For example, as our analysis indicates, ambiguity affects an organization's absolute and relative performance, and not only over specific current decisions but also over how the firm chooses to alter the contexts of such kinds of decisions in the future.

We address irreducible ambiguity when no other approach does; every other approach reduces it to a risk-equivalent form in order to identify an optimal action. We address it by estimating its potential costs, which is useful when considering the budget for possible actions to take, and by identifying some of those actions that can make the possible outcome from a decision involving irreducible ambiguity better. (One way is to focus on the unambiguous parts of the decision and optimize an alternative goal, such as minimizing the possible downsides, while another way is to focus on altering those unambiguous parts [e.g., payoffs] in order to decrease the impact of the ambiguity. These are new approaches to this context.) We illustrate how such ambiguity can affect strategy in different contexts, which provides insights into how altering the given unambiguous parts can affect the impact of the ambiguity. Additionally, we provide an analysis of how ambiguity can be used as a competitive weapon.

In order to explain the various strategic costs of irreducible ambiguity, we need to explain our modeling of the basic decision-making phenomena. In order to accomplish this, we first need to provide background with an overview of related literature and some grounding in definitions and information theory.

1.1. Related Literature

Decision-making quality is affected by the quality of information available. Hence, a low level of information (e.g., an ambiguous level of information where factors are unknown) is likely to lead to a poor outcome. Given the importance of some decisions to organizational performance, it is no surprise that ambiguity (and how to address it) has been of significant interest to strategists and pursued along several research streams.

One stream takes ambiguity 'as is' and focuses on how real decision-maker behaviors are affected by it. That reaction is captured in the concept of 'ambiguity aversion' (e.g., Ellsberg 1961), often measured by how much a decision-maker will pay to avoid ambiguity in a simple choice problem. That aversion is separate from 'risk aversion' in lab studies, indicating that people experience being confronted by an unknown probability over a set of possible bounded outcomes differently than being confronted by a lottery. In other words, ambiguity is its own real informational problem.

Another stream takes ambiguity and reduces it to a risk-equivalent factor so that any decision confronting it can be optimized. This stream is not behavioral, but instead is conceptual. It applies models of preference and utility functions, subjective probability beliefs, and bounded or point estimates of outcomes in order to collapse an unknowns associated with an initial ambiguity into a form where expected values for every choice over the decision can be calculated precisely (e.g., Bhattacharjee 2022; Rich 2021). This stream exists to avoid dealing with unknowns and to provide a means to make decisions 'as if' unknowns were known and discuss 'what then' (which is more publishable than admitting 'we don't know'). An offshoot of this stream tunes the utility functions or heuristics so that observed biases (the ambiguity aversions noted in lab studies) are 'explained' (e.g., Klibanoff et al. 2005). This sub-stream provides alternative possible value adjustments to outcomes that decision-makers use when confronted by well-defined ambiguity. This vibrant stream seeks to explain human behavior, with models based on conservatism and variational preferences (e.g., Ghirardato et al. 2004; Gilboa and Schmeidler 1989; Klibanoff et al. 2005; Maccheroni et al. 2006). This may help us understand what people are willing to pay to avoid ambiguity, but not what they decide when they cannot (and how costly that can be).

Another stream focuses on reducing ambiguity through managerial actions over time rather than 'what if' suppositions (e.g., Cerreia-Vioglio et al. 2013; and, in terms of more general emergent strategy-making—(Mintzberg 1978)). This is a more pragmatic approach that assumes the decision can be delayed until more information is gathered and makes some of the unknowables more known (e.g., through information pull actions such as search and experimentation, and information push actions such as marketing) so that a better decision can be made. This stream highlights contrasting managerial attitudes and possible approaches to ambiguity relative to risk and other forms of uncertainty (e.g., Courtney et al. 1997; Forbes 2007).

Related research has used simulations to model ambiguity's effects (e.g., over resource redeployment—(Sakhartov 2018); and over competitive interactions—(Arend 2022)), experiments to study different dynamic strategies to mitigate ambiguity (e.g., Kuechle et al. 2016), and mathematical analysis of hypothetical decisions of managers-as-agents facing ambiguous investment decisions to suggest improved directions from their principals (e.g., Arend 2020). Our analysis differs in its robustness, covering all four main strategic decision combinations, in its focus on the relative costs of ambiguity (and when those occur), and subsequently, in several of its findings.

Returning to the overarching theme that information quality affects decision quality, the main theory we draw upon is the *value of information* (VoI). VoI is a joint product of the uncertainties involved in a decision and their economic impacts on the focal entities (Howard 1966). VoI research has considered both the private and social impacts of the access to greater levels of relevant and true data prior to decision-making (aka *foreknowledge*—(Hirshleifer 1971)). We define VoI as *the change in expected value arising from the use of the best strategy given a new, more precise level of information* (e.g., Raiffa and Schlaifer 1961). In the real world, that new level of information is usually not free, but obtained at a substantive cost (e.g., Runge et al. 2011). Therefore, the assumption is that information has value. The fewer the unknowns, and the more precise the knowns, the better the expected outcome from a focal decision (assuming it is made rationally and exploiting what is known). This is why intelligence, and competitive intelligence specifically, is valued in the real world. Logically, then, a lack of information is costly. This is precisely what we want to explore, where that 'lack' is in the specific form of an 'ambiguity' level of information, and its costs depend on the strategic contexts involved (e.g., static versus dynamic).

1.2. Conceptualizing Ambiguity

To be clear about our definition of ambiguity, we consider *ambiguity to consist of a known range of possible future outcomes allocated by one or more unknowable distributions*. Such ambiguity implies an *im*precision not normally considered in conceptual decision-making analyses, where we are accustomed to precision. We are accustomed to knowing the limited sets of the possible outcomes and choices and the probability distribution(s) of possible outcomes related to our available choices so that we can calculate the expected value of each choice under each outcome. We almost always have at least one precise comparison choice in the form of an *opportunity cost* alternative from which to calculate our 'extra' payoffs. With ambiguity, however, we step into *un*familiar territory. In theory, ambiguity implies a context where no mathematics or logical reasonings can convert the non-precise information into the precise. We are stuck with a range of outcomes that may occur with unknown likelihoods rather than a calculable mean outcome value and its variance. While that may be frustrating to imagine, it is worthwhile to do so in order to better understand irreducible ambiguity. We provide that better understanding through a structured conceptual analysis of the impacts of such a state of limited information, an analysis that provides insights *un*available from alternative perspectives, that is generalizable, and that allows us to consider how ambiguity would work in strategic situations we have yet to observe.

We define ambiguity as a range of possible future values for a factor of interest. For example, it could represent the possible gross market demand for a new product (spanning from a lower limit of \$1M to an upper limit of \$1B) where no distribution information is provided for that range, nor information of the distribution of its possible governing distributions. The definition of ambiguity-as-range is consistent with its experimental conceptualization by Ellsberg (1961) and with its theoretical characterization in Knight's (1921) uncertainty idea as not knowing the probabilities of all possible future outcome-states of the world; it is the primary standard in the relevant literatures[4]. (That said, we acknowledge a stream of literature on ambiguity-as-equivocality, -as-multiple-meanings, or -as-haziness that is unrelated to the issues here and yet has strategic interest [e.g., Lund (2019); Townsend et al. (2018)].)

Our goal is to analyze how to better understand and deal with this kind of ambiguity head-on. To do so clearly, we define the related aspects of decision-making in simple and stark terms as well. We assume that the focal manager-as-decision-maker is a payoff maximizer who holds *no* subjective beliefs and acts solely on objective, given information (therefore, we do not model utility functions in our analysis). Thus, there are only a few objective preference rules that apply to the relevant ranges here: a higher lower and upper range boundary, all else being equal, is preferred, as the manager's preference is for higher objective payoffs, primarily in absolute terms, and secondarily in relative terms. We have now defined what ambiguity is in a decision and what the objective function is to maximize in a decision. Now, let us consider when such decisions are strategically interesting so that we can model them, analyze them, and discuss the implications.

When Is Ambiguity Strategically Interesting?

Not every decision in the real world, or in the theoretical world, involves ambiguity. Moreover, not every situation involving ambiguity holds strategic interest. Strategically interesting decisions encompass irreversible resource commitments that significantly affect firm performance for more than the short-term (Ghemawat 1991). In terms of ambiguity-as-range, this *restricts* strategically interesting decisions to those where: (i) the range of possible outcomes as corresponding payoff values spans the zero-point [e.g., the point equal to the level of opportunity costs] such that *the range includes both possible gains and possible losses* as outcomes, and (ii) the *actual values of the outcome and its mean*, if the distribution was known, *each also lie within the range*. We focus on these decisions in the remainder of the paper[5].

Conceptually, we have defined where ambiguity is of strategic interest in decisions. Others have defined how important the analysis of ambiguity is in our field. For example, Rumelt et al. (1994, p. 2) note that the fundamental strategic question of how organizations behave involves decision-making under uncertainty and, specifically, when traditional economic approaches do not work (the phenomenon we analyze here). They further detail how the field of strategy focuses on decisions that have a critical influence over success and failure, often leveraging crucial firm heterogeneity that can emerge through the action of ambiguity (pp. 9, 43). Similarly, Leiblein et al. (2018, p. 570) describe uncertainty as a core construct in theories of the firm, and as a linchpin and lever in strategic tensions, such as those involving investment irreversibility. They also define strategic decisions as defying simple calculation or optimization (p. 559). We contribute to this important part of entrepreneurial management with the first estimates of the costs of ambiguity in its main strategic contexts, in addition to identifying when the worst costs are likely to happen, what the source of those costs are, and what some tactics are to minimize them.

1.3. Structuring the Analysis along Game Theoretic Dimensions

How does one boil down the myriad of strategic decision contexts to a finite set that covers and illustrates what kinds of impacts ambiguity can have? One way is to apply a standard conceptualization of decisions to break out the dimensions that could define that set. We apply a game theoretic conceptualization. There are five dimensions in that conceptualization that define any decision, and holding some constant (because they are defined by what ambiguity is in our model), we can let the other dimensions change in order to provide the set of alternative decision contexts to cover. That resulting set has four variants, and we move from the simplest to the most complex in our analyses.

Specifically, a game is defined by five main dimensions: information, strategic choices, their associated payoffs, players, and game flow. Our focus on ambiguity defines two of these five dimensions already, i.e., the *payoffs* span a range that includes both potential gains and losses, and the *information* available is ambiguous. The model we describe below defines the third dimension by delineating the strategic *choices* available. That leaves only two dimensions to investigate in order to perform an analysis that covers all of the main strategic decisions, *players*, and *game flow*. In terms of analyzing players, we consider the other player being either nature or a second sentient party. In terms of game flow, we consider contexts that are static and contexts that are dynamic. We consider these variants below, starting with the simplest model, i.e., a static game against nature. We refer to this base model throughout the analysis in order to provide a fundamental illustration of the possible costs of ambiguity. We also explain how more complex decision contexts could work. Figure 1 illustrates the structuring of our analysis in 2-by-2 form, starting at the base game and then moving through decisions-as-games of added complexity. From the base game, we then move to playing against nature over time; this change in assumption allows us to explore how the decision-maker may reduce the ambiguity by taking actions to learn more about the unknown factor. We then move back to a static game, but change the assumption of who is being played against, considering both a competitive and a cooperative 'other' decision-maker. This allows us to explore the asymmetries of information affecting more than just the decision-maker. Finally, we change the assumption to playing against another decision-maker over time, which allows us to explore the ways in which asymmetries in information can be used to manipulate behaviors. These four contexts have possible (and different) costs of only having an ambiguity level of information, and that helps provide a full picture of its impact.

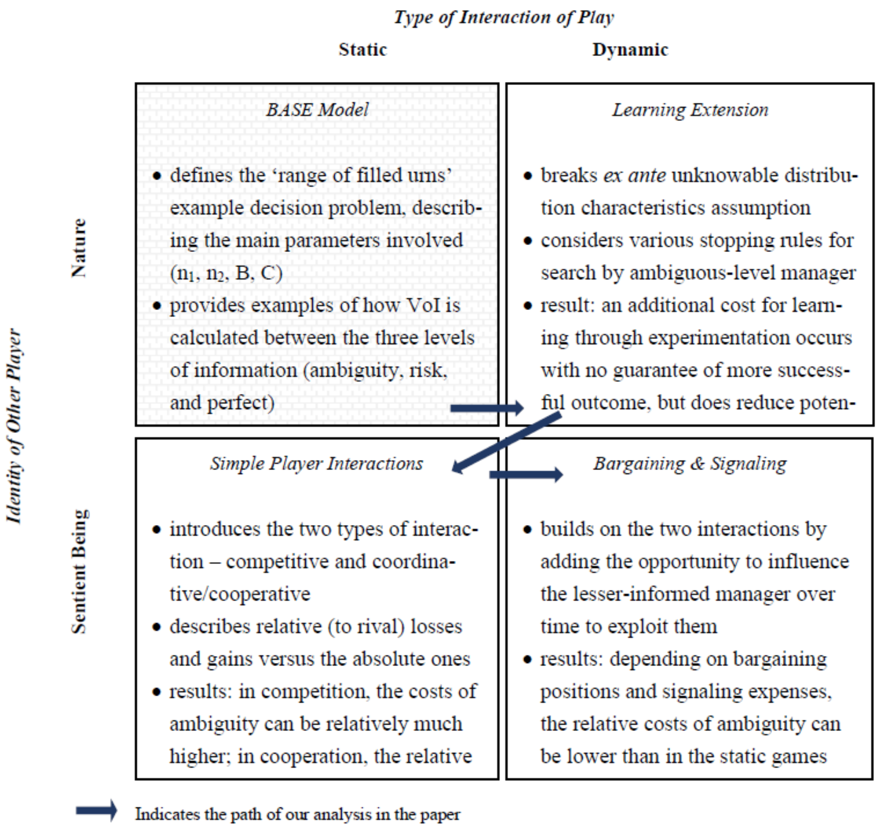

Figure 1. Outline of the Analysis of the Costs of Ambiguity in Strategic Decisions.

1.4. A Simple Model of Ambiguity for Analysis

We used the following simple decision example as a reference. Consider this static realistic problem: a decision-maker faces a one-shot investment choice over a set of possible opportunities, some of which will pay out and some of which will not. For example, a firm's manager faces the choice of R&D investments in a set of possible pharmaceutical products. Each lab costs an amount C (>0). If successful, it provides a gross revenue benefit of B (>C), and if unsuccessful, it provides zero gross benefit. There are n_2 (>1) opportunities, only n_1 ($\leq n_2$) of which will provide a successful outcome. Accurate information about B, C, and n_2 is freely known. The organization's opportunity cost is normalized to 0.

In the language of ambiguity experiments, we draw from n_2 urns, ordered in some identifiable way, where each urn either contains one ball or it does not. Only n_1 of the urns holds a ball. It costs C to draw from an urn, and zero to pass over that urn and not draw from it. When an urn is chosen to be drawn from and contains a ball, the prize is worth B, and if it is empty, then there is no prize. The prizes have objective monetary value. The manager then decides how many urns to draw from in order to maximize the (objective) value[6].

We began with this model, where the decision-maker plays against nature in a static (i.e., one-shot) game. It is the simplest variant of the relevant possible games and provides the most understandable reference point. The model is representative of *strategic* decisions because the payoff range involved includes both potential gains as well as potential losses. In the model, the manager is challenged by a problem that involves significant stakes in terms of resource commitments, as indicated by the sacrifice of available opportunity costs, and in terms of potential outcomes, as indicated by the potential payoffs relative to the opportunity costs. The model describes an important investment decision that affects the firm's future. For example, the decision could represent, in pharma, investing in a set of unproven alternative drugs; or in technology, it could represent investing in a set unproven new products; or in oil discovery, investing in a set of untapped drilling sites.

To reiterate, we analyze the model under the standard assumption of neutrality in our maximizations. Unless otherwise specified, the manager is maximizing payoffs (expected or known) and is holding *no* subjective beliefs. The manager bases her decision solely on what information is objectively given. And, because the purpose in this paper is to cover all of the main strategic decision-making issues as affected by ambiguity-as-range in one place, we have chosen to illustrate our analyses with this simple, generalizable model rather than with specific examples[7]. We believe that this approach provides clearer (and potentially more generalizable) insights to the effects of the focal factor of ambiguity. We understand the costs to realism involved in this choice and that alternative approaches, such as observing specific real-world decisions are also valuable. We leave the analysis of such behaviors under particular contexts and constraints for future work.

1.5. The Value of Information and Its Levels

The *value of information* is illustrated below, for the three levels of information relevant to our model:

- *Ambiguity* is the range (from 0 to n_2) that n_1 could occupy, where the distribution of the values over that range is unknowable, at least ex ante. Ambiguity implies a range of possible payoffs. At worst, it is harmful and costs '$(n_2 - n_1) \cdot C$' if one were to choose every empty urn to draw from and no others. At best, it is beneficial and pays off '$n_1 \cdot (B - C)$' if one were to draw only from full urns. However, for ambiguity, n_1 is *un*known, and so the possible payoff range faced by the entrepreneurial manager effectively spans from '$-n_2 \cdot C$' to '$+n_2 \cdot (B - C)$', from the case where she tries all urns and they are all empty to the case where they are all full.
- *Risk* is often defined as knowing, objectively, all of a factor's future outcome-states *and* their probabilities beforehand (e.g., Knight 1921). It is equivalent to knowing n_1 (or the distribution characteristics from which n_1 is drawn) in this model. The expected payoff is precise; it is simply '*max* $[0; n_1 \cdot B - n_2 \cdot C]$'. It is straightforward to prove that

the best strategy to play when '$n_1 \cdot B - n_2 \cdot C > 0$' is *all-in*, and otherwise, it is *all-out*[8]. Thus, the expected payoff is, in the worst case, the opportunity cost and, in the best case (i.e., when $n_1 = n_2$), is '$n_2 \cdot (B - C)$'.

- *Perfect Information* is often defined as knowing the future with accuracy (i.e., knowing which outcome will occur, with certainty). This implies not only knowing n_1, but also knowing each specific location that will provide success (i.e., knowing which specific urns contain a ball), *ex ante*. The value of perfect information is equal to '$n_1 \cdot (B - C)$'. If we assume that n_1 can take on any number in the range, then this certain payoff also includes the opportunity cost level (i.e., when $n_1 = 0$) in addition to the maximum payoff '$n_2 \cdot (B - C)$' level (i.e., occurring when $n_1 = n_2$).

2. Model Analysis

There are *three* types of costs that can arise from holding only the ambiguity-level of information (relative to holding higher levels). The first cost stems from not being able *to make a more precise decision in a given context*. The second cost stems from not being in a position *to take a better path* towards future investments through actions that alter the given context. The third cost stems from not being able *to exploit a less-informed party* in multi-period interactions. The first and second costs of ambiguity can be identified and explored using the base model of our game. The third cost of ambiguity requires a more complex form of our model; its understanding needs to be considered in a dynamic game against a sentient rival. Below, we analyze our model, in its variants, in order to explore these three costs of ambiguity in ways that have *not* been explored prior in the literature.

2.1. The First Cost in Terms of the Maximum Willingness to Pay for Better Information

What is a manager willing to pay in order to have more precise knowledge? According to our definition, information only has value when it alters a managerial decision. So, when $n_1 = 0$, the manager with risk-level information would *not* pay to have perfect information. However, whenever the information can stop a manager from making a costly mistake (e.g., going all-in when '$n_1/n_2 < C/B$') or from missing out on beneficial opportunities (e.g., going all-out when '$n_1/n_2 > C/B$'), then such information would be worth paying for.

What is the optimal manager's willingness to pay for risk-level information when they only has ambiguity-level information? Given the imprecision involved, the best we can do is to identify the *maximum* willingness to pay. In order to do that, we can calculate the maximum benefits lost by not participating in the opportunity when the firm should have, as well as the maximum costs saved by not participating in the opportunity when the firm would have. The amount of maximum benefits lost occurs when '$n_1 = n_2$' and is '$n_2 \cdot (B - C)$'. The amount of maximum costs saved occurs when '$n_1 = 0$' and is '$n_2 \cdot C$'. Note that: (i) these amounts differ when '$B \neq 2 \cdot C$'; and, (ii) given that the maximum value of the context is '$n_1 \cdot (B - C)$', it is actually possible that the amount of maximum costs saved *can be larger* than the value of the context, i.e., when '$1 + n_2/n_1 > B/C$', which is likely for small n_1. In other words, ambiguity involves an *unusual asymmetry* in that a manager's willingness to pay for better information may even *exceed* the actual stakes of the context because of the specter of making a very costly error exists. This is a new and unusual insight to this literature.

Note that we can do more than identify the maximum willingness to pay, we can also identify *when* those specific cases occur. When the expected value (i.e., n_1) is located very near the range's lower boundary (i.e., 0), then it is more likely that the manager will incur a costly mistake (i.e., go all-in); and, when the mean is located very near the upper boundary (i.e., n_2), then it is more likely that the manager will fail to incur possible benefits. Given we do *not* have an evaluation function that translates a given range into a decision for the strategic context described (i.e., a context where the expected value of the outcome lies between the boundaries, and possible gains and losses can occur within that range), we *cannot* identify what a manager would do when presented with such a given range. All we can do is hypothesize that ranges with greater bias towards gains are more likely to sway the manager to exploiting the opportunity than avoiding it, given, at the limit (i.e., when the range almost *only* includes gains) that is what should occur. Armed with such reasoning, we can see why ambiguity-as-range is so debilitating to strategic decision-making[9]. One reason is that when the range does *not* indicate to the manager any sense of where the expected value might be positioned, the potential losses from mistakes are unreasonably high (e.g., they can be greater than the possible value from the opportunity itself).

Obtaining risk-level information removes the unreasonable possible downside. An interesting question remains, though, whether there is something between the ambiguity-level and the risk-level of information that is of strategic value. The answer is yes; there are contractions of the full range to consider that may constitute potentially valuable intermediary levels of information precision. There are three such cases of interest to consider where such incremental additional precision (precision that does not reach the risk-level) still constricts the range of n_1 in a valuable way:

1. when incremental information increases the lower bound to rule out any potential losses;
2. when incremental information decreases the upper bound to rule out any potential gains; and
3. when incremental information constricts the range to decrease either the maximum potential losses or the maximum potential gains. In case (i), to the manager who would have gone all-out, the additional information is worth, at most, '$n_2 \cdot (B - C)$', which is the maximum potential gain that would have been missed. In case (ii), to the manager who would have gone all-in, the additional information is worth, at most, '$n_2 \cdot C$', which is the maximum potential loss avoided.

In case (iii), any increment (Δ) inwards from the range boundary is worth, at a maximum, the amount '$\Delta \cdot C$' in value [until a critical value is reached, as in case (i) or (ii)]. In other words, *more precise information is always conditionally worth something* (i.e., conditioned on altering the manager's decision), even information that is incrementally more precise than ambiguity as the full range, but remains *less* precise than the risk level. Note that this clear delineation of a meaningful informational space *between* ambiguity and risk is also new to the literature.

It may be useful to set the willingness-to-pay amounts computed thus far in some relevant perspective. To do that, consider the manager's willingness to pay to move from the risk-level of information to the perfect-level of information as a contrast. That willingness to pay depends on whether the manager was going to be all-out or all-in with only risk-level information. In the former case, the maximum willingness to pay is simply '$n_1 \cdot (B - C)$' [i.e., the value of missed benefits] and, in the latter case, it is '$(n_2 - n_1) \cdot C$' [i.e., the value from avoiding unnecessary costs]. Note that the value of the avoidable costs does not ever exceed the value potential of the context, as it did in the ambiguity analysis. This is because, with risk-level information, to get to all-in requires that '$n_1 > n_2 \cdot C/B$'. So, to put the willingness-to-pay levels in perspective, we have a ratio difference of '$n_2/n_{1'}$ for missed benefits, and '$n_2/(n_2 - n_1)$' for missed costs [i.e., where ratios indicate the willingness to pay to go from ambiguity to risk versus from risk to perfect information]. The maximum of the minimums across the-two ratios is two. Therefore, we can now state the cost of ambiguity in relative terms as well: *the cost of ambiguity is, at a minimum, twice as much as the cost of the next standard level of informational precision* for the measure of maximum willingness-to-pay for the next level of information.

Note that the maximum willingness to pay for more precise information *decreases* as the level of informational precision increases, and that is based on both potential avoidable costs and potential missed gains. For a given and fixed strategic decision problem landscape, like our set of urns, where some parts of the landscape provide net benefits while others provide net losses, the choice set of actions is also fixed (e.g., search an urn or pass over it, or search all or none). When this is the case, the maximum values of benefits and costs possible in this landscape are also fixed. When more precise information is provided that can alter a manager's choice of action, the maximum incremental values of benefits and costs possible in the landscape are consequently reduced. The strategic pie shrinks for any value added by further information. Therefore, the next similar increase in informational precision can only provide a fraction of the absolute value in that landscape, as we noted in our example. Thus, it is not unreasonable to propose that increases in the level of precision of information will be worth increasingly less, for a given strategic landscape.

Proposition 1. *There are decreasing returns from the addition of substantial levels of informational precision for a given fixed strategic landscape.*

To be clear, we are assuming that the precision is *truthful*, so that greater precision acts to rule out more and more of the potential inefficiencies and mistakes. For example, knowing n_1 rules out the punishing potential downside from going all-in, which then restricts the incremental value of additional information, reducing the potential value remaining. It is when the added precision does *not* provide added choices and does not allow a manager to alter the given strategic landscape that the pie shrinks.

2.2. The Second Cost in Terms of How Information Directs Action

For each level of informational precision, a *different* direction for how to improve the context emerges. In the simplest analysis, *comparative statics* provides the direction for best improving future payoffs. In the real world, this would translate into possible R&D investments and entrepreneurial actions directed at shifting the context to *improve potential payoffs to the firm by altering the values of the known factors* (e.g., McMullen and Shepherd 2006). Under an exogenously given information level, it is the identification of which investments give the firm its best returns for adjusting the current values of known initial factors (e.g., B) *if* those factors were alterable.

A game theoretic approach is useful for exploring ways in which the given game could be altered for at least one player's benefit; we do so here to illustrate how such alterations provide hypothetical benefits at different rates depending on the information level available to the decision-maker. This is a novel variation on that exploration. In this case, let us assume that a firm's C and B are alterable. Specifically, a firm can be made more efficient and robust [e.g., by increasing flexibility and identifying contingencies] in order to reduce its costs (C) here, when drawing from empty urns, and a firm can be made more efficient and exploitative [e.g., by identifying synergies and accessing complementary assets] in order to increase its benefits (B) here, when drawing from non-empty urns. In that context, for managers with the ambiguity-level of information, the best direction (conditional on entering the opportunity) is to lower a firm's potential downsides, which translates into lowering C (at a theoretical rate of return of n_2)[10]. For managers with the risk-level of information, the best direction (conditional on entering the opportunity) is to increase a firm's potential expected net payoffs, which is accomplished by decreasing C (at theoretical rate of return n_2). Note that the potential to lower C *can also affect the decision to enter the opportunity* for managers with only ambiguity- or risk-level of information; that effect is stronger on the risk-level informed manager, and so illustrates another potential cost [or missed potential gain] to having only the ambiguity-level information. For managers with perfect information, the best direction (which is not conditional because entry is always beneficial) is to increase its net payoffs, which is accomplished by increasing B or by decreasing C (at rate of return n_1).

The results of this directional analysis of the effects of increasing the manager's informational precision entail more complexity than the results of the willingness-to-pay analysis. While the latter identified a simple pattern of diminishing returns on additional information, the former identifies outcomes that are not so simple. The main complication is that there exist conditions where the returns to investments in altering a controllable factor (i.e., B or C) can *in*crease with additional informational precision, making it possible for the [informationally] rich to get even richer.

There are three clear conditions for this. First, it may be that being more informed also brings access to cheaper or better ways to alter a focal factor (e.g., the investment function is more attractive to the more-informed so that the *net* benefit from, say, decreasing C for a manager with risk-level information is greater than that for a manager with ambiguity-level information). Second, it may be that the parameter levels in the given context leverage the conditionality of such directional investments to favor the more-informed managers. If it is inexpensive to increase 'B', and '$n_1/n_2 < C/B$' remains after the increase, then managers with the highest information precision may enjoy the best relative return on investment for their directed investments. In those circumstances, the relative rate of return from increasing 'B' is positive when upgrading from the risk-level to perfect information, but zero when upgrading from the ambiguity-level to the risk-level. Third, it may be that with very high levels of information that there are new options available to invest in (e.g., that exploit synergies in specific urn combinations that can be identified as the opportunity is entered, a possibility that is *only* open to managers with perfect information) that have sufficiently high net returns[11].

Thus, it is not unreasonable to propose that for some decision–problem landscapes, specifically, for some landscapes that *can be altered* through actions and investments, that *in*creasing return to (additional) informational precision levels is possible.

Proposition 2. *There exists the possibility for increasing returns from the addition of substantial levels of more precise information when the landscape is alterable.*

2.3. What Happens When Dynamics Occur?

Our base model is a static (i.e., one-period) game. We now consider what occurs when the game is dynamic (i.e., involves more than one period of play). Note that new tactics for addressing information imprecision arise in a multi-period game. One such tactic, a tactic focused on reducing ambiguity, involves *experimenting* on the given context in order to learn more about it (e.g., through rapid prototyping, or by test-marketing ideas through crowdfunding, e.g., on *kickstarter.com* Dimov 2010; Johannisson and Monsted 1997; Sull 2004). This tactic is aimed at gleaning new, more precise knowledge about the focal unknown factors (e.g., n_1) and the potential outcomes from decisions. When that tactic is successful, an ambiguous context may be converted into a more familiar risky one (e.g., where the mean of the initially unknown distribution becomes known). Note that this version of the game *violates* the initial assumption of *ex ante* irreducible ambiguity insofar, as it may be reduced prior to the commitment to go all-in or all-out with whichever urns are left after the experimentation is done.

Recall that our benchmark model was a one-period game, so no learning was possible. Extending the model with dynamics makes the analysis more complex. This is because we then need to add details of a learning process (e.g., by describing the possible confidence levels, distribution characteristics, and stopping rules involved in playing the modified game).

To illustrate the added complexity, consider the learning problem faced by a manager with only ambiguity-level information when our base game is altered to allow for multiple periods of play. We formalize the problem as follows. Assume that the known variable values (n_2, C, B) are at levels such that the manager is attracted to learning more about the opportunity rather than avoiding it. The manager can learn more about the focal unknown, n_1, by drawing from the set of urns, say, one or more at a time, and then updating their beliefs (e.g., about the distribution of possible payoffs) after each period of such experimentation. Assume for simplicity that this is done *without* any discounting for time and *without* any informational processing costs. The objective is to maximize value for the manager's firm by choosing a way to learn about the unknowns (e.g., the level of n_1, or its minimum level) in order to choose an action that applies to the rest of the urns (i.e., in order to go all-in or all-out for the remainder of this opportunity).

This problem is addressed by identifying the best *stopping rule*—a rule that indicates when the manager should stop drawing from urns because the opportunity has turned from potentially beneficial to expectedly costly. There are many such rules to choose from: The simplest stopping rule is a *limit rule* that specifies not to draw from the next urn if it might mean exceeding a pre-set maximum net loss amount. For such a rule, there is *no* learning in terms of updating beliefs about n_1; rather, there is simply an updating of the net loss tally. A more complex rule involves a learning process where the manager updates their beliefs about the value of n_1, given an initial guess of its value, and a limit for how long the updated belief can remain below a pre-set threshold, perhaps drawing on a Bayesian updating process to form posterior probability beliefs. An even more complex rule involves the manager modeling what is being learned (e.g., the distribution of the value of n_1) and setting of a confidence level for choosing when to stop. For the more complicated process-based rules, note that n_2 would have to be relatively large in order to have sufficient learning take place that could provide a high statistical confidence level.

Given the fact that we are dealing with ambiguity-as-range, it is possible for even *incremental* additions of informational precision to provide significant value regardless of how confident a manager is in her initial beliefs about the value of n_1. For example, every successful draw decreases the maximum downside of the original opportunity, and possibly gets the manager to the critical point, whereas going all-in for the remainder of the draws is guaranteed to provide net benefits to the firm (where that point occurs when the net value of the urns already explored exceeds the gross costs of exploring the remainder of the urns). In other words, under some conditions, only a small increment of learning is needed to guarantee net benefits rather than the large amount of learning required to be sufficiently confident the manager has attained the risk-level of information precision.

However, when managerial learning efforts are *not* free, then there is a cost to only having the ambiguity-level of information precision in this game. That is a *new cost* that arises from having a dynamic game. However, it is a cost that is expected to be compensated for by new benefits arising from learning more about n_1 and using that knowledge to make better decisions about the remaining investments[12].

2.4. The Third Cost in Terms of Playing against a Second Sentient Party

Thus far we have analyzed VoI in the context of playing against nature, in both one-period (static) and multi-period (dynamic) games. We now consider the remaining two extensions to our base game—playing against a sentient rival in a static and in a dynamic context. With the costs of ambiguity already described in contexts when playing against nature, we now complete the analysis of the main strategic costs of holding only ambiguity-level information by describing the costs that arise when playing with another sentient party.

In the VoI perspective, it is implicitly assumed that the valuation of information is from the focal manager's perspective, calculated relative to the payoffs arising from alternative sets of knowledge, and *ignoring any costs and benefits to others*. In an alternative perspective, where there are costs of informational disadvantage (CoID), this is *not* the case. When one party has an informational advantage over a second party, the value should account for the non-focal party's change in performance as well. As with VoI, CoID only occurs when the new information alters the decision of the affected party. Playing against a second sentient party has effects in both a static context and a dynamic context. The dynamic context is more interesting because the possibility arises of influencing the less-informed party *prior* to their decision. We analyze that context after we consider the static one.

2.4.1. The Analysis of Static Games Involving a Rival

We formalize the analysis of these one-period two-player games in the following way: We assume that one player has ambiguity-level information while the other holds a more precise level of information, and that each player knows that as well as its own level of information. This asymmetry assumption (along with the known player-type assumption) allows a clearer analysis of the relative costs of ambiguity.

We modify our base urn game (i.e., the static game against nature) so that now two firms interact with each other either competitively or cooperatively. The competitive game unfolds in the following way: players simultaneously choose whether or not to exploit the opportunity. Then, the urns are allocated and the payoffs revealed. If both players decide to exploit the opportunity, then the urns are randomly assigned; however, if only one enters, then they can draw from the full set of urns. In other words, we assume that the total original prize is halved when both firms enter. The cooperative game unfolds in a different way because we assume that *both* parties' resources are required to exploit any urn in order to capture the cooperation element. So, if both players exploit, then they split the benefits (and costs), but if either chooses *not* to exploit then there is no benefit or cost to either party. The goal for each firm's manager in either situation is to maximize value: absolute value first and relative value second.

Consider the competitive game first. From the perspective of the manager at an informational *dis*advantage (i.e., having only the ambiguity-level of information), the following holds. If the manager enters mistakenly and the other does not, the maximum damage done is not only '-$n_2 \cdot C$' in absolute terms, it is that in relative terms as well (because the rival manager does not enter and receives the opportunity cost of 0). If the manager mistakenly does not enter and the rival correctly does (given that manager's more precise information), the maximum lost net gains are '$n_2 \cdot (B - C)/2$' in absolute terms and double that in relative terms (as the rival then has full market access). In the other two possible outcomes (i.e., both enter correctly, or both do not enter correctly), the relative costs are zero. Thus, one outcome has changed compared to the base game of playing against nature; in one of the bad outcomes (i.e., where the manager with ambiguous-level information does something that a more informed manager would not by not entering when they should), the *relative* costs of ambiguity has doubled. Thus, in this competitive context, we have identified another (new-to-the-literature) cost of ambiguity that can arise in the form benefits lost to the firm and transferred to the more informed rival.

Now consider the cooperative game. Of the four possible outcomes, nothing changes when each manager correctly and simultaneously enters or does not enter. When the manager with only the ambiguity-level information mistakenly enters, they are saved from potential losses because the more-informed manager correctly does not enter the opportunity. However, when the less-informed manager mistakenly does not enter the opportunity when the more-informed manager does, *both* firms lose out on the benefits. In terms of relative costs compared to the other player, there are none because of the interdependence of action assumed. Moreover, relative to the playing against nature, the costs are *reduced* substantially because there is no cost for mistaken entry, although the lost benefit from mistaken non-entry remains the same in *comparative* absolute terms. From this analysis, the optimal strategy for the manager with only ambiguity-level information *is* actually identifiable in this context where they partner with a manager with at least risk-level information; it is to *always* try to exploit the opportunity. That strategy eliminates any potential lost benefits to each party[13].

The takeaway from our analysis of simple static games against rivals is that *while competition increases the potential costs of ambiguity* through spillovers when the informationally disadvantaged firm mistakenly does not enter and gives away the entire market to the informationally advantaged firm, *cooperation decreases it* through spillovers when the informationally disadvantaged firm mistakenly does enter but is saved from costs because its more informed partner does not.

2.4.2. The Analysis of Dynamic Games Involving a Rival

We now consider the strategic context where the manager with only ambiguity-level information plays against a more-informed manager in a multi-period game. The most interesting version of CoID occurs when one party exploits its informational advantage against the other in *a manner that alters the latter's decision to the former's benefit*. As with the static case, we choose to explore this asymmetric structure because it provides the most direct effects of being in a less-informed position. This structure is common in contract theory, where informational asymmetries lead to costly shifts in the decisions of relatively uninformed principals such that they settle for second-best rewards that incentivize their agents (Anderson and Smith 2008). Those informational costs of are often reflected in the expensive factors added by second-best solutions (e.g., quality signals, warranties, investments in reputation, increased monitoring). As with standard economic analysis, here we consider only fully legal forms of exploiting informational advantages and avoiding illegal forms such as deception and fraud (that occur in the real world, e.g., Kane 2004).

We formalize the analysis of these dynamic games involving other sentient players in the following way. We take the perspective of the manager holding the informational advantage. The value from that advantage occurs when the more-informed manager can persuade the less-informed manager (i.e., the one with ambiguity-level information) to make a decision that benefits her directly or indirectly. Thus, we assume that managers can send signals to each other in the earlier time-periods, possibly prior to each making their decision to exploit the opportunity. As before, we assume no discounting and no processing costs. As before, we will use our modified urn game in either its previously described competitive or cooperative versions. Additionally, as before and to be consistent, we assume that the goal for any firm's manager is to maximize value: absolute value first and relative second.

There are many ways to model the dynamics and to model what *can* occur over the multiple periods of play. We will focus on the simplest version here. We assume that in the first period, players can try to influence each other, and in the second period, players decide on whether to act on the opportunity. Under this simple modeling of dynamics, there are only three interesting cases where a manager can leverage an informational advantage. Case (i) involves the cooperative game, where the more-informed manager can try to gain cheap access to the less-informed player's resources. Cases (ii) and (iii) involve the competitive game, where the more-informed manager can either try to deter the less-informed manager from entering into a good opportunity or try to lure the less-informed player into exploiting a bad opportunity.

Case (i) assumes that to draw from any urn requires the *combined* assets of two firms. This is consistent with the static cooperative game that we discussed previously. Given our more-informed manager knows n_1 but the other manager does not, the problem is to identify the best strategy to exploit that advantage in a context where they have a period to influence the less-informed manager. That problem translates into a *bargaining game*, where the outcome depends on the bargaining power of each party involved. Bargaining power normally depends on how many potential partners exist, what information each has, and what their intentions are (e.g., whether they were going to exploit or avoid the opportunity based on their information level) *before* entering the bargaining period. In the best case (i.e., where the less-informed player has no bargaining power and the n_1 level indicates to the more-informed manager that there are net benefits to gain from exploiting the opportunity) they should be able to pay the less-informed party its opportunity costs and then take the residual as their payoff, essentially shifting value from the less-informed party to their firm. Thus, in that variant of the analysis, the maximum relative cost of ambiguity is the difference between what the more-informed manager earns as a result of bargaining, less their costs to bargain, and what the less-informed manager received in the bargain [i.e., for a maximum of '$n_2 \cdot (B - C)$' in this game, but an expected value of 0 under the assumptions and rational play, just as in the static cooperative case][14].

Cases (ii) and (iii) assume that it is rational to deter or entice entry (i.e., that the benefits from deterring entry less its costs must be greater than the costs of allowing entry, or that the relative benefits from enticing mistaken entry, less its costs, must be positive). In the two-period game that we have assumed, the problem is to determine the best signal to send to persuade the less-informed manager to make the mistake desired (i.e., not enter when they should or enter when they should not). While it may be possible in the real and noisy world filled with non-rational decision-makers to send such a signal, it is *not* in the theoretical one; there is *no* credible signal that can be sent (from a rival in a competitive game like this) that a rational, less-informed manager would be persuasively influenced by to act in the intended direction (it is simply 'cheap talk', e.g., Farrell 1987). As such, because signaling does not work, the same possible costs of ambiguity occur in the dynamic competitive case(s) as in the static one(s).

In sum, our analysis of dynamic games involving another sentient player reveals similar worst-level costs of ambiguity as the *static* cooperative and competitive games did. That said, the relative costs are likely reduced in the dynamic cases for two main reasons: there may be added bargaining costs that the more-informed firm absorbs and there may be credible and truthful data and signals provided by the more-informed firm that the less-informed manager can learn from and exploit. We have only considered the simplest interesting dynamic game variants herein as formal manner[15]. We leave the analysis of the wider variety of possible specifications of dynamic games (e.g., that could include more periods of interaction) for future work.

This completes the identification of the various potential costs of ambiguity across *all* of the main strategic decision contexts, as illustrated by applying variations of a general, algebraic base model. We have described *the many ways* that ambiguity can be costly, estimated *how costly* it can be under various measures, and identified *when* it is costly. The addressing of such issues we believe constitutes a new contribution to the strategic decision-making under ambiguity (SDMUA) literature, and one that complements the experimental work there with novel insights emanating from an analytical approach that took ambiguity 'as is' (rather than reducing it to a probability through initial subjective beliefs).

3. Discussion

We have provided a structured conceptual analysis to address a gap in the ambiguity literature in order to answer the research question about the possible costs of holding only the ambiguity level of information in the main strategic decision-making contexts. We embodied ambiguity-as-range in a basic model (and three related variants) of strategic decision-making based on a game-theoretic analysis structure. We applied a logical rather than a behavioral approach to identifying, measuring, and assessing the costs of ambiguity. Our approach offers a complementary perspective on how to deal with ambiguity as given, illustrating the issues with a benchmark model that allows calculations of the VoIs involved. Our analysis reveals the main forms of the costs of ambiguity, when they occur, and how big they are (covering all of the main strategic decision-making contexts); those are new and important contributions to the literature. Figure 1 depicts the path of our analysis and the main results regarding the costs of ambiguity (expressed in VoI terms) in each strategic interaction confronted. Table 1 provides a list of the new insights that our analysis brings to the challenge of confronting ambiguity in its various strategic contexts.

Ambiguity is often debilitating to strategic decision-making because its existence makes determining precise managerial choices over strategic resource allocations impossible. However, when the manager can affect the context (e.g., by influencing the known factors, or by determining how to obtain more precise information about the unknown factor), they can play a non-trivial role in SDMUA. We have described what ambiguity is, when it matters, and why and how much it matters, all in an attempt to be one of the first studies to provide some guidance to managers on how to handle ambiguity-as-given in strategic situations.

Table 1. Summary of *New* Insights about Ambiguity and its Strategic Costs.

• Ambiguity-as-range can be captured in a robust model (game), where:
○ the ambiguity level information, risk level information, and perfect information can be clearly defined
○ the value of information (VoI) can be calculated in terms of a maximum willingness to pay (in both absolute and relative terms)
○ the cost of informational disadvantage (CoID) can be calculated for asymmetric extensions to the base model
• The VoI can be expressed in several components, as:
○ the net benefit (or loss avoided) from having a more precise level of information
○ the added benefit from higher rates of returns on (the different) investments in altering the context, available to more precise levels of information
○ the added benefit from being able to exploit a less-informed rival from having a more precise level of information
• The first estimates of the costs of ambiguity in strategic contexts can be calculated (in absolute and relative terms), where:
○ the conditions of the most costly contexts can be identified
○ the better (even optimal) tactics for managers to take can be identified
• The maximum willingness to pay (to gain more precise information) may exceed the 'maximum value' obtainable from the opportunity (in cases where large avoidable losses loom)
○ this possibility may underlie a new (separable) ambiguity-related bias (or reason for observing the bias)
• Intermediate levels of information precision (lying between the 'standard' levels of ambiguity and risk) may exist and provide value in specific contexts
• It can be formally proposed that, when managers cannot adjust their landscapes (e.g., alter C), the returns from adding information precision levels are diminishing
• It can be formally proposed that, when managers can adjust their landscapes, that it is possible the returns from adding information precision levels are increasing
• There can be additional costs to ambiguity in the form of learning efforts when the context is dynamic and it appears possible to the manager that characteristics about the unknown given distribution (underlying the ambiguity) can be uncovered through experimentation (e.g., limited sampling) over time
○ the learning algorithms applied in such cases resemble 'stopping rules'
• Extensions to the base model problem that involve playing against a (better-informed) rival include both cooperative and competitive possibilities, and the possibility that ambiguity also entails added relative costs (i.e., where the effects on the rival are accounted for as well)
○ in competitive versions, the relative costs can double the original costs of ambiguity
○ in cooperative versions, the relative costs may be zeroed under favorable assumptions over how and when partner assets are applied and combined
○ in dynamic competitive versions, lasting over two-periods, and with restrictive per-period capacity constraints, and free and true observations of rival moves, it is likely that the costs of ambiguity are reduced as credible data and signals from the rival improve a less-informed manager's ability to learn and choose well.

While we have provided several contributions to the SDMUA literature, we also understand that our approach involved several limitations. For example, we did not model ambiguity-related utility functions, although we expect that different people, and people in different roles, will differ in their attraction to a given range of future possibilities. We only considered the strategically interesting problems for ambiguous contexts here; therefore, that selection of cases may overstate the importance of ambiguity-as-range in the real world[16]. We limited our consideration of ambiguity to its costly forms[17], choosing to focus on its monetary effects in our analysis. Ambiguity that extends to non-payoff-related dimensions, or even to *ex ante* unknown dimensions, may be interesting to explore in the future.

Given the limitations of our conceptual study, we understand that its insights complement rather than substitute for what has been revealed in alternative types of studies on SDMUA. In practice, managers often fall back on a number of different approaches to rational decision-making when confronted by ambiguity, such as using subjective probabilities (e.g., Schmeidler 1989), delaying the decision until more information is known (e.g., Gader et al. 1995), or taking a system-monitoring approach (e.g., March 1994). A literature exists that considers addressing ambiguity in ways different from our approach, such as through mitigating the downsides of the ambiguous issue, identifying the maximin alternative, seeking insurance, or taking an option to exit (e.g., Cerreia-Vioglio et al. 2013; Gollier 2014; Machina 2014). Another branch of the ambiguity literature seeks to model how decision-makers act behaviorally and applies forms of subjective expected utility (SEU) theory (e.g., Savage 1954) to do so. There are several functional forms of SEU put forth in that research, with most appearing as variants of prospect theory (e.g., Kahneman and Tversky 1979; Tversky and Kahneman 1992) where individuals place subjective beliefs on events, apply weights to those probabilities depending on the levels and outcomes involved, and combine those weighted subjective probabilities with a utility function that is most often also weighted by contextual information.

Given the results of our analysis (and other analyses) that identify the many costs of ambiguity, we can understand why approaches for reducing it are of high interest. For example, we can appreciate the attraction of policies that support entrepreneurial actions by decision-makers who are more tolerant of ambiguity (e.g., Heath and Tversky 1991) when their activities reduce the ranges of focal ambiguous outcomes, or at least shift the lower range bound upwards. This is because those outcomes improve the precision of knowledge for the rest of society to leverage. From our results, we can also see danger for any new industry or technology when it entails a very unbalanced ambiguity range, especially when biased towards potentially big gains. This perhaps explains some of the outcomes from the US boom-to-bust experience with the Web 1.0 in the late 1990s.

Our analysis also provides opportunities for future work (some of which can address the limitations listed). There are standard paths to take in a deep, forward-looking research agenda based on our analysis. Along the *theoretical* path: the propositions could be tested logically and then transcribed into hypotheses by specifying appropriate empirical indicators (Dubin 1969); contingencies could be added to the models for missing factors; and several further variants of the four game types (e.g., variants of the dynamic competitive games against rivals) could be specified and analyzed in order to obtain more precise and wider-ranging estimates of the costs of ambiguity. Along the standard *empirical* path: the propositions-as-hypotheses could be tested in the field using surveys of firms' past strategic decisions and actions and their outcomes. Along the *experimental* path: every game we modeled could be tested with human subjects in a lab setting to identify the better choices and heuristics (e.g., to find the stopping rule with the highest performance in the dynamic game against nature) and the conditionality of those behaviors on game factors (e.g., on the range of the ambiguity, on the asymmetry of the range, on the relative values of C and B, and so on), and to estimate the actual willingness to pay for more precise information at differing levels of information. Additionally, the games could be used to identify utility functions, preferences, and heuristics that real decision-makers apply when facing ambiguity (e.g., in order to tease out whether there exists the catastrophic-loss avoidance bias that we suggest). Along the *practice-oriented* path: case studies could identify what problems firms consider to be ambiguous (versus risky), what they estimate it costs them, and the options that they use to address the challenge that ambiguity poses to their standard decision-making processes.

Regardless of the specific path taken though, initiating further research projects that address these limitations and that model particular examples of the challenges of strategic decision-making under ambiguity is sure to provide insights for significantly increasing entrepreneurial, organizational, and economic efficiencies.

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Notes

1 In this paper, ambiguity is the Ellsbergian (1961) type. It is a form of Knightian uncertainty where only the *range* of possible future outcome-states is known; nothing else about the distribution of probabilities for any outcome-state is known.

2 This perspective differs from the mainstream, experimental lens that instead seeks to explain human behavior in lab settings, focusing on the heuristics used and biased revealed. The approach we use here could be labeled more as 'rational'—borrowing from the game theory versus behavioral theory dichotomy, although that would depend on one's definition of rational (Gilboa and Schmeidler 2001).

3 Note that we are not suggesting that every ambiguity problem is represented in our basic model, just as not every game is represented by a prisoner's dilemma model; what we are suggesting is that new and non-specific insights can emerge from analyzing a single decision model, sufficient to make a contribution to the literature, just as so many game theory papers based on the analysis of single games have in the past.

4 Ambiguity-as-range is a more realistic modeling of what highly uncertain business decisions look like than alternatives. For example, assuming an *un*bounded distribution of outcome payoffs is less credible because infinite losses and gains do not exist in the real world (e.g., due to bankruptcy laws and budget constraints). Assuming either unknowable possible outcomes or their payoff levels is also less realistic, as their modeling is equivalent to the unbounded distribution alterative. (Note that other ambiguity range types can exist, but probability is the most researched—Aggarwal and Mohanty 2021).

5 Other strategic decision-making problems exist under ambiguity that we do not consider here, given the focus here is on costs of ambiguity. For example, if *all* players in a competitive game were equally ignorant about the outcomes, then there are standard heuristics that work effectively for the player currently in the lead. The *sailing tactic* of imitating every move of the second place player, who is also making adjustments to the same unpredictable shifting winds, often works to keep the lead.

6 Perhaps the best-known reference to ambiguity is the Ellsberg paradox (Ellsberg 1961), where an urn contains 90 balls, 30 red and the rest (60) either blue or green in color, and experimental subjects are asked to make choices over possible bets on the outcomes of drawing a ball from the urn. The ambiguity is the range from 0 to 60 balls that the non-red balls could span. It should be noted, however, that the paradox actually relies upon a very specific construction of ambiguity—i.e., that a given ambiguity (say, about the green-colored balls) has an *available complement*—the one provided by the blue-colored balls—providing a way to reduce the ambiguity to certainty (e.g., bet on blue-or-green). In this note, we have explicitly assumed that the given ambiguity is irreducible.

7 Note that there is a near-infinite number of alternative specifications of decision problems that involve ambiguity. This is because the range of parameter values across the possible relevant dimensions (e.g., of the payoffs, the outcome states, and the possible choices) of such a problem is large. And that number does not even consider the large array of possible ways to model the bounded rationality of players, or their possible initial beliefs, incentives and decision-making processes.

8 The optimization problem that indicates what probability (p) to play in order to maximize payoffs when playing against nature (which sets its probability of a full urn to 'n_1/n_2') leads to the strategy to set p equal to 1 when '$n_1/n_2 > C/B$', and to 0 otherwise.

9 Such an asymmetry (i.e., of the possibility of a relatively catastrophic loss) may be the basis for an *additional* ambiguity-avoidance bias in human decision-making behavior (e.g., Ellsberg 1961). Besides the standard ambiguity-avoidance bias arising from the discomfort of not knowing the mean of the distribution, and the bias in over-weighting potential regular losses relative to gains arising from risk (e.g., Kahneman and Tversky 1979), and possibly the asymmetrical weighting under ambiguity described by the decision weight model in venture theory (e.g., Hogarth and Einhorn 1990), there may be this additional bias that could be teased out in future lab experiments regarding potential losses that are greater than the apparent stakes.

10 The manager faces a gross cost of '$n_2 \cdot C$' whenever the opportunity is entered but only a gross benefit of '$n_1 \cdot B$'. Given '$n_2 > n_1$', the highest rate of return on altering one of the known factors (either C or B) to the manager with ambiguity-level information comes from reducing C. Note that this investment would *only* be made when the manager believed entering is a good choice.

11 Additional informational precision can provide added benefits in the real world as well. For example, when two receptive product markets or two gushing oil-drilling locations (i.e., two 'full' urns) are geographically close and known ahead of time, then many costly redundancies can be eliminated by sharing activities across the locations (e.g., marketing campaigns, training and refining facilities, transportation hubs). Other scale, scope and synergy economies may also be possible when the locations that pay off (i.e., the 'full' urns) are known ahead of striking them.

12 The learning functions we describe involve 'triggers' that are activated by updated information, mostly to call for a stop to further experimentation. In our simple dynamic model, there is *no time value of money* or scale economies or threats of rivals competing in the landscape. When that is *not* the case, a manager may wish to accelerate the learning (e.g., by making multiple urn-draws per period). This possibility provides a basis for a real-world prediction that a minimal level of consistent initial success in an ambiguous context will more likely lead to an acceleration of *entrepreneurial* action.

13 Lost benefits *to the partner* in cooperative interactions are another potential cost of ambiguity. These occur when the less-informed manager does *not* play the optimal strategy.

14 As with the static cooperative game, there exists an apparent identifiable *optimal* strategy for the less-informed manager to take when there are *only* two firms in the game as we have assumed; and, that is to refuse to take any bargain and then to go all-in. That assures the less-informed manager half of whatever net payoff exists when the more-informed manager correctly enters the opportunity. However, *if* there is more than one less-informed firm in the pool for the one more-informed manager to bargain with, then the more-informed manager is likely to be successful at bargaining and in shifting the less-informed firm's resource benefits to her firm.

15 Ambiguity is always the most costly information level in games against sentient rivals, absolutely and relatively, with those maximum potential relative losses increasing in the size of n_2 and most often in the size of B as well.

16 We have assumed that ambiguity involves a range that is big. Future work could examine at what point the range becomes small enough to be considered 'non-strategic'. This is interesting because the range is actually a collapsible construct that, in its shrinking limit, must end in point precision (i.e., compressing into the discrete expected factor value).

17 Such work may provide further examples of where ambiguity is beneficial. For example, for some specifications of *ignorance*, predictable mistakes that are made based on *not knowing* can actually increase payoffs, relative to knowing more and not making those mistakes.

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