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Using Mixed Probability Distribution Functions for Modelling Non-Zero Sub-Daily Rainfall in Australia

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Abstract: Probabilistic models for sub-daily rainfall predictions are important tools for understanding catchment hydrology and estimating essential rainfall inputs for agricultural and ecological studies. This research aimed at achieving theoretical probability distribution to non-zero, sub-daily rainfall using data from 1467 rain gauges across the Australian continent. A framework was developed for estimating rainfall data at ungauged locations using the fitted model parameters from neighbouring gauges. The Lognormal, Gamma and Weibull distributions, as well as their mixed distributions were fitted to non-zero six-minutes rainfall data. The root mean square error was used to evaluate the goodness of fit for each of these distributions. To generate data at ungauged locations, parameters of well-fit models were interpolated from the four closest neighbours using inverse weighting distance method. Results show that the Gamma and Weibull distributions underestimate and lognormal distributions overestimate the high rainfall events. In general, a mixed model of two distributions was found better compared to the results of an individual model. Among the five models studied, the mixed Gamma and Lognormal (G-L) distribution produced the minimum root mean square error. The G-L model produced the best match to observed data for high rainfall events (e.g., 90th, 95th, 99th, 99.9th and 99.99th percentiles).

Keywords: sub-daily rainfall; ungauged catchment; statistical modelling; probability distribution

1. Introduction

Rainfall data have been used as basic inputs in hydrological, agricultural, ecological and other environmental models and have applications in planning, design and management of water resources [1–5]. Better estimates of runoff and soil infiltration and drainage were achieved when sub-daily, rather than daily rainfall intensities were used as model inputs [6,7]. Studying characteristics of sub-daily rainfall helps understanding incidence of extremely intense events causing flash-flood [8,9]. Sub-daily rainfall is also important in biogeochemical and nutrient cycle modelling [10]. Because of inadequate data availability, the use of sub-daily rainfall is limited. The rain gauge network lacks in continuous long-term records, spatial representativeness and climate homogeneity [11,12]. For example, Australia has only 184 pluviograph stations with coverage exceeding thirty years [13].

Probabilistic models fitted to rainfall data are useful in imputing missing records in terms of length and spatial coverage [14].

Theoretical probability distributions have been used for decades for modelling rainfall data aggregated at coarse scale (seasonal or monthly). The presence of dry events with continuous rainfall demands complex models for even seasonal or monthly data from arid regions [15]. Modelling finer scale (daily to sub-daily) data faces greater challenges because of the presence of a larger number of dry events coupled with a highly skewed distribution of the amount of rainfall in wet events. In contemporary literature, the Gamma, Exponential, Kappa, Wakeby, Generalized Extreme Value (GEV) and Weibull distributions have been used to model daily non-zero rainfall intensities [16–18]. Distributions within the Tweedie family were fitted to model both components (occurrence and amounts of rainfall) simultaneously [19,20]. Gamma for rainfall intensities (three hours) [21,22]. However, a single distribution is found inappropriate for modelling the rainfall intensities at finer timescales, such as daily or sub-daily. Previous studies showed that mixed distributions fit significantly better than a single model [23–25]. However, the possibility of generating rainfall data at ungauged locations using these models were not explored.

The aim of this research is to achieve appropriate theoretical probability distribution to sub-daily rainfall totals from Australian stations. The possibility of using the observed distributions to generate distributions for ungauged locations were also explored [26]. This was done by fitting single (Lognormal, Gamma and Weibull) distributions as well as mixed distributions. Fit of the distributions were compared using the Root Mean Square Error (RMSE), which has the same unit as the observed rainfall. Statistical properties of the observed rainfall amount of studied stations were compared with those obtained through generated data with interpolated parameters from neighbouring stations. The outputs of the research have implications in understanding rainfall characteristics and generating data for ungauged locations and may have potential applications in ecological, hydrological and flood forecasting models.

2. Materials and Methods

2.1. Data

This study considered sub-daily (six-minute interval) rainfall data from 1467 Australian pluviograph stations (Figure 1). Approximately 16.9% of the stations have data for less than one year, whereas, 22.5% of stations have data for more than ten years. A detailed analysis was performed on nine case study stations obtained from both inland and coastal regions. Grey dots on Figure 1 show the rainfall stations included in this study and the black squares shows the locations of case study stations.

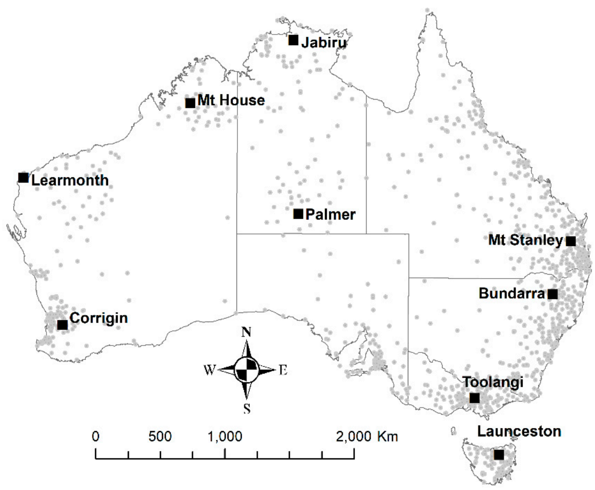

Figure 1. Location of the studied stations (grey dots). Case study stations are named and represented by squares.

Statistical properties of rainfall intensities of the case study stations are presented on Table 1. Column 1 of the table represents the percentages of dry events. The other statistics, such as, the median, mean, maximum, coefficient of variation and skewness were estimated for non-zero rainfall events. Reasonable variations exist among the case study stations in terms of centre (mean, median), extreme values (percentages of non-rainfall events, maximum rainfall), variations (co-efficient of variation) and skewness of the data. For example, among the case study stations, located at the centre and west coast of Australia, Corrigin, Palmer and Learmonth are relatively dry in terms of percentages of dry events (approximately 96%). Stations located at tropical north Australia, Jabiru and Mt House are relatively wet stations in terms of mean (0.49 mm and 0.43 mm) and median (0.10 mm and 0.13 mm) rainfall amounts. The stations were very similar in terms of variations, as measured by coefficient of variation (CV) and skewness, however, the maximum rainfall from a single event is higher in Jabiru (30.89 mm) than Mt House (17.17 mm). Though, the distribution of rainfall amounts from wet events are highly positive skewed, higher mean in rainfall produces lower skewness. As expected, higher differences in mean and median produced higher skewness and also higher coefficient of variation. Launceston, the only case study station from Tasmania (an island located at the south-east Australia) has very similar mean rainfall amounts to the mean for those from east-coast. However, Launceston has fewer extreme events, lower variation and the distribution is less skewed than the other three.

Table 1. Statistical measures of sub-daily rainfall totals for the case study stations.

	% Zero	Median	Mean	Maximum	Coefficient of Variation	Skewness
Corrigin	96.35	0.02	0.12	8.93	286.22	10.05
Mt Stanley	91.56	0.04	0.19	17.53	300.73	9.66
Bundarra	90.56	0.05	0.18	14.89	238.53	9.99
Toolangi	81.87	0.07	0.15	25.04	175.05	11.64
Launceston	89.27	0.09	0.16	9.18	150.72	6.11
Learmonth	95.96	0.06	0.25	14.27	229.61	6.77
Mt House	94.90	0.13	0.43	17.17	224.58	5.59
Jabiru	90.98	0.10	0.49	30.89	243.44	5.58
Palmer	96.98	0.03	0.15	12.35	316.18	11.00

2.2. *Probability Distribution Models*

This study used the Lognormal, Gamma and Weibull distributions as well as mixed Lognormal and Gamma (L-G), Gamma and Weibull (G-W) and (Lognormal and Weibull) distributions. The distribution functions and their properties are briefly described here.

The gamma distribution: Mathematical form of the gamma distribution is expressed as

$$f(x;k,\theta) = \frac{x^{k-1}e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)}, \text{ for } x > 0 \text{ and } k, \theta > 0. \tag{1}$$

where k is the shape parameter, θ is the scale parameter and $\Gamma(\ )$ is called the gamma function and is defined by $\Gamma(k) = \int_0^\infty x^{k-1}e^{-x}dx$. The shape parameter may govern the shape of the rainfall distribution and the scale parameter may determine the variation of rainfall amount series which is given in the same unit as the random variable X.

The gamma family of distributions is related to the exponential and χ^2 distribution of families. In particular, when $k = 1$, the gamma distribution is equivalent to an exponential distribution with parameter θ. Moreover, the $\Gamma\left(\frac{r}{2}, \frac{1}{2}\right)$ distribution follows the χ^2 distribution with r degrees of freedom.

The lognormal distribution: Mathematical form of the lognormal distribution is

$$f_X(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{(lnx-\mu)}{2\sigma^2}}, \ x > 0 \tag{2}$$

where μ and σ^2 are the mean and variance of the normal density for ln$X$ respectively. The natural logarithm transformation is taken as an attempt to symmetrize the data.

The Weibull distribution: Mathematical form of the Weibull distribution is

$$f(x;\alpha,\beta) \;=\; \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}} x \geq 0; \alpha,\beta > 0 \tag{3}$$

where α and β are the shape and scale parameters, respectively. Note that X^{α} is an exponential with mean β. If $\alpha < 1$, the Weibull distribution will have relatively 'heavier tails' than any exponential distribution. If $\alpha = 1$, then X is exponential with mean β. Finally, $\alpha > 1$, results in distributions with relatively 'lighter tails' than any exponential distribution.

2.3. The Maximum Likelihood Estimates (MLE)

Through simple algebra, the log-likelihood for an independently and identically distributed (IID) sample of size n from a gamma distribution can be expressed as:

$$l(k,\theta) = -nk\ln\theta - n\ln\{\Gamma(k)\} + \;(k-1)\sum_{i=1}^{n}\ln x_i - \frac{1}{\theta}\sum_{i=1}^{n} x_i$$

This gives the score equations as:

$$\frac{dl(k,\theta)}{dk} \;=\; -n\ln\theta - n\frac{\Gamma'(k)}{\Gamma(k)} + \sum_{i\,=\,1}^{n}\ln x_i \;=\; 0$$

$$\frac{dl(k,\theta)}{d\theta} \;=\; -\frac{nk}{\theta} + \frac{1}{\theta^2}\sum_{i\,=\,1}^{n} x_i = \; 0$$

The mean and variance of the distribution can be expressed as:

$$\hat{\mu}_{MLE} = k\theta, \quad \hat{\sigma}^2_{MLE} = k\theta^2$$

For the Lognormal distribution, the MLE of the parameters are as follows:

$$\hat{\mu}_{MLE} = \;\overline{\ln x} = \;\frac{1}{n}\;\sum_{i=1}^{n}\ln x_i$$

$$\hat{\sigma}^2_{MLE} = \;\overline{(\ln x)^2} - \;\left(\overline{\ln x}\right)^2 \;=\; \frac{1}{n}\;\sum_{i=1}^{n}(\ln x_i)^2 - \left(\frac{1}{n}\sum_{i=1}^{n}\ln x_i\right)^2$$

Through some simple algebra, the log-likelihood for an IID sample of size n from a Weibull distribution is $l(\alpha,\beta) \;=\; n\ln\alpha - n\ln\beta + (\alpha-1)\sum_{i=1}^{n}\ln x_i - \frac{1}{\theta}\sum_{i=1}^{n} x_i^{\alpha}$ This gives the score equations as:

$$\frac{dl(\alpha,\ \beta)}{d\alpha} = \;\frac{n}{\alpha} + \sum_{i=1}^{n}\ln x_i - \;\frac{1}{\beta}\;\sum_{i=1}^{n} x_i^{\alpha}\ln x_i \;= 0$$

$$\frac{dl(\alpha,\ \beta)}{d\beta} = \;-\;\frac{n}{\beta} + \;\frac{1}{\beta^2}\sum_{i=1}^{n} x_i^{\alpha} = 0$$

These equations do not have a closed form, and hence cannot be solved directly. Alternatively, complex computational algorithm is used to solve the equations.

The mixture distributions: The single distributions may not capture well the lower and heavy upper tails of observed rainfall totals at finer timescales (six-minute pluviograph). To capture both the tails of the distribution of positive sub-daily rainfall intensities, along with single models, mixture of the distribution with the following form have been fitted:

$$\phi\ (x) =\ q_1 f_1(x) +\ q_2 f_2(x)$$

where, q_1 and q_2 are respective weights for the two distributions with probability functions, respectively. One of the distributions is lognormal and the other is the gamma or the Weibull. The fit of the models has been evaluated using the RMSE. The RMSE measures the differences between the modelled values and the observed rainfall totals using the formula:

$$RMSE =\ \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_i -\ y_i)^2}{n}}$$

Once the optimal distribution(s) for modelling rainfall is decided upon, the study examined the performance of the mixture models (G-L and W-L) in terms of generating extreme rainfall events. For the purpose, 1000 samples each of the size of the observed data were generated using the G-L and W-L distributions. The statistics, 90th, 95th, 99th and 99.9th percentiles, for each of the samples were estimated as a presentation of extreme events. For data from a specific station, the median of each of the percentiles over the samples were estimated. These statistics from simulated data were compared with respective statistics from the observed dataset.

The possibility to utilize the parameters of optimal distribution(s) from neighbouring stations to generate data for ungauged locations were also explored. This is done as the contemporary literature indicates that, climate data from close neighbours are highly correlated and the degree of correlation becomes weaker as the distance increases [14]. The parameters of the distributions for the ungauged locations were interpolated from four closest neighbouring stations using inverse weighted distance method. The method works well only when the parameters are spatially consistent, and the targeted location has close neighbours. Figure 1 shows that, except for the middle and south-west parts of Australia, the rainfall gauges have close neighbours. However, the regions with less density are not of much hydrological or ecological importance. Hence, generating data for ungauged locations with the interpolated parameters form the neighbouring gauges is applicable for most regions of Australia with significant agriculture and economic activities. This was done by applying the method to generate data for the gauged locations and comparing these with the observed data for the location. This is done in four steps. First, the parameters were interpolated for the specific location with the estimated parameters from neighbouring locations using inverse weighting distance method. Then, using the interpolated parameters of G-L and W-L distributions, data were generated for the location. Similar percentiles as used in drawing the previous figure were estimated for the observed and generated data. Finally, the percentiles for observed and generated data were presented.

3. Results

The results from the gamma, Weibull and lognormal fit to positive rainfall intensities from nine Australian pluviograph stations have been presented on the plots in Figure 2. For better visualization, normalised probability is plotted against the logarithm of observed rainfall data and fitted data using the models mentioned above. Variations in the fit exist based on the choice of the model or nature of the case study station. For example, the gamma models fit relatively well to the data from wet stations of Northern Australia (Jabiru and Mt House). Whereas, among the three models considered in the study, log-normal distribution fits better to the sub-daily rainfall data from the case study stations located at east coast and Tasmania. For the dry stations, Palmer and Corrigin, gamma distribution overestimate and Weibull distribution underestimate the average rainfall intensities. In general, the gamma and Weibull distributions systematically underestimate extreme rainfall events. The Lognormal distribution fits reasonably well to most parts of the data; however, has a tendency of overestimating the extreme events. Hence, all the distributions considered in the study has specific limitation in modelling sub-daily rainfall intensities of Australian stations.

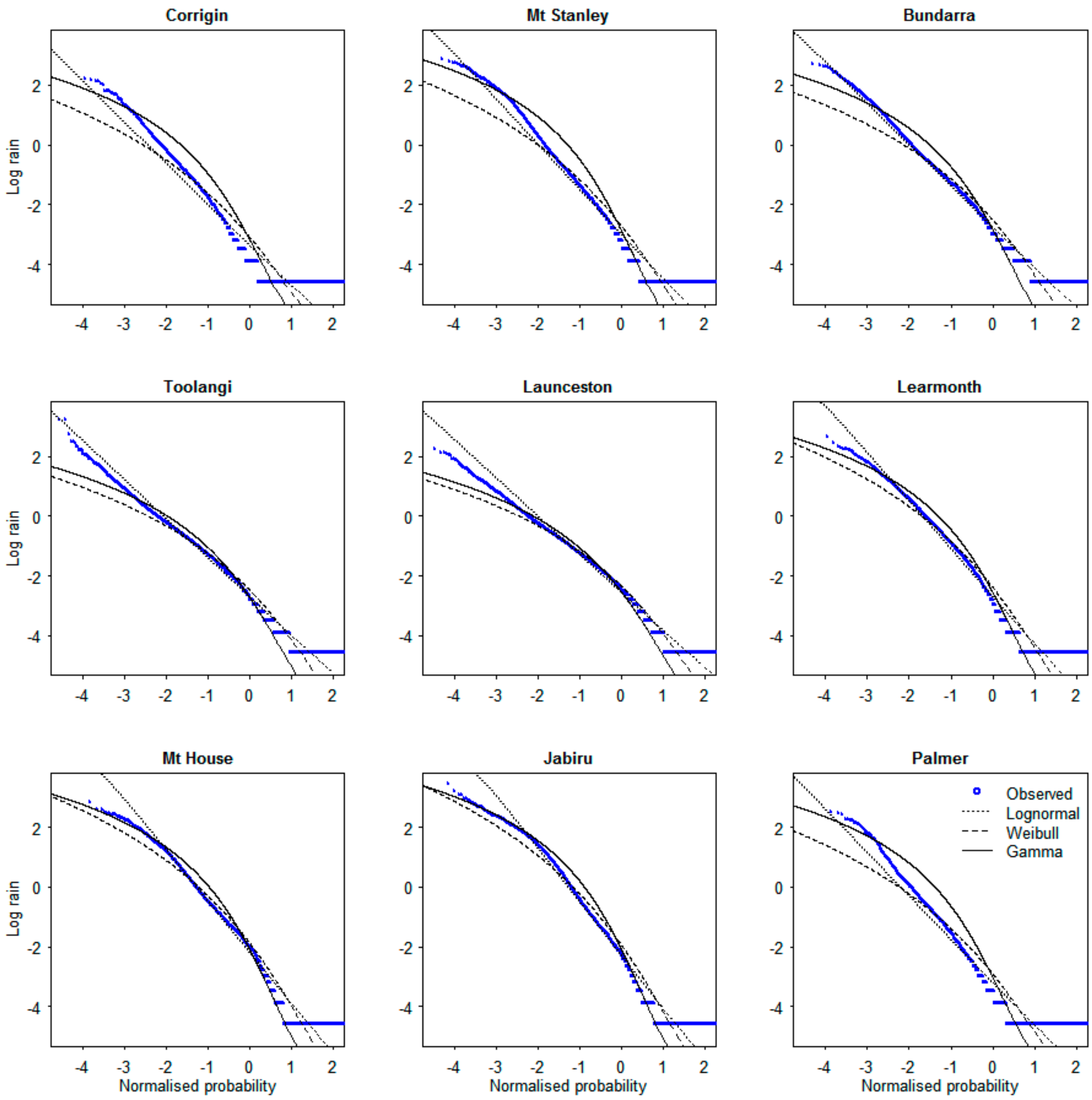

Figure 2. Logarithm of rainfall verses normalised probability of observed and modelled rainfall using Gamma, Weibull and Lognormal distributions.

To overcome the limitation, the study explored the possibility to utilize the mixtures of the distributions. For the purpose, the mixture of Weibull-lognormal (W-L) and Gamma-lognormal (G-L) distributions are used. The fit for all single (Lognormal, Gamma and Weibull) models is compared with the mixture (W-L and G-L) models. This is done for the sub-daily rainfall intensities from all the studied 1467 Australian pluviograph stations. To achieve the optimal mixture distributions, the single location parameter (as estimated from the fit of single model) was considered, however, ten percent variations in the dispersion parameter were allowed. The weights were considered from 0.1 to 0.9 with the sum of weights equals to one. Data were generated with each combination of the parameters and weights. Using the RMSE, generated data for each combination were compared with the observed data. The optimal combinations of parameters and weight were achieved for the minimum of the RMSE. Data of size equal to the number of rainfall events in the historical dataset were generated with the fitted parameters of the single distributions and optimal combination of the parameters for mixture distributions.

The generated data were compared with the observed data using the RMSE statistics. The RMSE for the data generated using various distributions from all 1467 studied stations were presented by the boxplots in Figure 3. Among the single models considered in the study, the gamma distribution had the highest (0.66) and the lognormal distribution had the lowest (0.29) median of RMSE. However, the both distributions produced larger variations (interquartile range (IQR) = 0.61 for gamma and 0.54 for lognormal) than the Weibull distribution (IQR = 0.21). The median RMSE for W-L and G-L models were 0.06 and 0.04, respectively. The IQR for the mixture distributions are 0.11 and 0.04 for W-L and W-L distributions, respectively.

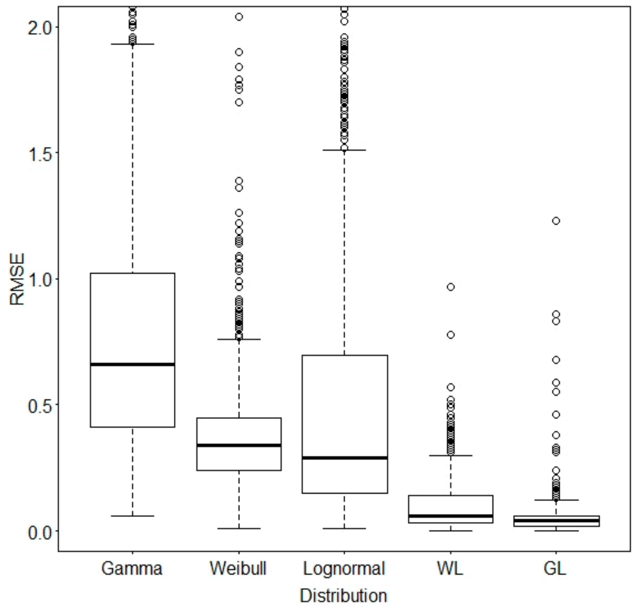

Figure 3. Root Mean Square Error (RMSE) of observed versus simulated rainfall using single and mixed distributions. The boxes represent the interquartile range (IQR), the dotted lines are extended to 1.5 times of IQR, the circles represent observations outside the dotted lines (extreme observations).

The fit of the two mixed models has been compared based on their performance in generating data with similar statistical properties. The observed and simulated statistics for all the 1467 stations were presented in the scatterplots in Figure 4. Two scatterplots compare the performance of G-L (left panel) and W-L (right panel) models. The solid lines in the figures represent the exact fit, that is when the generated data produce the same statistics as those of the observed data. Points close to the dotted line indicate that the statistics obtained from the generated and observed data are very similar. The left panel of Figure 4 represents the relationship between the statistics from observed data and the data generated using G-L model. A higher spread in the points have been observed for the W-L model (the right panel). This indicate that the G-L model simulates non-zero, sub-daily rainfall intensities better than the W-L model. The G-L model fit is equally good in generating data for all the percentiles considered in this study. The exception is for the middle values for which the model overestimates the observed rainfall intensities. Alternatively, the W-L distribution showed a curvilinear relationship. The model overestimated the lower to middle and systematically underestimate the higher rainfall intensities. The results indicate that the G-L model is better than the W-L model in generating non-zero sub-daily rainfall intensities.

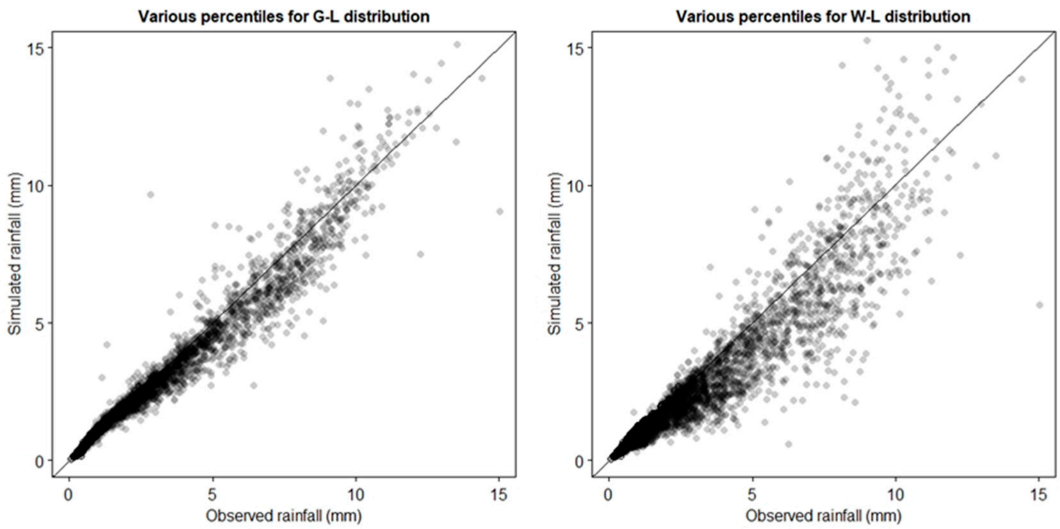

Figure 4. The scatter plot presenting various percentiles of observed and simulated (by fitted parameters) sub-daily rainfall.

Finally, the possibility of generating parameters at ungauged locations has been explored. For this purpose, parameters were generated for the locations having pluviograph station. The observed percentiles are then compared with similar percentiles of data generated with the parameters obtained through spatial interpolation. The performance of spatial interpolation of parameters for respective mixture distributions are presented in the scatterplots of Figure 5. From the figure, we observe that, the percentiles of generated data using the G-L distribution are close to those from observed data. No systematic variation in the scatterplot was observed. This result indicates that G-L model can be used to generate reasonably well sub-daily rainfall data for ungauged locations. Right panel of the figure representing the percentiles of observed data and simulated data using W-L data showed relatively higher scatter. The W-L model systematically underestimates various percentiles representing the extreme rainfall events. Hence, within the models considered in the study, the G-L models were proven ability to generate sub-daily rainfall totals of the Australian stations.

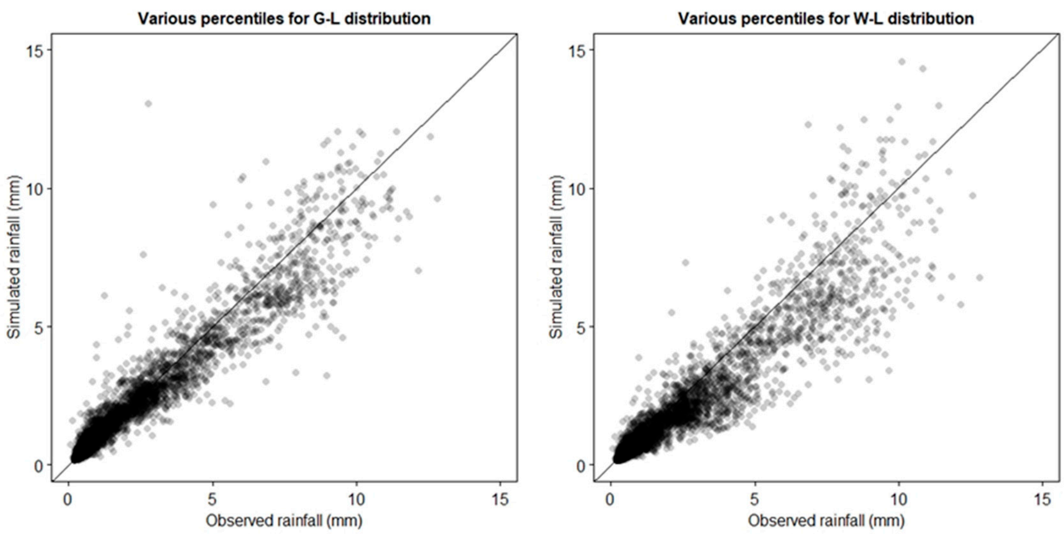

Figure 5. The scatter plot presenting various percentiles of observed and simulated (with interpolated parameters) sub-daily rainfall.

4. Discussion

The Gamma, Lognormal and Weibull are commonly used theoretical distributions in modelling skewed data with heavy tail at the right [17,18]. None of these single distributions, however, are capable of modelling both lower and upper parts of the non-zero sub-daily rainfall totals; though the models fit relatively better for the high rainfall stations. One key finding is that the Gamma and Weibull models systematically underestimate and the Lognormal overestimates the extremely high rainfall events. In case of low rainfall events, the Gamma and Weibull models overestimate and the Lognormal model underestimates sub-daily rainfall. This is one of the main limitations of individual probability model in predicting sub-daily rainfall and the results are supported by the previous studies [27].

The mixed models (e.g., W-L, G-L) fits observed rainfall better compared to any single model. While there are not many differences between mixed models, the G-L model performs slightly better compared to the W-L model. The performances of mixed models in predicting sub-daily rainfall for ungauged catchments using parameters from neighbouring catchments are reasonably good. Again, the G-L model was found to be the best to generate sub-daily rainfall for ungauged catchments. It is important to note that shorter data period may lead to bias in the estimated parameters. Moreover, there should be a common data period to compare the fitted parameters between stations.

5. Conclusions

The research is designed to search for well-fit models for sub-daily rainfall data (skewed with long tail at the right) using single and mixed probability distribution functions and to evaluate the fitted models for predicting rainfall in ungauged catchments. The study used data from 1467 rainfall stations across the Australian continent and tested three commonly used probability distribution functions (Gamma, Lognormal and Weibull). Results show that none of the single distribution models are capable of modelling tail end rainfall totals (both low and high rainfall events). The predictions are, however, reasonably good for the gauges with high rainfall totals. In general, Gamma and Weibull models underestimate and Lognormal model overestimates extremely high rainfall events, and the Gamma and Weibull models overestimate and the Lognormal model underestimates low rainfall events. In comparison to a single probability model, all mixed models (e.g., G-L, G-W, W-L) produce better fit to historical rainfall data. The parameters obtained from the fitted model can be used to characterize rainfall across the region. The framework developed as a part of this study, could be used in generating sub-daily rainfall at ungauged locations. However, while generating data at ungauged locations using information from neighbouring stations, it is important to consider the distances of neighbouring stations and their intercorrelations. If the locations of the neighbours are far enough to invalidate their relation, the generating data may be misleading. For the rainfall gauges located at middle and south-west parts of Australia do not have neighbours close enough to generate reasonably valid rainfall data for a station. For other parts of Australia, the stations have close neighbours to interpolate the parameters to the ungauged locations form the gauged locations, and consequently, to generate data to ungauged locations.

The research has developed a knowledge base to understand the probabilistic features of rainfall in Australia at finer timescale. The understanding may be transferred to the physical properties of rainfall that have implications in understanding catchment level hydrology, ecology and occurrence of extreme events. The model outputs can be used basic inputs in a wide range of environmental models. They may have applications in planning, design, and management of water resources.

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