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Supplemental Information

## Coordination of Candida Albicans Invasion and Infection Functions by Phosphoglycerol Phosphatase Rhr2


(A)

(B)

Figure S1. (A) Stress-strain curves for the four different polyacrylamide hydrogel substrate. Four different hydrogels were polymerized where acrylamide was used at $12 \%, 18 \%, 21 \%$ and $24 \% \mathrm{w} / \mathrm{v}$ while bisacrylamide was used at a constant concentration of $0.5 \mathrm{w} / \mathrm{v}$. The legend on right shows markers corresponding to different hydrogels made by using different $\%$ of acrylamide. The horizontal error-bars mark standard deviation in strain values. (B) Young's moduli for the four different hydrogels. Young's moduli were computed as the stress/strain ratios from the data displayed in Figure 1(A). The error-bars mark standard deviation in moduli values.


Figure S2. Fungal burden in intra-abdominal abscesses. Abscesses $>1 \mathrm{~mm}$ in size were used for CFU determination. The plot shows $\operatorname{logCFU} / \mathrm{g}$ of abscesses for the indicated C. albicans strains for abscesses that were harvested 3, 7 and 14 days after infection.

## Expansion of a Cylindrical Tube or Borehole in an Elastic Medium under Internal Pressure

A long cylindrical borehole or tube in a concentric elastic cylinder or an infinite elastic solid can be represented in the relaxed state $(P=0)$ by concentric cylindrical shells in which the innermost shell has internal radius $\mathrm{R}_{\mathbf{0}}$, and external radius $\mathrm{R}_{\mathbf{1}}$ (Figure S 3 ). Upon pressurization of the borehole, the inner radius grows to $r_{0}$ and the external radius grows to $r_{1}$, with purely radial displacement (U) of every material point ( $R \geq R_{\mathbf{0}}, r \geq R$ ). If the material is incompressible, $U(R)$ is completely defined by geometry:

$$
\begin{equation*}
\mathrm{U}(\mathrm{R})=\mathrm{r}(\mathrm{R})-\mathrm{R}=\left(\mathrm{R}^{2}+\mathrm{r}_{0}^{2}-\mathrm{R}_{0}^{2}\right)^{1 / 2}-\mathrm{R} \tag{1}
\end{equation*}
$$

$U(R)$ is greatest at the inner radius, where it equals $r_{0}-R_{0}$, and decreases asymptotically as $1 / R$.


Figure S3. Elastic medium in relaxed state (left) $\mathrm{P}=0$, and in expanded state (right) $\mathrm{P}>0$. In the relaxed state, $\mathrm{R}_{0}$ is the inner radius of the tube.

In the simplest case of an incompressible rubber-like material, Treloar [1-3] derived the principal stresses in terms of the local principal strain ratios. In the case of expansion of a cylindrical tube, it can be shown that the three Treloar principal stresses are the diagonal elements of the stress tensor in which the three principal directions are radial, circumferential, and axial. For the infinite elastic solid, the three principal stresses are:

$$
\begin{align*}
& \sigma_{11 \infty}=-\mathrm{G}+\mathrm{GR}^{2} / \mathrm{r}^{2}=-\mathrm{G}\left(1-\mathrm{R}^{2} / \mathrm{r}^{2}\right)  \tag{2}\\
& \sigma_{22 \infty}=-\mathrm{G}+\mathrm{Gr}^{2} / \mathrm{R}^{2}=\mathrm{G}\left(\mathrm{r}^{2} / \mathrm{R}^{2}-1\right)  \tag{3}\\
& \sigma_{33 \infty}=-\mathrm{G}+\mathrm{G}=0 \tag{4}
\end{align*}
$$

Radial stress is negative since $\mathrm{R}<\mathrm{r}$ for all $\mathrm{P}>0$. Circumferential stress is always positive, for the same reason. Surprisingly, pressurization of the tube causes no axial stress.

From the boundary condition of hydrostatic pressure P on the internal surface, the relation between $\mathrm{R}_{0}, \mathrm{r}_{0}$ and P is determined:

$$
\begin{equation*}
-\mathrm{P}=-\mathrm{G}+\mathrm{GR}_{0}{ }^{2} / \mathrm{r}_{0}{ }^{2} \tag{5}
\end{equation*}
$$

Solving for $\mathrm{r}_{0}$ :

$$
\begin{equation*}
\mathrm{r}_{0}=[\mathrm{G} /(\mathrm{G}-\mathrm{P})]^{1 / 2} \mathrm{R}_{0} \tag{6}
\end{equation*}
$$

This result shows that, for any pressure less than the modulus of elasticity $(P<G)$, a finite expansion will be obtained for a tube of any nonzero internal radius in the relaxed state. If $\mathrm{P} \rightarrow \mathrm{G}$, the expanded inner radius can be made arbitrarily large, implying that runaway inflation may occur. Practically, expansion would be halted by the elastic limit of the material, or freed by material failure. In principle, a "borehole" may be initiated even when $\mathrm{R}_{\mathbf{0}}=0$ by any mechanism that can generate $\mathrm{P}=\mathrm{G}$. Conversely, it is shown by Equation (5) that $\mathrm{P}=\mathrm{G}$ for any finite $\mathrm{r}_{\mathbf{0}}$, when $\mathrm{R}_{\mathbf{0}}=0$. Therefore, for invasion of an elastic solid, generation of internal pressure equal to the modulus $(\mathrm{P}=\mathrm{G})$ allows expansion from $\mathrm{r}=0$ to any internal radius, $\mathrm{r}_{0}$. This is, of course, a reversible (equilibrium) result. Expansion or invasion at a finite rate would require $\mathrm{P}>\mathrm{G}$.

## Invasion Progress after Tube Initiation by a Hyphus

If we consider the hyphus as a cylinder of uniform diameter which grows by extension of the tip cell after each mitosis, we can apply a model originally derived for plant cell growth [4-6] in which the cell wall has an elastic modulus E and a yield stress for irreversible plastic extension, $\mathrm{P}_{\mathrm{c}}$. If the hyphus is invading an elastic solid, it also must expand against the modulus G . As shown above, this requires a pressure $\mathrm{P}=\mathrm{G}$. Under this condition, the invasive growth rate depends on the turgor pressure in excess of $P_{c}+G$;

$$
\begin{equation*}
P_{\text {eff }}=P-\left(P_{c}+G\right) \tag{7}
\end{equation*}
$$

In the case of steady growth under non-nutrient-limited conditions (such as in our eperiments), we make the simplifying assumption that hyphal cells maintain a relatively constant osmolarity and resulting turgor pressure through the cell cycle. In this case, the volumetric cell growth equation developed by Lockhart [4] and extended by Ortega [5,6] can be simplified by dropping the elastic term:

$$
\begin{equation*}
(1 / \mathrm{V}) \mathrm{dV} / \mathrm{dt}=\phi \mathrm{P}_{\mathrm{eff}} \tag{8}
\end{equation*}
$$

In the cell growth equation, $\phi$ is the linear cell wall extensibility above the yield stress. Since hyphal growth occurs essentially along one axis, $(1 / \mathrm{V}) \mathrm{dV} / \mathrm{dt}=(1 / \mathrm{L}) \mathrm{dL} / \mathrm{dt}$ where L is the tip cell length:

$$
\begin{equation*}
\mathrm{dL} / \mathrm{dt}=\phi \mathrm{P}_{\mathrm{eff}} \mathrm{~L} \tag{9}
\end{equation*}
$$

in which case cell elongation is exponential:

$$
\begin{equation*}
L(t)=L_{0} \exp \left[\phi P_{\text {eff }} t\right] \quad t \geq 0 \tag{10}
\end{equation*}
$$

Our observations with invading hyphae show that the invading hyphae have septa, thus they have undergone mitosis. We assume that during invasion, the tip cell elongates, then stops, undergoes mitosis and septates to form a new tip cell of length $\mathrm{L}_{0}$. The new tip cell then continues the growth process. Under our tacit assumption, all the cells in a hyphal chain, except the one at the tip, go to a $G_{0}$ state. Hence, the growth curve for a single hyphal chain would contain a series of exponential "spurts", each of duration equal to the cell-cycle period. An overall plot of hyphal length $v s$. time would look similar to a series of exponential "spurts" arranged in tandem. The mean slope would represent the change in hyphal length per cell cycle period, $t_{c c}$, which is the mean speed of invasion $\left(v_{i}\right)$ :

$$
\begin{gather*}
\boldsymbol{v}_{i}=\left\{\mathrm{L}\left(t_{c c}\right)-\mathrm{L}_{\mathbf{0}}\right\} / t_{c c}  \tag{11}\\
=\mathrm{L}_{\mathbf{0}}\left\{\exp \left[\phi \mathrm{P}_{\text {eff }} t_{c c}\right]-1\right\} / t_{c c} \tag{12}
\end{gather*}
$$

which can be linearized under the growth conditions used in these experiments:

$$
\begin{equation*}
\boldsymbol{v}_{i} \approx \mathrm{~L}_{0} \phi \mathrm{P}_{\mathrm{eff}}=\mathrm{L}_{0} \phi\left(\mathrm{P}-\left(\mathrm{P}_{\mathrm{c}}+\mathrm{G}\right)\right) \tag{13}
\end{equation*}
$$

In our invasion experiments, the invasion depth observed at time " tg " $(=48 \mathrm{~h})$ for the wild-type strain is very nearly equal to the embedded hyphal length:

$$
\begin{equation*}
\mathrm{L}_{\mathbf{h}}=\boldsymbol{v}_{i} \mathrm{t}_{\mathrm{g}}=\mathrm{L}_{\mathbf{0}} \phi\left(\mathrm{P}-\mathrm{P}_{\mathbf{c}}-\mathrm{G}\right) \mathrm{t}_{\mathrm{g}} \tag{14}
\end{equation*}
$$

With a fixed growth period $\left(\mathrm{t}_{\mathrm{g}}=48 \mathrm{~h}\right)$, this relation is a linear equation in $G$, the external elastic modulus:

$$
\begin{equation*}
L_{\mathbf{h}}=\left(\mathrm{L}_{\mathbf{0}} \phi\left(\mathrm{P}-\mathrm{P}_{\mathbf{c}}\right) \mathrm{t}_{\mathbf{g}}\right)-\left(\mathrm{L}_{0} \phi \mathrm{t}_{\mathbf{g}}\right) \mathrm{G} \tag{15}
\end{equation*}
$$

which shows that increasing the modulus $G$ should decrease the observed invasion depth, $\mathrm{L}_{\mathbf{h}}$, and predicts that invasion will fail when $G=P-P c$.

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