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Flexure Strengthening and Analysis Using CFRP Composite and Reactive Powder Concrete

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Abstract: There are many cases of carbon fiber-reinforced polymer (CFRP) and reactive powder concrete (RPC) in structural repair and reinforcement, but there are not many related theoretical and experimental discussions. Therefore, the purpose of this research is to focus on the experiment and theoretical calculation of the flexural strengthening of concrete beam specimens. The study was primarily separated into two parts. In the first part, the laboratory tests of flexure strengthening by using CFRP and two reactive powder concretes (RPC1 and RPC2) are used to evaluate the effect of strengthening concrete beams. Secondly, the test value of the maximum flexure failure load of the strengthening specimen using RPC or CFRP and its theoretical value calculated by the transform section method or the ultimate strength method are discussed and compared with their reinforced effect. The test results show that the RPC and CFRP display excellent repair and retrofit potential. The RPC reinforcement material with a thickness of 2 cm and a steel fiber content of 1.0% is approximately equal to the reinforcement effect of three layers of CFRP. In particular, the RPC reinforcement effect is good, and the bonding interface is not damaged. The transform section method could be used to analyze and calculate the maximum flexure failure load of the RPC strengthening concrete beam. The transform section method and ultimate strength method cannot accurately analyze the maximum flexure failure load of the CFRP-strengthening concrete beam.

Keywords: repair and retrofit; CFRP; reactive powder concrete; beam; theoretical value; flexure strengthening

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1. Introduction

For more than thirty years, with the development of concrete technology, traditional concrete has been greatly improved. France, the United States, and Canada are actively developing ultra-high-performance concrete (UHPC)—Reactive Powder Concrete (RPC) [1–4]. Because RPC does not include coarse aggregates, it was also called high-activity powder mortar in Japan in 1991, or "ultra-high-performance cement-based composite material". The main reason is that this material not only has a compressive strength of 200 MPa, but also a compressive strength of 800 MPa after a special curing process, as well as ultra-high toughness and durability [5–7]. Because the strength, toughness, and durability of RPC far exceeds traditional concrete, it is expected that RPC will be widely used in civil construction and many special projects in the future [8–10].

Taiwan is located in an area with frequent earthquakes. Many structures are affected by the earthquake force that causes concrete cracks or spalling, resulting in serious damage to the structures. In addition, many old structures such as bridge structures and nuclear power plants are affected by environmental factors. The impact caused rapid deterioration of the structure, and it was about to face the problem of demolition, reconstruction, or repair and reinforcement, so the maintenance and reinforcement of concrete structures became more and more important; therefore, the demand for repair and reinforcement of concrete structures is expected to increase rapidly in the future.

In recent decades, CFRP has been widely used in structural reinforcement, mainly because of its light weight, relatively ease of installation, high tensile strength, superior chemical properties, corrosion resistance, and fatigue performance, making it an attractive material in engineering structures [11–14]. In addition, RPC has also shown superior performance in structural reinforcement in recent years [15]. RPC is used as a reinforcing material, and attention must be paid to the bonding between RPC and concrete substrates. Therefore, the roughening treatment of the bonding interface is particularly important. According to the research results of literature [16,17], the shape, arrangement direction, length-to-diameter ratio of steel fibers, and the amount of steel fibers added will affect the strength of concrete. Therefore, it should be ensured that the steel fibers can be evenly distributed during mixing to reduce the entanglement of steel fibers. According to the literature [18–20], the test results of the durability of RPC show that it has excellent wear resistance, chemical corrosion resistance, and neutralization resistance. Especially in harsh environments, RPC is used as a reinforcing material, and its strength will not be greatly affected [21–24].

Reinforced concrete structures or bridge pier structures in Taiwan are often damaged by external forces or the concrete deteriorates due to ambient temperature, humidity, and chemical damage, resulting in insufficient structural bearing capacity. Generally, the reinforced concrete reinforcement method is commonly used to enlarge the column section to increase the bearing capacity, but the quality control of ordinary concrete reinforcement is not easy, and the large internal pores are prone to dry shrinkage and deformation, which reduces the effect of reinforcement and increases the column fracture. In addition, enlarging the section of the column will cause an unsightly appearance and inconvenience in space use [12,13]. Although there are many cases of CFRP and RPC in structural repair and reinforcement, there are not many related theoretical and experimental discussions. In particular, experimental and theoretical studies on flexure strengthening of concrete beams.

Our previous study [15] only evaluated and compared RC beams retrofitted with RPC or CFRP. The main test items and methods are as follows: (1) The feasibility of RPC as a reinforced material was evaluated using basic mechanical tests. (2) Discuss the retrofit effects of making small concrete beams and using RPC or CFRP. The results of the study show that the ultimate load increases by 35% and 56.6% when retrofitting reinforced concrete beam members with RPC or CFRP, respectively. However, theoretical analysis and verification have not been performed. Therefore, the purpose of this study is to perform experiments and basic theoretical calculations on the flexural strengthening of concrete beam specimens reinforced with CFRP and RPC. This continuous research can provide a reference for the reinforcement technology and design of concrete structures.

2. Materials and Methods

This study uses CFRP composite or RPC material for the flexural strengthening of concrete beams. The main test was divided into two parts. Firstly, the flexural strength properties of RPC with different fiber contents are tested. Second, ordinary concrete beam specimens undergo flexural strengthening with two kinds of RPCs (fiber content 0.5% and 1.0%) or CFRP patches. More specifically, this was done to test their flexural strength, compare the strengthening effect of the two reinforcing materials, and discuss the calculation and experimental value of the theoretical value of the flexural strength.

2.1. Raw Material

The cements used in RPC in this study are Portland Type I and Type II cements produced by Taiwan Cement Company. The heat generation rate is relatively slow. In Taiwan's highly corrosive environment, Type II cement has the characteristics of moderate sulphate resistance, which contributes to the durability of the material. Its chemical composition and properties are shown in Table 1. The silica fume is imported BIAXIS SF, with a particle size of 0.1–0.2 µm. Its chemical composition and physical properties are shown in Table 1. Portland Type II cement has a lower heat of hydration than Type I cement, as well as low alkali content and moderate sulphate resistance. Silica fume, a fine powder with high silicon dioxide content (SiO_2 content above 90%), can be used as a high-strength concrete admixture to replace part of the cement.

Table 1. Chemical composition and properties of cement and silica fume.

Chemical Composition and Properties	Cement I	Cement II	Silica Fume
SiO_2	20.1	19.3	90
Fe_2O_3	2.4	2.3	1
Al_2O_3	5.4	4.3	1
CaO	62	61.1	0.4
MgO	2.6	2.4	1
SO_3	5.0	3.0	2
L.O.I	2.3	0.9	3
Specific surface area (m^2/g)	0.37	0.37	20
Specific gravity	3.15	3.15	2.2

The reinforced concrete beam specimens were ordinary concrete (PC) with a design compressive strength of 20 MPa at 28 days of age. PC uses type I cement, and RPC1 and RPC2 use type II cement. The Mix design of RPCs and PC is shown in Table 2. The steel fiber content of RPC1 and RPC2 is 0.5% and 1.0% of the volume content, respectively. The mechanical behavior of RPC is closely related to the amount of steel fiber content. Its flexural strength tends to increase significantly with the increase of steel fiber content. However, the higher the fiber content, the higher the cost and tendency to cause balling. Therefore, choosing 0.5% and 1.0% content is more economical and can maintain the workability. The copper-plated fiber on the surface of the steel fiber is golden yellow, and it is a straight steel fiber. It adopts Dramix OL 13/.20HC steel fiber produced in Belgium, with a density of 7850 kg/m^3, a diameter of 0.25 mm, a length of 13 mm, and an aspect ratio of 65.

Table 2. Mix design of PC and RPC (kg/m^3).

Mix	Cement	Coarse Aggregate	Fine Aggregate	Silica Fume	Quartz Powder	Steel Fiber	Superplasticizer	Water
PC	342 (1)	926	785	0	0	0	0	222
RPC1	720 (2)	0	900	216	252	40	71.3	133.7
RPC2	720 (2)	0	860	216	252	80	71.3	133.7

Note: (1) Portland Type I Cement, (2) Portland Type II cement.

A company in Taichung provided the CFRP used in this test and the model was ordinary AEC200. The physical properties and tests of CFRP materials are shown in Table 3. CFRP material is a commonly used forward-looking repair and strengthening material. The elastic modulus and tensile strength of the CFRP sheet are 230,456 MPa and 470 MPa, respectively. A company in Taichung also provided epoxy resin. Epoxy resin is divided into two types: main agent and hardener. Its physical properties and test indicators are shown in Table 2. Epoxy resin was prepared according to the weight ratio of the main agent:curing agent = 2:1, stirred with a mixer, made into the two mixes evenly and applied on the concrete surface, and pressed closely on the carbon fiber board to strengthen the concrete components.

Table 3. Mechanical properties of CFRP sheet and epoxy resin.

Materials Properties	Epoxy	CFRP
Areal weight (g/m^2)	-	200
Thickness (mm/ply)	-	0.11
Ultimate tensile strain (%)	-	1.7
Consistency	Non-sag	-
Viscosity (25 °C)	4725 mPa·s	-
Compressive strength	94 MPa	-
Elasticity modulus	2258 MPa	23,456 MPa
Tensile strength	42 MPa	470 MPa

2.2. Test Specimens and Methods

The preparation and manufacture of the concrete beam specimens in this study are complicated. Firstly, after the concrete beam specimen was cured for 28 days, the PC beam specimen was reinforced with RPC and CFRP materials, respectively. Among them, RPC adopts two mixed proportions (RPC1 fiber content 0.5% and RPC2 fiber content 1.0%). Seven days after the reinforcement was completed, the bending test of the concrete beam specimen was carried out. At the end of the experiment, the calculation and experimental value of the theoretical value of the flexural strength of the reinforced material can be checked. The composite concrete sample after reinforcement was calculated by the conversion section method or ultimate stress method of material mechanics. It can withstand the maximum damage load and compare the difference between the calculated maximum damage load theoretical value and the experimental value.

2.2.1. Test Plan

The test plan of this study is mainly divided into two parts. The first is PC concrete sample production. After the PC concrete sample is cured, it will be reinforced with CFRP and RPC materials.

The second is to carry out the concrete beam flexural test, and to reinforce the RPC and CFRP materials with different contents. Bending loads tested the effect of PC concrete samples modified with RPC and CFRP materials. The test flowchart of the beam retrofitted with CFRP and CFRP is shown in Figure 1. Samples of three different composite laminates named CFRP1, CFRP2, and CFRP3 were tested. They are respectively coated with 1 to 3 layers of CFRP sheet with a unit area weight of 200 g/m^2 and coated with Guosen epoxy resin on both sides. Each of these layers is approximately 0.5 mm thick.

2.2.2. Fabrication of Concrete Samples Retrofitted with RPC or CFRP

The fabrication of concrete samples retrofitted with RPC or CFRP can be explained in the following:

(1) Fabrication of concrete samples retrofitted with CFRP

The concrete sample is made by PC concrete into a 10 $\times$ 10 $\times$ 35 cm bending test specimen. After removing the formwork, it is cured in saturated lime water for 14 days. After curing, the test specimen is placed in the air for more than 24 h, and the specimen is dried. After that, the CFRP patch can be applied; the flexural test specimen is covered with CFRP patch as shown in Figure 2, which is divided into one layer, two layers, and three layers of reinforcement. The test age of the specimen was 14 days after the CFRP coating was completed, and the flexural strength test was carried out.

(2) Fabrication of concrete samples retrofitted with RPC

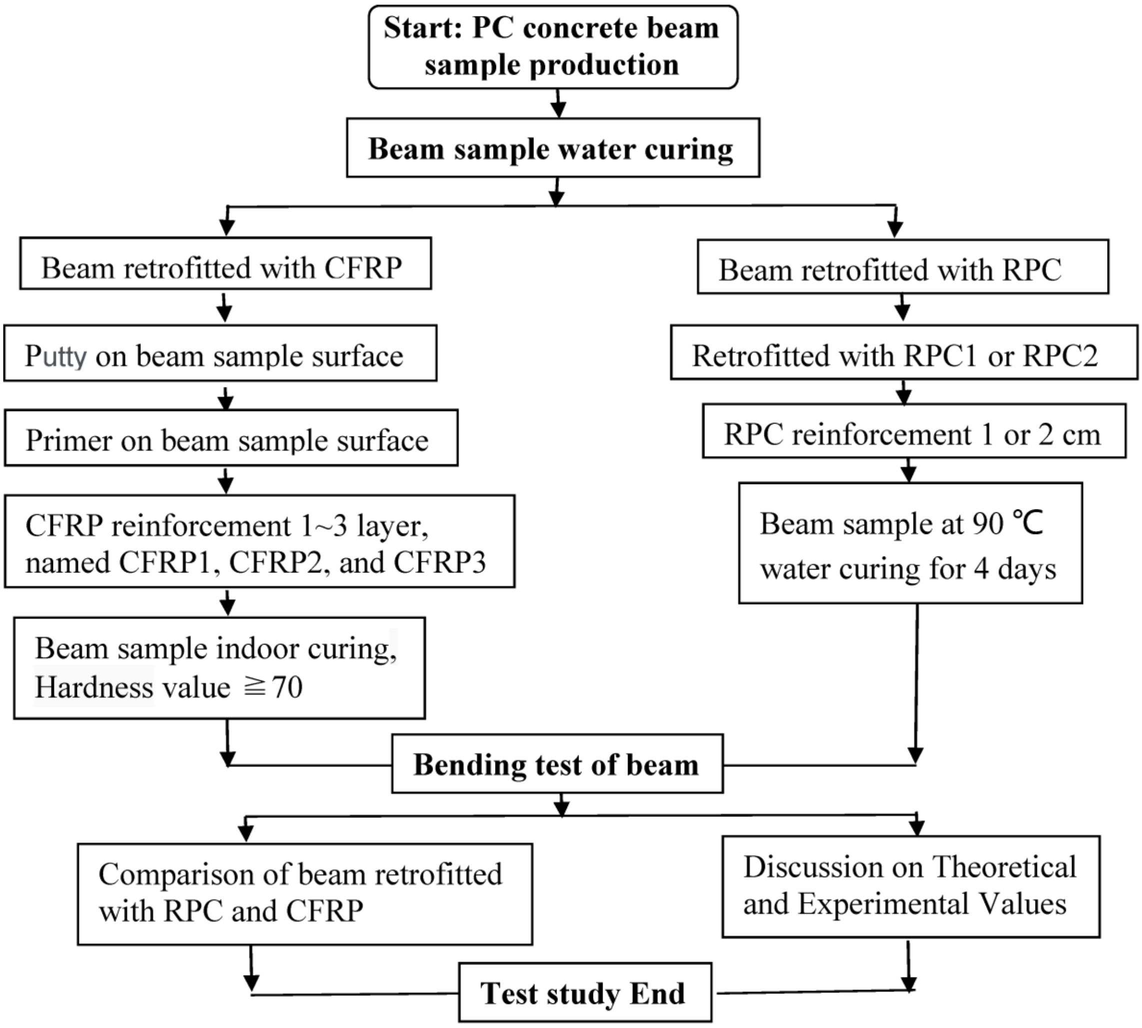

Figure 1. Test flowchart of beam retrofitted with RPC and CFRP.

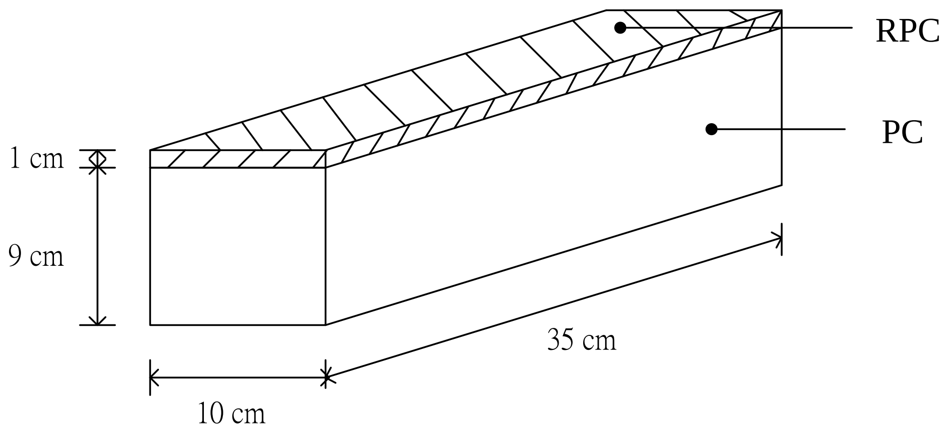

Figure 2. Reinforced concrete sample retrofitted with RPC or CFRP.

In the production of the concrete sample retrofitted with RPC, the specimen of 8 × 10 × 35 cm or 9 × 10 × 35 cm is poured with PC concrete. After the formwork is removed, it is cured in saturated lime water for 14 days. A layer of 2 × 10 × 35 cm or 1 × 10 × 35 cm RPC1 or RPC2 was poured on top of the specimen to bond it into a 10 × 10 × 35 cm concrete sample, as shown in Figure 2.

After removing the mould from the hard solid, it was put in water at 90 °C for curing. The test age of the test specimen is 14 days after the RPC pouring is completed, and then the flexural strength test is carried out.

3. Results and Discussion

The test results and discussion of this study include (1) RPC flexural strength test, (2) RPC and CFRP concrete beam reinforcement tests, (3) theoretical calculation and experiment of bending strength of reinforced PC beams and discussion of the differences in their reinforcement effects, as well as checking the hardness value of early CFRP above 70.

3.1. Flexural Strength of RPC

Figure 3 shows the flexural strength test results of RPC concrete with 0%, 0.5%, 1%, and 1.5% steel fibers respectively. It can be clearly seen from the figure that the flexural strength tends to increase significantly with the increase of steel fiber content. When the steel fiber content increases to 1.5%, its flexural strength is about 2 times higher than that without steel fiber, which shows that the addition of steel fiber can effectively improve the tensile strength and toughness of concrete. The randomly dispersed steel fibers in the RPC concrete matrix disperse the stress and confine the concrete matrix. Possibly due to this mechanism, steel fiber reinforced concrete has improved flexural strength and ductility, and increased energy absorption capacity. The steel fiber parameters that affect the mechanical properties of RPC include volume fraction, size, shape, orientation and distribution, flexural strength and tensile strength, etc., which deserve further study [17].

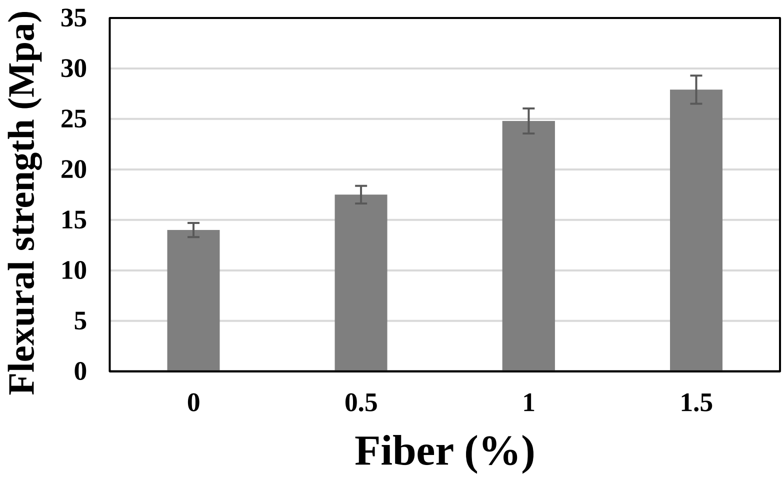

Figure 3. Flexural strength of RPC concretes.

When the ordinary concrete specimen is subjected to the three-point bending test, the specimen will be broken into two parts from the middle loading point, while the RPC concrete increases the toughness due to the addition of steel fibers. There will be cracks, the cracks will slowly extend upwards, and the opening of the cracks will gradually become larger. At this time, you can clearly see the situation where the steel fibers are pulled out in the cracks. When the bending test specimen is damaged, the steel fiber has the binding effect, so that the test body will not be broken into two parts suddenly. Therefore, it can be clearly seen from the results of the bending test that the improvement of the brittleness of concrete by adding steel fibers can be achieved. This can improve the flexural strength of concrete, avoid concrete cracks, and increase the toughness and earthquake resistance of structures. In this study, due to the consideration of the construction property and the cost of steel fiber materials, the fiber content of 0.5% and 1% of the RPC concretes was used in the concrete reinforcement test, which is RPC1 and RPC2 in Table 2, respectively.

3.2. Concrete Beam Reinforcement

Concrete beam reinforcement test is to reinforce PC beam with CFRP patch and RPC, and then test the reinforcement effect of its flexural strength; CFRP patch reinforcement is divided into one layer, two layers, and three layers of CFRP patch on PC beam specimen. For RPC reinforcement, 1 cm and 2 cm thick RPC1 or RPC2 were directly poured on the PC beam specimen, as shown in Figure 2.

3.2.1. Concrete Beams Reinforced with CFRP

It can be seen from Table 4 that the flexural strength of the beam specimen reinforced with one layer of CFRP is about 65% higher than that of the unreinforced (control group), and the flexural strength of the reinforced two-layer CFRP is about 110% higher than that of the unreinforced, the flexural strength of the reinforced three-layer CFRP is about 160% higher than that of the unreinforced one. It is obvious that flexural strength increases with the increase of the number of reinforced layers. When the flexural specimen reinforced by CFRP is damaged, there will be cracks at the loading place of the beam specimen. When the crack becomes larger, it will continue to extend to the bond interface between the CFRP and the concrete, and finally break from the weaker bond interface, causing the specimen to break into two pieces. The beam specimen on the other side is still tightly bonded to the CFRP, while the broken concrete specimen only had damage at the bonding interface with the CFRP, and the CFRP patch did not break, as shown in Figure 4.

Table 4. Flexural strength of concrete beam reinforced with RPC and CFRP at 28 days age.

Reinforcement Type and Sample Number	#1	#2	Mean (Coefficient of Variation)
	(MPa)		
PC control group	4.82	5.67	5.24 (11.5%)
RPC1-reinforced 1 cm	7.55	7.06	7.31 (4.7%)
RPC1-reinforced 2 cm	9.59	10.39	9.99 (5.7%)
RPC2-reinforced 1 cm	8.48	9.18	8.83 (5.6%)
RPC2-reinforced 2 cm	11.12	13.10	12.11 (11.5%)
CFRP-reinforced 1 layer	8.99	8.56	8.78 (3.4%)
CFRP-reinforced 2 layers	11.67	11.24	11.45 (2.6%)
CFRP-reinforced 3 layers	14.00	13.32	13.66 (3.5%)

Figure 4. Failure of CFRP-strengthened beam specimen.

3.2.2. Concrete Beams Reinforced with RPC

The strength development of the beam concrete test specimen reinforced with RPC can be seen from Table 4. With the increase of steel fiber content and repair thickness, its strength increases significantly. Among them, the reinforcement of RPC2 with a thickness of 1 cm is about equal to that of a coating layer of CFRP, reinforced with 2 cm thick RPC1 is also approximately equal to covering two layers of CFRP. Reinforcing with 2 cm thick RPC2 is also approximately equal to covering three layers of CFRP, showing that RPC-reinforced concrete can also obtain good reinforcement effect, as shown in Figure 5.

Figure 5. Failure of RPC-strengthened beam specimen.

When the CFRP-reinforced compression test specimen fails, it will produce brittle failure without warning, and with the increase of the number of reinforcement layers, the brittle failure of the test specimen is more obvious [11–14]. Therefore, this phenomenon should be avoided in the structural reinforcement design. Concrete reinforcement mainly focuses on the bonding problem of the interface between new and old materials and the effect of reinforcement. After observing the damage of the flexural test specimen reinforced with CFRP, it was found that the first and second layers of CFRP reinforcement were finally damaged due to cracks at the bonding interface. Therefore, When CFRPs are used to reinforce beam members, the ends of CFRPs can be anchored with chemical bolts or stiffeners to increase the effect of reinforcement. While RPC is used to strengthen the flexural specimen, it may be broken into two pieces, but RPC can exert excellent bond strength with the concrete matrix, so using RPC as a reinforced material for concrete components can indeed achieve reinforcement effects.

3.3. Theoretical Calculation and Analysis of Bending Strength of Reinforced PC Beams

Here, we calculate the maximum failure load that the reinforced composite beam can withstand by the conversion section method or ultimate stress method of material mechanics and compare the difference between the calculated maximum failure load theoretical value and the experimental value. See Appendix A for the theoretical formulas of material mechanics.

3.3.1. Comparison of Experimental and Theoretical Values of PC Beams Reinforced with RPC

Reinforce the 10 × 10 × 35 cm PC beam with 1 cm thick RPC1, and the span length L = 30 cm. Calculate the maximum failure load P when the PC beam is subjected to a three-point bending test. The basic properties of concrete substrates and reinforcing materials are as follows:

- PC beam: elastic modulus $E_1 = 2.41 \times 10^5$ kg/cm^2, compressive strength $\sigma_1 = 347$ kgf/cm^2, flexural strength $\sigma_2 = 53$ kgf/cm^2, oblique shear strength $\tau = 117$ kgf/cm^2
- RPC1: elastic modulus $E_2 = 3.01 \times 10^5$ kg/cm^2, flexural strength $\sigma_3 = 201$ kgf/cm^2

The shear force diagram and bending moment diagram of a simply supported beam subjected to a P force are as shown in Figure 6.

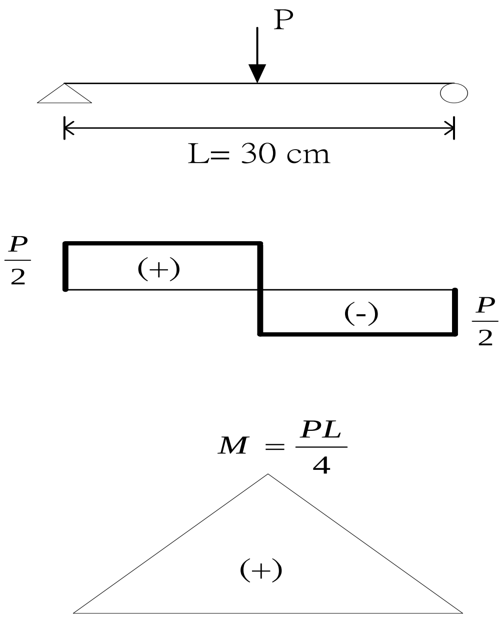

Figure 6. Shear force diagram and bending moment diagram of a simple beam.

The converted section method (shown in Figure 7) is used to calculate the internal stress of the composite beam as follows:

$$n = \frac{E_2}{E_1} = \frac{3.01 \times 10^5}{2.41 \times 10^5} = 1.25 \quad (1)$$

$$\begin{aligned} \overline{y} &= \frac{E_1 h_1^2 + 2E_1 h_1 h_2 + E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)} \\ &= \frac{(2.41\times10^5)\times(9)^2 + 2(2.41\times10^5\times9\times1) + (3.01\times10^5)\times(1)^2}{2(2.41\times10^5\times9 + 3.01\times10^5\times1)} = 4.89 \text{ cm} \end{aligned} \quad (2)$$

$$I = \frac{1}{3} \times 10 \times (5.11)^3 + \frac{1}{3} \times 12.5 \times (4.89)^3 - \frac{1}{3} \times 2.5 \times (3.89)^3 = 882.9 \text{ cm}^4 \quad (3)$$

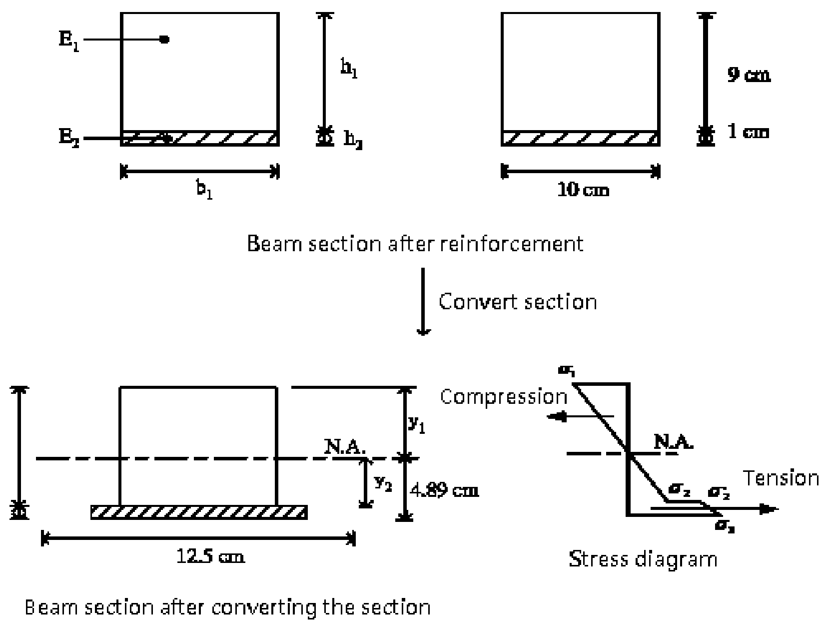

Figure 7. RPC convert section method.

PC concrete subjected to maximum compressive stress: $\sigma_1 = \frac{M_1 y_1}{I}$

$$M_1 = \frac{\sigma_1 I}{y_1} = \frac{347 \times 882.9}{5.11} = 59,954.3 \text{ kg} \cdot \text{cm} \tag{4}$$

PC concrete subjected to maximum tensile stress: $\sigma_2 = \frac{M_2 y_2}{I}$

$$M_2 = \frac{\sigma_2 I}{y_2} = \frac{53 \times 882.9}{3.89} = 12,029.2 \text{ kg} \cdot \text{cm} \tag{5}$$

RPC1 concrete subjected to maximum tensile stress: $\sigma_3 = \frac{n M_3 \overline{y}}{I}$

$$M_3 = \frac{\sigma_3 I}{n \overline{y}} = \frac{201 \times 882.9}{1.25 \times 4.89} = 29,032.8 \text{ kg} \cdot \text{cm} \tag{6}$$

Shear stress at the bonding interface: $\tau = \frac{VQ}{Ib}$

$$117 = \frac{{}^{P}\!/_{2} \times 54.875}{882.9 \times 10} \Rightarrow P = \frac{117 \times 882.9 \times 10 \times 2}{54.875} = 37,648.9 \text{ kg} \tag{7}$$

The maximum failure load is controlled when the PC concrete is subjected to the maximum tensile stress.

$$M = \frac{PL}{4} = 12,029.2 \tag{8}$$

Theoretical value $P = \frac{4M}{L} = \frac{4 \times 12,029.2}{30} = 1603.9$ kg

The experimental value P = 1658.94 kg.

RPC1 reinforced 1 cm increases the strength by 34.9% compared with the unreinforced theoretical value, however, the strength of the experimental value increased by 39.5%.

The above results show that the theoretical maximum breaking load of the reinforced bending test body is P = 1603.9 kgf, which is 34.9% higher than that of the unreinforced bending strength. The bending strength increases by 39.5%, and the maximum failure load of the experimental value is very close to the calculated theoretical value.

In addition, the calculation results are shown in Table 5 for the increase ratio of the maximum failure load theoretical value of the 2 cm thick RPC1 reinforced PC beam, the 1 cm thick RPC2 reinforced PC beam, and the 2 cm thick RPC2 reinforced PC beam. Since the RPC-reinforced concrete is analyzed by the internal stress of the beam in material mechanics, the theoretical value and the experimental value of the maximum failure load can obtain similar results. Therefore, this method is suitable for evaluating the PC beams reinforced with RPC.

Table 5. Increase the ratio of theoretical calculation and experiment of maximum bending strength of reinforced PC beams.

Reinforcement Type	Experimental Value	Theoretical Calculation Value	
		Conversion Section Method	Ultimate Stress Method
RPC1 reinforced 1 cm	39.5%	34.9%	-
RPC1 reinforced 2 cm	90.7%	92.7%	-
RPC2 reinforced 1 cm	68.5%	70.4%	-
RPC2 reinforced 2 cm	131.2%	140.6%	-
CFRP1 reinforced 1 layer	67.3%	2.8%	104.9%
CFRP2 reinforced 2 layers	118.4%	6.4%	310.8%
CFRP3 reinforced 3 layers	160.8%	10.3%	618.2%

3.3.2. Comparison of Experimental and Theoretical Values of PC Beams Reinforced with CFRP

The theoretical value of the maximum failure load of the CFRP-reinforced PC beam was calculated using the conversion section method and the ultimate stress method of material mechanics. A 10 × 10 × 35 cm PC beam is reinforced with a layer of CFRP patch, and the span length L = 30 cm. The maximum failure load P is calculated when the PC beam is subjected to a three-point bending test. The basic properties of concrete substrates and reinforcing materials are as follows:

- PC beam: elastic modulus $E_1 = 2.41 \times 10^5$ kg/cm^2, compressive strength $\sigma_1 = 347$ kgf/cm^2, flexural strength $\sigma_2 = 53$ kgf/cm^2, oblique shear strength $\tau = 117$ kgf/cm^2
- CFRP1: elastic modulus $E_2 = 7.4 \times 10^5$ kg/cm^2, flexural strength $\sigma_3 = 7800$ kgf/cm^2, oblique shear strength $\tau = 100$ kgf/cm^2, thickness $t = 0.3$ mm.

The shear force diagram and bending moment diagram of a simply supported beam subjected to a P force are as shown in Figure 6.

(1) Using the converted section method (shown in Figure 8) to calculate the internal stress of the composite beam is as follows:

$$n = \frac{E_2}{E_1} = \frac{7.4 \times 10^5}{2.41 \times 10^5} = 3.07 \tag{9}$$

$$\begin{aligned} \overline{y} &= \frac{E_1 h_1^2 + 2E_1 h_1 h_2 + E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)} \\ &= \frac{(2.41\times10^5)\times(10)^2 + 2(2.41\times10^5\times10\times0.03) + (7.4\times10^5)\times(0.03)^2}{2(2.41\times10^5\times10 + 7.4\times10^5\times0.03)} = 4.98 \text{ cm} \end{aligned} \tag{10}$$

$$I = \frac{1}{3} \times 10 \times (5.05)^3 + \frac{1}{3} \times 30.7 \times (4.98)^3 - \frac{1}{3} \times 20.7 \times (4.95)^3 = 856.3 \text{ cm}^4 \tag{11}$$

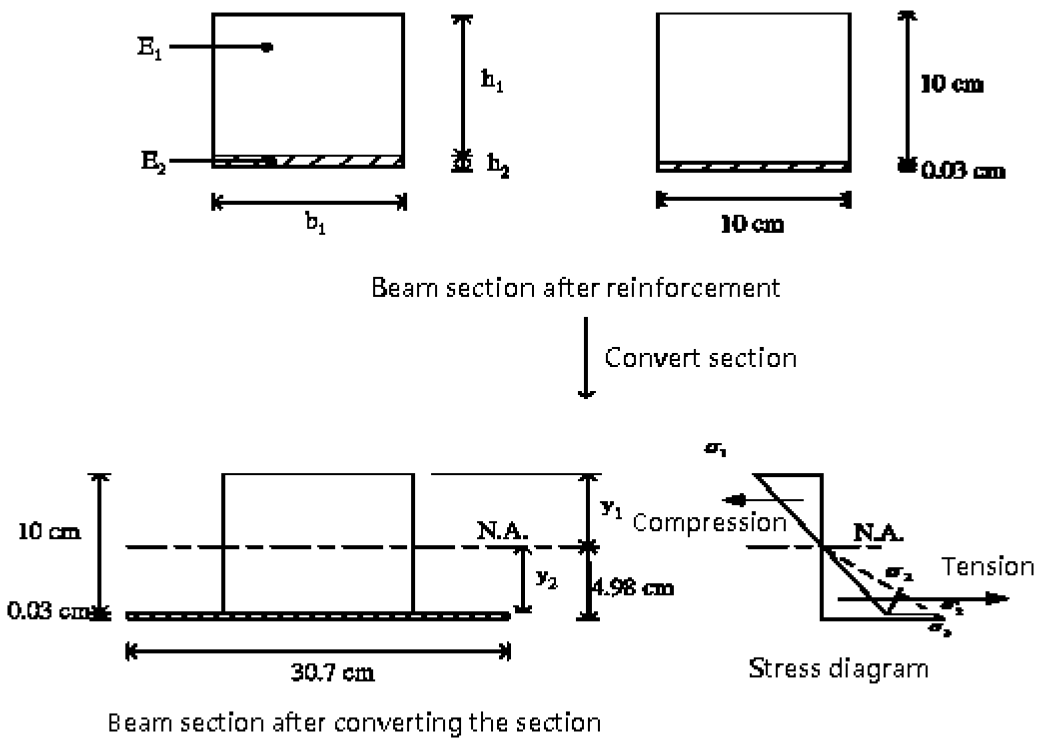

Figure 8. CFRP convert section method.

PC concrete subjected to maximum compressive stress: $\sigma_1 = \frac{M_1 y_1}{I}$

$$M_1 = \frac{\sigma_1 I}{y_1} = \frac{347 \times 856.3}{5.05} = 58,838.8 \text{ kg} \cdot \text{cm} \tag{12}$$

PC concrete subjected to maximum tensile stress: $\sigma_2 = \frac{M_2 y_2}{I}$

$$M_2 = \frac{\sigma_2 I}{y_2} = \frac{53 \times 856.3}{4.95} = 9168.5 \text{ kg} \cdot \text{cm} \tag{13}$$

CFRP1 subjected to maximum tensile stress: $\sigma_3 = \frac{n M_3 \overline{y}}{I}$

$$M_3 = \frac{\sigma_3 I}{n\overline{y}} = \frac{7800 \times 856.3}{3.07 \times 4.98} = 436,870.6 \text{ kg} \cdot \text{cm} \tag{14}$$

Shear stress at the bonding interface: $\tau = \frac{VQ}{Ib}$

$$100 = \frac{P/2 \times 4.57}{856.3 \times 10} \Rightarrow P = \frac{100 \times 856.3 \times 10 \times 2}{4.57} = 374,748.4 \text{ kg} \tag{15}$$

The maximum failure load is controlled when the PC concrete is subjected to the maximum tensile stress.

$$M = \frac{PL}{4} \tag{16}$$

Theoretical value $P = \frac{4M}{L} = \frac{4 \times 9168.5}{30} = 1222.5$ kg.
Experimental value P = 1990.3 kg.

CFRP1 reinforced 1 layer increased the strength by 2.8% compared to the unreinforced theoretical value, however, the strength of the experimental value increased by 67.3%.

(2) The ultimate stress method (shown in Figure 9) calculates the internal stress of the composite beam:

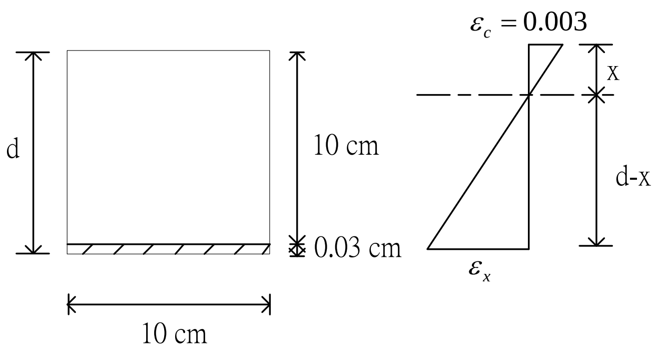

Figure 9. CFRP ultimate stress method.

Assume that the concrete cannot withstand tensile stress. Concrete strain $\varepsilon_c = 0.003$. Find the neutral axis position:

$$\frac{0.003}{x} = \frac{\varepsilon_x}{d - x} \tag{17}$$

$$\varepsilon_x x = 0.003d - 0.003x \tag{18}$$

$$x = \frac{0.003d}{\varepsilon_x + 0.003} = \frac{0.003d}{\sigma/E + 0.003} = \frac{0.003 \times 10.03}{7800/7.4 \times 10^5 + 0.003} = 2.22 \text{ cm} \tag{19}$$

$$T = \sigma A_s = 7800 \times 10 \times 0.03 = 2340 \text{ kgf} \tag{20}$$

$$10.03 - 2.22 = 7.81 \text{ cm}$$

$$\text{M} = 7.81 \times 2340 = 18{,}275.4 \text{ kgf}\cdot\text{cm}$$

$$M = \frac{PL}{4} \tag{21}$$

Theoretical value $P = \frac{4\times 18275.4}{30} = 2436.7$ kgf.

Experimental value P = 1990 kgf.

CFRP1 reinforced 1 layer increased the strength by 104.9% compared to the unreinforced theoretical value, however, the strength of the experimental value increased by 67.3%. The theoretical calculation and experimental results of CFRP reinforcement are shown in Table 5. It is obvious that the theoretical values of one-layer, two-layer, and three-layer CFRPs are all overestimated. The analysis of the internal stress of the beam by the conversion section method and the ultimate stress method cannot correctly evaluate the effect of CFRP reinforcement. There is such a large difference between the theoretical value and the experimental value, which may be due to: (a) The CFRP patch has a higher tensile strength than the bonded interface. (b) When the concrete has cracks at the loading point, the concrete can hardly bear the tensile force at this time, and the tensile force is borne by the bonding interface between the CFRP patch and the concrete. (c) The test load is jointly borne by the CFRP and the concrete, and the CFRP can withstand larger deformation than the concrete. (d) When the tensile force is greater than the bond strength that the interface can withstand, the bonding interface between the concrete and the CFRP will fall off and be damaged, but the CFRP will not be torn [15,25]. The interface bonding between CFRP and concrete plays an important role in the reinforcement of concrete members using CFRP [25]. In addition, factors such as bending capacity, ductility, and maximum deflection should be considered. An important parameter in these analyses is the value of the elongation at failure of the CFRP composite, which can lead to premature failure of the "beam-bond-reinforcement" system [26].

The theoretical value obtained by the conversion section method is because the concrete is damaged by tension, and the strength provided by the bonding interface between the CFRP and the concrete is not considered after the concrete is damaged. Therefore, the calculated theoretical value will be significantly lower than the experimental value. The theoretical value obtained by the ultimate stress method is based on the assumption that the concrete cannot bear the tensile force and the CFRP patch has reached a subdued state, which is different from the phenomenon observed in the experiment, so the theoretical value obtained by the ultimate stress method will be higher [13]. Xiang et al. [27] related study on the calculation method of flexural strength of damaged reinforced concrete beams strengthened by CFRP sheets, and the section analysis method was used to analyse the flexural strength of damaged beams. After the effective strain equation of the CFRP sheet is recommended, the experimental results can be verified.

ACI PRC 440.2 [28] provides guidance for the selection, design and installation of FRP systems for external reinforcement of concrete structures. Ross et al. [29] also proposed a practical design method, which agrees well with the experimental results. To evaluate the strength enhancement provided by the FRP panels, an inelastic section analysis procedure was developed that can accurately predict the load–displacement response of the modified beam.

4. Conclusions

In this study, the reinforced PC beam test was carried out for RPC and CFRP materials, the flexural strength was tested, and the theoretical and experimental values of the flexural strength of different reinforced materials were discussed. The main research results are as follows:

(1) According to the results of the bending test, the flexural strength of RPC concrete has a tendency to increase significantly with the increase of steel fiber content. When the

steel fiber content increases to 1.5%, its flexural strength is about 2 times, so adding steel fiber can effectively improve the tensile strength and toughness of concrete.

(2) The flexural strength of CFRP-reinforced concrete beam specimens can be increased by 67.3% with one layer of CFRP, 118.4% with two layers of CFRP, and 160.8% with three layers of CFRP. It shows that CFRP-reinforced concrete can obtain a good reinforcing effect, and the flexural strength is significantly improved with the increase of CFRP-reinforced layers.

(3) The flexural strength of the concrete beam specimen reinforced with RPC increases significantly with the increase of steel fiber content and repair thickness. Among them, RPC reinforcement with a thickness of 1 cm and a steel fiber content of 1% is approximately equal to a layer of CFRP, and an RPC reinforcement with a thickness of 2 cm of 0.5% steel fiber is also approximately equal to a two-layer CFRP coating. The RPC reinforcement with a thickness of 2 cm and a steel fiber content of 1.0% is approximately equal to that of three layers of CFRP.

(4) The RPC reinforcement can be used to analyze the internal stress of the beam using the conversion section method of material mechanics. The theoretical value and experimental value of the maximum failure load obtained can obtain similar results. Therefore, this method is suitable for evaluating the effectiveness of RPC-reinforced concrete beams.

(5) Analyzing the internal stress with the conversion section method and ultimate stress method of material mechanics cannot correctly evaluate the effectiveness of CFRP-reinforced concrete beams.

(6) Due to the high cost of experimental testing, future research directions could consider developing methods to accurately predict the behavior of unreinforced and strengthened composite beams.

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Appendix A

Theoretical formulas:

$$n = \frac{E_2}{E_1} \tag{A1}$$

$$\underline{y} = \frac{E_1 h_1^2 + 2E_1 h_1 h_2 + E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)} \tag{A2}$$

$$I = \frac{1}{3}bh^3 + \frac{1}{3}bh^3 - \frac{1}{3}bh^3 \tag{A3}$$

$$\sigma_1 = \frac{M_1 y_1}{I} \tag{A4}$$

$$M_1 = \frac{\sigma_1 I}{y_1} \tag{A5}$$

$$\sigma_2 = \frac{M_2 y_2}{I} \tag{A6}$$

$$M_2 = \frac{\sigma_2 I}{y_2} \tag{A7}$$

$$\sigma_3 = \frac{nM_3\underline{y}}{I} \tag{A8}$$

$$M_3 = \frac{\sigma_3 I}{n\underline{y}} \tag{A9}$$

$$\tau = \frac{VQ}{Ib} \tag{A10}$$

$$M = \frac{PL}{4} \tag{A11}$$

$$P = \frac{4M}{L} \tag{A12}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_x}{d-x} \tag{A13}$$

$$\varepsilon_x x = 0.003d - 0.003x \tag{A14}$$

$$x = \frac{0.003d}{\varepsilon_x + 0.003} = \frac{0.003d}{\frac{\sigma}{E} + 0.003} \tag{A15}$$

$$T = \sigma A_s \tag{A16}$$

Nomenclature

n	elastic modulus ratio
E_1	elastic modulus of PC
E_2	elastic modulus of reinforcing material
b	width of beam rectangular section, mm
h	height of beam rectangular section, mm
h_1	height of PC beam section, mm
h_2	height of reinforcing material beam section, mm
y	distance from neutral axis to outer surface of beam, mm
σ_1	maximum compressive stress of PC
σ_2	maximum tensile stress of PC
σ_3	maximum tensile stress of reinforcing material
τ	shear stress at the bonding interface
x	depth of the equivalent rectangular stress block, mm
ε_c	concrete strain
ε_x	strain of reinforcing material
I	moment of inertia
M_1	bending moment of maximum compressive stress of PC
M_2	bending moment of maximum tensile stress of PC
M_3	bending moment of maximum tensile stress of reinforcing material
As	total area of tensile reinforcement (CFRP), mm^2

References

1. Dugat, J.; Roux, N.; Bernier, G. Mechanical properties of reactive powder concretes. *Mater. Struct.* **1996**, *29*, 233–240. [CrossRef]
2. Lee, M.; Chiu, C.; Wang, Y. The Study of Bond Strength and Bond Durability of Reactive Powder Concrete. *J. ASTM Int.* **2005**, *2*, 104–113. [CrossRef]
3. Richard, P.; Cheyrezy, M. Composition of reactive powder concretes. *Cem. Concr. Res.* **1995**, *25*, 1501–1511. [CrossRef]

4. Lee, M.; Wang, Y.; Chiu, C. A preliminary study of reactive powder concrete as a new repair material. *Constr. Build. Mater.* **2007**, *21*, 182–189. [CrossRef]
5. Bonneau, O.; Lachemi, M.; Dallaire, E.; Dugat, J.; Aitcin, P.C. Mechanical Properties and Durability of Two Industrial Reactive Powder Concretes. *ACI Mater. J.* **1997**, *94*, 286–290.
6. Graybeal, B.A. Compressive behavior of ultra-high-performance fiber-reinforced concrete. *ACI Mater. J.* **2007**, *104*, 146–152.
7. Wu, Z.; Shi, C.; Khayat, K.H. Investigation of mechanical properties and shrinkage of ultra-high performance concrete: Influence of steel fiber content and shape. *Compos. Part B Eng.* **2019**, *174*, 107021. [CrossRef]
8. Skazlic, M.; Bjegovic, D.; Serdar, M. Influence of test specimens' geometry on compressive strength of ultra-high-performance concrete. In Proceedings of the 2nd International Symposium on Ultra-High-Performance Concrete 2008, Kassel, Germany, 5–7 March 2008; pp. 295–301.
9. Magureanu, C.; Sosa, I.; Negrutiu, C.; Heghes, B. Mechanical properties and durability of ultra-high-performance concrete. *ACI Mater J.* **2012**, *109*, 177–183.
10. Kang, S.T.; Choi, J.I.; Koh, K.T.; Lee, K.S.; Lee, B.Y. Hybrid effects of steel fiber and microfiber on the tensile behaviour of ultra-high-performance concrete. *Compos. Struct.* **2016**, *145*, 37–42. [CrossRef]
11. Lee, M.; Huang, Y.; Kan, Y.; Wang, Y.; Chen, Y.; Kao, S. Experimental study on durability of CFRP-strengthened wood members. *J. Mater. Res. Technol.* **2023**, *24*, 3704–3716. [CrossRef]
12. Qiang, X.; Chen, L.; Jiang, X. Experimental and theoretical study on flexural behavior of steel–concrete composite beams strengthened by CFRP plates with unbonded retrofit systems. *Compos. Struct.* **2023**, *309*, 116763. [CrossRef]
13. Zhang, Y.; Duan, L.; Liu, H.; Lu, J.; Huo, Y. Experimental and numerical study on multi-impact performance of pre-damaged beams strengthened with CFRP. *Eng. Struct.* **2023**, *285*, 116034. [CrossRef]
14. Guo, R.; Li, C.; Xian, G. Water absorption and long-term thermal and mechanical properties of carbon/glass hybrid rod for bridge cable. *Eng. Struct.* **2023**, *274*, 115176. [CrossRef]
15. Huang, Y.; Lee, M.; Kan, Y.; Wang, W.; Wang, Y.; Pan, W.-B. Reinforced concrete beams retrofitted with UHPC or CFRP. *Case Stud. Constr. Mater.* **2022**, *17*, e01507. [CrossRef]
16. Nguyen, D.L.; Ryu, G.S.; Koh, K.T.; Kim, D.J. Size and geometry dependent tensile behaviour of ultra-high-performance fiber-reinforced concrete. *Compos. Part B-Eng.* **2014**, *58*, 279–292. [CrossRef]
17. Wang, W.; Huang, Y.; Lee, M.; Wang, Y.; Kan, Y.; Hsiao, Y.; Chang, H. Case study for ditch cover with ultra-high-performance concrete. *Case Stud. Constr. Mater.* **2022**, *17*, e01647. [CrossRef]
18. Lee, M.; Wang, W.; Wang, Y.; Hsieh, Y.; Lin, Y. Mechanical Properties of High-Strength Pervious Concrete with Steel Fiber or Glass Fiber. *Buildings* **2022**, *12*, 620. [CrossRef]
19. Kazemi, S.; Lubell, A. Influence of specimen size and fiber content on mechanical properties of ultra-high-performance fiber-reinforced concrete. *ACI Mater. J.* **2012**, *109*, 675–684.
20. Elsayed, M.; Tayeh, B.A.; Elmaaty, M.A.; Aldahshoory, Y. Behaviour of RC columns strengthened with Ultra-High Performance Fiber Reinforced concrete (UHPFRC) under eccentric loading. *J. Build. Eng.* **2022**, *47*, 103857. [CrossRef]
21. Ren, G.; Wu, H.; Fang, Q.; Liu, J. Effects of steel fiber content and type on static mechanical properties of UHPCC. *Constr. Build. Mater.* **2018**, *163*, 826–839. [CrossRef]
22. Akeed, M.H.; Qaidi, S.; Faraj, R.H.; Mohammed, A.S.; Emad, W.; Tayeh, B.A.; Azevedo, A.R. Ultra-high-performance fiber-reinforced concrete. Part I: Developments, principles, raw materials. *Case Stud. Constr. Mater.* **2022**, *17*, e01290. [CrossRef]
23. Li, P.; Yu, Q.; Brouwers, H. Effect of coarse basalt aggregates on the properties of Ultra-high Performance Concrete (UHPC). *Constr. Build. Mater.* **2018**, *170*, 649–659. [CrossRef]
24. Huang, Y.; Grünewald, S.; Schlangen, E.; Luković, M. Strengthening of concrete structures with ultra high performance fiber reinforced concrete (UHPFRC): A critical review. *Constr. Build. Mater.* **2022**, *336*, 127398. [CrossRef]
25. Zhang, J.; Li, H.; Liu, S.; Zhang, X.; Yang, C.; Zhang, R. Bond behavior of the CFRP-concrete interface under combined sustained load and sulfate erosion. *Structures* **2022**, *35*, 551–564. [CrossRef]
26. Bakalarz, M.M.; Kossakowski, P.G. Application of Transformed Cross-Section Method for Analytical Analysis of Laminated Veneer Lumber Beams Strengthened with Composite Materials. *Fibers* **2023**, *11*, 24. [CrossRef]
27. Xiang, K.; Wang, G. Calculation of Flexural Strengthening of Fire-damaged Reinforced Concrete Beams with CFRP Sheets. *Procedia Eng.* **2013**, *52*, 446–452. [CrossRef]
28. ACI Committee 440. *ACI PRC 440.2-17: Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures*; American Concrete Institute: Farmington Hills, MI, USA, 2002.
29. Ross, C.A.; Jerome, D.M.; Tedesco, J.W.; Hughes, M.L. Strengthening of reinforced concrete beams with externally bonded composite laminates. *ACI Struct.* **1999**, *96*, 212–220.