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Review

A Review of Simplified Numerical Beam-like Models of Multi-Storey Framed Buildings

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Abstract: Modern computational techniques have greatly influenced the numerical analyses of structures, not only in terms of calculation speed, but also in terms of procedural approach. In particular, great importance has been given to structural modelling, that is, the process by which a structure and the actions to which it is subjected are reduced to a simplified scheme. The use of a simplified calculation scheme is necessary since the structures are, in general, considerably complex physical systems whose behaviour is influenced by a large number of variables. The definition of a structural scheme that is at the same time simple enough to be easily computable as well as sufficiently reliable in reproducing the main characteristics of the behaviour of the analysed structure is, therefore, a crucial task. In particular, with reference to multi-storey framed buildings, the extensive use of three-dimensional finite element models (FEM) has been made in recent decades by researchers and structural engineers. However, an interesting and alternative research field concerns the possibility of studying multi-storey buildings through the use of equivalent beam-like models in which the number of degrees of freedom and the required computational effort are reduced with respect to more demanding FEM models. Several researchers have proposed single or coupled continuous beams to simulate either the static or dynamic response of multi-storey buildings assuming elastic or inelastic behaviour of the constitutive material. In this paper, a review of several scientific papers proposing elastic or inelastic beam-like models for the structural analyses of framed multi-storey buildings is presented. Considerations about limits and potentialities of these models are also included.

Keywords: modelling of framed structures; multi-storey buildings; beam-like models; elastic models; inelastic models

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1. Introduction

The evaluation of the dynamic response of multi-storey buildings, particularly when subjected to seismic loading, has represented one of the most important tasks of structural engineering research studies in the last century, and is still the objective of studies and improvements. The advancements in computational procedures and parallel processing in the recent years have enhanced the accurate dynamic analysis and seismic vulnerability of multi-storey buildings. These analyses can be carried out at the building or urban level; simplified or high-fidelity models, as well as approximate or sophisticated large-scale simulations, are currently adopted. However, when analysing entire urban areas or when performing several simulations aiming to provide sufficient data for expressing probability failure maps, a convenient balance between accuracy and computational burden is needed. In this precise context, beam-like models play an important role.

Analyses developed by means of three-dimensional (3D) numerical models aim to provide an accurate representation of the main characteristics of the dynamic behaviour of real structures and, for this reason, must be detailed and based on reliable data. Therefore, a rigorous and accurate evaluation of the dynamic response of a multi-storey building subjected to seismic excitation requires adequate structural expertise and great computational burden. On the other hand, the need for a sufficiently accurate seismic vulnerability assessment of a large number of existing buildings in seismic areas, in particular at an urban scale, has stimulated a significantly increased interest in simplified but sufficiently accurate models able to represent multi-storey buildings.

Since multi-storey buildings may exhibit a great number of degrees of freedom, especially if several deformability parameters of the structural members are taken into account, simplified multi-degree-of-freedom (MDOF) models are usually considered. In this regard, some models have been presented in the scientific literature for the simulation of linear as well as non-linear dynamic responses of multi-storey buildings. The most important goal of these simplified models is to reduce the number of degrees of freedom of the 3D model, preserving the main features of its dynamic response.

A new and renovated interest has recently grown in beam-like models, which were introduced in the last century. Beam-like models, which are based on the equivalence of multi-level structures to flexural–shear coupling continuum beams, aim to simulate the dynamic behaviour of multi-level buildings and meanwhile reduce the computational burden. Several authors have demonstrated interest in beam-like models and proposed suitable simplified approaches for the dynamic analysis of multi-storey structures.

In this paper, several elastic and inelastic beam-like models proposed in the literature, together with their use in practical applications, are considered and reviewed. In particular, elastic and inelastic models are described in Sections 2 and 3, respectively, adopting opportune classifications. Some considerations regarding the limitations and the possible improvements of these models are also reported.

It is worth pointing out that this review paper does not pretend to compare the performance of each single model, it rather aims to contribute to the comprehension of multi-storey framed structure convenient modelling by means of promising simplified beam-like schemes.

2. Elastic Beam-like Models

The analysis of multi-storey buildings has been performed in recent decades by means of equivalent beam-like models. These models are represented by continuum mono-dimensional elements whose static and dynamic responses are strictly related to the assumed hypotheses concerning their geometry, their deformability, the material behaviour and the kind of developed analysis (linear or non-linear).

Most of the studies in the scientific literature concerning beam-like models deal with elastic behaviour. Each of these models aim to reproduce the static or dynamic response of complex structures, taking into account their prevailing elastic deformability. Therefore, different beam-like models having shear-only, flexural-only or coupled shear–flexural deformability are reviewed and discussed below. Some beam-like models are equivalent to plane structures, whilst others are able to represent the three-dimensional behaviour of a building. The presence of twisting effects may or may not be considered. All these characteristics of the beam-like models lead to different choices of the degrees of freedom in order to define a system able to reproduce the investigated behaviour with the established kinematics. The approaches also differ in the assumption hypothesis about the mass of the models which can be concentrated or distributed along the beam.

The main advantage of some beam-like models present in the literature is their adoption for the study of the static or dynamic behaviour of real buildings, as shown in the applicative section of the following cited papers. Furthermore, beam-like models can be base-isolated systems or include soil–structure interaction by means of an appropriate modelling of the base restraints.

Several classifications can be made in the attempt to illustrate the great number of scientific contributions in this field. In the following, we choose to classify the scientific literature by separating the models of a single equivalent beam from those in which beams with different characteristics are coupled to each other, or in which single beams are coupled to walls or frames. For the first category, a further distinction will be made between planar and three-dimensional systems.

In order to make the paper easier to read, the adopted classification has been graphically illustrated in Figure 1.

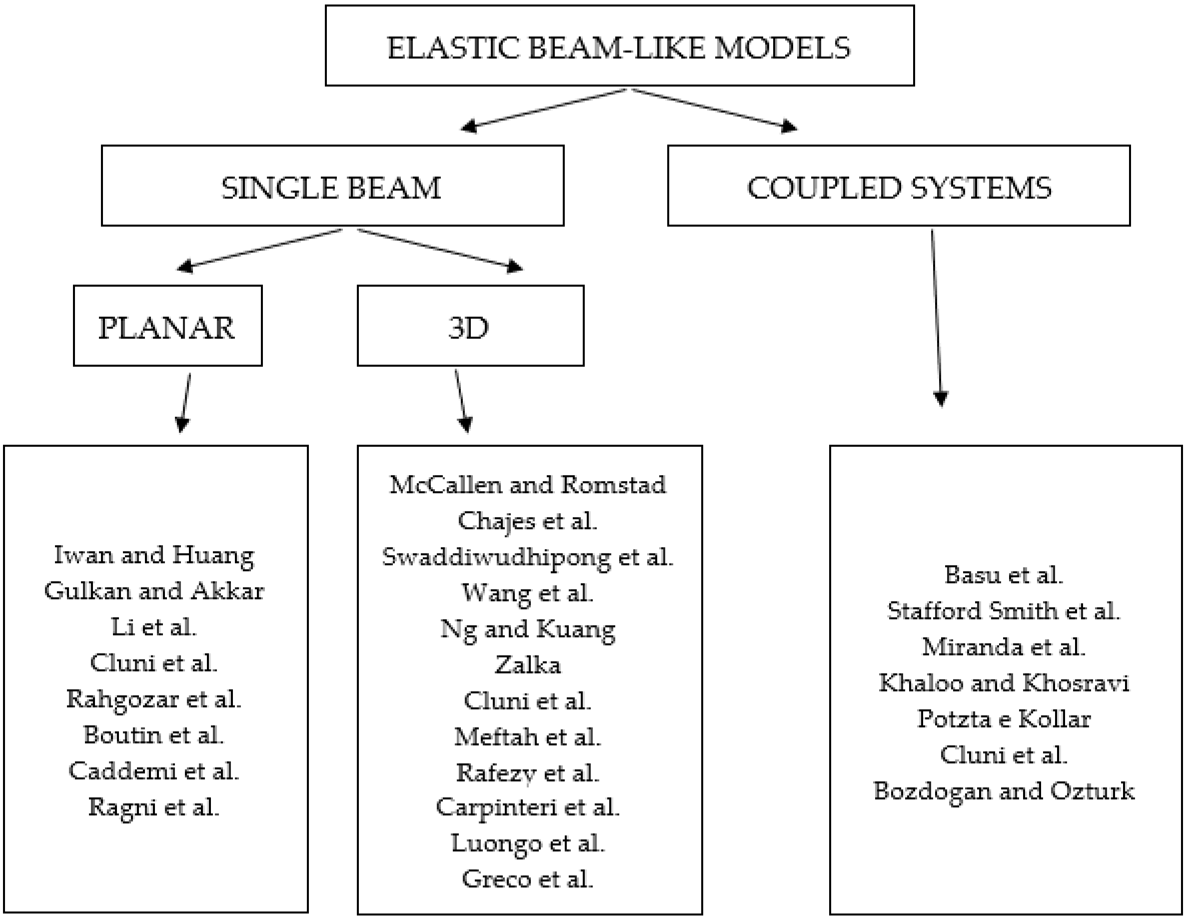

Figure 1. Classification of elastic beam-like models.

In the sections of the following review, the fundamental characteristics of each model presented in the literature are outlined, underlining their areas of application.

2.1. Single Beams

2.1.1. Planar Systems

Among the studies concerning planar beam-like structures presented in the scientific literature in the last 50 years, one of the first ones was developed by Iwan [1] and Huang [2], who proposed a continuous uniform shear–beam model to predict elastic storey drift demand on structures due to near-field earthquake ground motions. Noticing that the continuous model predicts the inter-storey drifts more accurately than an equivalent single-degree-of-freedom (SDOF) system, they suggested the use of a drift spectrum instead of the well-known response spectrum.

The evaluation of the maximum inter-storey drifts of a building by means of the first-mode shape of a uniform shear beam was the aim of a later study proposed by Gulkan and Akkar [3]. In 2005, they modified the equivalent shear beam model in order to take into account the general moment-resisting frame (MRF) behaviour of the structure by introducing some empirical coefficients into the maximum ground storey and inter-storey drift expressions [4].

Beam-like structures, due to their inherent characteristics, have great application in modelling tall buildings. The in-plane free vibration of tall buildings was studied in 2000 by Li et al. [5] by means of an equivalent flexural multi-step cantilever beam with stiffness, mass and axial loads distributed according to a power or exponential law of variation. The approximated solution of this complex problem was obtained using the exact solution of a one-step bar with a variable cross-section together with the transfer matrix method. This approach was extended in [6] by Rahgozar et al. for an equivalent Timoshenko multi-step beam, and in [7] for a sandwich beam, the latter defined according to Zalka's, Potza's and Kollar's approach described in the following.

Additionally, Cluni et al. adopted two equivalent beam models for studying the dynamic response of tall buildings under wind loads [8]. The equivalent beam models had uniform flexural and shear stiffness, linked together in series or in parallel. The equivalence criterion was based on the minimization of the difference of static and dynamic response features obtained by means of the equivalent beam and the FEM model. The mechanical parameters used to describe the beam varied sensibly with respect to the real ones, but the main interest was to find an equivalent beam which could accurately describe the response of the slender building, regardless of any physical meaning. In 2020, the model was extended in order to describe the flexural, shear and torsional behaviour of uniform tall buildings with an asymmetrical plan subjected to wind or earthquake loads [9]. The mechanical and dynamic features of the equivalent beam were first evaluated, imposing the equivalence of the deformation energy between the equivalent sandwich beam and the sub-structures of the tall building, and then calibrated by minimizing a function which takes into account natural frequencies and static displacements. The limitation of this model consists of its unsuitability in representing asymmetrical buildings. The results of the analyses performed on regular and symmetrical buildings with uniform stiffness distribution are shown in terms of natural frequencies and mode shapes and in terms of displacements under static or dynamic wind or earthquake loads.

Tall buildings were also modelled by means of an equivalent beam by Rahgozar et al. in 2010. Basing the study on previous research [10], they proposed a continuum model for predicting the stress distribution and the displacement profile for a combined system of different structural elements, but only under specific load patterns [11]. The model consisted of an equivalent uniform cantilever beam with flexural and shear deformability and rotational springs simulating the belt trusses (Figure 2). This model was applied to estimate the natural frequencies and mode shapes of the tall buildings [12,13], obtaining acceptable errors if compared to finite element models.

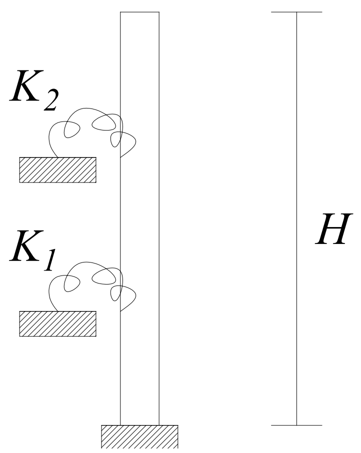

Figure 2. Schematization of the equivalent beam model proposed by Rahgozar et al.

Beam-like models have also been applied for the analysis of repetitive framed buildings. For example, in [14,15], Boutin et al. adopted the homogenization method of periodic discrete media (HPDM) for the analysis of repetitive reticular structures composed of interconnected elements (beams or plates) with the purpose of deducing the modal characteristics of the repetitive framed buildings. The homogenized continuum model can be defined as a shear-only beam, a Timoshenko beam, or a Euler–Bernoulli beam, and it provides the main structural characteristics. The first phase of the HPDM is the discretisation of the dynamic balance of the structure under harmonic vibrations, followed by the actual homogenisation procedure, through which a continuum model is elaborated from the discrete description. For the homogenization process, a scale parameter, measuring or the size of the basic cell of the structure or the characteristic size of deformation of the structure under vibrations, is introduced. Since this parameter has to be sufficiently small, the method is limited to the first frequencies and mode shapes, with wavelengths much larger than cell size. In [16], the continuum model obtained by means of the HPDM was adopted for studying the local resonance in reticulated frames.

The application of the HPDM is limited to periodic structures where the structural elements are slender enough to behave as Euler–Bernoulli beams. Franco et al. in [17] overcame this limitation by substituting an analytical part of the procedure with a numerical static analysis of a unit cell performed by means of a detailed finite element model. The authors state that this homogenization procedure can be applied to building structures if two fundamental conditions hold: periodicity and scale separation. The periodicity refers to the repetition of same-storey properties all along the building height whilst the scale separation ratio, which relates the height of one storey to the height of the entire building, must be sufficiently small. The beam-like model has been used to perform time–history analyses with natural seismic records of an existing 13-storey reinforced concrete frame building, namely, the Grenoble City Hall [18].

An important aspect in the beam characterization lies in the possibility of considering a variable cross-section in order to better simulate the non-uniform characteristics of the equivalent building.

Models of stepped Euler–Bernoulli or Timoshenko beams in the presence of deflection and rotation discontinuities along the span are presented by Caddemi et al. [19,20]. The proposed models rely on the adoption of Heaviside's and Dirac's delta distributions to model abrupt and concentrated—both flexural and shear—stiffness discontinuities of the beam that lead to exact closed-form solutions of the elastic response in the presence of static loads. Based on the latter solutions, a beam element suitable for the analysis of frame structures with an arbitrary distribution of singularities is proposed [19]. The exact explicit dynamic stiffness matrix of damaged frames structures is derived in [20], allowing the exact evaluation of the frequencies and the corresponding vibration modes, consistent with the distributed parameter model, through the application of the Wittrick–Williams's algorithm.

Beam-like structures have also been adopted taking into account dissipative devices. For example, Ragni et al. [21] proposed a displacement-based method, particularly devoted to seismic design steel frames equipped with dissipative braces, by using an equivalent continuous beam-like model where flexural deformability and shear deformability are related, respectively, to columns and diagonals of the bracing system. The design method is appealing since analytical expressions of the required flexural and shear stiffness distributions are obtained and conveniently adopted in the preliminary design of dissipative diagonal braces and columns of steel frames.

2.1.2. Three-Dimensional Systems

One of the pioneering studies concerning the use of three-dimensional beam-like structures for the analysis of buildings has been provided by McCallen and Romstad. In [22,23], they proposed a simple uniform continuum model for the analysis of lattice structures (which are repetitive reticular structures). The equivalence between the lattice structure and the continuum model was established in terms of deformation energies for three assumed global deformation modes (axial, shear, bending). The continuum model was then analysed by means of a traditional finite element approach. Geometrical non-linearities are accounted for through updated Lagrangian coordinate transformation for each continuum finite element. The translational and rotational inertias are lumped at the end or midpoint nodes of the finite elements of the discretised continuum.

The approach was extended to three-dimensional buildings with rigid floors. Therefore, the global model has only three degrees of freedom, two orthogonal translations and one rotation at each floor level. By applying horizontal forces only, the rotational and axial degrees of freedom of the continuum can be condensed out. Later, the condensed continuum model of an individual frame can be transformed to global coordinates and added into the global stiffness (Figure 3).

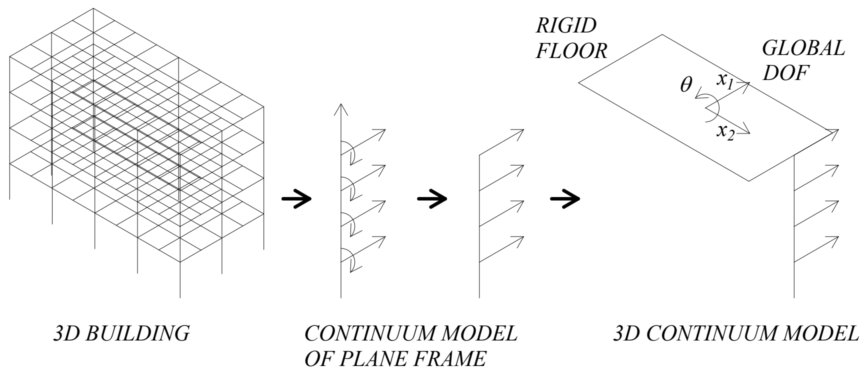

Figure 3. Construction of a 3D building model using continuum models for each plane frame according to McCallen and Romstad.

The continuum is used for the free vibration analysis of planar frames or buildings with uniform stiffness distribution along the height and the results are shown in terms of natural frequencies and mode shapes.

The previous study proposed by McCallen and Romstad inspired further developments. In particular, in 1993, Chajes et al. [24] extended the method to the determination of displacements and member forces in two-dimensional frames with reticular elements. In [25,26], the equivalent continuum model was applied to predict the measured seismic response of two existing buildings (a reinforced concrete one and a steel one) during the Loma Prieta earthquake (using the continuum model discretized with a number of finite elements equal to the number of floors).

Axial deformation of the equivalent beam-like model has been considered by Swaddiwudhipong et al. [27,28]. They proposed a uniform shear–flexural cantilever beam for 3D free vibration analysis of frame–core wall buildings. The solution in terms of natural frequencies and mode shapes is obtained using the Galerkin's technique, adopting an exponential shape function.

Three-dimensional beam models are mandatory in order to take into account the torsional behaviour of buildings, and several researchers have focused their studies on this subject.

Among others, Wang et al. proposed an approximate method to estimate the first two periods of vibration of multi-storey uniform buildings with an asymmetrical plan [29]. The coupled natural frequencies of the multi-storey structure, due to the asymmetric distribution of the structural members with respect to the floor plan, were expressed in terms of uncoupled lateral frequency, uncoupled fundamental torsional-to-lateral frequency ratio and the eccentricity ratio. However, this approach is valid only for proportionate structural systems whose centres of stiffness lie on a vertical line.

Another study which deals with the application of three-dimensional equivalent beams for the evaluation of the torsional behaviour of buildings has been provided by Ng and Kuang [30,31]. In the cited references, they study the modal analysis for the coupled flexural–torsional vibration of asymmetric uniform tall buildings by means of an equivalent flexural–torsional cantilever, whose stiffness is determined as a simple sum of the stiffness of the vertical members. The equivalent beam can be coupled with a shear cantilever in order to take into account the shear deformability. However, further studies [32,33] have been limited to the free vibration analysis considering only an equivalent Euler–Bernoulli beam, or an equivalent shear cantilever beam, respectively.

Applying the concept of an "equivalent column", Zalka proposed simple formulas in 2001 [34] for evaluating the natural three-dimensional frequencies of the buildings braced by frameworks, coupled shear walls, shear walls and cores. The method considered the local bending of the single vertical elements, the global bending of the frames/shear walls (associated with the axial deformation of the vertical elements) and shear deformations of the frames/shear walls (Figure 4). The pure torsional frequency is obtained by means of an analogy with respect to bending. Approximate formulas were provided to account for the interaction between translational and torsional modes for non-symmetrical buildings. The equivalent column was applied for the analysis of regular buildings with double planar symmetry and in-plane rigid floors. The same author proposed approximated closed-form solutions for studying tall buildings subjected to uniformly distributed static horizontal loads for both symmetrical [35] and non-symmetrical [36] systems in 2009 and 2014, respectively.

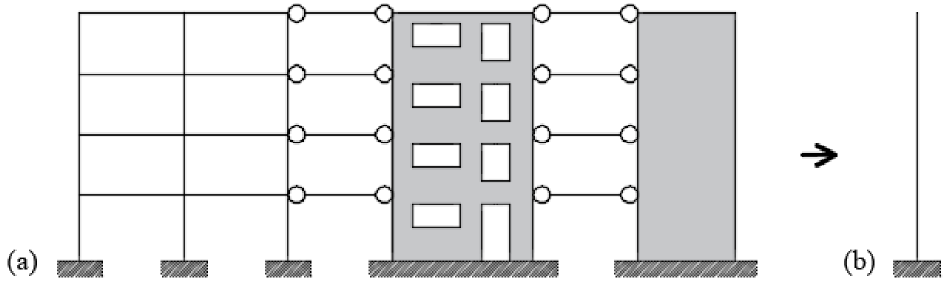

Figure 4. Model for lateral vibration analysis according to Zalka: (**a**) bracing system consisting of frames, coupled shear walls, shear walls and cores; (**b**) equivalent column.

In [37,38], Meftah et al. propose an approximate hand method for the seismic analysis of an asymmetric building structure with constant properties along its height, stiffened by a combination of shear walls and thin-wall open-section structures. The governing equations of free vibration of the equivalent flexural–torsional cantilever are derived on the base of the continuum method and D'Alembert principle, and the solution is determined by applying Galerkin's technique in [37], or an analytical method in [38]. The internal forces of the building subjected to an earthquake are also derived using the acceleration response spectrum and combining the modal responses by means of the square root of sum of squares (SRSS) method.

In addition to taking into account torsional effects, due to asymmetric building plans, some beam-like structures are also able to reproduce non-uniform stiffness distribution along the height. This result can be achieved by opportunely dividing the beam in different segments representing the building inter-storeys.

With this aim in [39] and in [40], Rafezy et al. propose a stepped shear–torsional cantilever model and a stepped Timoshenko cantilever model, respectively, for the calculation of the natural frequencies of an asymmetric three-dimensional frame or wall–frame structures by means of the Wittrick–Williams algorithm. Each beam segment is representative of a number of uniform storeys, variable from one to the total number of storeys of the building in the case this is uniform throughout its height. The stiffness of each beam segment is calculated as the sum of the stiffness of each frame in the considered direction with reference to the corresponding storeys. The equations of motion refer to each beam segment of the original, asymmetric, three-dimensional wall–frame structure. The entire original structure can then be modelled by assembling the substitute beams corresponding to each segment in the usual way.

The variation in the stiffness at each inter-storey of the considered building is also considered in the construction of an equivalent beam-like model by Carpinteri et al. They propose an equivalent shear–torsional beam model for estimating the response of tall buildings under static horizontal loads in the initial design phase [41]. The equivalent shear beam has the stiffness of each inter-storey segment equal to the sum of stiffness of the columns of the corresponding floor and is subjected to elastic rotational springs simulating the stiffness of the floor slabs and to horizontal springs simulating the diagonal bracings (Figure 5). Therefore, the model is able to consider a non-uniform stiffness distribution. With N being the number of floors of the building, the total number of degrees of freedom is 3N: two translations and one rotation for each floor. The results are shown in terms of displacements and internal forces due to static wind loads. In [42,43], the model was extended in order to consider second-order torsional effects (Vlasov's theory), allowing the computation of the natural frequencies and mode shapes of a uniform building by means of the equivalent beam with shear and torsional deformability.

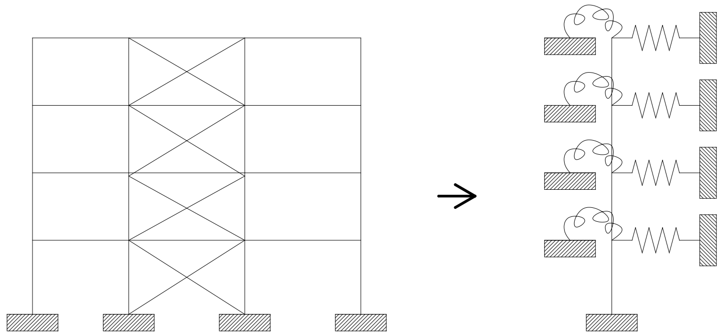

Figure 5. Model of a braced frame equivalent to a shear wall according to Carpinteri et al.

An equivalent beam model, deformable in shear and torsion and capable of approximately reproducing the dynamic behaviour of the three-dimensional shear-type structures, was introduced by a research team coordinated by Luongo [44,45], and its aeroelastic instability to wind excitation was analysed [46,47]. The homogenization process is based on the equivalence of the strain energy of the building cell with the corresponding beam segment. The model was used for the modal analysis of periodic buildings with symmetrical and asymmetrical plans. The model was also adopted in other papers in 2019 [48,49] for studying the linear and non-linear elastic behaviour of periodic tower buildings under the assumption that the beam is internally constrained, so that it is capable of experiencing shear strains and torsion only. The elasto-geometric and inertial characteristics of the beam are directly identified from a discrete model of a three-dimensional frame via a homogenization process. A more refined Timoshenko beam-like model suitable for the dynamic analysis of periodic buildings has recently been proposed [50]. The homogenization process is still based on the equivalence of the strain energy of the building cell, between two rigid floors with structural elements such as columns and shear walls, and the corresponding beam segment. In [51–53], the classical rigid-floor assumption is overcome in the calibration procedure, which is based on FE analysis results, by means of the introduction of opportune correction coefficients that take into account the out-of-plane floor deformability due to the shear–torsional behaviour. The results of the analyses performed on buildings with uniform stiffness distribution are shown in terms of natural frequencies, mode shapes, lateral displacements and axial forces of the structural elements under static loads. The model can also be adopted for the buckling analysis [54], taking into account the soil–structure interaction, and for studying the dynamic response [55,56] of tower buildings. A summary of the study of these latter models and their various applications is reported in [57].

A contribution to the evaluation of the elastic response of non-uniform buildings by means of equivalent beam-like models is also given by the present authors in [58]. In the cited study, we presented a 3D shear–torsional cantilever beam suitable for the schematization of real buildings that do not have a uniform mass and stiffness distribution along their height and are characterised by unsymmetrical plans causing not negligible torsional effects (Figure 6). The equivalent beam has a non-uniform stepwise cross-section and each portion represents a building inter-storey whose shear and torsional stiffness are initially approximately evaluated according to a geometrical consistent reference model. Stiffness contributions due to the beams and floors of the building can be neglected (rigid floor hypothesis) or considered by means of appropriate stiffness reduction coefficients. The linear dynamic behaviour of the non-uniform beam-like element is evaluated by discretizing the continuous model according to a Rayleigh–Ritz approach based on an appropriate number of modal shapes of a uniform beam having only shear and torsional deformability.

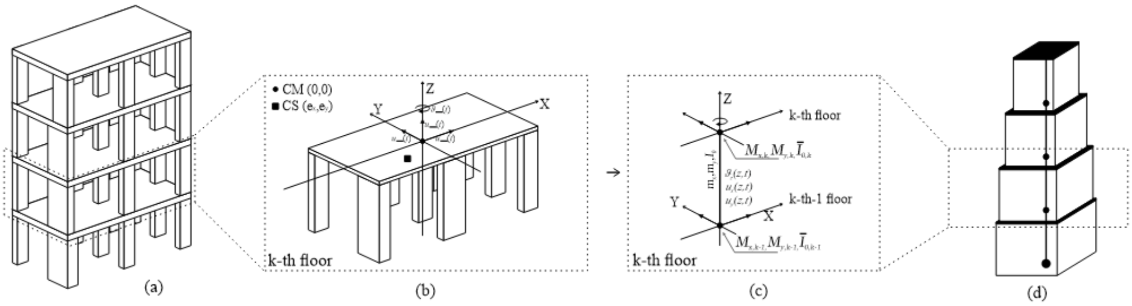

Figure 6. Concept of representation starting from (**a**) the 3D structure and (**b**) the generic k-th floor (or inter-storey) up to the (**c**) sub-beam element and the (**d**) proposed beam-like model. Source: own figure [58].

2.2. Coupled Beams

With regard to high-rise buildings, the discretization of multi-floor frames with elastic coupled beams was first introduced by Basu et al. in 1982 [59,60]. In these studies, a fixed-base multi-storey building was idealized as an equivalent coupled shear wall connected in series to an equivalent frame. The coupled wall was modelled as a continuum of uniform properties and the frame as a uniform shear beam; the solutions were then obtained by treating the structure as a lumped parameter system. The proposed approach was limited to the principal three modes of vibrations for the in-plane behaviour. The obtained modal features were used in [61,62] to evaluate the design forces of the building by means of seismic response spectra.

A similar approach was used by Stafford Smith et al. in [63,64] for the determination of the period of free vibration and the earthquake design forces of a building by means of equivalent coupled shear–flexure cantilevers. The stiffness of the equivalent beams was determined as a simple sum of the stiffness of the vertical members. The approach was limited to symmetrically loaded buildings with uniform properties along the height and a symmetrical plan.

An approximate method based on an equivalent uniform continuum model that linearly combines a flexural and a shear deformable cantilever beam, was introduced by Miranda in 1999 [65] for estimating the maximum lateral displacement in multi-storey buildings. The lateral displacements of the building were given by the combination of shear and bending deformations and the seismic response was evaluated in terms of maximum roof displacement and maximum inter-storey drift ratio. The evaluated dynamic response was inherently approximated since a uniform distribution of mass and stiffness along the height of the building was assumed for the beam model and, furthermore, only the first mode of vibration, whose lateral displacements were given by a closed form solution of a static problem under a fixed load pattern, was considered. The formulation was successively generalized in [66,67], where the limitation of uniform lateral stiffness distribution along the building was removed, assuming linear or parabolic variations. The uniform continuum model was subsequently used to find closed-form solutions capable of approximating the dynamic characteristics of the non-uniform buildings (for example, mode shapes, periods and modal participation factors) [67]. An estimate of the ground acceleration request on the structures that respond linearly to the seismic motions was determined in [68] for planar models, by considering up to the first three modes of vibration in the dynamic analysis.

Later, Miranda and Akkar merged their studies and used the simplified continuous model of Miranda for the evaluation of generalised inter-storey drift spectrum in [69].

A combination of uniform shear and flexural cantilever beams to estimate the elastic structural response of a tall building was also proposed in 2008 by Khaloo and Khosravi [70]. Extending the formulation proposed by Stafford Smith and Miranda, they investigated the multi-mode effects of tall buildings subjected to near-field ground motions, assuming the linearity of the system. The location along the height and the spectra of the maximum inter-storey drift ratio were also calculated.

In 2003, Potzta e Kollar [71] modelled buildings with an equivalent uniform sandwich beam that was defined by three types of stiffness derived from the resistant elements: global bending stiffness, local bending stiffness and shear stiffness. The deformation energy of the equivalent beam was deduced from the generalization of Timoshenko's theory for spatial problems, introducing separate contributions between global flexural stiffness and shear stiffness, on one hand, and between global flexural stiffness and local flexural stiffness on the other. In any case, the shape of the displacements under sinusoidal horizontal loads had to be fixed (i.e., sinusoidal and cosinusoidal form) in order to obtain the replacement stiffness of the equivalent system, leaving the choice of a certain length (a sort of free length of element inflection) corresponding to the best equivalence. Using the obtained equivalent stiffness, an approximate expression was proposed for estimating the buckling load and the natural frequencies of symmetrical structures, whilst in unsymmetrical structures the lateral–torsional vibration modes were determined by an eigenvalue problem, assuming uniform mass distribution along the height. The model was applied to study doubly symmetrical or unsymmetrical buildings with uniform stiffness distribution along the height. The results are shown in terms of natural frequencies and the buckling load of the structure. Later, Tarjan and Kollar applied the proposed model to estimate the basic internal forces [72]. Unfortunately, the procedure for calculating the stiffness of the equivalent model was rather complicated and not entirely automatic. The procedure proposed by Potzta and Kollar in the sandwich beam [73] was later adopted by Cluni et al., who refined the definition of their previous equivalent beam model described in the previous section.

Recently, Bozdogan and Ozturk modelled a uniform building by means of an equivalent shear beam model, taking into account the axial deformability of structural elements by introducing a shear stiffness correction coefficient [74]. The coupled shear wall was modelled as a flexural beam. Mechanical and geometric characteristics were assumed uniform along the height of the building (Figure 7).

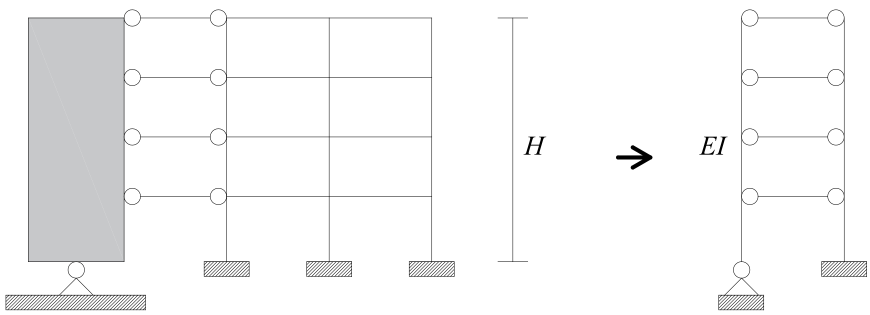

Figure 7. Equivalent model of frame hinged shear wall structures according to Bozdogan.

Introducing the appropriate boundary conditions, natural frequencies and mode shapes of the equivalent beam can be determined. By means of the response spectra, it is possible to obtain the maximum deflection, maximum relative displacement, maximum shear force and maximum bending moment of the system.

The proposed model was extended to beams with stiffness distribution, which varied according to a fixed law along the height and with uniform mass distribution [75]. Equations of motion were solved by means of the "differential transform method". Since only planar frames were considered, the torsional problem was neglected.

2.3. Summary and Considerations about Elastic Beam-like Models

As the previous sections clearly show, the scientific contributions in the field of elastic beam models are numerous and very varied.

Table 1 summarizes the main characteristics of each of the previously discussed models taking into account: out of plane deformability, spatial frames, non-uniform stiffness distribution, and eccentricity between CM and CS. The symbol √ indicates the presence of a hypothesis/feature/analysis (in the corresponding column) into the beam-like model of the specific author (in the corresponding row).

Table 1. Summary of the literature review about linear elastic beam-like models.

Author	Beam's Model	Out-of-Plane Floor Deformability	Non-Uniform Stiffness	3D Systems	CM $\neq$ CS	Modal Analysis	Seismic Response
Iwan and Huang	S	-	-	-	-	√	√
Gulkan, Akkar	SF	-	-	-	-	√	√
Li, Rahgozar et al.	FT	-	√	-	-	√	-
Cluni et al.	T	-	-	√	-	√	√
Rahgozar et al.	T	-	-	-	-	√	-
Boutin et al.	T	√	-	-	-	√	√
Ragni et al.	T	-	√	-	-	-	-
McCallen, Chajes et al.	SF	√	-	√	-	√	√
Swaddiwudhipong et al.	T	-	-	√	-	√	-
Wang et al.	SF	-	-	√	√	√	-
Ng and Kuang	SF	-	-	√	√	√	-
Zalka, Potzta, Kollar	SF	√	-	√	√	√	-
Meftah et al.	F	-	-	√	√	√	√
Rafezy et al.	ST	-	√	√	√	√	-
Carpinteri et al.	S	√	√	√	-	√	-
Luongo et al.	T	√	-	√	√	√	-
Greco et al.	S	√	√	√	√	√	√
Basu et al.	SF	-	-	-	-	√	-
Stafford Smith et al.	SF	-	-	-	-	√	-
Miranda	SF	-	√	-	-	√	√
Khaloo, Khosravi	SF	-	-	√	-	√	√
Bozdogan	SF	-	√	-	-	√	-

Legend: S = shear beam; F = flexural beam; T = Timoshenko's beam.

As this excursus shows, the definition of an equivalent beam has not always been simple and immediate, especially in the case of three-dimensional, torsional coupling behaviour; reducing complex structural systems to equivalent beam models, even in the elastic field, is still an open challenge of great interest.

3. Inelastic Beam-like Models

All the papers presented in the previous section focus exclusively on the dynamic behaviour of multi-storey buildings obtained by exploiting equivalent elastic beam-like models. Nevertheless, as is very well known, the dynamic responses of real structures under seismic loads exhibit significant excursions into the plastic regime which cannot be taken into account by elastic models.

The models currently present in the literature rarely focus on non-linear responses and, when considered, they do not include flexural–torsional coupling in the inelastic field.

Following a well-established procedure, consistent with actual seismic codes, the assessment of the seismic vulnerability of buildings is nowadays generally performed, rather than by means of the non-linear dynamic analysis of detailed 3D FEM models, by identifying the seismic demand of each building through the inelastic behaviour of the SDOF system assumed equivalent to the 3D structural model. The inelastic behaviour of equivalent SDOF oscillators is inferred by considering the results of non-linear static analyses performed on detailed 3D FEM models. Well-known examples of such procedures, computationally less demanding with respect to the dynamic non-linear analysis performed on 3D FEM models, are given by the N2 method [76,77], the capacity spectrum method (CSM) [78,79], uncoupled modal response history analysis (UMRHA), modal pushover analysis (MPA) [80], and incremental dynamic analysis (IDA) [81].

Recently, some authors proposed the adoption of simplified non-linear MDOF models, that can be considered as inelastic beam-like models, for the analysis of multi-storey buildings in order to describe their kinematics more accurately than with an SDOF system. The presence of additional degrees of freedom accounts for a partial collapse mechanism, often related to the contribution of higher modes or triggered by geometrical irregularities. Furthermore, a beam-like model allows the evaluation of some engineering demand parameters, such as inter-storey drifts and floor acceleration demands, since the model provides the time histories at all levels of the building.

Among the inelastic beam-like models presented in the literature, some deal with a discrete number of degrees of freedom, while others consider continuum systems. In the following, first, general inelastic discrete systems are discussed in order to show the difficulties arisen in the definition of an inelastic simplified MDOF model equivalent to a building, and then inelastic continuum models are addressed.

In order to make the paper easier to read, the adopted classification is graphically illustrated in Figure 8.

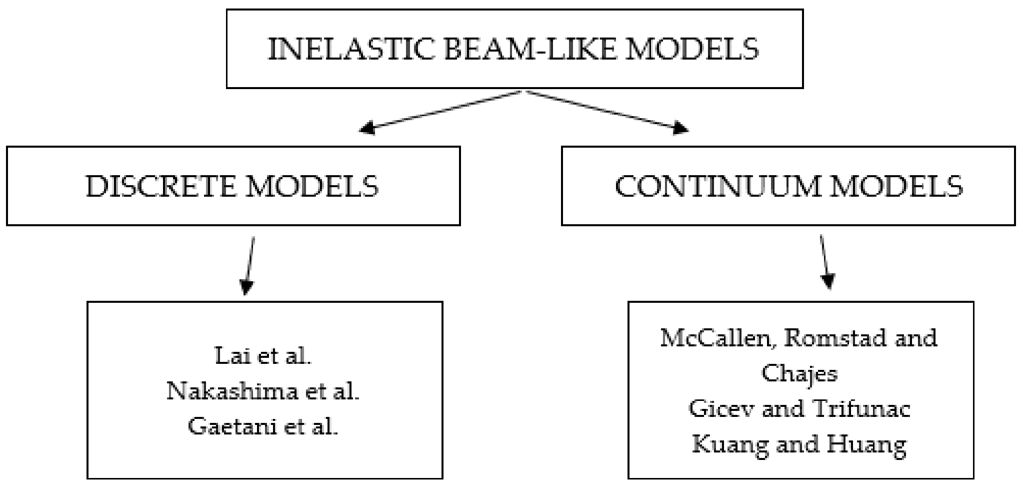

Figure 8. Classification of inelastic beam-like models.

3.1. Discrete Models

The definition of a simplified inelastic beam-like model equivalent to a building involves the adoption of correct kinematics and appropriate constitutive laws. This task is not so easy to achieve considering the different structural typologies and materials adopted in building construction.

One of the first studies in this context was by Lai et al. [82], who proposed a multi-rigid-body model with material non-linearity for the earthquake response analysis of shear-type structures in 1992. The model assumes that structural deformation concentrates totally on the nodes of the rigid elements. A damper and a spring are attached at each joint, and the stiffness of the spring incorporates the material non-linearities in accordance with each storey (Figure 9). The chosen non-linear models of restoring forces are bi-linear type and tri-linear degrading type. No further details were provided for the calibration strategy.

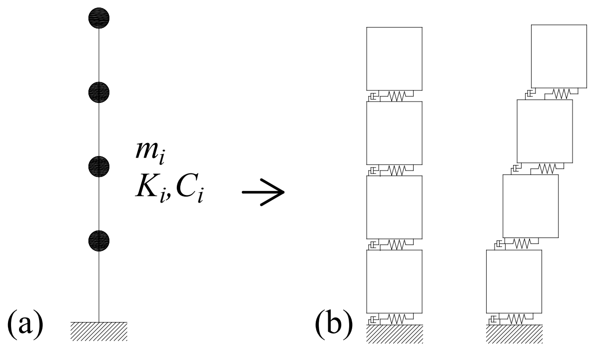

Figure 9. The multi-rigid-body model proposed by Lai et al. (**a**) shear-type model; (**b**) multi-rigid-body discrete model and displacements of the shear-type structure described by multi-rigid-body model.

Among the equivalent MDOF models, the fishbone model, or the generic frame in its first conceptualization presented by Nakashima et al. [83] in 2002 (Figure 10b), is adopted for the seismic analysis of steel moment-resisting frames. It consists of a single column with beams at every floor level extending halfway towards an adjacent column with a roller supporting each beam at midspan. The inelastic behaviour is taken into account by means of plastic hinges located at the ends of each member, representing the sum of the effects of the plastic hinges of the original frame in the corresponding position (Figure 10a). Precisely, in [83], the authors substituted the two half-beams and their plastic hinges with a rotational spring at each storey level (Figure 10b). Further investigations into the damage assessment of buildings by means of the fishbone model can be found in [84]. The fishbone model has been developed by several researches in order to take into account the flexural deformation of moment frames due to the axial elongation and contraction of columns in [85] (Figure 10c), considering tall buildings or irregular frames such as braced frames and/or moment-resisting frames with non-regular span-length [86], and it has also been improved for the seismic analysis of reinforced concrete frames into the substitute frame model [87] and the improved fishbone model [88]. The successive modification of the fishbone model is described by Soleimani et al. in [87] and reported in Figure 10.

The prediction of the seismic performance of buildings by means of the use of simplified models was the objective of the study by Gaetani et al. in 2020 [89]. The proposed model, named Stick-I (stick model for infilled frames), is a multi-degree-of-freedom (MDOF) system consisting of a series of lumped masses connected by means of non-linear shear link elements. The shear link behaviour is suitably calibrated, adopting a multi-objective genetic algorithm procedure that employs the results of non-linear cyclic pushover analyses performed on refined non-linear FEM.

The model is adopted for the evaluation of engineering demand parameters in infilled moment-resisting frames subjected to seismic loadings such as inter-storey drift ratios and peak floor accelerations. Another model, denoted as the Stick-IT model, is defined as a function of few geometric and mechanical parameters in order to make it suitable for building typologies and applied to the seismic vulnerability assessment at a large scale.

The discussed simplified MDOF models constitute a first step toward the definition of a continuum inelastic beam-like model, which requires appropriate kinematics and suitable constitutive law to predict possible plastic local deformations.

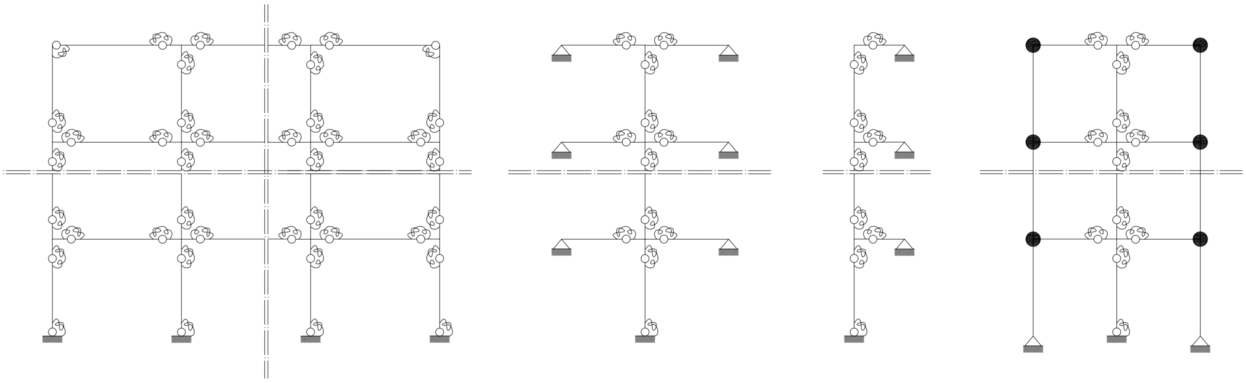

Figure 10. (**a**) Frame building (**b**) fishbone model (**c**) generic frame model (**d**) modified fishbone model.

3.2. Continuum Models

In the present section, we intend to present alternative beam-like continuum models which consider the inelastic behaviour of multi-storey buildings.

The first contribution in the definition of inelastic beam-like continuum models is due to McCallen, Romstad and Chajes [90–92], who updated their elastic continuum model (discussed in the previous section) in order to also consider material non-linearities. They assumed that the structural elements of the lattice (which is a repetitive reticular structure) were characterised by the Ozdemir model's elasto-plastic behaviour with kinematic hardening [93], and derived the instantaneous stiffness matrix of the continuum finite element. The inelastic continuum model was adopted for predicting the static and dynamic non-linear analysis of planar lattice frames.

Elasto-plastic material properties were also assumed by Gicev and Trifunac [94] in the evaluation of horizontal shear deformations in a 1D building supported by a half-space and excited by a vertically propagating shear wave.

In line with the previous studies described in this section, Kuang and Huang [95] also took into account only flexural and shear deformation in defining their equivalent beam-like model. In particular, they modelled a wall–frame structure with uniform stiffness as an equivalent continuum system consisting of a combination of a flexural cantilever and a shear cantilever beams. The model is based on the equivalent continuum system proposed by Miranda et al., but it is discretized by one flexural and one shear deformation element at each storey, as shown in Figure 11. Furthermore, in the proposed model, the deformation compatibility constraints are set at the floor levels where the storey mass is lumped. In this case, a bilinear hysteretic model is used for the material properties of flexural and shear cantilevers.

It is worth pointing out that, in the previously described beam-like models, no attention on the shear locking effect has been paid. In Kuang and Huang's work, shear and bending deformation were considered separately; however, this separation can introduce further difficulties in the calibration strategies if the model has to be considered equivalent to a building structure.

Some more examples of the application of beam-like models are given by the studies of Esteghamati et al. [96] and Barkhordari et al. [97], who analysed buildings coupled with shear walls and used a beam element to represent the non-linear behaviour of the shear-walls.

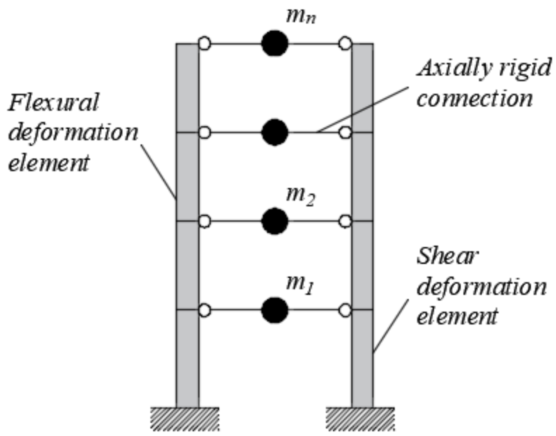

Figure 11. Mass distribution, flexural and shear deformation elements and floor links in the model proposed by Kuang and Huang.

3.3. Summary and Considerations about Inelastic Beam-like Models

As the previous section clearly shows, the scientific contributions in the field of inelastic beam-like models are very limited with respect to elastic models, showing the difficulties of researchers in defining an appropriate inelastic simplified model equivalent to a building.

Table 2 summarizes the main characteristics of each of the previously discussed models taking into account: spatial frames, non-uniform stiffness distribution, and structural analysis performed. The symbol √ indicates the presence of a hypothesis/feature/analysis (in the corresponding column) for the beam-like model of the specific author (in the corresponding row).

Table 2. Summary of literature review about inelastic beam-like models.

Author	Discrete or Continuum Model	Beam's Model	Non-Uniform Stiffness	3D Systems	Modal Analysis	Seismic Response
Lai et al.	Discrete	S	-	-	-	√
Nakashima et al.	Discrete	F	√	-	-	√
Gaetani et al.	Discrete	S	√	-	-	√
McCallen, Romstad and Chajes	Continuum	SF	-	-	√	√
Gicev and Trifunac	Continuum	S	-	-	-	√
Kuang and Huang	Continuum	SF	-	-	√	-

Legend: S = shear beam; F = flexural beam.

4. Limits and Future Developments of Beam-like Models

The adoption of beam-like models as an alternative to spatial FEM ones to simulate the static or dynamic behaviour of multi-storey buildings allows a drastic reduction in required computational effort, and is therefore very promising. Nevertheless, the limitation of beam-like models clearly consists of the small number of degrees of freedom with respect to detailed 3D models and, therefore, the related results, although sufficiently reliable, are certainly more approximate than FEM ones.

Furthermore, a great main limit of the beam-like models presented in the literature is the prevalent adoption of an elastic behaviour. Although some inelastic beam-like models have been proposed, there is still a long way before the definition of a correct simplified inelastic beam-like model is considered to be equivalent to a building. In fact, the first problem concerns the correct definition of the inelastic cyclic constitutive law for each inter-storey of the building, which can be also different from one floor to the other, and which should maybe be differentiated according to the typology of the structure (steel, reinforce concrete, masonry, etc.).

With reference to the spatial behaviour, further potential problems arise in coupling shear and torsional deformation fields. However, this is the main goal to achieve in order to define an inelastic beam-like model suitable for the 3D non-linear structural analysis of a building. Therefore, future studies should focus on the definition of an appropriate calibration procedure of the inelastic constitutive law of the spatial equivalent beam-like model.

Furthermore, the calibration of the beam-like models is based on the geometrical and mechanical characteristics of the building, which are mainly used to define the stiffness of the equivalent simplified model. However, an important step toward the improvement of beam-like models could consist of the adoption of dynamic identification techniques which may allow the definition and calibration of the equivalent beam-like model based solely on information on the external dimensions of the building. With reference to the calibration of the inelastic behaviour, an interesting extension could be represented by the data-driven beam-like models. Some important contributions to data-driven models and hybrid models that integrate both mechanics-based and data-driven techniques are represented by the studies by Gentile and Galasso [98], Guan et al. [99] and Soleimani-Babakamali Esteghamati [100].

5. Conclusions

The evaluation of the static or dynamic response of multi-storey buildings requires the adoption of opportune mechanical models which, on one hand, have to be sufficiently accurate and reliable but, on the other, must be simple enough to be easily implemented in numerical codes.

The use of sophisticated and extremely detailed models would in fact require great expertise and a huge computational effort. Moreover, oversimplified models could provide unreliable results leading to misleading results and wrong decisions, and economic and social impact could also be relevant.

The approaches for the seismic vulnerability assessment of buildings, currently proposed also in technical codes, consist of performing pushover analyses on 3D FEM models and successively adopting an equivalent SDOF systems assumed to be representative of the non-linear global seismic behaviour of the building. This approach partially nullifies the efforts associated with the construction of complex three-dimensional models. Furthermore, the adoption of an SDOF equivalent system would be exact only if the structure vibrates in a single mode with constant deforming shape over time whilst, in the case of high-rise or irregular buildings, the higher modes' effects cannot be neglected.

In order to improve the schematization of multi-storey buildings, increasing the degrees of freedom with respect to the SDOF system, several simplified MDOF models have been proposed in the scientific literature.

Among these models, the adoption of beam-like structures considered equivalent to multi-storey buildings have aroused great interest. Several authors have proposed different beam-like models and shown that their structural responses can be considered sufficiently accurate in reproducing the more computationally demanding responses of 3D models. This class of simplified models drastically reduces the computational burden and can, therefore, find a very useful field of application in seismic assessment at an urban scale where a great number of buildings have to be analysed and, therefore, the adoption of simple although reliable models is mandatory.

In this paper, a review of both elastic and inelastic beam-like models proposed in the scientific literature is presented. Due to the great production in this field, the present review does not pretend to be exhaustive, it rather aims to provide an input towards the inception of a common forum helpful for researchers willing to retrieve information on the subject and stimulate precious contributions for the adoption of these very interesting and promising structural models.

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