buildings

Article

Physics-Based Shear-Strength Degradation Model of Stud Connector with the Fatigue Cumulative Damage

Xiao-Wei Zheng [1,2,*], Heng-Lin Lv [1,2], Hong Fan [3] and Yan-Bing Zhou [3]

[1] Jiangsu Key Laboratory of Environmental Impact and Structural Safety in Engineering, School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou 221116, China
[2] Jiangsu Vocational Institute of Architectural Technology, Jiangsu Collaborative Innovation Center for Building Energy Saving and Construct Technology, Xuzhou 221116, China
[3] Key Laboratory of Urban Safety Risk Monitoring and Early Warning, Shenzhen Urban Public Safety and Technology Institute Co., Ltd., Shenzhen 518000, China
* Correspondence: xwzheng@cumt.edu.cn or xwz217@163.com

Abstract: In the whole lifetime of structures, fatigue damage accumulation will exist in the shear connector of steel–concrete composite beams. It is essential to determine the residual mechanical properties of shear connectors under long-term fatigue loads, e.g., the vehicle load on bridges. In this regard, a shear-strength degradation model is proposed for shear connectors. The Bayes theorem is used to develop posterior estimates of the unknown parameters in the degradation model based on the collected pushout test data of pre-damaged stud connectors caused by high-cycle fatigue loads. In addition, according to the proposed shear-strength degradation model, the service reliability assessment is performed with a composite bridge beam. The results indicate that (1) There is a large diversion in the traditional strength degradation model under the action of fatigue cumulative damage. More importantly, this proposed physics-based degradation model can effectively reduce uncertainty. (2) The effects of steel type and test specimen size can be well considered in the proposed shear-strength degradation model, which is beneficial for improving the reliability of risk assessment for fatigued bridges.

Keywords: strength degradation; fatigue cumulative damage; Bayes theorem; stud connector; composite beam

Citation: Zheng, X.-W.; Lv, H.-L.; Fan, H.; Zhou, Y.-B. Physics-Based Shear-Strength Degradation Model of Stud Connector with the Fatigue Cumulative Damage. *Buildings* **2022**, *12*, 2141. https://doi.org/10.3390/buildings12122141

Academic Editors: Oldrich Sucharda and Harry Far

Received: 10 October 2022
Accepted: 2 December 2022
Published: 5 December 2022

1. Introduction

The steel–concrete composite beams are widely adopted in buildings and infrastructures, in which stud connectors are commonly used to transfer the shear force across the steel–concrete interface [1–4]. The fatigue damage accumulation may cause shear capacity degradation of stud connectors. Much effort has been devoted to the fatigue behavior of stud connectors in steel–concrete composite beams by the pushout experiments [5,6]; however, only a few studies on investigating the residual mechanical properties of pre-fatigued stud connectors have been presented [7].

The experimental approach is commonly used to investigate the residual strength degradation of materials and components under high-cycle fatigue loads. Bro et al. [8] studied the fatigue life of the intact stud connector and the residual strength of pre-fatigued connectors. The results indicated that shear strength will decrease as the fatigue damage degree goes. Wang et al. [3] investigated the mechanical property degradation law of pre-fatigued stud connectors and indicated that the shear strength of pre-damaged stud connectors will nonlinearly decrease along with the loading ratio (i.e., the ratio between the cyclic stress amplitude and tensile strength of intact material) and the cycle ratio (i.e., the ratio between cycle times under given loading stress and the fatigue life corresponding to the same stress level). Other work also focused on the residual strength of pre-damaged stud connectors under high-cycle fatigue loads [9–12]. Wang et al. [3,13] investigated the residual bearing capacity of steel–concrete composite beams considering the effect of high-cycle fatigue cumulative damage through experiments and theory analysis. Furthermore, researchers developed vibration-based models to identify the residual strength capacity, damage level, and potential failures of composite steel-reinforced concrete structures (components), such as Fang et al. [14] and Pranno et al. [15].

Significant efforts indicated that the strength degradation model of fatigued components can be expressed as a function of loading ratio, cycle ratio, and other variables [16]. Based on the traditional strength degradation model, Wang et al. [17] presented a modified model with respect to the loading ratio and cycle ratio. However, the uncertainty associated with the unknown model parameters is commonly neglected in the widely used strength degradation model under high-cycle fatigue loads. In addition, the size effect has been verified to have a significant impact on the shear capacity of shear connectors [18], which is commonly ignored in the traditional strength degradation model. In light of these shortcomings, this study develops a physics-based shear-strength degradation model of fatigued stud connectors, considering the effects of fatigue load mechanism and diameter of the stud.

The rest of this paper is organized as: Section 2 presents 26 monotonic pushout tests of pre-fatigued stud connectors collected from previous publications. Section 3 discusses the widely used traditional strength degradation model of materials and components under fatigue loads. In Section 4, a physics-based shear-strength degradation model is proposed for fatigued stud connectors, in which, the Bayesian updating rule is used to develop the posterior probability distribution of the unknown model parameters based on the collected experimental data. In Section 5, this physics-based strength degradation model is applied for the reliability analysis of a representative composite beam under fatigue loads. The main conclusions drawn from this study are summarized in the final section.

2. Experimental Data Collection

The static strength of stud connectors that have been loaded to different cycle ratios under a given stress level was determined by experiments [7,19]. In this study, the coupled pre-fatigue test and static test approach were used to obtain the residual strength of the pre-fatigued stud connector. The testing procedure was conducted in the following steps:

(1) the static strength, $P(0)$, of the stud connector specimen was obtained by the static pushout test;
(2) the loading ratio (=$P_{\max}/P(0)$), a random variable, was used to determine the maximum value of the fatigue load $P_{\max}$;
(3) the fatigue life (N) of the stud connector specimen under a certain loading level was got by the fatigue test;
(4) the intact stud connector was loaded to different cycles n (such as 1×10^4, 5×10^4, 10×10^4 and 25×10^4) under the same loading level that corresponds to the fatigue life N, in which, n/N was defined as the cycle ratios. Upon completing the loading process, there will be pre-fatigued damage existing in the stud connector, which was called a pre-fatigued specimen.
(5) finally, the static pushout test was performed to obtain the residual strength of the pre-fatigued stud connectors.

The collected experimental data are summarized in Table 1, in which, $P_{\max}$ denotes the amplitude of cyclic load, $n \leq \mathrm{N}$ is the loading times and $P(n)$ is defined as the residual strength when a given stress level attains a certain loading time. Moreover, $n/\mathrm{N} = 0$, $P(n)$ represents the strength of the undamaged stud connector specimen; $n/\mathrm{N} = 1$, $P(n)$ denotes the fatigue strength of the specimen. In Table 1, a variable $k = d/d_{\max}$ is introduced to normalize the specimen size in the collected data, which can account for the impacts of different specimen sizes of a stud on the proposed degradation model. $d_{\max}$ denotes the maximum diameter of a stud in the collected experimental data. In this study, 26 experimental data have been collected from publications.

Table 1. Summarization of experimental data.

Sources	Variable k	Loading Ratio	Fatigue Life N ($\times 10^3$)	Loading Cases					
Oehlers [19]	0.59	0.3	1379	n/N	0.18	0.36	0.54	0.74	0.91
				$P(n)/P(0)$	0.85	0.80	0.74	0.55	0.49
Hanswille et al. [7]	1.0	0.3	6400	n/N	0.19	0.73	–	–	–
				$P(n)/P(0)$	0.59	0.6	–	–	–
		0.44	6200	n/N	0.32	0.70	–	–	–
				$P(n)/P(0)$	0.75	0.63	–	–	–
		0.44	5100	n/N	0.24	0.69	–	–	–
				$P(n)/P(0)$	0.66	0.61	–	–	–
		0.71	3500	n/N	0.29	0.72	–	–	–
				$P(n)/P(0)$	1.0	0.86	–	–	–
		0.71	1200	n/N	0.32	0.7	–	–	–
				$P(n)/P(0)$	0.95	0.84	–	–	–
Wang et al. [17]	0.59	0.6	2705	n/N	0.19	0.37	0.56	0.75	0.93
				$P(n)/P(0)$	0.98	0.91	0.83	0.77	0.64
Ahn et al. [20]	0.73	0.25	2495	n/N	0.2	0.4	0.6	–	–
				$P(n)/P(0)$	0.909	0.875	0.787	–	–
Bro et al. [8]	1.0	0.138	4900	n/N	0.082	0.204	0.245	–	–
				$P(n)/P(0)$	0.929	0.905	0.893	–	–

3. Traditional Strength Degradation Models

A Chinese specification [21] indicates that the shear strength of stud connectors is proportional to the strength of the material for fabricating the stud. Thus, the shear-strength degradation model of stud connectors, considering the fatigue cumulative damage, can be determined based on the strength degradation model of the materials [17]. Brountman and Sahu [22] proposed that the residual strength of the materials under fatigue loads can be directly calculated by the loading ratio and cycle ratio, given by

$$\frac{S(n)}{S(0)} = 1 - (1 - \frac{S_{\max}}{S(0)})\frac{n}{N} \tag{1}$$

where $S_{\max}$ denotes the maximum value of the fatigue loads or fatigue stresses; $S(n)/S(0)$ is defined as the strength degradation coefficient, namely, the ratio between the residual strength and the strength of the intact material. As indicated in the Chinese specification [21], the strength degradation coefficient of stud connectors is the same as the degradation coefficient of the steel material for fabricating the stud. In the following text, the strength degradation coefficient of stud connectors is directly written as $P(n)/P(0) = S(n)/S(0)$. Li [23] proposed a strength degradation model with respect to the loading ratio and cycle ratio, which is applied to depict the strength decay attenuation law of 45# steel under the action of high-cycle fatigue cumulative damage, that is

$$\frac{P(n)}{P(0)} = \frac{P_{\max}}{P(0)} + \left[1 - \frac{P_{\max}}{P(0)}\right](1 - \frac{n}{N})^{\theta} \tag{2}$$

in which, $P(n)$ denotes the residual strength of the stud connector after loading n cycles; $P(0)$ is the static strength of the stud connector obtained by the pushout test; $P_{\max}$ is the maximum value of the fatigue load; $P_{\max}/P(0)$ denotes the loading ratio; and n/N is the loading cycle ratios.

Referring to the characteristics of the fatigue cumulative damage and the existing strength degradation models, Zhang et al. [24] proposed a modified strength degradation model under high-cycle fatigue loads, written as

$$\left[\frac{P(n)}{P(0)}\right]^{1/\theta} = 1 - \left[1 - \left(\frac{P_{\max}}{P(0)}\right)^{1/\theta}\right]\frac{n}{N} \tag{3}$$

It should be stressed that the frequency inference (e.g., least square method) is commonly used to determine the unknown coefficients involved in the strength degradation model. In other words, the traditional degradation model is treated as deterministic without considering the epistemic uncertainty in the unknown model parameters. Thus, this paper applies the Bayes theorem to develop the posterior probability distribution of the unknown model parameters, and the uncertainty can be considered in the structural reliability analysis based on the full probability theory.

4. Physics-Based Shear-Strength Degradation Model

As previous discussions, various variables, such as the size of the testing specimen, fatigue load mode, and stress ranges, may have significant impacts on the strength degradation law of pre-damaged stud connectors under fatigue loads. It might cause vast errors in evaluating the residual strength of the pre-fatigued stud connector without the uncertainties associated with these variables, and the probability model of the variables can well quantify their corresponding uncertainties. Referring to the basic concept of the capacity model of the bridge piers for the seismic fragility estimates presented in the study of Gardoni et al. [25], a physics-based shear-strength degradation model of fatigued stud connectors can be expressed as

$$\frac{P(n|\mathbf{\Theta})}{P(0)} = \mathrm{f}(\mathbf{X}|\theta_{\mathrm{m}}) + \gamma + \sigma\varepsilon = \mathrm{f}(\mathbf{X}|\theta_{\mathrm{m}}) + \sum_{i=1}^{N_{\mathrm{h}}} \theta_{\mathrm{h},i}\mathrm{h}_i + \sigma\varepsilon \tag{4}$$

where the first term in the right of this equation is the deterministic term that can be expressed as Equations (1), (2), or (3); Vector $\mathbf{X}$ represents variables in the deterministic term, e.g., the loading ratio and cycle ratio, and θ_{m} is a vector of coefficients in the deterministic term; the second term γ is the error correction term, $\theta_{\mathrm{h},i}$ is the coefficient in the correction term, and h_i denotes the normalized variables that can impact the residual strength of stud connectors under fatigue loads, e.g., material type, specimen size, and loading mechanism; N_{h} is the size of variables in the correction term; the unknown model parameters $\mathbf{\Theta}$ = ($\mathbf{\theta_m}$, $\theta_{\mathrm{h},1}$, $\theta_{\mathrm{h},2}$, $\cdots$, $\theta_{\mathrm{h},i}$, σ); $\sigma\varepsilon$ denotes the model error following a normal distribution with a mean value of 0 and a standard deviation of σ. To capture a potential bias in the model that is independent of the variables $\mathbf{X}$, h_1 is set to 1 [25]. Applying the Bayesian updating rule [25,26], the posterior probability density function (PDF) of $\mathbf{\Theta}$, f($\mathbf{\Theta}$), is written as [27]

$$\mathrm{f}(\mathbf{\Theta}) = \mathrm{cL}(\mathbf{\Theta})\mathrm{p}(\mathbf{\Theta}) \tag{5}$$

in which, the normalized coefficient c = $[\int \mathrm{L}(\mathbf{\Theta})\mathrm{p}(\mathbf{\Theta})\mathrm{d}\mathbf{\Theta}]^{-1}$ is used to guarantee the integration of f($\mathbf{\Theta}$) equaling 1; L($\mathbf{\Theta}$) = $\prod$f(Data | $\mathbf{\Theta}$) denotes the likelihood function, and f(Data | $\mathbf{\Theta}$) is the PDF of the observations given $\mathbf{\Theta}$; p($\mathbf{\Theta}$) is the prior PDF of $\mathbf{\Theta}$. When only one variable is in the model, the prior PDF of $\mathbf{\Theta}$ can be written as

$$\mathrm{p}(\mathbf{\Theta}) \propto \frac{1}{\sigma} \tag{6}$$

For the n-dimensional model, the correlation between the unknown model parameters should be considered in the prior PDF of $\mathbf{\Theta}$, given by [25]

$$\mathrm{p}(\mathbf{\Theta}) \propto |\mathbf{R}|^{-(n+1)/2} \prod \frac{1}{\sigma} \tag{7}$$

where $\mathbf{R}$ = $[\rho_{ij}]$ denotes the correlation matrix among $\mathbf{\Theta}$; n is the number of variables in the model.

Herein, only the specimen size is considered in the collected experimental data. Equations (1)–(3) are, respectively, selected as the deterministic term, and this physics-based model is defined as Model I, Model II and Model III, respectively. This paper uses a

Markov Chain Monte Carlo (MCMC) sampling [28] to obtain the posterior estimates of the unknown parameters in the strength degradation models, as shown in Figure 1.

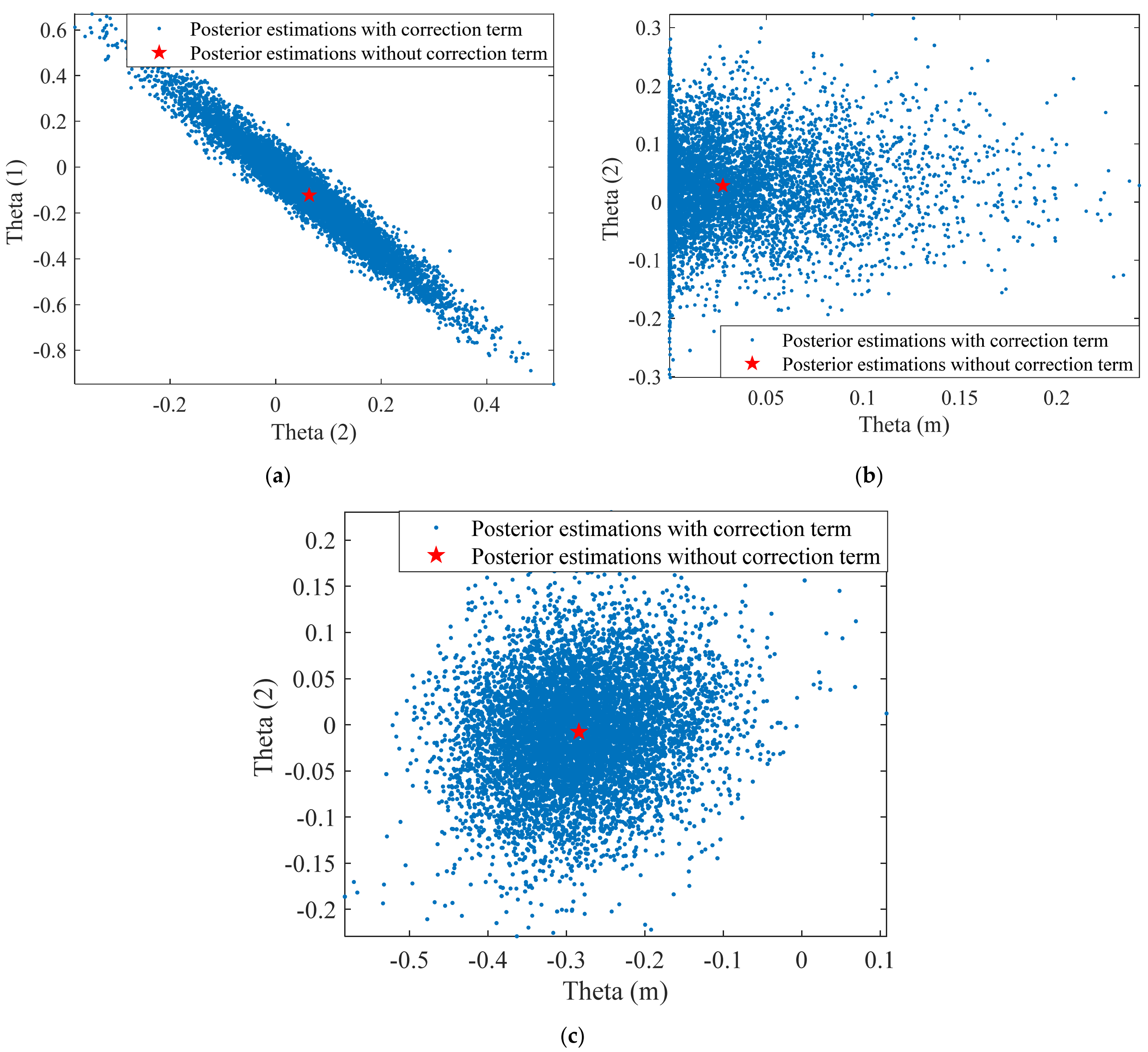

Figure 1. The posterior estimates of the unknown model parameters. (**a**) Model I. (**b**) Model II. (**c**) Model III.

Since there are no unknown parameters in the deterministic term of Model I, Figure 1a shows θ_1 and θ_2 of the correction term, conversely, Figure 1b,c illustrate the unknown parameter of the deterministic term and θ_2 in the correction term. It can be observed that the posterior estimates of the unknown parameters in the shear-strength degradation models are centered around their mean values, and there is a large variation in the posterior estimates. Thereby, uncertainties associated with the unknown model parameters should be carefully considered in the service reliability assessment of the fatigued bridges. In addition, as shown in Figure 1c, the mean values of the unknown parameters in Model III are contradicting the actual situation. The mean value of θ_m in the deterministic term of Model III is a negative value indicating a rising trend in the shear strength of the connectors as the loading ratio increases. Thus, Model I and Model II will be used in the following analysis. The posterior statistics of the unknown model parameters in the shear-strength degradation models are listed in Table 2.

Table 2. Posterior statistics of unknown parameters in the strength degradation model.

Models	Parameters	Mean	Standard Deviation
I	θ_1	−0.125	0.222
	θ_2	0.067	0.122
	σ	0.260	0.038
II	θ_m	0.025	0.038
	θ_1	−0.248	0.136
	θ_2	0.025	0.074
	σ	0.156	0.024

A comparison between the prior and posterior PDFs of the model errors (σ) is shown in Figure 2. It can be observed that there are significant differences between different model errors. Additionally, Figure 2 shows that compared to the prior distribution of the model errors, there are narrow ranges of the posterior probability distribution. This result indicates that information in the experiment data can concentrate the distributions of the unknown model parameters and reduce their corresponding uncertainty.

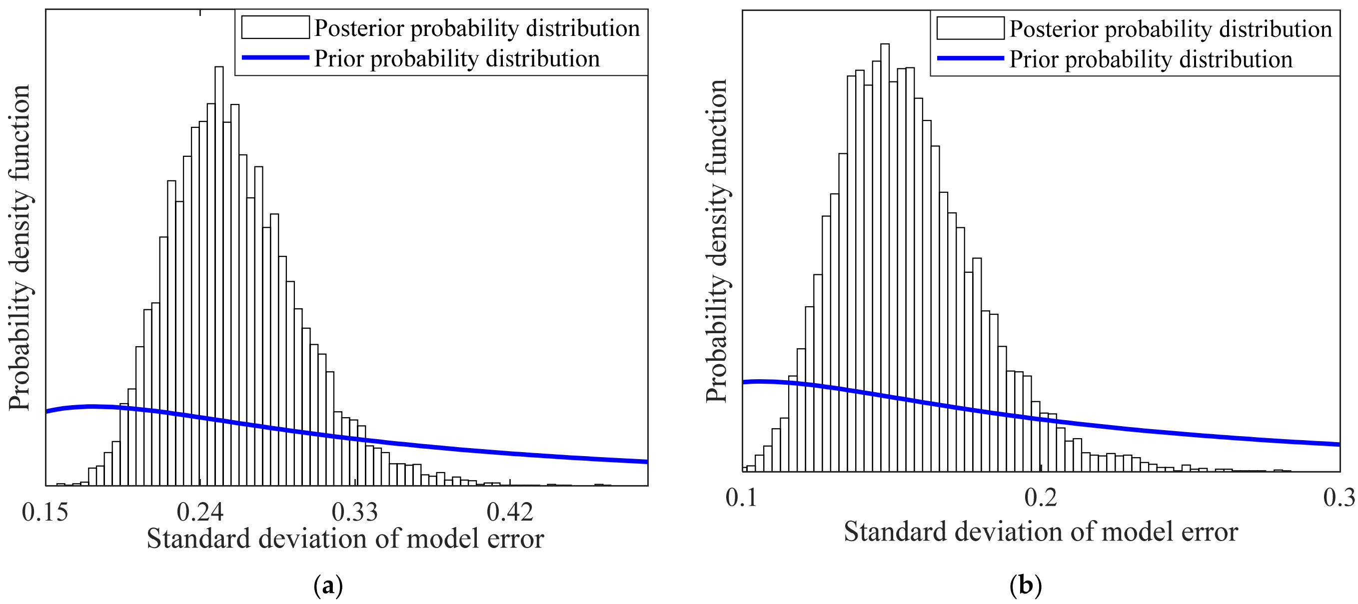

Figure 2. The prior and posterior density of model errors. (**a**) Model I. (**b**) Model II.

To further illustrate the decay attenuation law of the shear strength of stud connectors, the degradation distributions of Model I and Model II under different cases are shown in Figures 3 and 4.

When the cycle ratio is 0.3, the contours of Model I concerning the loading ratio and variable k are shown in Figure 3a. It can be observed that the loading ratio and k both have significant influences on the strength degradation of stud connectors. Figure 3b shows the strength degradation distribution versus the loading ratio and cycle ratio with k equaling 1.5. It is shown that the shear strength of stud connectors will vastly decrease as the cycle ratio increases.

In addition, when the loading ratio equals 0.3, Figure 4a illustrates the contours of the shear-strength degradation model concerning the cycle ratio and variable k. Similarly, the cycle ratio and k both have significant impacts on the degradation model, and the strength degradation coefficient increases as k increases. In contrast, the distribution of Model II versus the loading ratio and cycle ratio with k of 1.5 is shown in Figure 4b. It indicates that when the loading ratio is less than about 0.9, the shear-strength degradation model of stud connectors is insensitive to the loading ratio, which is inconsistent with the actual situation. Thus, the physics-based strength degradation model with a deterministic term of Equation (1) will be used in the service reliability analysis of the deteriorated girder bridges caused by high-cycle fatigue cumulative damage.

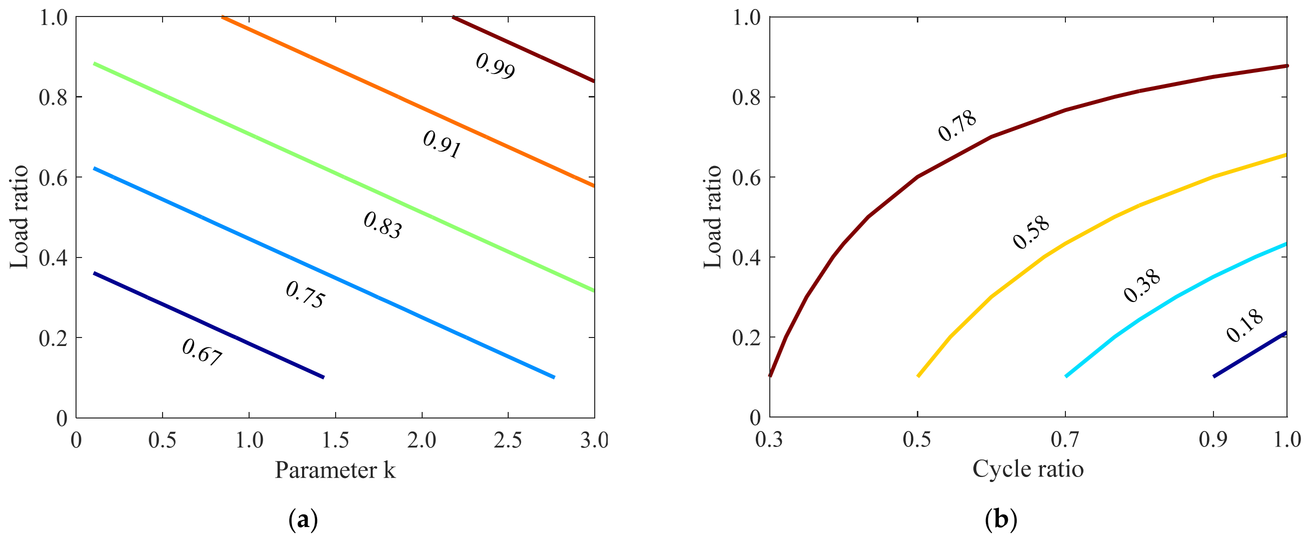

Figure 3. Contours of physics-based Model I. (**a**) Cycle ratio = 0.3. (**b**) Parameter k = 1.5.

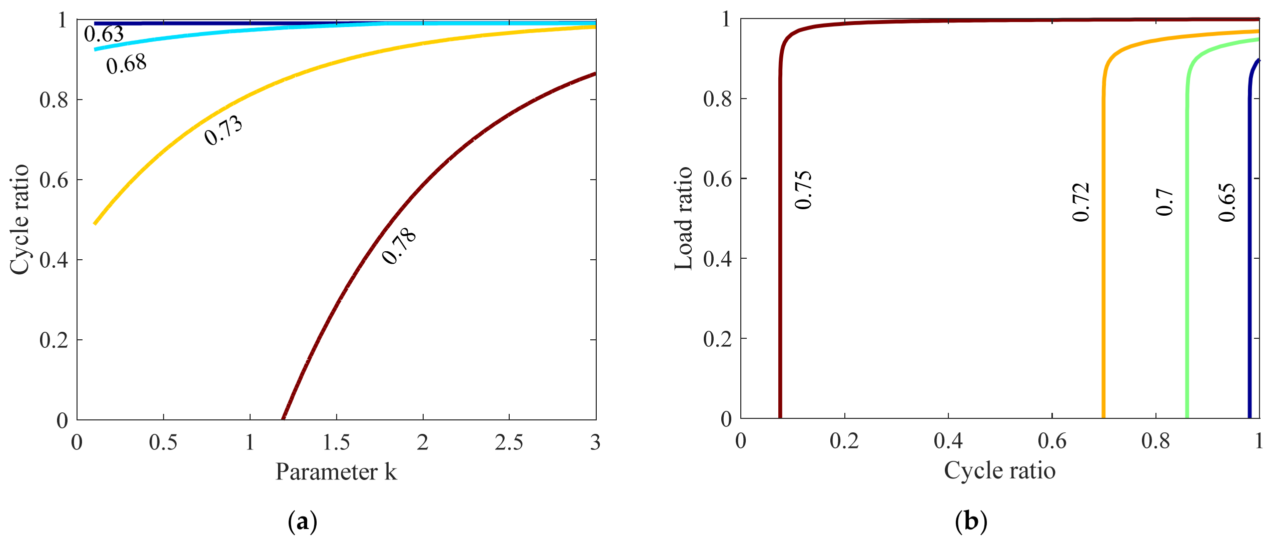

Figure 4. Contours of physics-based Model II. (**a**) Load ratio = 0.3. (**b**) Parameter k = 1.5.

It should be stressed that variable k (=d/d_{max}) is not considered in the traditional strength degradation model, which is one of the main defects of the traditional model compared to this proposed physics-based strength degradation model of the fatigued stud connectors.

5. Analytical Derivation for Service Reliability of Composite Girder

In this study, a cantilever I-shape steel-concrete composite bridge beam is used to assess the reliability considering the shear-strength degradation of fatigued stud connectors, as detailed in Figure 5. In the composite beam with a length (l) of 6 m, the interval (Δl) between the adjacent studs is designed as 150 mm, and there are 80 (=2 $\times$ 6000/150) studs. The diameter (d) of the stud is 20 mm, which is treated as a random variable following a Normal distribution with a coefficient of variation (COV) of 0.1 [29,30]. For the cross-section of I-shape steel, the thicknesses of the flange and web are 10 mm and 6 mm, respectively.

In addition, the steel types of Q345 and ML-15 are, respectively, selected for I-shape steel and studs. The reinforced concrete deck of this composite beam is formed with the C50 concrete and HPB300 reinforcement bars. The mechanical properties of the steel and concrete materials are treated as random variables herein, and their corresponding detailed information is summarized in Table 3. It should be pointed out that the shear strength of this composite bridge beam is only related to the strength of the corresponding materials. Thus, the strain values of the steel and concrete materials are not presented in Table 3.

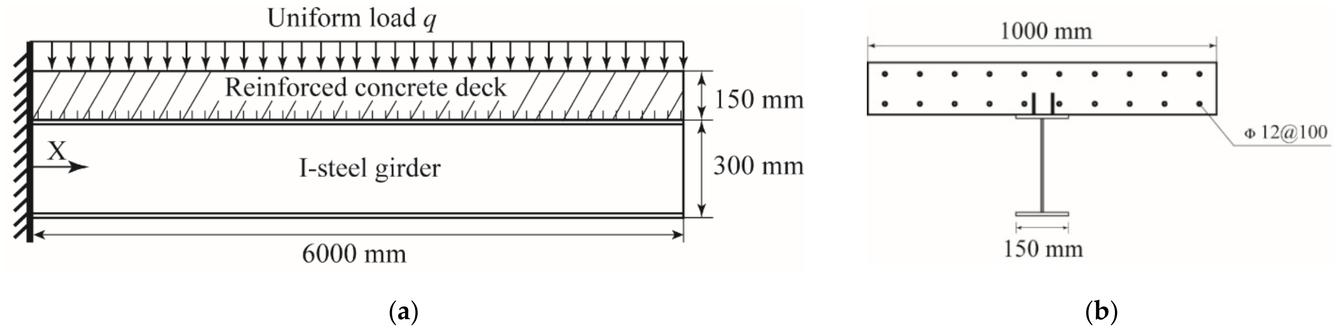

Figure 5. Schematic diagram of steel–concrete composite girder. (**a**) Evaluation view. (**b**) Cross-section profile.

Table 3. Probabilistic description of the materials.

Name	Mean (MPa)		COV/%	Distribution	Upper Level	Lower Level	References
Q345	f_{y1}	352	5	Lognormal	$1.1 f_{y1,mean}$	$0.9 f_{y1,mean}$	Zheng et al. [31]
	f_{u1}	495	5	Lognormal	$1.1 f_{u1,mean}$	$0.9 f_{u1,mean}$	Zheng et al. [31]
	E_1	2.06×10^5	3.3	Lognormal	$1.1\ E_{y1,mean}$	$0.9\ E_{y1,mean}$	Barbato et al. [32]
C50	f_c	44.8	20	Lognormal	$1.4 f_{c,mean}$	$0.6 f_{c,mean}$	Barbato et al. [32]
	E_c	$4733\sqrt{f_c}$	12	Normal	$1.2 f_{c,mean}$	$0.8 f_{c,mean}$	Xu et al. [33]
ML-15	f_{y2}	442	5	Lognormal	$1.1 f_{y2,mean}$	$0.9 f_{y2,mean}$	Melchers [34]
	f_{u2}	525	5	Lognormal	$1.1 f_{u2,mean}$	$0.9 f_{u2,mean}$	Zheng et al. [35]
	E_2	2.0×10^5	3.3	Lognormal	$1.1\ E_{y2,mean}$	$0.9\ E_{y2,mean}$	Barbato et al. [32]
HPB300	f_{y3}	300	5	Lognormal	$1.1 f_{y2,mean}$	$0.9 f_{y2,mean}$	Zheng et al. [36]
	E_2	2.0×10^5	3.3	Lognormal	$1.1\ E_{y2\ mean}$	$0.9\ E_{y2,mean}$	Zheng et al. [37]

In Table 3, $f_{y,i}$, $f_{u,i}$ and E_i denote the yield strength, ultimate strength, and Young's modulus of different types of steel materials, respectively. Moreover, f_c and E_c are the compressive strength and Young's modulus of concrete. By the internal force analysis, the beam's inner force at the *i*-th section corresponding to the *i*-th stud is shown in Figure 6. It can be observed that the axial forces and moments at the reinforced concrete deck and steel girder meet the following Equations (8) and (9).

$$N_c = -N_a = N \tag{8}$$

and

$$M(x) = M_c(x) + M_a(x) + \frac{Nh}{2} \tag{9}$$

where h represents the total depth of the beam equaling the height (h_c) of the reinforced concrete deck plus the height (h_a) of the steel girder. To compute the beam inner force, the strain at the *i*-th section of the reinforced concrete deck and steel girder is calculated by

$$\begin{cases} \varepsilon_1 = \frac{M_c(x)h_c}{2E_cI_c} - \frac{N}{E_cA_c} \\ \varepsilon_2 = \frac{N}{E_aA_a} - \frac{M_a(x)h_a}{2E_aI_a} \end{cases} \tag{10}$$

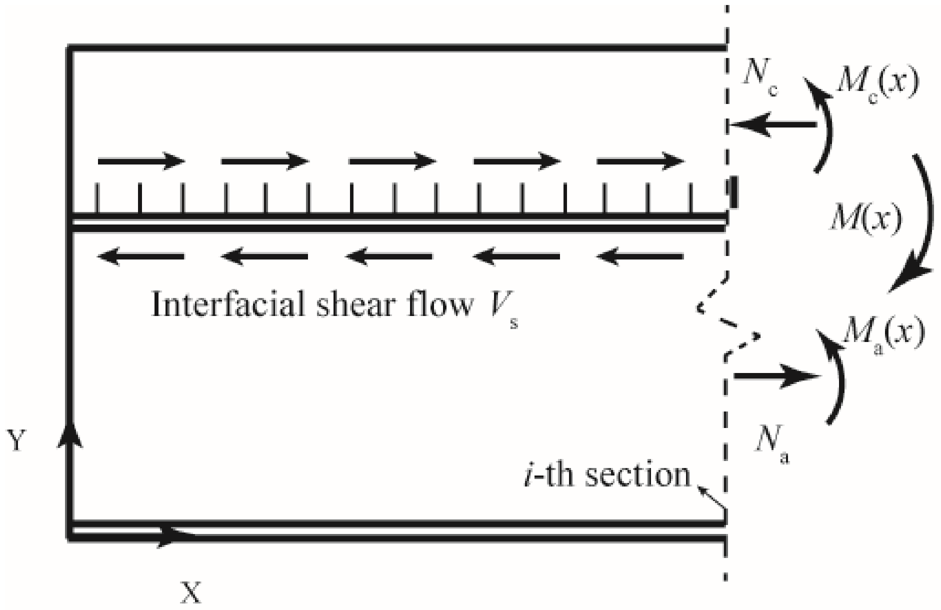

Figure 6. Internal force analysis for the composite beam.

In addition, considering the deformation coordination condition (i.e., $\varepsilon_1 = \varepsilon_2$), the different layers of the composite beams have the same axial elongation, which is expressed as

$$\int_{l-i\cdot\Delta l}^{l-(i-1)\cdot\Delta l} \varepsilon_1 \mathbf{d}\boldsymbol{x} = \int_{l-i\cdot\Delta l}^{l-(i-1)\cdot\Delta l} \varepsilon_2 \mathbf{d}\boldsymbol{x} \tag{11}$$

Because the i-th section of the different layers have the same curvature (i.e., $M_c(x)/E_cI_c = M_a(x)/E_aI_a$), combined with Equation (9), the moments at the i-th section of the different layers are calculated as

$$\begin{cases} M_c(x) = \frac{E_cI_c}{E_cI_c+E_aI_a}\left[M(x) - \frac{Nh}{2}\right] \\ M_a(x) = \frac{E_aI_a}{E_cI_c+E_aI_a}\left[M(x) - \frac{Nh}{2}\right] \end{cases} \tag{12}$$

in which, E_c and I_c are the elastic modulus and section moment of inertia for the reinforced concrete deck, respectively. Conversely, E_a and I_a are the elastic modulus and section moment of inertia for an I-shape steel girder, respectively.

For a cantilever beam under the uniform load q, the moment caused by the uniform load at the i-th section is written as

$$M(x) = \frac{q(l-x)^2}{2} \tag{13}$$

Combined with Equations (11)–(13), the axial force at the i-th section is calculated as

$$N_i = \frac{(3i^2 - 3i + 1)q\Delta l^2}{3h + \frac{12(E_cI_c+E_aI_a)(E_cA_c+E_aA_a)}{E_cA_cE_aA_ah}} \tag{14}$$

where A_c and A_a are the cross-section area of the reinforced concrete deck and steel girder, respectively. Uniform q includes the design dead load q_1 and live load q_2, and for the highway of Class I, q_2 is set to 10.5 kN/m [38]. Similarly, the axial force at the $(i-1)$-th section can be expressed as

$$N_{i-1} = \frac{\left[3(i-1)^2 - 3(i-1) + 1\right]q\Delta l^2}{3h + \frac{12(E_cI_c+E_aI_a)(E_cA_c+E_aA_a)}{E_cA_cE_aA_ah}} \tag{15}$$

Thus, the shear force on a single stud at the i-th section under the uniform load is given by

$$V_{s,i} = \frac{N_i - N_{i-1}}{2} = \frac{iq\Delta l^2}{h + \frac{4(E_c I_c + E_a I_a)(E_c A_c + E_a A_a)}{E_c A_c E_a A_a h}} \quad (16)$$

If the failure of this composite beam is controlled by the concrete crush, the design shear load at a single stud is calculated as $V_{s,d} = 0.43\eta A_s\sqrt{f_c E_c} = 148.0$ kN, in which, η (=$0.016\Delta l/d + 0.8$) denotes the reduction factor of the group stud effect [39], and A_s is the cross-area of a stud. In contrast, when the failure is due to the cut of the studs, the design force is computed as $V_{s,d} = 0.7A_s f_u = 115.4$ kN [38]. Therefore, the failure mode of the stud connector is controlled by the stud cutting. For the intact stud connector, the maxima of the mean shear force at the fixed end of this beam, which is the most dangerous section, is computed as 93.7 kN < 115.4 kN. In other words, the composite beam is safe without considering the shear-strength degradation of stud connectors. To investigate the impacts of fatigue-induced strength degradation, the Monte Carlo simulation is used to calculate the failure probability of stud connectors. The elastic modulus of the concrete and steel, as well as the ultimate strength and diameter of the stud, are treated as random variables.

The calculated failure probability results under the different cases are illustrated in Figure 7. It can be observed that the stud connector is not always safe considering the effects of the shear-strength degradation caused by the fatigue cumulative damage, even though this composite beam is just subject to the combined action of the self-weight and the design live load. This proposed physics-based model can be used to estimate the residual strength of the studs under fatigue loads given the loading ratio and cycle ratio. Furthermore, similar to the degradation law of the shear strength of the stud connector, the loading ratio, cycle ratio, and specimen size have significant impacts on the failure probability. Thus, in the performance assessment and design of the composite beam bonded with the stud connector or other shear connectors, the shear capacity degradation effects should be carefully considered.

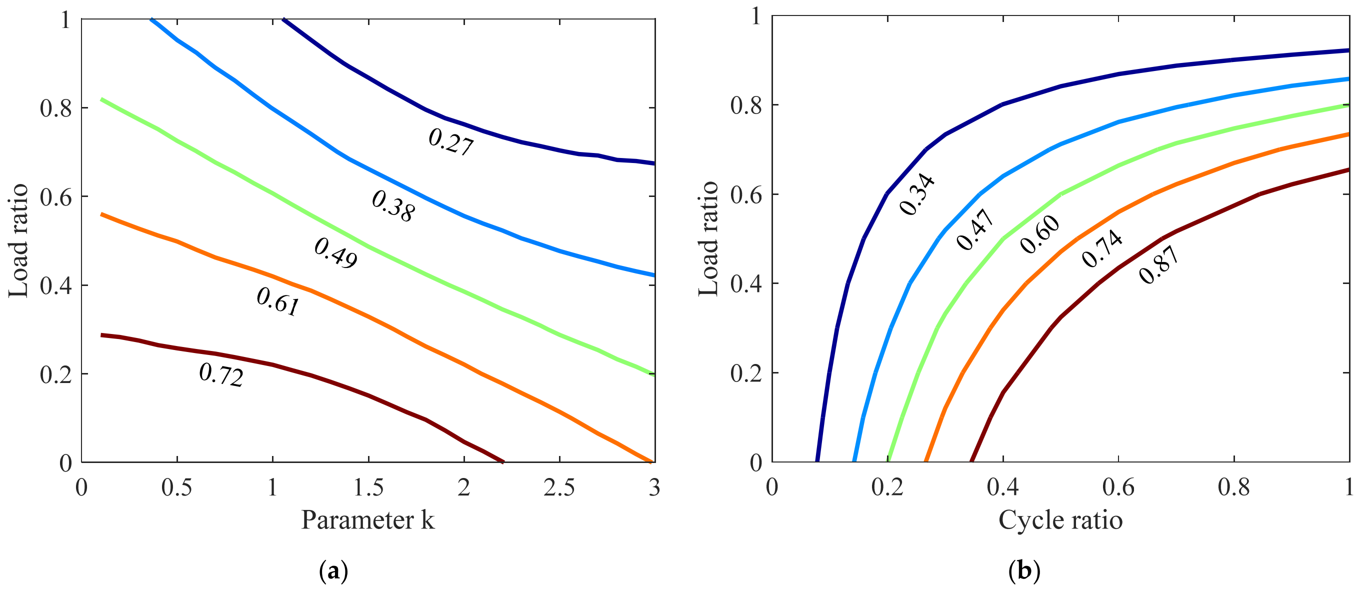

Figure 7. Failure probability of this composite bridge beam. (**a**) Cycle ratio = 0.3. (**b**) Parameter $k = 1.5$.

6. Conclusions

This paper proposed a physics-based shear-strength degradation model of stud connectors, and the Bayes theorem is used to determine the posterior probability distribution of the unknown model parameters based on the collected experimental data. The main conclusions are as follows:

(1) There is a large variation in the traditional strength degradation model under the fatigue load, and the epistemic uncertainty in the unknown model parameters should be carefully considered;
(2) For the same test results, there are significant differences among various strength degradation models and lack of necessary mathematical and physical background. The proposed physics-based degradation model can well fill up this shortcoming and consider the effects of various variables, such as the specimen size and loading mechanism;
(3) Considering the shear-strength degradation of stud connectors, the composite beam may fail under the combined action of the self-weight and the design live load, which should be accounted for in the structural design phase.

It should be stressed that this physics-based strength degradation model of stud connectors was developed based on the experimental data under shear loading conditions. Thus, this proposed strength degradation model may not be well suitable for the torsional loading condition. However, this degradation model can be updated if new data under various loading conditions are obtained.

Author Contributions: Conceptualization, methodology, resources, analysis and writing: X.-W.Z.; revision of writing: H.F. and Y.-B.Z.; constructive comments: H.-L.L. All authors have read and agreed to the published version of the manuscript.

Funding: The authors sincerely appreciate the financial support of the National Natural Science Foundation of China (No. 52208511) and the Youth Doctor Funds of Jiangsu Collaborative Innovation Center for Building Energy Saving and Construct Technology (SJXTBZ2115).

Data Availability Statement: Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare that there is no conflict of interest.

References

1. Saleh, S.M.; Majeed, F.H. Shear Strength of Headed Stud Connectors in Self-Compacting Concrete with Recycled Coarse Aggregate. *Buildings* **2022**, *12*, 505. [CrossRef]
2. Fang, Z.; Fang, S.; Liu, F. Experimental and Numerical Study on the Shear Performance of Short Stud Shear Connectors in Steel-UHPC Composite Beams. *Buildings* **2022**, *12*, 418. [CrossRef]
3. Wang, B.; Huang, Q.; Liu, X.; Ding, Y. Study on stiffness deterioration in steel-concrete composite beams under fatigue loading. *Steel Compos. Struct.* **2020**, *34*, 499–509. [CrossRef]
4. Qin, X.; Yang, G.T. Elastic stiffness of stud connection in composite structures. *Steel Compos. Struct.* **2021**, *39*, 419–433. [CrossRef]
5. Xu, X.; Liu, Y. Analytical prediction of the deformation behavior of headed studs in monotonic push-out tests. *Adv. Struct. Eng.* **2019**, *22*, 1711–1726. [CrossRef]
6. Xu, C.; Sugiura, K.; Su, Q. Fatigue Behavior of the Group Stud Shear Connectors in Steel-Concrete Composite Bridges. *J. Bridge Eng.* **2018**, *23*, 04018055. [CrossRef]
7. Hanswille, G.; Porsch, M.; Ustundag, C. Resistance of headed studs subjected to fatigue loading—Part II: Analytical study. *J. Constr. Steel Res.* **2007**, *63*, 485–493. [CrossRef]
8. Bro, M.; Wilde, M. Influence of Fatigue on Headed Stud Connectors in Composite Bridges. Master's Thesis, Lulea University of Technology Sweden, Luleå, Sweden, 2004.
9. Veljkovic, M.; Johansson, B. Residual static resistance of welded stud shear connectors. In Proceedings of the 5th International Conference on Composite Construction in Steel and Concrete V, Berg-En-Dal, South Africa, 18–23 July 2004; p. 524.
10. Feldmann, M.; Hechler, O.; Hegger, J.; Rauscher, S. New investigations on the fatigue behavior of composite beams made of high strength materials with two different kinds of shear connection. *Stahlbau* **2007**, *76*, 826–844. [CrossRef]
11. Kim, S.H.; Kim, K.S.; Park, S.; Aim, J.H.; Lee, M.K. Y-type perfobond rib shear connectors subjected to fatigue loading on highway bridges. *J. Constr. Steel Res.* **2016**, *122*, 445–454. [CrossRef]
12. Alsharari, F.; El-Sisi, A.E.D.; Mutnbak, M.; Salim, H.; El-Zohairy, A. Effect of the Progressive Failure of Shear Connectors on the Behavior of Steel-Reinforced Concrete Composite Girders. *Buildings* **2022**, *12*, 596. [CrossRef]
13. Wang, B.; Liu, X.L.; Zhuge, P. Residual bearing capacity of steel-concrete composite beams under fatigue loading. *Struct. Eng. Mech.* **2021**, *77*, 559–569. [CrossRef]
14. Fang, L.; Zhou, Y.; Jiang, Y.; Pei, Y.; Yi, W. Vibration-Based Damage Detection of a Steel-Concrete Composite Slab Using Non-Model-Based and Model-Based Methods. *Adv. Civ. Eng.* **2020**, *2020*, 8889277. [CrossRef]

15. Pranno, A.; Greco, F.; Lonetti, P.; Luciano, R.; De Maio, U. An improved fracture approach to investigate the degradation of vibration characteristics for reinforced concrete beams under progressive damage. *Int. J. Fatigue* **2022**, *163*, 107032. [CrossRef]
16. Philippidis, T.P.; Passipoularidis, V.A. Residual strength after fatigue in composites: Theory vs. experiment. *Int. J. Fatigue* **2007**, *29*, 2104–2116. [CrossRef]
17. Wang, B.; Huang, Q.; Liu, X.L. Deterioration in strength of studs based on two-parameter fatigue failure criterion. *Steel Compos. Struct.* **2017**, *23*, 239–250. [CrossRef]
18. He, J.; Lin, Z.; Liu, Y.; Xu, X.; Xin, H.; Wang, S. Shear stiffness of headed studs on structural behaviors of steel-concrete composite girders. *Steel Compos. Struct.* **2020**, *36*, 553–568. [CrossRef]
19. Oehlers, D.J. Deterioration in strength of stud connectors in composite bridge beams. *J. Struct. Eng.* **1990**, *116*, 3417–3431. [CrossRef]
20. Ahn, J.-H.; Kim, S.-H.; Jeong, Y.-J. Fatigue experiment of stud welded on steel plate for a new bridge deck system. *Steel Compos. Struct.* **2007**, *7*, 391–404. [CrossRef]
21. *GB50017-2017*; Standard for Design of Steel Structures. China Machine Press: Beijing, China, 2017.
22. Broutman, L.J.; Sahu, S. A New Theory to Predict Cumulative Fatigue Damage in Fiberglass Reinforced Plastics. In *Composite materials: Testing and Design (Second Conference), Proceedings of the Second Conference on Composite Materials: Testing and Design, Anaheim, PA, USA, 20–22 April 1971*; ASTM International: West Conshohocken, PA, USA, 1972.
23. Li, L. Research on the Law of the Stiffness Degradation and Strength Degradation of the Bolt under Fatigue Loads. Master's Thesis, Northeastern University, Shenyang, China, 2013.
24. Zhang, L.; Ji, W.; Zhou, W.; Li, W.-L.; Ren, C.-X. Fatigue cumulative damage models based on strength degradation. *Trans. Chin. Soc. Agric. Eng.* **2015**, *31* (Suppl. S1), 47–52.
25. Gardoni, P.; Der Kiureghian, A.; Mosalam, K.M. Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations. *J. Eng. Mech.* **2002**, *128*, 1024–1038. [CrossRef]
26. Zheng, X.-W.; Li, H.-N.; Gardoni, P. Probabilistic seismic demand models and life-cycle fragility estimates for high-rise buildings. *J. Struct. Eng.* **2021**, *147*, 04021216. [CrossRef]
27. Sobieraj, J.; Metelski, D. Quantifying Critical Success Factors (CSFs) in Management of Investment-Construction Projects: Insights from Bayesian Model Averaging. *Buildings* **2021**, *11*, 360. [CrossRef]
28. Gelman, A.; Carlin, J.B.; Stern, H.S.; Dunson, D.B.; Vehtari, A.; Rubin, D.B. *Bayesian Data Analysis*, 3rd ed.; Chapman & Hall: New York, NY, USA, 2020.
29. Papakonstantinou, K.G.; Shinozuka, M. Probabilistic model for steel corrosion in reinforced concrete structures of large dimensions considering crack effects. *Eng. Struct.* **2013**, *57*, 306–326. [CrossRef]
30. He, Z.Q.; Ou, C.X.; Tian, F.; Liu, Z. Experimental Behavior of Steel-Concrete Composite Girders with UHPC-Grout Strip Shear Connection. *Buildings* **2021**, *11*, 182. [CrossRef]
31. Zheng, X.-W.; Li, H.-N.; Li, C. Damage probability analysis of a high-rise building against wind excitation with recorded field data and direction effect. *J. Wind. Eng. Ind. Aerodyn.* **2019**, *184*, 10–22. [CrossRef]
32. Barbato, M.; Gu, Q.; Conte, J.P. Probabilistic Pushover analysis of structural and soil-structure systems. *J. Struct. Eng.* **2010**, *136*, 1330–1341. [CrossRef]
33. Xu, Q.H.; Shi, D.D.; Shao, W. Service life prediction of RC square piles based on time-varying probability analysis. *Constr. Build. Mater.* **2019**, *227*, 116824. [CrossRef]
34. Melchers, R.E. *Structural Reliability Analysis and Prediction*; John Wiley: New York, NY, USA, 1999.
35. Zheng, X.-W.; Li, H.-N.; Gardoni, P. Life-cycle probabilistic seismic risk assessment of high-rise buildings considering carbonation induced deterioration. *Eng. Struct.* **2021**, *231*, 111752. [CrossRef]
36. Zheng, X.-W.; Li, H.-N.; Gardoni, P. Reliability-based design approach for high-rise buildings subject to earthquakes and strong winds. *Eng. Struct.* **2021**, *244*, 112771. [CrossRef]
37. Zheng, X.-W.; Li, H.-N.; Yang, Y.-B.; Li, G.; Huo, L.-S.; Liu, Y. Damage risk assessment of a high-rise building against multihazard of earthquake and strong wind with recorded data. *Eng. Struct.* **2019**, *200*, 109697. [CrossRef]
38. *JTG-D64-2015*; Specifications for Design of Highway Steel Bridge. China Communications Press: Beijing, China, 2015.
39. *GB50917-2013*; Code for Design of Steel and Concrete Composite Bridges. China Planning Press: Beijing, China, 2013.