

Review

Structural Optimization in Civil Engineering: A Literature Review

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Abstract: Since tremendous resources are consumed in the architecture, engineering, and construction (AEC) industry, the sustainability and efficiency in this field have received increasing concern in the past few decades. With the advent and development of computational tools and information technologies, structural optimization based on mathematical computation has become one of the most commonly used methods for the sustainable and efficient design in the field of civil engineering. However, despite the wide attention of researchers, there has not been a critical review of the recent research progresses on structural optimization yet. Therefore, the main objective of this paper is to comprehensively review the previous research on structural optimization, provide a thorough analysis on the optimization objectives and their temporal and spatial trends, optimization process, and summarize the current research limitations and recommendations of future work. The paper first introduces the significance of sustainability and efficiency in the AEC industry as well as the background of this review work. Then, relevant articles are retrieved and selected, followed by a statistical analysis of the selected articles. Thereafter, the selected articles are analyzed regarding the optimization objectives and their temporal and spatial trends. The four major steps in the structural optimization process, including structural analysis and modelling, formulation of optimization problems, optimization techniques, and computational tools and design platforms, are also reviewed and discussed in detail based on the collected articles. Finally, research gaps of the current works and potential directions of future works are proposed. This paper critically reviews the achievements and limitations of the current research on structural optimization, which provide guidelines for future research on structural optimization in the field of civil engineering.

Keywords: critical review; structural optimization; optimization strategy; metaheuristic algorithm



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1. Introduction

Civil engineering is defined as a discipline dealing with the design, construction, operation, and maintenance of buildings and infrastructures including a variety of works such as residence, bridges, and roads [1]. However, the architecture, engineering, and construction (AEC) industry is often considered as an industry with high labor intensity, low efficiency, and considerable environmental impacts [2,3] while it accounts for a large part of the economy. According to a report by Horta et al. [4], the global construction industry makes up approximately 9% of the world's gross domestic product (GDP). Another survey from Xu and Wang [5] pointed out that in 2017, the construction industry was the second-largest energy consumption sector in China, accounting for about 20% of the total energy consumption, about 23% of the total electricity consumption, and about 30% of the total CO₂ emissions, which had considerable impacts on the environment. Therefore, there has been growing interests in improving the social, economic, and environmental performance of civil engineering projects. Since the 20th century, with the advent and development of computational methods for structure design and analysis, optimization methods based on mathematical programming techniques have been proposed and adopted in the field of civil engineering in the past few decades [6].

Optimization refers to acquiring the best outcome under specific conditions [7]. In the field of civil engineering, optimization can be executed in each step of a project life cycle such as design, construction, operation, and maintenance. One of the most commonly used types of optimization is structural optimization. In this study, “structural optimization” refers to an optimization which aims to find the best arrangement of structures or structural components to achieve certain objectives under prescribed conditions [8], while ignoring the properties of adopted materials. Material is a critical part of civil engineering structures, which significantly affects their performance. Concrete based composite materials are most commonly used in buildings and civil engineering infrastructures [9], including plain concrete, reinforced concrete, pre-stressed concrete, etc. [1,10]. Although some civil engineering structures which contain different types of materials, structures that only contain a single type of material are normally considered in terms of structural optimization due to the computational difficulty when considering material distribution of structures. Structural optimization can be divided into the following four categories [11]:

1. Size optimization: also known as sizing optimization, which treats the cross-sectional areas of structures or structural members as the design variables;
2. Shape optimization: also known as configuration optimization, which treats the nodal coordinates of structures as the design variables;
3. Topology optimization: focuses on how nodes or joints are connected and supported, aiming to delete unnecessary structural members to achieve the optimal design;
4. Multi-objective optimization: simultaneously considers two or more of the above optimization objectives for better optimization results; an optimization involving size, shape, and topology at the same time is also known as layout optimization.

At the early stage, researches on structural optimization in the field of civil engineering only involves mathematical theorems and programming techniques based on simple structures as benchmarks. With the development of computational and construction techniques, structural optimization has become increasing popular and has been applied to larger and more complex civil engineering structures, especially topology optimization. For example, topology optimization based on iterative 3D Extended Evolutionary Structure Optimization (EESO) algorithms were implemented during the design process of the Qatar National Convention Centre (QNCC) in Doha in order to minimize the structural compliance, which is one of the largest civil engineering structures created by generative tools based on topology optimization [12]. Another example of structural optimization applied on a large-scale civil engineering structure is the Shenzhen CITIC Financial Center in Shenzhen, China. Through topology optimization assisted design, the optimized exoskeleton truss layout improved the material efficiency while ensured the overall stiffness of the structure [13].

One of the principal objectives of structural optimization is minimizing the total cost of the structure [14]. In construction projects, a lower cost is always desired on the premise of satisfying the requirements of structural performance. Many studies have been reported to reduce the total cost by minimizing the total weight of the structure. Recently, with the increasing attention on the environmental issue and sustainable development, reducing environmental impacts has become another significant objective of structural optimization because of the considerable amount of CO₂ emissions in the civil engineering industry [3]. In addition, some research articles on structural optimization focus on improving certain structural performance [15] such as mechanical behavior, aerodynamic performance, and dynamic seismic performance in order to adapt the structures to different environments.

To achieve the abovementioned objectives, many optimization methods have been proposed and developed. Recently, metaheuristic methods have become one of the most popular optimization methods in civil engineering structural optimization research because they are suitable for combinatorial optimization problems [16]. However, these metaheuristic methods also have some shortcomings such as high complexity [17] and inadequacy for high-dimensional problems [18]. Therefore, there has been increasing studies that focus on improving the performance of optimization methods, either to enhance the existing metaheuristic methods or to propose novel optimization methods. For example, Mor-

tazavi [19] proposed an auxiliary fuzzy decision mechanism to improve the performance of interactive search algorithm (ISA) for structural size and topology optimization. The combined algorithm, namely the fuzzy tuned interactive search algorithm (FTISA), achieves a lower computational cost and a higher solution accuracy. Degertekin [20] proposed two improved harmony search algorithms (i.e., efficient harmony search algorithm and self-adaptive harmony search algorithm) for size optimization of truss structures. Based on the experimental results from several cases, the new algorithms are proved to have lower computational cost, higher convergence speed, and better optimization results than the traditional harmony search algorithm. Furthermore, Zheng et al. [21] presented an explicit topology optimization method, namely transformable triangular mesh (TTM) method, for structural topology optimization, which is able to obtain the optimal solution more effectively compared with other state-of-the-art algorithms.

These abovementioned studies in the field of structural optimization presented the achievements and potential of structural optimization to improve the efficiency and sustainability of the civil engineering industry. However, although a substantial number of studies as well as survey reports were published in this domain, none of them achieved a comprehensive review of the research developments on structural optimization. Therefore, this paper aims to comprehensively review the state-of-the-art literature on structural optimization in the field of civil engineering, including the analysis of the optimization objectives and their temporal and spatial trends, analysis of the optimization processes with four major steps, and the discussions of research limitations and recommendations of future works.

The rest of this paper is organized as follows. Section 2 demonstrates the methodology used for literature retrieval and Section 3 presents a statistical analysis of the selected articles. Then, the optimization objectives of the selected articles are categorized and analyzed regarding the temporal and spatial trends in Section 4. Next, Section 5 provides an exhaustive review and analysis of the structural optimization process in four aspects, including structural analysis and modelling, optimization problem formulation, optimization methods, and computational tools and design platforms. Section 6 indicates the limitations of the current research and based on which elaborates the potential future works. Finally, conclusions are drawn to conclude and summarize this study in Section 7.

2. Methodology

A holistic approach was adopted in this paper to critically analyze the state-of-the-art literature and present a comprehensive review on structural optimization in the field of civil engineering. Figure 1 elaborately depicts the overall methodology of this study, including literature retrieval and selection from a digital database, statistical analysis of the selected literature, review of optimization objectives with temporal and spatial trends, review of optimization process, limitations and future work recommendation, and conclusion. The details of literature retrieval are presented in Section 2.1, and a brief introduction of the keywords utilized for literature retrieval is presented in Section 2.2.

2.1. Literature Retrieval

The database used for literature retrieval in this study is Google Scholar, which contains most of the academic literature published. Several search keywords including structural optimization, size, shape, topology, layout, optimal design, civil engineering structures, and metaheuristic algorithms, were adopted to facilitate the literature retrieval. Then, the most relevant literature was manually selected from the search results. Through the above literature retrieval method, a total of 196 papers were selected, including 154 research articles, 19 conference abstracts, 12 book chapters, 7 review articles, and 4 theses. Although structural optimization has a history of more than one hundred years, it is first applied in aerospace industry and the application in civil engineering industry is much later [22]. Moreover, with the advent of information technology, the optimization methods used in recent studies have changed a lot compared with early studies. Therefore, the range

of publication year of the selected papers is set from 1970 to 2020 in order to analyze and summarize the latest achievements of research on civil engineering structural optimization.

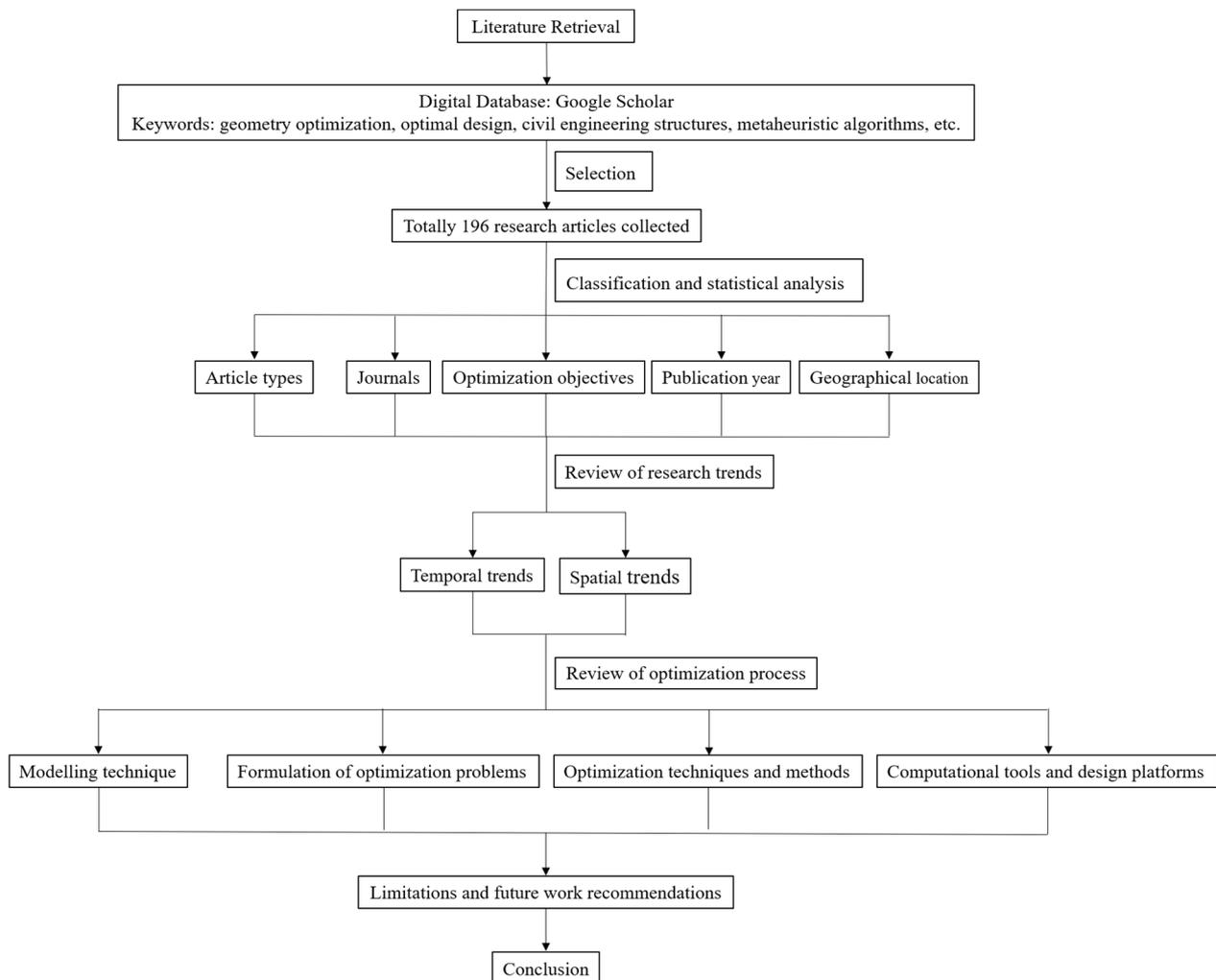


Figure 1. The research methodology of this study.

2.2. Keywords for Literature Retrieval

As mentioned above, eight keywords are utilized for searching relevant publications. During the process of choosing suitable keywords, “civil engineering structural optimization” is firstly searched in Google Scholar. Then several review articles are selected to find the state-of-the-art techniques in this field and based on which some other keywords such as topology optimization and metaheuristic algorithms are summarized for literature retrieval. Through these steps, a set of keywords which are sufficient to cover most of the articles in this domain could be obtained.

3. Statistical Analysis of Selected Literature

The distribution of papers regarding the publication years is presented in Figure 2, which divides the timeline into five time periods. It is obvious that the number of papers on structural optimization experiences a significant increase over the years. Among all the selected papers, 88% of them were published after 2000, and 73% of them were published after 2010, indicating that this theme has attracted increasing attention from researchers.

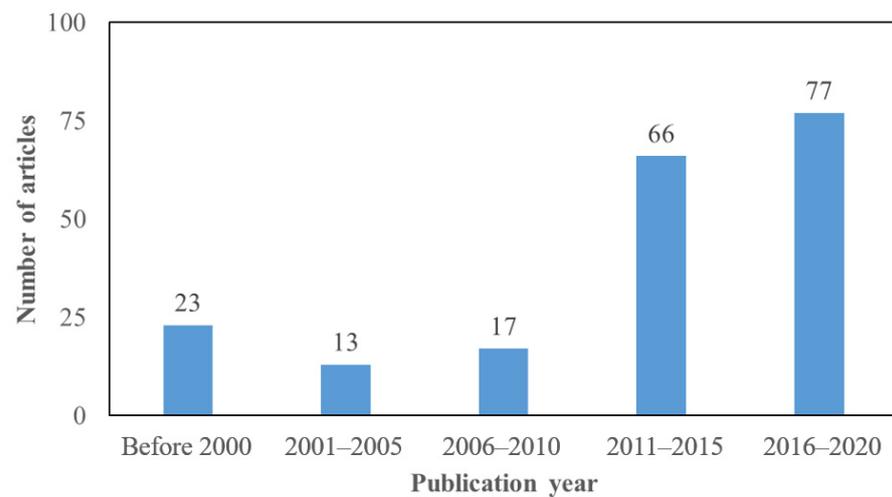


Figure 2. Distribution of the selected articles regarding the publication year.

These selected papers are also analyzed based on the journals they are published in. Figure 3 shows the top ten journals that publish the most articles in the field of civil engineering structural optimization. The top ten journals have published a total of 82 papers. The journal of *Computers and Structures* is ranked first with 21 papers published, followed by *Structural and Multidisciplinary Optimization* and *Engineering Structures*, each of which also published more than 10 papers.

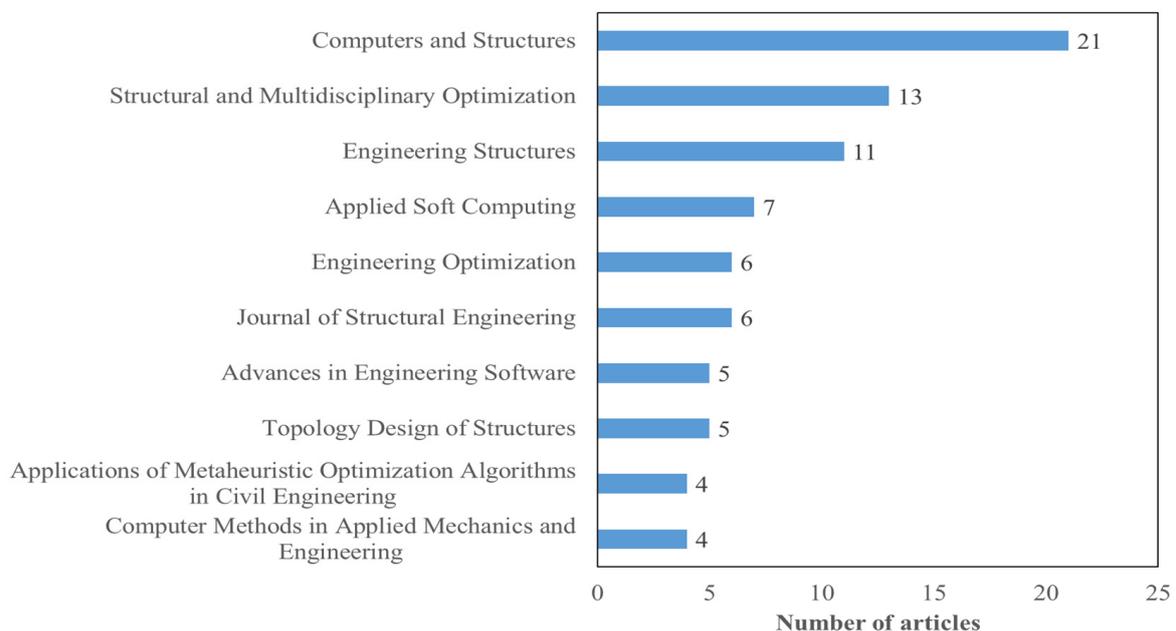


Figure 3. Number of selected articles published in the top ten journals.

Moreover, the retrieved articles are classified according to the geographical location of the first author's research institution, and the distribution of geographical locations is shown in Figure 4. The top three continents are Asia, Europe, and North America, which have published 79, 66, and 36 papers, respectively, which altogether account for 92% of the total number of the collected papers.

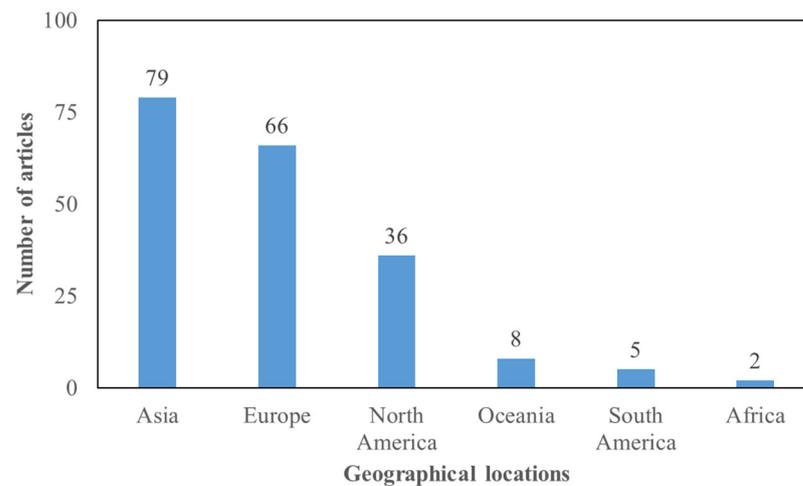


Figure 4. Distribution of selected articles by the continent of the first author's research institution.

4. Objectives of Structural Optimization

4.1. Categories of Optimization Objectives

The optimization objectives of the selected papers on structural optimization can be divided into the following four categories:

1. **Cost minimization:** the objective of the structural optimization design is to minimize the total cost, which is usually achieved by reducing the weight or the volume of the structure;
2. **Structural performance improvement:** the objective of the structural optimization design is to improve certain properties of the structure such as mechanical behavior, aerodynamic performance, dynamic seismic performance, in order to meet the requirements in different environments;
3. **Environmental impact minimization:** the objective of the structural optimization design is to reduce the greenhouse gas emission or energy consumption to improve the environmental performance of the structure;
4. **Multi-objective:** the objective of structural optimization contains more than one of the above three objectives.

Table 1 presents a summary of the four categories of optimization objectives and some relevant literature from the selected papers for further analysis.

Table 1. Summary of the four categories of optimization objectives with their respective relevant articles in the field of structural optimization.

Optimization Objectives	Description	Relevant Articles
Cost minimization	Optimization for minimizing the total cost of civil engineering structures, which is usually achieved by reducing structure weight or volume	Barbieri, Cinquini [23]; Lin, Che [24]; Zhou and Rozvany [25]; Liang, Xie [26]; Ghasemi and Dizangian [27]; Ho-Huu, Nguyen-Thoi [28]; Zhao, Xu [29]
Structural performance improvement	Optimization for improving certain properties of civil engineering structures in order to adapt functional requirements	Rahmatalla and Swan [30]; Natke and Soong [31]; Achtziger [32]; Wang [33]; Guest and Moen [34]; Uroš, Gidak [35]; Martin and Deierlein [36]
Environmental impact minimization	Optimization for reducing the environmental impacts of civil engineering structures, such as greenhouse gas emission and energy consumption	Yi and Malkawi [37]; Brown and Mueller [38]; Penadés-Plà, García-Segura [39]; Mayencourt and Mueller [40]
Multi-objective	Optimization considering more than one of the above objectives	Bremicker, Chirehdast [41]; Ohsaki and Swan [42]; Paik and Raich [43]; Munk, Vio [44]; Choi, Oh [3]; Xia, Langelaar [45]

Figure 5 presents the percentage of the selected articles for each optimization objective. It is found that most researchers are from the standpoint of project stakeholders and focus on the objective of cost minimization, accounting for 62% of the selected articles. Another 22% of the collected articles aiming to improve the structural performance based on structural optimization, whereas 14% of the articles engage in structural optimization for more than one goal. Few studies concentrate on reducing the environmental impact of civil engineering structures alone, which only account for 2% of the selected articles. The reason might be that reducing greenhouse gas emissions and embodied energy consumption will result in a reduction of the total cost of structures at the same time [39]. Therefore, a more common approach is to consider cost minimization and environmental impact minimization simultaneously, which is categorized as multi-objective optimization in this study.

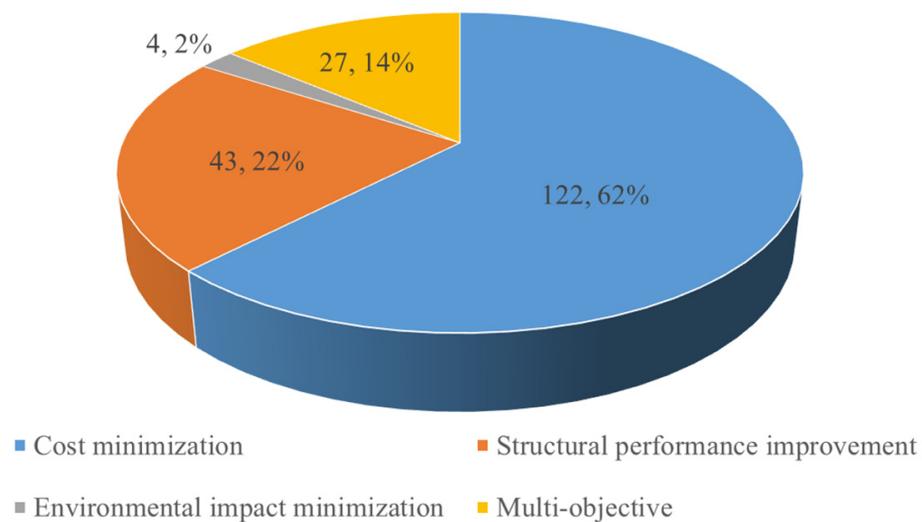


Figure 5. Number and proportion of articles for each optimization objective.

4.2. Temporal Trends of Optimization Objectives

Over the years, the overall tendency of research on civil engineering structural optimization experiences an upward trend, while the proportion of articles with each objective has been constantly changing. To analyze the changes of research trends in the field of structural optimization over time, the number and proportion of the selected papers with each optimization objective in the five time periods are shown in Figure 6a,b, respectively.

Before 2000, most of the articles focus on cost minimization. There are 16 papers related to this theme, which account for 70% of the total number of papers before 2000. Since weight or volume of a structure constitutes a considerable part of the cost [46], cost reduction was achieved by reducing the total weight or volume of the structure in all of these early studies collected. In addition to cost minimization, a few studies also concentrate on structural performance improvement and multi-objective optimization, which account for 17% and 13% of the selected articles, respectively. All of the articles aiming at structural performance improvement collected in this period are based on topology optimization, in which design optimization is achieved by eliminating subsystems with negligible contributions to structural performance while satisfying prescribed criteria [31]. It is worth noting that there was not a uniform structural performance indicator in these studies, and various performance indicators such as compliance [32,47], and maximum displacement or moment [48] were applied for optimization. For multi-objective structural optimization, the earliest study retrieved in this paper conducted a mean compliance minimization and a weight minimization separately, and then combined these two types of optimizations [41]. A more common form of multi-objective optimization is to consider two objectives at the same time. Some researchers proposed a multiplier to convert multi-objective problems to

single-objective problems [49] and others adopted a Pareto solution for achieving multiple optimization objectives simultaneously [50].

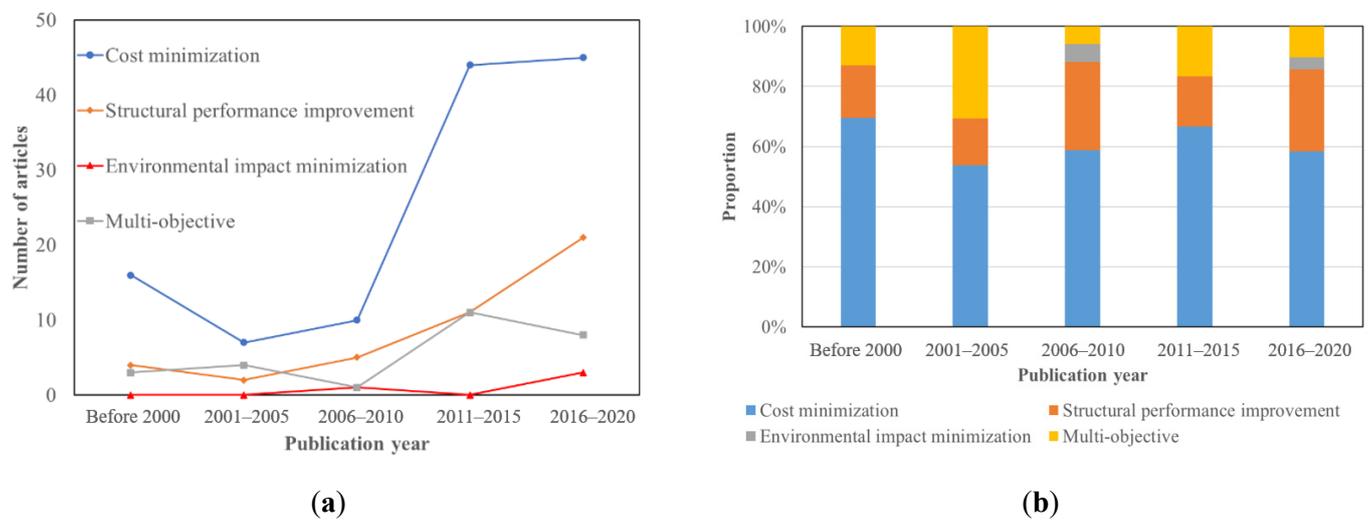


Figure 6. Number and proportion of articles with each objective in each time period: (a) number of articles, and (b) proportion of articles.

The number of articles in the field of structural optimization increases rapidly after 2000, especially in the fourth time period, from 17 articles between 2006 and 2010 to 66 articles between 2011 to 2015, while the proportion of articles with each objective has been changing over the years. According to Figure 6a,b, it is obvious that cost minimization has always been the hottest research theme, and the number of articles on this theme keeps increasing in the four time periods after 2000. However, the proportion of this theme in the last four time periods is less than that in the first period (before 2000), with 54%, 59%, 67%, 58% respectively. Structural performance improvement is the second hottest topic and accounts for 22% of the total selected articles. The number of articles with this objective has also been increasing in all of the four time periods after 2000. This type of structural optimization is particularly important in case the safety and serviceability of a structure system are more important than cost (weight) reduction [48]. In addition, some researchers focus on simultaneously achieving several different objectives through structural optimization. These objectives are usually incommensurable and in competition with each other [43,51]. However, due to some limitations such as computational complexity [50] and uncertainty of the solution [51], the number of studies relevant to this theme is limited, which only accounts for 14% of the total selected articles. According to Figure 6a, the number of articles with multi-objective has been fluctuating in the time periods after 2000. Moreover, one thing that must be pointed out is that although only four articles (2%) aim to reduce the environmental impact of the structures, three out of the four articles were published from 2016 to 2020, which suggests that this theme may become more popular in the future with increasing concern on sustainability in the field of structural engineering [39].

4.3. Spatial Trends of Optimization Objectives

Generally, more funding resources from the government or private institutes lead to a relatively larger number of research articles in specific directions [52]. In a previous review work, a term called “geographical scope” is defined for the division of geographical locations [53].

In this study, continent was chosen as the geographical scope at first, and the distribution of collected articles in each continent is shown in Figure 5, where Asia, Europe, and North America are the top three continents with relatively more articles. Furthermore, Figure 7 presents the distribution of collected articles with each optimization objective in each continent. It is observed that cost minimization is the theme with the highest

proportion in all the continents. However, in Europe and North America, the proportion of this theme is slightly lower than that in Asia. Another difference is that research on environmental impact minimization has been found in Europe and North America, while there are no articles with this objective found in Asia. In terms of the other three areas (Africa, South America, and Oceania), only two, five, and eight articles were retrieved from these continents, respectively, which means that research relevant to structural optimization is relatively elementary in these regions.

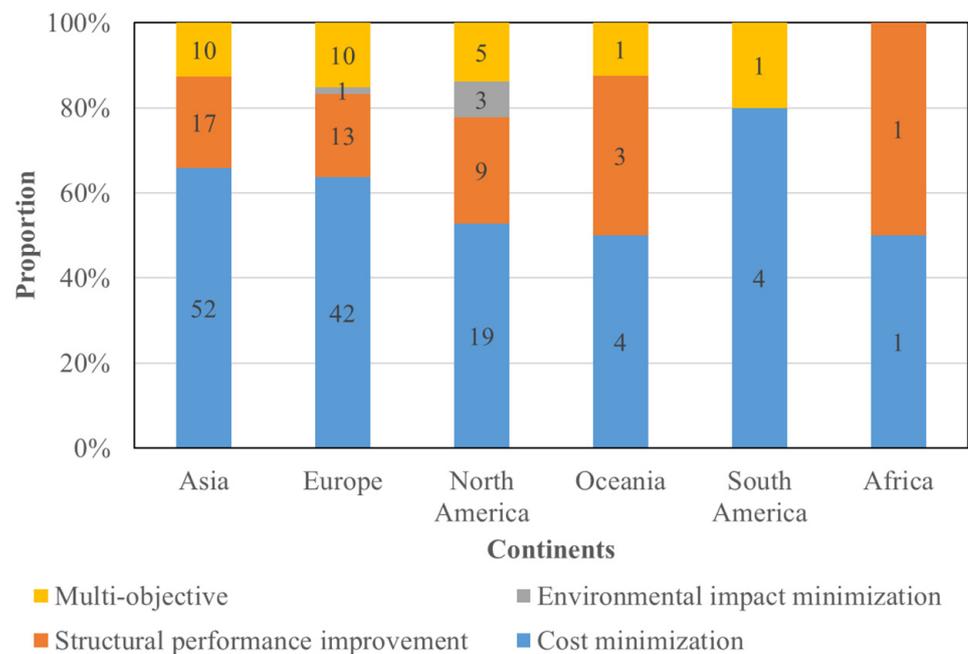


Figure 7. Distribution of articles with each objective in each continent.

Then, individual country was used as the geographical scope to further analyze the geographical trends of research in the field of civil engineering structural optimization. The 196 articles collected in this study are contributed by a total of 37 countries. Figure 8 presents the top 12 countries with the most collected articles and the number of their contributed articles. The top four countries with relatively more articles retrieved are Iran, the USA, China, and Turkey. A total of 117 papers are published by researchers from these four countries, accounting for 60% of the total number of collected articles. In terms of the distribution of optimization objective, generally speaking, the different research objectives are more evenly distributed in countries with a better economy (e.g., the USA, China, Australia, Greece, Italy, South Korea, the UK, Germany, and Spain), whereas in countries with a relatively poor economy (e.g., Iran, Turkey, and Brazil), the research mainly focuses on cost minimization. Besides, it is also found that more articles related to structural performance improvement come from countries that fall in seismically active zones, including the USA, China, and South Korea. This finding suggests that geographical and environmental factors can affect the research directions [10]. In these regions, researchers are probably more motivated to pay attention to improving the structural behavior under dynamic seismic loads.

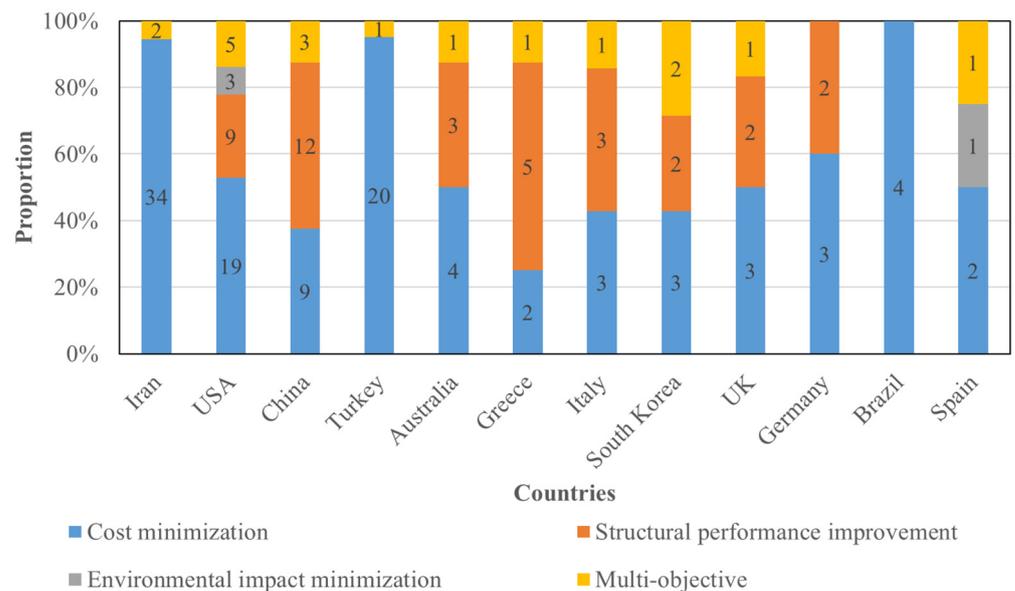


Figure 8. Distribution of articles with each objective in major countries.

5. Process of Structural Optimization

In the field of civil engineering, there is a wide range of structural optimization objects, including columns, beams, rods, etc., among which the most commonly used object for structural optimization is skeletal structures such as trusses and frames. Generally, there are four major components that should be taken into account in the process of structural optimization:

1. Modelling technique for structural analysis and design, based on which structural optimization can be divided into discrete optimization and continuum optimization;
2. Formulation of optimization problem, including the definitions of variables, objective function(s), and constraints;
3. Optimization method, referring to the mathematical programming methods applied to achieve structural optimization;
4. Computational tool and design platform, referring to the software platforms used to run the optimization codes and conduct structure design.

The following Sections 5.1–5.4, will provide a detailed review and discussion of the state-of-the-art literature according to the abovementioned four aspects.

5.1. Modelling Techniques for Structural Analysis and Design

Structural optimization is an iterative process in nature. During the optimization process, structural analysis needs to be repeated many times to evaluate the improvement of each design until convergence is reached, which is an enormous computational task. Therefore, it is necessary to select a structural analysis method that is computationally inexpensive, especially for large, complex civil engineering structures. Generally, structural analysis is conducted based on finite element method (FEM), and crude finite element models are more commonly used compared with detailed finite element models to reduce computational cost. Another method to reduce this cost is to combine structural analysis with structural design, which is called simultaneous analysis and design approach [22,54]. According to the modelling techniques used in the early stage of the design process, structural optimization can be divided into two broad categories, namely discrete optimization and continuum optimization. In discrete optimization, the structure system is modelled with discrete structural elements, while the structure system is treated as a solid continuum with variable topology in continuum optimization [42].

Since structures are broken down into several sections in discrete optimization method, it is convenient to select the cross-sectional properties and the nodal locations. Therefore,

this method is usually used in size and shape optimization with predefined and fixed topology. Structural optimization focusing on size and shape is also known as no topological or pre-defined topological optimization [1]. In other words, the aim of topological optimization is to create the shape of a structure, while shape optimization only has to tune the shape of the structure in order to improve the desired properties of the structure (usually mass or weight). Studies reported in [55–58] are some examples of the pre-defined topological optimization. For topology optimization, the discrete optimization method often focuses on the connectivity of structural elements. The overall optimal design can be obtained by combining the optimal design of each section.

In terms of continuum optimization, this method is usually applied in topology optimization, dealing with material distribution problems [42]. To some extent, the continuum optimization method has better performance than the discrete optimization method because the optimal design does not necessarily lead to truss-like or beam-like structural elements. Nonetheless, the application of the continuum method in the field of civil engineering is relatively limited because the optimization problem is more complex and the programming process is more difficult than treating the structure as discrete components [47].

5.2. Formulation of Optimization Problems

Problem formulation refers to determining the three fundamental components of an optimization problem in the problem search space, namely the design variables, objective function(s), and constraints [10]. When conducting structural optimization, it is presupposed some freedom to change the attributes of the structure. The parameters used to represent the change of these attributes are usually called design variables and they are usually denoted by a vector. Design variable can be divided into two categories according to its value, namely continuous design variable and discrete design variable. The values of continuous design variables fluctuate within a certain range, while discrete design variables only have isolated values. Objective function refers to a function or a set of functions that can be used as a measure of the optimization result. Constraints refer to the safety and serviceability requirements that must be satisfied during the optimization process. According to the form of the expression, constraints can be divided into two categories, namely equality constraints and inequality constraints. They can be interconverted to satisfy the requirements of different optimization methods. For example, an equality constraint $h(\mathbf{X}) = 0$ can be replaced by two inequality constraints $h_1(\mathbf{X}) \geq 0$ and $h_2(\mathbf{X}) \leq 0$ [22]. In addition, constraints could be combined into the objective function as penalty functions to convert the constrained objective function to an unconstrained one [59]. The range of design variables is called search space or design space, which could be further divided into feasible domain and infeasible domain. Feasible domain contains design points that satisfy all of the constraints, while design points that violate at least one constraint constitute infeasible domain. The general form of an optimization problem can be defined as follows [56]:

$$\begin{aligned} & \text{Minimize/Maximize : } f(\mathbf{X}); \\ & \text{Subject to : } g_i(\mathbf{X}) \leq 0, \quad i = 1, 2, 3, \dots, m; \\ & \quad \quad \quad h_j(\mathbf{X}) = 0, \quad j = 1, 2, 3, \dots, p; \\ & \quad \quad \quad \mathbf{X} \in S. \end{aligned}$$

where \mathbf{X} is usually a vector $\mathbf{X} = [x_1, x_2, x_3, \dots, x_n]$ and represents the set of design variables, in which n is the number of design variables; $f(\mathbf{X})$ is the objective function; $g_i(\mathbf{X})$ and $h_j(\mathbf{X})$ refer to inequality and equality constraints; m and p are the number of constraints; and S is the search space of the optimization problem.

As mentioned above, there are four different types of objectives in structural optimization. Therefore, the problem formulation will be discussed according to the type of objective. Defining the objective function refers to finding a quantification of the desired result for an optimization problem while satisfying some requirements. Therefore, the parameter representing the objective function is sometimes different from the optimization

objective. For example, cost minimization, which is the most commonly adopted objective in structural optimization, is usually quantified as the total weight of the structure to set up the objective function [60]. As a result, the optimal design is achieved by minimizing the total weight of the structure. However, using weight to represent cost is often criticized by structural designers because a structure design with minimum weight does not necessarily lead to the minimum cost [61]. Therefore, some objective functions are proposed to deal with the minimization of the cost, but only a small fraction of articles in the field of civil engineering structural optimization focus on this topic because of the uncertainties and fuzziness encountered [46]. In terms of size optimization, the structure system is usually divided into several structural elements, and the cross-sectional areas are chosen as the design variables because the total weight of the structure is directly relevant to the cross-sectional properties of each structural element [60]. Since the distribution of different materials is not considered in these studies, the objective function can be defined as Equation (2) [62]:

$$\text{Minimize : } W = \sum_{i=1}^n \gamma g A_i L_i$$

where W is the total weight of the structure; γ is the density of the material; g is the acceleration of gravity; the set of design variables $\mathbf{X} = \{A_1, A_2, A_3, \dots, A_n\}$ represents the cross-sectional areas of structural elements; and L_i is the length of each structural member. For shape optimization, the nodal coordinates are used as the design variables. This type of structural optimization is often combined with size optimization for weight minimization [6,63,64]. In terms of topology optimization, this type of structural optimization focuses on finding the optimal connectivity among nodes (joints), that is, determining whether there should be structural elements between the nodes or not [65]. Topology optimization generally starts from a predefined dense structure with a lot of structural members, which is called the ground structure. In the optimization process, unnecessary elements are progressively eliminated and eventually the optimal design with minimized weight is obtained [19]. Similarly, a vector is used as the set of topology variables. There are two values of these variables, namely 1 and 0. If the value of a topology variable is 1, the structural element represented by this variable can be removed, while 0 means the element cannot be removed [66]. Structural topology optimization is also commonly combined with size optimization for structure weight minimization because the structural elements with very small cross-sectional areas are regarded as unnecessary and can be removed [66]. In structural optimization with the goal of cost minimization, stress and displacement constraints are usually adopted, and the specific design requirements depend on the regional specifications applied instead of the type of optimization. Some most commonly used regional specifications include the ACI Codes for Concrete, Eurocodes 2, AASHTO, and British Standards [10].

Another commonly adopted objective for structural optimization is improving structural performance. However, there is not a uniform parameter to quantify the structural performance. Many performance indexes such as stiffness [31], compliance [67], strain energy [68], and static displacement [69] are used to construct the objective function in the collected literature. Most of the articles aiming at structural performance improvement reviewed in this paper are based on topology optimization. The reason may be that topology optimization leads to optimal structural size in principle, and can be further refined by size and/or shape optimization methods [67]. In this type of structural optimization, compliance minimization is generally set as the objective function to maximize the stiffness of structures. The objective function can be expressed as Equation (3) [70]:

$$\text{Minimize : } C = \mathbf{F}^T \times \mathbf{u}(x)$$

where C is the compliance of the structure; F represents the load vector applied on the structure; and u refers to the displacement vector. The constraints of structural optimization for structural performance improvement are more diverse than that for weight minimization

because of the various design requirements of structural properties. For example, when considering the dynamic response of structures, natural frequency is always constrained to avoid destructive effects of dynamic loads [71]. Based on the design requirements, many mechanical constraints such as displacement, stiffness, stress, and buckling loads are adopted in this type of structural optimization. In addition, material weight or volume is often constrained to control the structural cost [68].

The third objective of structural optimization is environmental impact minimization, which has received little attention from researchers. Only four articles on this theme are collected. In these articles, the environmental impact of civil engineering structures is quantified as CO₂ emissions or embodied energy consumption, and environmental impact minimization is achieved by reducing the material consumed [39]. Similar to the optimization for cost minimization, safety and serviceability constraints are adopted to satisfy the design requirements.

In the civil engineering industry, there is always a common criterion to evaluate a project, which is minimizing the cost while maximizing safety and serviceability. However, these goals may conflict with each other, which means that improving one of them implies worsening another one [1]. Therefore, increasing studies have focused on balancing the competing objectives (usually two objectives [72]) in the field of structural optimization. This type of structural optimization is usually termed as multi-objective optimization, which is defined as the last type of structural optimization objective in this paper. One of the major differences between multi-objective optimization and the aforementioned single-objective optimization is that more than one objective function are considered in multi-objective optimization. For example, researchers may consider minimizing weight and deflection simultaneously [73]. Obviously, multi-objective optimization problems are more complex and require more advanced computational methods [72]. Another major difference is that there is not a unique solution that reaches the optimum of all objectives simultaneously, which is determined by the intrinsic feature of multi-objective optimization [74]. Generally, a multi-objective optimization problem can be formulated as Equation (4) [1]:

$$\begin{aligned} & \text{Minimize the vector function} \\ & f(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), f_3(\mathbf{X}), \dots, f_k(\mathbf{X})]^T \\ \text{Subject to : } & g_i(\mathbf{X}) \geq 0, i = 1, 2, 3, \dots, m; \\ & h_j(\mathbf{X}) = 0, j = 1, 2, 3, \dots, p; \\ & \mathbf{X} \in S. \end{aligned}$$

where $f(\mathbf{X})$ is the set of objective functions; $g_i(\mathbf{X})$ and $h_j(\mathbf{X})$ refer to the inequality and equality constraints; $\mathbf{X} = [x_1, x_2, x_3, \dots, x_n]$ is the set of design variables; and S is the search space of the solution. As mentioned above, no unique optimal solution can be obtained from the formulation, and the optimization result consists of a series of trade-off solutions in fact. These solutions are known as no-dominated solutions and the set of these solutions is known as Pareto optimal set [74]. The Pareto optimal set, when it is plotted in the coordinate system considering the design criteria, is referred to as the Pareto front [74], which is a useful tool to display the result of multi-objective optimization. It is convenient for the designer to make trade-off decisions between the competing objectives. The constraints for multi-objective optimization is a combination of the constraints for each objective, including mechanical constraints, volume constraints, deflection constraints, etc.

Problem formulation plays a crucial role in structural optimization. It defines the variables, objectives, constraints, and solution scope of an optimization problem. Subsequently, computational techniques and methods are adopted in the process of optimization to find the optimal solution(s) in the search space.

5.3. Optimization Techniques and Methods

Structural optimization is one of the most intensively investigated research domains in engineering in the 20th century [75]. In this field, one of the milestones is proposed by Kuhn

and Tucker in 1951 [76], which presented some fundamental mathematical programming techniques for structural optimization, including Lagrange multiplier method, equivalence theorem, etc. These techniques are widely used in the subsequent researches. In recent years, mathematical programming and numerical search techniques have become one of the most commonly applied approaches in the field of structural optimization to search for the optimal solution in an efficient manner. The optimal solution searching process normally starts from an initial design and gradually improves the value of the objective function by iteration until convergence is achieved [22]. Generally, two categories of optimization methods are widely used in civil engineering structural optimization, namely gradient-based approaches and heuristic approaches [61].

Gradient-based approaches require a predefined search direction, which is called gradient, before searching for the optimal solution. This type of optimization approach can be further divided into four categories: linear programming methods, non-linear programming methods, optimality criteria methods, and feasible direction methods [61]. The linear programming methods refer to optimization methods with linear objective functions and constraints. When at least one of these functions are non-linear, the optimization methods are termed as non-linear programming methods. The optimality criteria methods involve developing efficient algorithms for the optimization of structures subject to stiffness constraints based on statically determinate or indeterminate structures and structural dynamics principles [77]. The optimality criteria methods are usually used for calculating the Lagrange multipliers, which are used for finding local minima/maxima of a function subjected to equality constraints, with stress and displacement constraints, as well as satisfying the certain optimality criterion. In the feasible direction methods, searching optimum starts from a point that satisfying all the constraints. Then the point is moved to a better point based on the following iterative scheme:

$$X_{i+1} = X_i + \lambda S_i$$

where X_i and X_{i+1} are the start point and the end point of the i th iteration; S_i is the direction of movement; and λ is the distance of movement, whose value is always predefined to make X_{i+1} falls within the feasible region. S_i determines the search direction and is found based on two principles: (1) a small move without violating constraints, and (2) a direction that reduces the value of the objective function. Therefore, the optimal solution can be reached after a number of iterations [78]. Sometimes researchers may incorporate approximate techniques in these gradient-based optimization approaches to reduce computational cost. These techniques construct an approximation of the structural design problem at first based on structural analysis and then solve the approximate problem by optimization methods. The optimal solution of the approximate problem is used as a basis for performing further analysis and refine the design [22].

These gradient-based optimization methods are also termed as conventional methods and are widely used in the early studies on civil engineering structural optimization. For example, Chan [6] used a linear programming algorithm for the optimization of structures subject to multiple loading. Dobbs and Felton [79] adopted a steepest descent nonlinear programming algorithm for truss geometry optimization to minimize the structure weight. Lin et al. [24] proposed a bi-factor α - β algorithm, which is an effective iteration algorithm and belongs to feasible direction methods, for minimal weight design of structures under static and dynamic constraints. However, despite the extensive applications, many limitations of these gradient-based algorithms have been pointed out by previous studies. These limitations can be divided into three categories in general:

1. Although these gradient-based algorithms have good performance in some cases of civil engineering structural optimization, the convergence to the global optimum is difficult to ensure [6]. In fact, a structural optimization problem generally has more than one local optimum, so these gradient-based algorithms may converge to one of these local optimum if the initial design and the search direction are not well defined.

In other words, these algorithms are usually trapped into a local optimum instead of reaching the global optimum.

2. The requirement of computation gradients makes them difficult to implement and inefficient [80]. Therefore, these gradient-based methods cannot efficiently handle the optimization problem of large structures with highly nonlinear, implicit, and discontinuous constraints;
3. Few of the gradient-based algorithms contain comprehensive optimization constraints [6], which limits the scope of their applications.

To address the limitations of the gradient-based algorithms, a new type of mathematical programming methods is proposed and adopted to meet the requirements in structural optimization, which is known as heuristic methods.

Heuristic methods refer to problem-solving approaches that obtain the solution by trial and error. This kind of optimization methods include lots of machine learning techniques, such as artificial neural networks [81] and support vector machines [82], that aim to improve the accuracy of solutions by iterations [83]. Although heuristic methods are relatively easy to program with high computational speed, these methods are problem-dependent and may be trapped in local optimum [84]. Therefore, researchers have proposed further developed heuristic methods, namely metaheuristic methods, for better optimization results. Metaheuristic methods are problem-independent and utilize certain trade-off randomization to move from local search to global search. This type of optimization method has become increasingly popular in research on structural optimization in the past few decades [83].

Metaheuristic methods are normally inspired by natural or man-made phenomena such as ant colony, water flow, and ensemble of musicians [17]. Some examples of the metaheuristic methods include genetic algorithm (GA) [85], harmony search (HS) [86], firefly algorithm (FA) [87], artificial bee colony (ABC) [88], differential evolution (DE) [89], Tabu search (TS) [90], teaching–learning based optimization (TLBO) [91], particle swarm optimization (PSO) [92], bat algorithm (BA) [93], cuckoo search (CS) [94], and Jaya [95]. Some taxonomies based on certain attributes of the algorithms are adopted to classify the metaheuristic algorithms [96], such as nature-inspired against non-nature inspired, population-based against trajectory-based, and dynamic objective function against static objective function. However, despite these differences, all metaheuristic algorithms have two main components: exploitation and exploration [16]. Exploration aims to generate diverse solutions for comparison, while exploitation is used to find the current optimal solution. Eventually, the global optimal solution can be effectively achieved by a good combination of exploitation and exploration.

The metaheuristic algorithms have many advantages compared to conventional deterministic and stochastic optimization methods, which can be concluded as the following four aspects [16]. First, the metaheuristic algorithms are suitable for combinatorial optimization problems with both continuous and discrete design variables. Second, the metaheuristic algorithms do not require the gradient information. Third, the metaheuristic algorithms do not require the convexity or an explicit relationship between the objective function and constraints. Fourth, the metaheuristic algorithms can find a global optimum more effectively. In structural optimization, there have been some successful applications of the metaheuristic methods. For example, Kociecki and Adeli [97] proposed a two-phase GA for size and topology optimization of frame structures with rectangular hollow structural sections to minimize the total weight of the structure. Mortazavi and Toğan [71] proposed an integrated PSO algorithm for size and layout optimization of truss structures to improve the structural dynamic characteristics and minimize the weight of structures. Bekdaş et al. [62] applied a recently developed metaheuristic algorithm, namely the flower pollination algorithm, in size optimization to minimize the weight of truss structures.

Despite the advantages and the successful applications mentioned above, the metaheuristic algorithms also have some drawbacks and limitations according to previous studies. For example, Sörensen [17] claimed that the metaheuristic algorithms are usually

very complicated and are only tested on a few samples with small structural size. Although the metaheuristic algorithms can achieve excellent results, it cannot be concluded that they are better than constructive heuristic algorithms. Saka et al. [16] indicated that one of the disadvantages of the metaheuristic algorithms is that they are computationally expensive, particularly in large and complex structures under several load cases. According to Mahdavi et al. [18], the main deficiency of the standard metaheuristic algorithms is that they cannot well handle high dimensional problems because of the high landscape complexity and large search space. Therefore, many recent studies in structural optimization involve further improvements of the existing optimization methods. These algorithm improvement methods aim to enhance the optimization efficiency based on the characteristics of each metaheuristic algorithm. For example, Cheng et al. [98] proposed a hybrid HS algorithm, which utilized the PSO search and neighborhood search instead of the randomization function for global optimum while retaining the harmony memory and pitch adjustment functions in the traditional HS algorithm. This hybrid algorithm is proved to perform better in solution accuracy as well as convergence rate compared with traditional metaheuristic algorithms. Arjmand et al. [99] presented a hybrid algorithm that combines the improved dolphin echolocation algorithm with the ant colony optimization algorithm. The hybrid algorithm enhances the efficiency of the improved dolphin echolocation algorithm by using the positive attributes of ant colony optimization. Moreover, Cao et al. [100] concluded four methods to improve the performance of the traditional PSO algorithm: (1) balancing the local search and the global search, (2) replacing traditional global topology with different neighborhood topologies, (3) combining PSO with other metaheuristic methods, and (4) combining traditional gradient-based algorithms with PSO. These methods expand the exploration ability of the traditional PSO algorithm to obtain global optimum and enhance its exploitation ability to accelerate the convergence rate as well as improve the accuracy of solutions. Table 2 presents a summary of some other studies on structural optimization that involve improved metaheuristic algorithms. Although these variants of traditional metaheuristic algorithms have many different forms, each of them focuses on improving certain ability (abilities) of the original algorithm. Therefore, it is of great importance to select the most suitable algorithm for a specific optimization problem in order to obtain the best optimal design while minimizing computational cost.

In addition to improving the performance of algorithms, an alternative approach to enhancing the optimization efficiency is to reduce the time-consuming evaluations of optimization objective or constraint function in the optimization process [100]. However, this approach may lead to an optimization result that deviates from the optimization objective, and thereby this approach is not discussed in this paper.

There is another special type of optimization methods apart from gradient-based methods and heuristic methods, namely the reliability-based design optimization (RBDO) methods. RBDO aims to seek for the best compromise between structural cost and safety by considering the uncertainties of the structural system, including dimension, material, model, loads, etc. [101]. Therefore, this type of optimization methods ensure a minimum level of reliability, which provide a prior for the designers [102]. There are three main categories of RBDO methods, namely the two-level approach, the single loop approach and the decoupled approach [101]. Despite the aforementioned advantages, these RBDO methods also have some drawbacks such as high computational cost caused by the reliability analysis in each iteration and the difficulty of computing the gradients of the probabilistic constraints and thus their application in civil engineering structural optimization is relatively limited [103].

Table 2. A summary of some recently proposed improved metaheuristic algorithms.

Year	Involved Algorithms	Inspiration	Reference
2012	Efficient harmony search algorithm (EHS) and self-adaptive harmony search algorithm (SAHS)	Proposed two improved harmony search algorithms to decrease the parameter-dependency of HS algorithm for size optimization	Degertekin [20]
2012	Enhanced GA with multiple populations (EGAwMP)	Enhance the exploitation and exploration capacities of the GA with multiple populations by a radial-basis neural network and a new design strategy to increase the convergence degrees of optimal designs	TALASLIOGLU [104]
2013	Accelerated firefly algorithm (AFA)	AFA improves the searching procedure of the standard firefly algorithm based on randomness reduction and scaling the random term to increase the convergence rate	Baghlani, Makiabadi [105]
2015	Improved ray optimization (IRO) algorithm	IRO changes the formulation of generating solutions and returns violated agents to feasible search space to make the original ray optimization algorithm more efficient	Kaveh and Ghazaan [106]
2016	Hybrid genetic algorithm and particle swarm optimization algorithm (HGAPSO)	Divide the population members into two equal groups based on their fitness values, using PSO algorithm for the best half while using the GA for the worst half	Maheri, Askarian [107]
2017	Colliding bodies optimization (CBO) and enhanced colliding bodies optimization (ECBO)	CBO is a new metaheuristic algorithm based on one-dimensional collision, while ECBO uses memory to improve the performance of CBO without increasing computational cost	Kaveh and Moradveisi [108]
2017	Integrated particle swarm optimization algorithm (IPSO)	IPSO is a particle swarm optimizer combined with the improved fly-back mechanism and the weighted particle concept, and it is used for weight minimization of structures with frequency constraints	Mortazavi and Toğan [71]
2017	Genetic algorithm with domain-trimming (GADT)	Enhance the global optimum searching capacity of GA through domain-trimming technique, and use the GADT for weight minimization design of support structures for offshore wind turbines	AlHamaydeh, Barakat [109]
2017	Whale optimization algorithm (WOA) and enhanced whale optimization algorithm (EWOA)	WOA is a metaheuristic algorithm inspired by the hunting behavior of whales; EWOA improves the formulation of the WOA in to improve solution accuracy, reliability, and convergence rate	Kaveh [110]
2018	Adaptive hybrid evolutionary firefly algorithm (AHEFA)	AHEFA utilize an automatically adapted parameter for an effective trade-off between the global and local search and use an elitist technique to select the best individuals	Lieu, Do [64]
2019	Discrete advanced Jaya algorithm (DAJA)	DAJA produces new trial designs and forms descent directions in the neighborhood of each design candidate to overcome the limitation of the Jaya algorithm: no utilization of algorithm-specific parameters	Degertekin, Lamberti [66]

5.4. Computational Tools and Design Platforms

In addition to structural analysis and modelling, optimization problem formulation, and optimization method, it is also important to choose suitable computational tools and design platforms to perform the optimization codes as well as to achieve the optimal design of the structures. In the past, structural design and analysis were carried out by manual calculations through trial and error, which was characterized by heavy workload and error-proneness. With the development of information technology, many computational tools and design platforms have been developed to provide an environment for structural modelling, analysis, and design. Some prominent software packages, such as ETABS and SAP, significantly improve the calculation speed and lead to better results [111]. However, not all software packages perform that well. Some existing software packages have been

proven to be less effective when dealing with large-scale structures [10]. Meanwhile, building information modelling (BIM) based software, which is often used for structure design and visualization, always faces the problem of low data interoperability [112].

In terms of structural optimization, it is crucial to select suitable software packages to carry out optimization because the software performance directly affects the efficiency of optimization. Generally, after determining the problem formulation and optimization method, the optimization process follows such a sequence: solution encoding, mathematical computing, and structural analysis and design. There are two encoding methods when using metaheuristic operators, namely natural encoding, which uses real values to represent the design variables, and binary encoding, which uses binary strings to represent the design variables. Because each algorithm has a different behavior, the selected encoding method depends on the adopted metaheuristic algorithm [1]. Then two types of software packages are involved in the structural optimization: computing software and design software. The former is used to perform the optimization codes, while the latter is used to conduct structural analysis and design. In the computing software, the iterative part of the optimization is carried out and each iteration will generate a set of values for the design variables. Then, the design variables are transferred into the design software to update the structure model with new geometric properties. After reaching the algorithm convergence, the range of the design variables (i.e., the minimum and maximum values) will be determined. The optimal design can be obtained through structural analysis based on some predefined criteria [113].

MATLAB is a commonly used computing software in structural optimization due to its excellent ability for computation and programming. For example, Zhou et al. [114] proposed a modified bidirectional evolutionary structural optimization (BESO) method for topology optimization, where they adopted a MATLAB program to implement this method. Zegard and Paulino [115] used the ground structure method for topology optimization, and the source code, which is known as GRAND3 (ground structure analysis and design in 3D) was implemented in MATLAB. In terms of structural analysis and design, some software packages such as SAP2000 and ETABS are often used because of their good performance. BIM software is another type of commonly used software for structural analysis, design, and visualization. However, the structure information from the BIM environment must be converted to finite element analysis tools such as ETABS, SAP, and ANSYS to obtain the optimal design [10].

In addition to using two types of software and conducting mathematical computation and structural analysis and design separately, some researchers have adopted a single integrated platform to conduct the entire process of structural optimization. For example, Sotiropoulos and Lagaros [113] proposed a platform for topology optimization of framed structures, namely the high-performance topology optimization computing platform (HP-TOCP). A wrapper was created in SAP2000 to call the *fmincon* function in MATLAB's toolbox. Through this method, computing software is not used in the optimization process, and it is not necessary to conduct information conversion.

Although computational tools and design platforms are not introduced in detail in most of the collected articles, they are of great importance because they can significantly affect the optimization efficiency. There is no doubt that the existing tools can satisfy the computational and design requirements. However, the development of new tools or integrated platforms are still desired to further improve the optimization capability, computational efficiency, and data interoperability.

6. Limitations and Future Work

In the past few decades, with the development of theories and technologies, researchers have made great progresses in the field of civil engineering structural optimization. Optimization can involve multiple aspects such as size, shape, and topology of structures at the same time. In addition, optimization objectives are diverse, extending from minimizing the total cost to minimizing environmental impact, improvement of

structural performance, and even multi-objective optimization. Moreover, the emergence and development of metaheuristic algorithms have significantly improved the efficiency and accuracy of structural optimization. Despite these achievements, there are still some limitations and research gaps, which need to be solved in the future. These research limitations with the recommendations of future works will be discussed in this section.

6.1. Weighting Criteria in Multi-Objective Optimization

As mentioned above, multi-objective optimization is an important and promising theme in the field of civil engineering structural optimization, which balances between more than one competing optimization objective and thus better satisfying the requirements of structure designers. However, some issues of the multi-objective structural optimization have yet been solved. There is not a definite solution in optimization considering two objectives simultaneously. Although a set of optimal solutions (Pareto set) can be obtained, it may be difficult to select one solution that best satisfies the design requirements. Moreover, all of the selected articles in this paper on multi-objective optimization only consider two objectives simultaneously. None of these studies considered three or more optimization objectives at the same time.

Researchers have made some efforts to deal with these issues faced by multi-objective optimization. An alternative to the Pareto optimality is the compromise solution method, which yields a single optimal solution [116]. In this method, an unattainable ideal point is proposed, and the optimal solution is obtained by progressively reducing the difference between the potential optimal point and the ideal point. However, it is difficult to represent the closeness between the two points mathematically unless the objective functions are dimensionless [116].

It is more common to incorporate the preferences of decision-makers to deal with multi-objective optimization. These methods use weights as the parameters to represent the preferences of decision-makers. According to the moment when the decision-makers' preferences are provided, these methods can be divided into three categories, namely priori approaches, interactive approaches, and posteriori approaches [117]. The priori approaches define the weight of each optimization objective before the optimal solution search. Many weighted criteria have been proposed to facilitate this process, such as linearly weighted criterion [118], weighted global criterion [116], and weighed scalar-valued performance criterion [119]. In the field of civil engineering structural optimization, Sanaei and Babaei [120] used the weighted sum method (WSM), which is the simplest and most common weighted criteria method, for simultaneous shape and topology optimization of continuum structures. This method uses a set of scalar values to define the weight of each optimization objective, and thus synthesize these objective functions into one single objective function. Then, the optimization problem can be solved following the process of single-objective optimization and a definite optimal solution can be obtained. The interactive approaches provide the decision-makers' preferences during the search. However, interactive approaches are not that common in the collected articles, the reason of which might be the inconsistency of the preference information provided by a decision-maker [117]. In terms of posteriori approaches, the decision-makers' preferences are involved after the search. The weighting criteria in the posteriori approaches may depend on the solutions obtained [116]. For example, Zavala et al. [1] have adopted the posteriori approaches in their review work on multi-objective structural optimization, where they provided the solutions to the decision-makers based on an approximation of the Pareto front and then incorporated the decision-makers' preferences into the solutions.

However, there is still a major problem for the preference-based methods in spite of these achievements. That is, no matter which criterion is adopted to weight the objectives, it is subjective more or less. In other words, it is difficult to determine whether a weighting criterion is suitable for a specific problem or not. Moreover, sometimes decision-makers cannot provide their preferences for each objective or to determine the most appealing solution among the solutions provided by the optimization results [116]. Hence, future

studies are necessary to propose a comprehensive optimization objective weighting system, which would provide the range of weight values based on the adopted algorithms, design specifications, and constraints for researchers to select relatively suitable weighting criteria. Although such a weighting system may not be able to cover all the different cases of structural optimization, this system would be sufficient to provide a reference when the decision-makers' preferences are not that explicit. Additionally, based on this weighting system, three or more objectives can be considered simultaneously by transforming multiple objective functions into a single objective function.

6.2. Quantification of Optimization Objectives

Generally, in order to achieve an optimization objective, researchers should find a mathematical quantification of this objective first, and then the optimal solution can be obtained through mathematical computation. There are some commonly used quantification methods in structural optimization. For example, the structural cost is often quantified as the weight of the structure. Compliance and total strain energy are often used as the quantification of structural stiffness. Theoretically, all of the structural properties even aesthetics attributes can be used as optimization objectives if these properties can be appropriately quantified [121].

However, it can be difficult to accurately quantify the objectives in some cases. Aldwaik and Adeli [61] pointed out that structure designers often questioned the feasibility of using structure weight to quantify the total structural cost. Although a design with the minimum weight or volume reduces the material cost, which accounts for a large part of the total structural cost, the total cost also contains other components such as transport cost and installation cost. Therefore, the total cost cannot be directly represented by structure weight. There have been some studies focusing on the quantification of structural cost. For example, Kaveh [122] combined the man-hours for fabrication, structure weight, and price of web cutting to construct the structural cost function for optimization of castellated beams. Sharafi et al. [123] used an objective function that minimizes the material cost and formwork cost simultaneously for the optimal design of continuous reinforced concrete beams. Some researchers also apply parametric mixed-integer non-linear programming (MINLP) approach in structural optimization for cost (or mass) minimization [124–126], which is a mathematical programming technique that simultaneously optimizes the discrete system structure and continuous parameters with nonlinear objective functions and constrains [127]. The algorithms for MINLP approach are usually designed for handling large-scale, highly combinatorial and highly nonlinear problems, such as outer approximation/equality-relaxation (OA/ER) algorithm and generalized Bender's decomposition (GBD) algorithm [127]. Based on MINLP approach and corresponding algorithms, a wide range of design parameters influencing the structural cost such as material unit prices, hourly labor costs, imposed loads, structure spans, steel and concrete grades [124] could be considered simultaneously to formulate the objective function and thus the optimization result might be better. However, MINLP problems are extremely complex to solve because they contain all the difficulties of their subclasses, namely the combinatorial nature of mixed integer programs (MIP) and the difficulty in solving nonconvex (and even convex) nonlinear programs (NLP) [127]. Therefore, the application of MINLP approach is more or less limited.

Despite the aforementioned achievements, there is not a widely accepted quantification of structural cost because researchers would consider different aspects of structural cost in different optimization problems. Therefore, future work is required to propose a comprehensive system for structural cost estimation, which would consider all the aspects related to the total structural cost, such as material cost, transport cost, as well as fabrication and construction cost that is related to the construction method (e.g., precast or cast in place) and the standardization rate of structural components. To establish such a system, it is necessary to collect structural cost data from existing projects, and build the cost estimation system based on in-depth analyses of existing project data. Additionally, it is promising to

propose more convincing quantifications of structural mechanical properties and aesthetic properties so that these properties can be treated as objectives of structural optimization.

6.3. Applicability of Optimization Algorithms

Many studies on structural optimization aim to propose novel algorithms with a higher convergence rate and better optimal solutions. However, there has not been a standard method to evaluate the performance of the optimization algorithms. For example, Kaveh et al. [128] conducted size optimization of two-dimensional frame structures with the goal of structure weight minimization to compare the performance of different metaheuristic algorithms. They used three types of frame structures as design examples, namely 1-bay, 10-story steel frame, 3-bay, 15-story steel frame, and 3-bay, 24-story steel frame. Seven population-based metaheuristic algorithms were compared in their study, including the artificial bee colony algorithm (ABC), big bang–big crunch algorithm (BB-BC), cyclical parthenogenesis algorithm (CPA), Cuckoo search algorithm (CS), thermal exchange optimization algorithm (TEO), teaching–learning-based optimization algorithm (TLBO), and water evaporation optimization algorithm (WEO). Their results suggest that these metaheuristic algorithms present different performance in different design examples. For the 1-bay, 10-story steel frame structure, the average structure weight of the optimal design of each metaheuristic algorithm is ranked as follows (in ascending order): TEO, CS, BB-BC, CPA, WEO, TLBO, and ABC. For the 3-bay 15-story steel frame structure, the average structure weights follow another sequence (in ascending order): TEO, WEO, TLBO, CPA, BB-BC, CS, and ABC. Furthermore, the results are also different for the 3-bay 24-story frame structure, where the order of the average weights is as follows (in ascending order): TEO, TLBO, WEO, BB-BC, CS, CPA, and ABC. It should be noted that all of the above results are obtained under 20,000 analyses. All of these algorithms have reached convergence under this condition. Since the optimization objective is weight minimization, smaller average structure weight of the optimal design means better performance of the algorithm. In terms of convergence rate, their study presented the convergence curve of each algorithm instead of comparing the convergence rate directly. According to their results, TEO, TLBO, and WEO have higher convergence rates in general. However, the convergence rates of these three algorithms also vary with different design examples. For the 1-bay, 10-story steel frame structure, the convergence rate of TEO is much faster than other algorithms, while for the 3-bay 15-story steel frame structure and the 3-bay 24-story frame structure, TEO, TLBO, and WEO reach convergence almost at the same time.

The above results show that the metaheuristic algorithms have limited applicability. Each algorithm may only have good performance for a specific optimization problem. If a novel optimization algorithm is proposed to solve a specific optimization problem, the performance of this algorithm for other optimization problems cannot be ensured even if it displays better performance than other algorithms for this optimization problem. In addition, all the newly proposed algorithms are tested on different structures, making it difficult to compare the performance of these novel algorithms.

Therefore, future work should be focused on the establishment of a benchmarking system for optimization algorithm comparisons, in order to facilitate the development of new metaheuristic algorithms with higher applicability for structural optimization. To facilitate the comparisons of algorithms, the structural optimization problems could be classified into different categories based on the structure types, scales, or other characteristics. For each category of optimization problems, a few standardized structural optimization problems could be established as the benchmark test problems. Meanwhile, traditional metaheuristic algorithms with relatively better performance for each category of optimization problems could be used as the benchmark algorithms. Afterwards, the performance of any newly proposed algorithm can be verified by comparing with the benchmark algorithms using the benchmark test problems for the respective category of optimization problem. Based on the benchmarking system, it is expected that novel optimization algorithms could be

developed to address a category of optimization problems with better performance rather than a specific optimization problem.

7. Conclusions

This review work comprehensively analyzed the previous literature on structural optimization in the field of civil engineering. After retrieval and selection, 196 relevant articles from Google Scholar were collected. The publication time period of these collected articles spans from 1970 to 2020. These articles were analyzed statistically regarding the publication year, article type, journal, geographical location, and optimization objective. The temporal and spatial trends of the optimization objectives were also discussed in detail. In general, the number of research articles in this field has experienced an upward trend over the years, especially in the regions where government could provide sufficient funding. Cost minimization is the most popular optimization objective, while research on structural performance improvement and multi-objective have increased rapidly in recent years.

In addition, the process of structural optimization was discussed in detail in this paper. Based on the optimization objectives, these collected articles were divided into four research themes, namely cost minimization, structural performance improvement, environmental impact minimization, and multi-objective. Four major steps in the structural optimization process were reviewed and discussed, including structural analysis and modelling, formulation of optimization problems, optimization techniques and methods, and computational tools and design platforms. According to the modelling techniques used in the early stage, structural optimization can be divided into discrete optimization and continuum optimization. Optimization formulation comprises of three main components: design variables, objective functions, and constraints. For structural optimization, the design variables include one or more of cross-sectional areas, nodal coordinates, and connectivity of structural elements. The objective function for structural optimization aims to minimize or maximize the quantifications of optimization objectives, such as the total weight of structure, compliance of the structure, and total strain energy. Constraints are usually the safety and serviceability requirements that must be satisfied during the optimization process, including stress, displacement, etc. Optimization techniques and methods refer to the way to perform the structural optimization and obtain the optimal design. Mathematical programming based on metaheuristic algorithms have been the most commonly used optimization method in the recent years, and a considerable part of collected articles aim to propose novel metaheuristic algorithms with better performance (higher convergence rate and more accurate optimal solution) than the conventional algorithms. Eventually, the optimization methods should be implemented in certain software packages or platforms. Generally, there are two types of software packages involved in structural optimization, namely computing software and design software. The computing software, such as MATLAB, is used to perform optimization codes and generate optimal solution through iterations. Then, the structural geometry data are transferred into the design software such as ETABS for structure design and analysis. Some researchers also adopt integrated platforms for structural optimization, where data transformation is not required.

Finally, limitations of the current research on structural optimization were identified, and future works were recommended to address the limitations. Although many research achievements have been made over the years, there are still three major research gaps, namely weighting criteria in multi-objective optimization, quantification of optimization objectives, and applicability of optimization algorithms. First, structural optimization considering two objectives simultaneously would generate a set of optimal solutions, which is called Pareto set, instead of a unique optimal solution and thus may not satisfy the requirements of designers. Future work may focus on proposing a comprehensive criterion to weight each objective and thus convert multi-objective optimization problem to single-objective optimization problem. Second, mathematical quantifications must be found to represent the optimization objectives appropriately in order to conduct structural optimiza-

tion. However, there has not been a standard method to evaluate the accuracy of objective quantifications yet, which is expected to be proposed in the future. Third, the metaheuristic algorithms have limited applicability. In other words, the performance of a metaheuristic algorithm can be different for different optimization problems. Therefore, the future work may focus on categorizing the optimization problems according to their characteristics, and proposing a benchmarking system for each category of optimization problem including benchmark test problems and benchmark algorithms. Based on the benchmarking system, novel optimization algorithms could be developed to address a category of optimization problems with better performance rather than a specific optimization problem.

In conclusion, there are four major contributions of this review article. First, this study comprehensively reviewed and summarized the available literature on civil engineering structural optimization in the past few decades. Second, this study statistically analyzed the collected literature regarding the temporal and spatial trends of research in this field. Third, this study discussed four major components of the optimization process in detail, including structural analysis and modelling, formulation of optimization problems, optimization techniques and methods, and computational tools and design platforms. Lastly, this study proposed limitations of the current research on civil engineering structural optimization and recommended corresponding future works. This paper has filled the gap that there is a lack of comprehensive review work in the field of civil engineering structural optimization, providing a useful reference and guidance for future works in this field.

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