Supplementary Material

Flexibility Characterization of Phenomenological Hardening Law for Automotive Sheet Metals Using Ordinary Differential Equation

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Supplementary Table

Table S1: Calibrated parameters of hardening laws for DP590 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	716.539	306.106	17.977			773.222	342.039	11.576		
Hockett-Shetby	748.401	356.650	9.395	0.814		821.406	442.410	4.852	0.674	
Chinh et al.	748.359	356.592	0.106	0.814		821.344	442.303	0.206	0.674	
Hollomon	922.876	0.146				928.306	0.148			
Ludwik	320.796	764.846	0.352			169.438	765.611	0.196		
Swift	992.010	0.00510	0.180			926.938	0.00183	0.150		
Hollomon/Voce	0.130					1.034				
Swift/Voce	0.460					0.719				
Swift/HS	0.072					0.726				
Modified Voce	601.612	201.310	30.545	701.956		614.466	212.719	28.237	610.396	
Double Voce	824.871	153.813	36.337	271.664	5.414	849.234	160.810	35.558	289.055	4.624
Ludwigson	1111.273	0.455	5.833	-2.282		945.199	0.349	5.672	-2.082	
Misiolek	784.940	0.110	0.772			902.750	0.140	0.058		
Pham and Kim	407.977	625.217	61.003	0.394		409.781	574.923	52.015	0.358	

Proposed 1	919.473	88.108	0.036	0.535		1225.417	346.237	0.023	0.231	
Proposed 2	389.719	346.632	13.126	0.741		371.508	423.504	7.202	0.559	
Proposed 3	398.143	17.957	0.920	1.077		392.376	21.538	0.671	1.299	
Proposed 4	398.965	26.036	3.748	0.609		399.584	25.312	3.407	0.620	

Table S2: Calibrated parameters of hardening laws for DP780 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	928.515	393.105	28.858			993.390	424.433	17.945		
Hockett-Shetby	1016.412	548.576	6.943	0.624		1039.833	578.262	5.905	0.592	
Chinh et al.	1016.142	548.159	0.144	0.624		1039.800	578.175	0.169	0.592	
Hollomon	1253.540	0.139				1170.400	0.117			
Ludwik	331.819	1055.744	0.257			-367.263	1532.257	0.083		
Swift	1304.623	0.00138	0.155			1170.133	0.00010	0.117		
Hollomon/Voce	0.405					0.743				
Swift/Voce	0.720					0.640				
Swift/HS	0.088					0.171				
Modified Voce	772.971	263.519	58.818	1394.896		828.179	307.338	42.917	834.606	
Double Voce	977.509	139.911	122.630	341.929	16.495	996.112	158.730	105.458	339.439	14.002
Ludwigson	1543.748	0.380	5.993	-3.892		1167.515	0.117	1.000	0.169	
Misiolek	1160.631	0.123	0.462			1170.399	0.117	0.000		
Pham and Kim	510.988	906.318	121.104	0.356		525.255	712.215	72.433	0.273	
Proposed 1	1500.969	367.945	0.009	0.213		1332.387	222.621	0.011	0.291	
Proposed 2	454.368	523.923	14.219	0.497		436.985	572.667	10.626	0.435	
Proposed 3	486.349	62.600	0.587	1.324		489.905	55.200	0.641	1.258	
Proposed 4	506.266	53.559	6.440	0.606		517.990	39.804	3.084	0.677	

Table S3: Calibrated parameters of hardening laws for DP980 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	1077.827	254.548	98.222			1106.80	259.99	64.21	0.00	
Hockett-Shetby	1103.301	317.915	23.485	0.691		1111.13	330.65	19.28	0.65	
Chinh et al.	1103.259	317.830	0.043	0.691		1111.13	330.63	0.05	0.65	
Hollomon	1340.952	0.067				1211.15	0.04			
Ludwik	587.606	874.833	0.174			-0.88	1212.02	0.04		
Swift	1383.736	0.00047	0.075			1211.15	0.00000	0.04		
Hollomon/Voce	0.347					0.53				
Swift/Voce	0.523					0.46				
Swift/HS	0.015					0.52				
Modified Voce	1008.844	198.723	158.642	1809.113		1039.60	223.19	125.04	941.20	
Double Voce	1094.606	90.707	355.827	203.557	63.065	1103.41	112.63	284.00	188.34	51.68
Ludwigson	1339.412	0.067	0.500	0.100		1210.07	0.04	0.10	-0.10	
Misiolek	1298.188	0.062	0.480			1400.76	0.07	-0.73		
Pham and Kim	810.914	644.254	250.729	0.274		817.90	454.45	169.33	0.18	
Proposed 1	1179.664	15.216	0.005	0.608		1139.26	3.68	0.01	0.92	
Proposed 2	775.149	318.020	60.532	0.540		758.65	346.94	46.83	0.45	
Proposed 3	799.053	119.000	0.837	0.537		802.98	105.17	0.97	0.50	
Proposed 4	806.699	131.404	11.051	0.328		812.27	107.44	5.00	0.36	

Table S4: Calibrated parameters of hardening laws for SPCC sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	379.363	202.833	14.272			390.272	210.024	12.528		
Hockett-Shetby	448.475	291.629	4.343	0.689		396.938	232.365	8.326	0.828	
Chinh et al.	448.708	291.873	0.231	0.689		396.934	232.352	0.120	0.828	
Hollomon	517.802	0.199				465.359	0.158			
Ludwik	129.675	491.781	0.397			-104.913	568.103	0.122		
Swift	564.985	0.00545	0.242			465.235	0.00019	0.159		
Hollomon/Voce	0.254					0.379				
Swift/Voce	0.774					0.245				
Swift/HS	-0.008					0.466				
Modified Voce	277.705	108.945	30.486	551.805		337.502	163.820	18.186	191.744	
Double Voce	399.911	35.116	105.314	202.423	10.484	390.094	29.374	161.043	200.600	11.980
Ludwigson	484.691	0.391	4.856	-0.097		475.005	0.251	4.302	-2.535	
Misiolek	413.503	0.147	0.987			545.777	0.209	-0.317		
Pham and Kim	171.643	464.127	76.757	0.479		178.889	301.679	30.261	0.280	
Proposed 1	694.230	216.827	0.032	0.258		417.782	6.808	0.118	1.681	
Proposed 2	154.041	267.203	6.836	0.605		161.101	231.951	10.295	0.724	
Proposed 3	161.107	26.387	0.535	2.036		170.671	15.190	1.085	1.338	
Proposed 4	167.868	28.164	6.111	0.714		173.542	18.011	1.179	0.961	

Table S5: Calibrated parameters of hardening laws for TRIP980 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	1207.540	574.531	21.054			1234.143	590.376	18.678		
Hockett-Shetby	1212.312	582.196	19.726	0.982		1238.252	615.242	15.347	0.922	
Chinh et al.	1212.305	582.185	0.051	0.982		1238.247	615.211	0.065	0.922	
Hollomon	1685.091	0.171				1453.460	0.120			
Ludwik	497.450	1603.354	0.390			1.000	1452.487	0.120		
Swift	1866.017	0.005	0.216			1453.326	0.00009	0.120		
Hollomon/Voce	-0.010					0.280				
Swift/Voce	0.049					0.185				
Swift/HS	0.009					0.574				
Modified Voce	1124.146	495.299	24.507	568.646		1136.406	507.014	23.938	479.587	
Double Voce	1283.269	437.553	25.771	216.780	6.630	1285.856	435.440	25.885	221.569	6.576
Ludwigson	1578.975	0.386	6.207	-0.109		1542.930	0.247	5.799	-3.396	
Misiolek	1401.262	0.131	1.045			1829.351	0.187	-0.559		
Pham and Kim	642.442	1005.660	45.938	0.306		641.967	823.605	35.555	0.211	
Proposed 1	1276.190	2.170	0.142	2.925		1261.879	1.246	0.165	3.464	
Proposed 2	631.639	577.350	20.803	0.988		623.818	612.512	17.736	0.894	
Proposed 3	633.211	17.500	1.411	1.030		633.184	17.500	1.408	1.031	
Proposed 4	628.576	22.614	0.829	0.870		628.487	22.662	0.852	0.869	

Table S6: Calibrated parameters of hardening laws for TRIP1180 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	2606.549	1679.989	1.463			1810.278	897.690	3.288		
Hockett-Shetby	3177.335	2254.693	0.985	0.956		1653.389	717.553	6.020	1.211	
Chinh et al.	3150.428	2227.165	0.996	0.960		1653.531	717.818	0.166	1.210	
Hollomon	1604.621	0.126				1989.108	0.229			
Ludwik	916.834	1770.579	0.871			857.206	1274.142	0.606		
Swift	2242.666	0.212	0.569			1987.016	0.07461	0.304		
Hollomon/Voce	0.049					0.368				
Swift/Voce	1.000					0.500				
Swift/HS	-0.008					0.481				
Modified Voce	946.931	82.116	288.943	2042.834		1810.249	897.639	3.288	0.101	
Double Voce	2930.995	75.923	501.529	1997.707	1.178	1842.304	868.283	3.349	61.464	0.768
Ludwigson	1749.988	0.869	6.821	-0.021		873.739	0.551	6.773	0.562	
Misiolek	1052.007	0.024	1.461			1285.885	0.068	0.704		
Pham and Kim	913.107	1779.230	412.246	0.874		841.946	1282.124	6037.887	0.594	
Proposed 1	3280.387	2105.845	0.892	0.994		2270.367	619.869	0.506	1.154	
Proposed 2	917.023	15387.850	0.086	0.875		934.134	772.594	4.675	1.251	
Proposed 3	931.589	2.167	2.043	1.111		929.095	3.993	12.790	0.644	
Proposed 4	926.687	0.046	4.645	0.572		902.561	0.047	13.354	0.304	

Table S7: Calibrated parameters of hardening laws for AA6016 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	313.363	186.250	10.618			330.635	198.235	8.616		
Hockett-Shetby	329.870	209.181	7.024	0.873		339.796	220.834	5.955	0.833	
Chinh et al.	329.870	209.180	0.142	0.873		339.787	220.816	0.168	0.833	
Hollomon	431.410	0.241				395.139	0.203			
Ludwik	88.761	411.847	0.421			0.000	395.139	0.203		
Swift	465.341	0.00745	0.286			394.952	0.00057	0.204		
Hollomon/Voce	0.112					0.537	395.139	0.203	330.635	198.235
Swift/Voce	0.327					0.374	394.952	0.001	0.204	330.635
Swift/HS	0.021					0.634	394.952	0.001	0.204	339.796
Modified Voce	245.525	121.713	16.394	262.976		263.610	138.810	14.243	184.166	
Double Voce	356.470	73.877	21.214	159.170	4.698	339.652	53.898	24.328	162.382	6.268
Ludwigson	484.547	0.468	4.531	1.453		394.170	0.204			
Misiolek	356.071	0.192	0.665			455.733	0.253	-0.243		
Pham and Kim	129.215	336.371	37.586	0.445		130.621	281.043	26.167	0.344	
Proposed 1	394.339	33.760	0.105	0.926		379.710	24.659	0.121	1.109	
Proposed 2	120.012	204.114	8.422	0.825		117.116	217.758	7.053	0.752	
Proposed 3	123.361	11.500	0.966	1.663		123.536	11.400	0.995	1.639	
Proposed 4	123.789	16.314	2.162	0.989		124.992	14.651	1.329	1.070	

Table S8: Calibrated parameters of hardening laws for AA6022 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	382.879	226.170	8.255			409.054	245.700	6.510		
Hockett-Shetby	408.156	260.389	5.361	0.862		431.576	287.825	4.146	0.798	
Chinh et al.	408.428	260.713	0.187	0.862		431.533	287.750	0.241	0.798	
Hollomon	504.295	0.248				486.354	0.231			
Ludwik	100.220	465.779	0.410			-1.000	487.321	0.231		
Swift	538.698	0.01017	0.293			486.155	0.00073	0.232		
Hollomon/Voce	0.145					0.741				
Swift/Voce	0.365					0.489				
Swift/HS	0.016					0.681				
Modified Voce	294.280	141.401	12.997	279.747		305.411	151.919	12.060	238.954	
Double Voce	423.707	63.048	19.577	208.564	4.629	437.516	76.702	17.924	208.655	3.905
Ludwigson	531.310	0.457	4.675	1.079		486.354	0.231	-12.256	9.540	
Misiolek	431.084	0.206	0.483			513.190	0.251	-0.085		
Pham and Kim	160.949	388.389	31.512	0.484		162.294	345.268	24.243	0.414	
Proposed 1	530.706	87.558	0.111	0.668		539.683	94.416	0.109	0.639	
Proposed 2	146.182	254.078	6.249	0.804		140.175	280.943	4.834	0.708	
Proposed 3	151.668	8.848	0.924	1.704		150.761	9.120	0.867	1.768	
Proposed 4	152.723	12.716	1.990	0.950		153.462	12.034	1.576	0.993	

Table S9: Calibrated parameters of hardening laws for AA7075 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	638.716	153.558	19.722			636.747	151.950	20.236		
Hockett-Shetby	658.001	176.032	11.977	0.893		637.362	153.224	19.307	0.985	
Chinh et al.	658.060	176.098	0.084	0.892		637.356	153.211	0.052	0.986	
Hollomon	697.154	0.062				742.129	0.080			
Ludwik	469.233	550.801	0.537			411.043	359.263	0.255		
Swift	824.204	0.01537	0.129			741.766	0.00472	0.085		
Hollomon/Voce	0.046					0.103				
Swift/Voce	0.413					-0.080				
Swift/HS	-0.011					0.627				
Modified Voce	591.882	107.487	26.240	363.290		636.747	151.949	20.236		
Double Voce	695.640	86.077	28.790	125.193	5.705	638.178	151.398	20.301	2.000	2.000
Ludwigson	523.134	0.526	6.150	-0.100		358.169	0.234	5.991	0.053	
Misiolek	575.928	0.026	1.641			660.565	0.051	0.368		
Pham and Kim	487.584	473.823	85.180	0.545		487.610	285.577	50.121	0.347	
Proposed 1	725.125	20.891	0.064	0.888		650.636	0.424	0.159	3.256	
Proposed 2	481.313	172.149	14.881	0.848		483.660	153.593	19.698	0.965	
Proposed 3	483.425	12.154	0.949	0.591		485.469	13.737	1.600	0.430	
Proposed 4	484.025	17.227	1.678	0.360		483.080	17.321	0.000	0.405	

Table S10: Calibrated parameters of hardening laws for AA5052 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	253.617	76.475	27.458			257.815	79.227	24.022		
Hockett-Shetby	252.747	75.170	30.219	1.024		258.680	82.573	19.007	0.921	
Chinh et al.	252.740	75.162	0.033	1.024		258.678	82.568	0.053	0.921	
Hollomon	304.747	0.090				299.570	0.085			
Ludwik	162.215	245.339	0.417			71.077	230.372	0.120		
Swift	345.545	0.00675	0.137			299.295	0.00084	0.086		
Hollomon/Voce	0.005					0.742				
Swift/Voce	-0.050					0.254				
Swift/HS	0.020					0.662				
Modified Voce	253.617	76.476	27.457	0.000		244.435	67.232	30.185	90.555	
Double Voce	253.622	76.478	27.452	0.000	2.125	265.015	76.453	25.128	10.447	2.326
Ludwigson	293.478	0.096	2.525	-0.101		336.605	0.390	5.003	-3.009	
Misiolek	251.445	0.053	1.538			300.915	0.086	-0.014		
Pham and Kim	177.766	87.839	32.959	0.077		178.597	128.032	50.352	0.254	
Proposed 1	264.751	0.226	0.100	2.587		292.856	14.357	0.017	0.526	
Proposed 2	177.625	75.391	28.496	1.037		175.846	82.277	22.263	0.875	
Proposed 3	177.766	19.300	1.698	0.549		177.283	19.200	1.404	0.593	
Proposed 4	176.140	25.655	1.073	0.494		177.442	22.170	0.082	0.537	

Table S11: Calibrated parameters of hardening laws for AA6021 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	347.189	198.799	12.968			344.862	197.264	13.344		
Hockett-Shetby	358.471	214.116	9.732	0.918		344.935	197.634	13.233	0.996	
Chinh et al.	358.460	214.091	0.103	0.919		344.934	197.627	0.076	0.996	
Hollomon	489.516	0.227				408.709	0.158			
Ludwik	113.687	492.307	0.447			-267.118	672.048	0.084		
Swift	544.722	0.00770	0.283			408.663	0.00010	0.158		
Hollomon/Voce	0.056					-0.092				
Swift/Voce	0.228					-0.054				
Swift/HS	0.011					0.544				
Modified Voce	289.658	143.294	17.611	269.909		344.862	197.265	13.345	0.000	
Double Voce	364.706	41.365	30.227	177.496	9.221	344.862	22.636	13.401	174.627	13.337
Ludwigson	485.381	0.443	4.729	-0.102		406.699	0.158	-0.212	-0.089	
Misiolek	378.414	0.166	1.087			551.210	0.252	-0.600		
Pham and Kim	151.760	388.420	42.769	0.442		147.597	241.357	20.308	0.159	
Proposed 1	493.111	69.868	0.065	0.587		374.717	8.792	0.081	1.311	
Proposed 2	143.969	210.765	11.182	0.886		146.946	197.963	13.260	0.988	
Proposed 3	146.204	12.700	1.068	1.505		149.854	11.700	1.579	1.274	
Proposed 4	146.319	17.529	1.894	0.986		148.411	14.290	0.106	1.190	

Table S12: Calibrated parameters of hardening laws for AA3004 sheets

Model	Common curve fitting method					Constrained curve fitting method				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
Voce	195.525	110.980	12.936			201.268	114.727	11.401		
Hockett-Shetby	221.848	146.159	5.105	0.741		204.700	125.891	7.984	0.844	
Chinh et al.	221.687	145.956	0.195	0.742		204.699	125.888	0.125	0.844	
Hollomon	272.254	0.221				240.748	0.172			
Ludwik	57.957	259.602	0.394			0.000	240.748	0.172		
Swift	293.336	0.00538	0.261			240.720	0.00010	0.172		
Hollomon/Voce	0.248					0.339				
Swift/Voce	0.611					0.237				
Swift/HS	0.002					0.508				
Modified Voce	146.558	65.141	23.629	235.334		173.632	90.390	16.146	92.326	
Double Voce	200.920	14.963	174.086	110.598	11.090	201.245	15.092	168.406	110.688	11.002
Ludwigson	253.488	0.363	3.958	-0.101		240.747	0.172	-10.788	0.009	
Misiolek	223.473	0.173	0.786			294.439	0.237	-0.384		
Pham and Kim	83.848	230.860	56.307	0.464		86.363	160.331	26.750	0.280	
Proposed 1	627.774	371.951	0.026	0.107		213.917	3.188	0.161	2.036	
Proposed 2	74.326	137.657	7.384	0.658		77.031	125.767	9.547	0.746	
Proposed 3	77.928	20.100	0.643	1.949		82.171	13.600	1.120	1.441	
Proposed 4	81.144	22.645	3.926	0.849		83.324	16.542	1.048	1.052	

Supplementary Figure

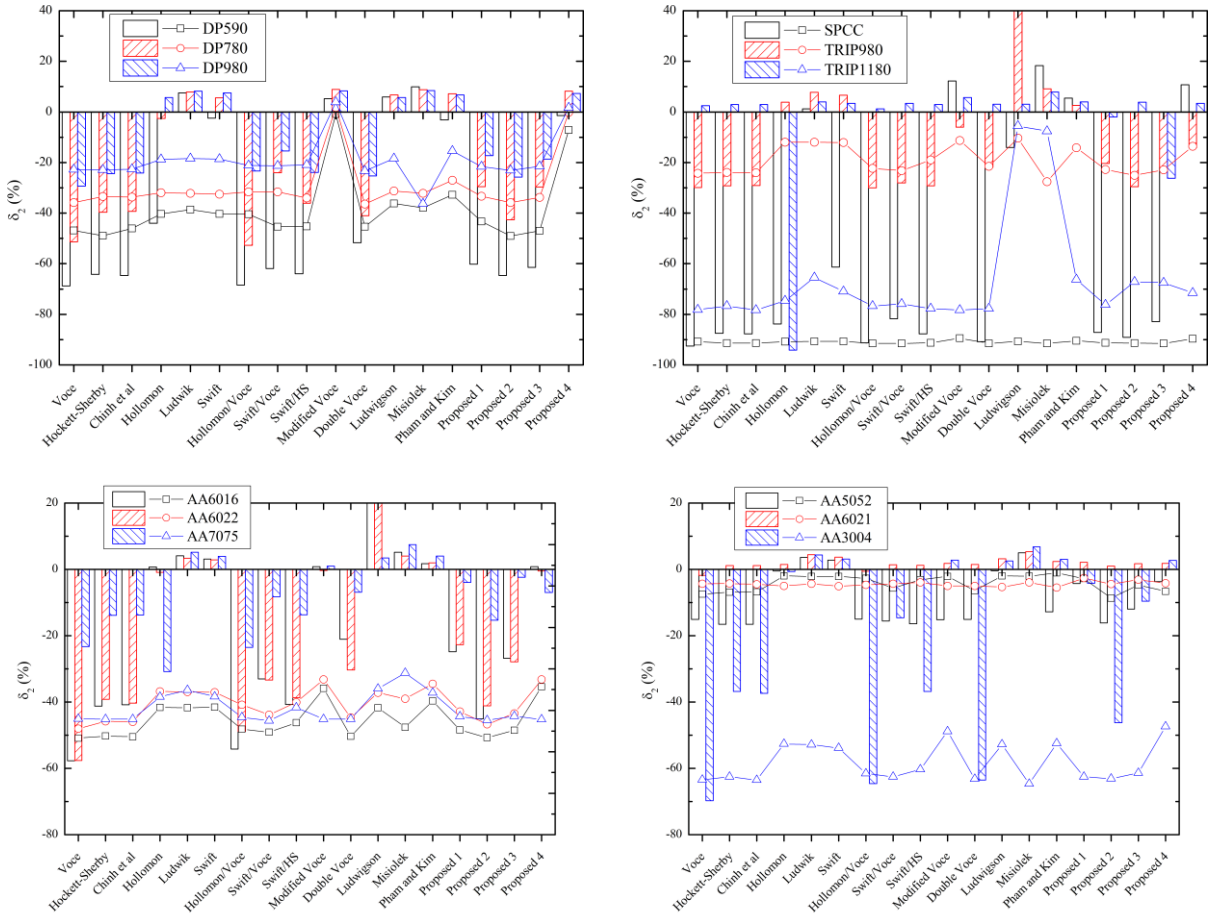

Figure S1. Calculated δ_2 of all identified hardening laws for tested materials. Column charts indicate the results of the common curve fitting method; opened-symbols indicate the results of the constrained curve fitting method.