

## Article

# Evaluation of Mass Transfer Coefficient during Scrap Melting

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**Abstract:** Mass transfer is a critical scrap melting step. Herein, mass transfer coefficients ( $k$ ) during scrap melting were calculated using laboratory-scale experiments. Correlation analysis and the entropy weight method were used to determine the effect of variables on  $k$ . The evaluation model under natural and forced convection was established. It was consistent with the experimental results. Under forced convection, at 1573 and 1673 K, when the rotation speed was increased from 141 to 423 r/min,  $k$  increased from  $7.50 \times 10^{-5}$  to  $1.54 \times 10^{-4}$  m/s and from  $8.42 \times 10^{-5}$  to  $1.72 \times 10^{-4}$  m/s, respectively. Furthermore, as the bath temperature was increased from 1573 to 1723 K, the  $k$  value of a stationary specimen increased from  $3.14 \times 10^{-5}$  to  $5.31 \times 10^{-5}$  m/s, respectively. Correlation analysis and the entropy weight method indicated that the effects of variables on  $k$  decreased as follows: molten pool stirring rate > bath temperature > scrap type. Moreover, the explicit functional relationships between  $k$  and the factors affecting  $k$  under natural and forced convection conditions were established, and the results were consistent with the experimental data. Our results can be used to determine the quantitative relationships between  $k$  and the factors affecting  $k$ .

**Keywords:** evaluation model; quantitative relationship; scrap melting; mass transfer coefficient; steelmaking process



**Citation:** Gao, M.; Gao, J.-T.; Zhang, Y.-L. Evaluation of Mass Transfer Coefficient during Scrap Melting. *Metals* **2021**, *11*, 1368. <https://doi.org/10.3390/met11091368>

Academic Editors: Thomas Echterhof, Ko-ichiro Ohno and Ville-Valtteri Visuri

Received: 15 July 2021

Accepted: 26 August 2021

Published: 30 August 2021

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## 1. Introduction

Scrap melting during the steelmaking process is the result of coupling heat and mass transfer. The evaluation of mass transfer, a critical scrap melting step, under different conditions is conducive to analyzing the melting process and optimizing the melting parameters [1,2].

Many recent studies have analyzed the changes in mass transfer coefficient ( $k$ ) during scrap melting. Yang et al. [3] melted  $\Phi 20$ –50 mm steel bars in a 250 kg induction furnace in a bath temperature ( $T$ ) range of 1573–1873 K and reported that the  $k$  values at 1573 and 1673 K were  $6.3 \times 10^{-5}$  and  $6.4 \times 10^{-5}$  m/s, respectively. Kosaka and Minowa [4] reported that the  $k$  value of a  $\Phi 10$  mm steel bar subjected to a rotation speed of 200 r/min increased from  $1.06 \times 10^{-5}$  to  $1.53 \times 10^{-5}$  m/s as  $T$  was increased from 1573 to 1673 K, respectively. Isobe et al. [5] performed a scrap melting experiment using a 5 t converter. Upon increasing the stirring energy density of the molten pool from 3500 and 12,000 W/t,  $k$  increased from  $8.3 \times 10^{-5}$  to  $19.4 \times 10^{-5}$  m/s. Wei et al. [6] performed scrap melting experiments at  $T = 1673$  K. The carbon content of the hot metal and the bottom-blowing flow rate ranged between 2.1 and 4.03 wt% and 3 and 7 L/min, respectively, and the calculated  $k$  values ranged between  $8.0 \times 10^{-5}$  and  $10.0 \times 10^{-5}$  m/s. In addition, the empirical formulas derived using the analogy method were used to analyze mass transfer [7,8]. Dimensionless correlations have been proposed for scrap melting under natural and forced convection conditions [4,9–12]. Wright [9], Kim and Pehlke [10], and Mineo and Susumu [11] calculated the dimensionless correlations during scrap melting under natural convection conditions. Conversely, Li [7], Don [8], Kim and Pehlke [10], and Mineo and Susumu [11] obtained the dimensionless correlations of the Colburn J-factor and Reynolds number ( $Re$ ) under forced convection conditions for  $Re > 100$ . However, the aforementioned correlations

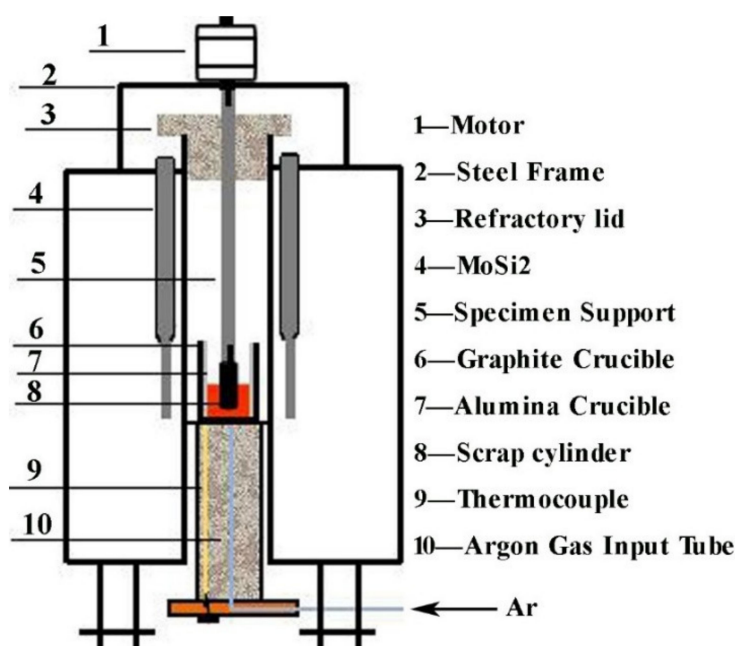
were empirical and presented limitations and constraints. To date, the changes in  $k$  values during scrap melting have not been systematically analyzed. Therefore, an evaluation model should be established.

To evaluate the quantitative effects of several factors on  $k$ , the  $k$  values under different scrap melting conditions were calculated using laboratory-scale experiments. Correlation analysis and the entropy weight method were used to evaluate  $k$  during scrap melting. Furthermore, we established a theoretical model for determining  $k$  under natural and forced convection conditions. We anticipate that the model will serve as a theoretical foundation for online predicting and evaluating the scrap melting step of the steelmaking process.

## 2. Experimental Determination of Mass Transfer Coefficient

### 2.1. Experimental Set-Up and Description

A vertical tube furnace (BCMT-1973K, China) was used for the experiments. Approximately 1 kg of pig iron for steelmaking was melted in a 65 mm × 70 mm × 100 mm (I.D. × O.D. × H) alumina crucible to ensure that the bath depth reached 50 mm (Figure 1). For the tests involving rotating specimens, the rotational speeds were 141, 282, and 423 r/min. The cylinder was rotated after immersion into the hot metal. The material composition is summarized in Table 1.



**Figure 1.** Schematic of experimental apparatus.

Before each experiment, the scrap cylinders were polished to ensure that their weights and diameters were identical. Argon gas was injected into the furnace to prevent oxidation of the hot metal. When the bath temperature was stabilized at the predetermined value, the scrap cylinders were immersed in the hot metal to a depth of approximately 30 mm without preheating and were held in the metal bath for different durations.

After removing them from the hot metal bath, the scrap cylinders were quenched in water. The quenched cylinders, especially the lower sections, contained frozen nodules, which transferred from the hot metal bath to the cylinders. Each specimen was cleaned to obtain a flat surface and subsequently weighed, and the final total length and diameters at the top, center, and bottom of the scrap cylinders were measured. Additionally, melt samples were collected before and after each test to determine their carbon concentrations using a carbon-sulfur analyzer (EMIA-920V2, Horiba, Kyoto, Japan).

**Table 1.** Material composition (mass%).

|                             | C     | Si        | P      | Mn        | S      |
|-----------------------------|-------|-----------|--------|-----------|--------|
| Q235 low-carbon steel rod   | 0.168 | 0.110     | 0.041  | 0.391     | 0.0227 |
| Q235 low-carbon steel plate | 0.182 | 0.17~0.37 | ≤0.030 | 0.35~0.65 | 0.0011 |
| 45# steel rod               | 0.491 | 0.27      | 0.035  | 0.65      | 0.035  |
| QT500-7 Ductile iron rod    | 3.69  | 1.653     | 0.0077 | 0.0825    | 0.0125 |
| Iron/carbon bath            | 4.61  | 0.42      | 0.17   | 0.33      | 0.04   |

## 2.2. Calculation of Mass Transfer Coefficient

During scrap melting, the melting point of scrap is higher than bath temperature ( $T$ ) owing to the low carbon content of scrap. The large carbon concentration gradient between the low-carbon scrap and the high-carbon molten pool promotes carbon mass transfer from the molten pool to the scrap surface. As the surface carbon content of scrap increases, the melting point of the scrap decreases. Therefore, carbon mass transfer is a critical step during scrap melting. Determining the  $k$  values under different conditions can provide a theoretical basis for improving melting efficiency. In previous studies,  $k$  (m/s) has been calculated using the melting rate ( $v$ ; m/s), as follows [4,10–12]:

$$v = -\left(\frac{dr}{dt}\right) = k \ln\left(1 + \frac{C_b - C_L}{C_L - C_s}\right) \quad (1)$$

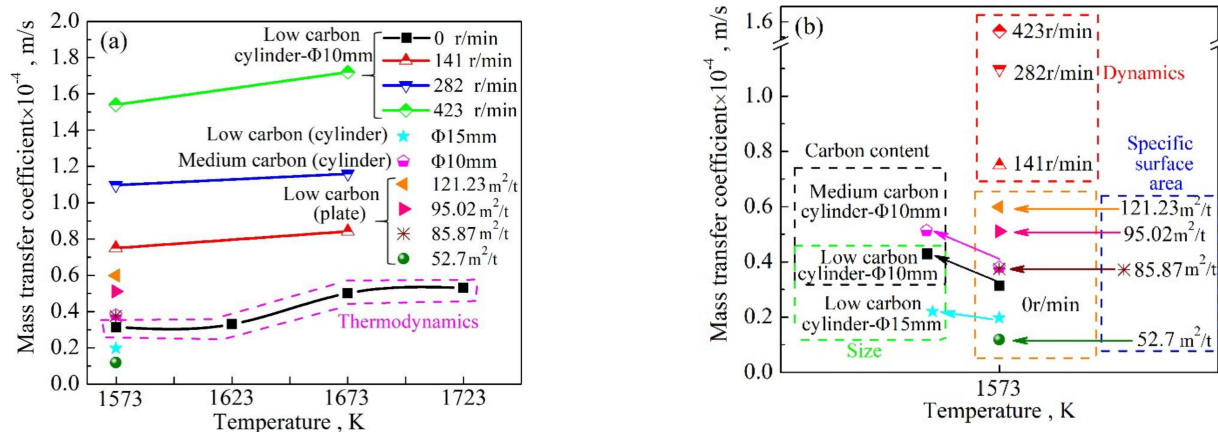
where  $C_b$  and  $C_s$  are the carbon contents of the hot metal and scrap, respectively (wt%), and  $C_L$  is the carbon concentration in the liquid pool at the solid–liquid interface (wt%).

In previous studies, the melting point of scrap was assumed to be the liquidus temperature. The equilibrium temperature was calculated using the heat balance between the high-temperature molten pool and solid scrap. The  $C_L$  value was determined via linear approximation using the Fe–Fe<sub>3</sub>C phase diagram [4,11–14]. The  $C_L$  values corresponding to the  $T$  values of 1573, 1623, 1673, and 1723 K were 2.972, 2.448, 1.87, and 1.293 wt%, respectively.

The  $v$  values under different conditions were substituted in Equation (1) to calculate the corresponding  $k$  values [15]. The effect of molten pool stirring on  $k$  was analyzed, and the results are presented in Figure 2a. The  $k$  values associated with rotating stationary scrap rods were larger than those associated with a stationary scrap rod and increased with increasing rotating speed. The  $k$  values increased by approximately 1.1 times when the rotating speed increased from 141 to 423 r/min. At  $T = 1573$  K,  $k$  increased from  $7.50 \times 10^{-5}$  to  $1.54 \times 10^{-4}$  m/s when the rotation speed was increased from 141 to 423 r/min. The corresponding  $k$  values at  $T = 1673$  K were  $8.42 \times 10^{-5}$  and  $1.72 \times 10^{-4}$  m/s, respectively. The effect of the rotation speed on  $k$  was the largest for the same  $T$  and amount of added scrap (Figure 2b, area enclosed by the red dotted line). Therefore, melting pool stirring played a major role in carbon mass transfer. This was ascribed to the great agitation caused by the high rotation speed promoting the convective mass transfer of carbon from the molten iron pool to the surface of the solid scrap [16,17].

The effect of  $T$  on  $k$  was also analyzed, and the results are presented in Figure 2a (area enclosed by the pink dashed line). For the stationary specimen, as  $T$  was increased from 1573 to 1773 K,  $k$  increased from  $3.14 \times 10^{-5}$  to  $5.31 \times 10^{-5}$  m/s, respectively. This was attributed to the high  $T$  value under natural convection conditions creating a large temperature gradient between the hot metal and the scrap and subsequently generating a large thermal driving force [16,17]. Therefore, heat exchange between the hot metal and solid scrap was promoted to enhance carbon diffusion.

The effect of scrap size on  $k$  was analyzed, and the results are illustrated in Figure 2b (areas enclosed by the black and green dashed lines). Under the same  $T$  and molten pool stirring conditions,  $k$  decreased from  $3.14 \times 10^{-5}$  to  $1.97 \times 10^{-5}$  m/s upon increasing the scrap size from  $\Phi 10$  to  $\Phi 15$  mm, respectively, and increased from  $3.14 \times 10^{-5}$  to  $3.799 \times 10^{-5}$  m/s upon increasing the carbon content of scrap from 0.1675 to 0.49 wt%, respectively.



**Figure 2.** (a) Temperature dependence of the mass transfer coefficient under different experimental conditions (b) Magnified area of the graph in (a).

The effect of the scrap specific surface area ( $S_{sp}$ ) on  $k$  was analyzed, and the results are presented in Figure 2b (area enclosed by the blue dashed line). Under the same  $T$  and molten pool stirring conditions,  $k$  increased from  $1.19 \times 10^{-5}$  to  $5.99 \times 10^{-5}$  m/s upon increasing  $S_{sp}$  from 52.7 to  $121.23 \text{ m}^2/\text{t}$ , respectively.

Using the aforementioned analysis results, we concluded that the rotation speed affected  $k$  the most, followed by  $T$ ,  $S_{sp}$ , and  $C_s$ . Therefore, the effect of each factor on  $k$  decreased as follows: molten pool stirring rate > bath temperature > scrap type. These results were consistent with those of the subsequent theoretical evaluation model.

### 3. Qualitative Evaluation of Factors Affecting Mass Transfer Coefficient

#### 3.1. Correlation Analysis of the Factors Affecting Mass Transfer Coefficient

The factors affecting  $k$  were used as explanatory (independent) variables, and  $k$  was used as the explained (dependent) variable. For the steelmaking process, the effects of top/bottom-blowing causing agitation in the molten pool during conventional steelmaking can be converted into mixing power. In this study, the relation between the imposed rotational speed and the speed induced in the metal bath has been explained as follows: firstly, the work generated by the rotation of the scrap rod can be determined using the formula for the rigid-body moment of inertia. The rotating speed was used to calculate the mixing power, as follows:

$$J = \frac{1}{2} m_s r^2 ; E = \frac{1}{2} J \omega^2 \quad (2)$$

where  $E$  is the work owing to rotation (W),  $J$  is the moment of inertia ( $\text{kg} \cdot \text{m}^2$ ), and  $r$  is the radius of the scrap metal rod (m). Furthermore,  $\omega$  is the angular velocity (rad/s), which can be calculated as follows:  $\omega = 2\pi n$ , where  $n$  is the rotating speed in the experiment (r/s), and  $m_s$  is the total mass of hot metal (1 kg in this study). The stirring energy per unit mass of molten steel ( $\varepsilon$ ; W/t) can be expressed as follows:

$$\varepsilon = E/m \quad (3)$$

where  $m$  is the total mass of the hot metal (t).

According to the above equations, the rotating speeds used in the previous experiment, that is, 141, 282, and 423 r/min, were converted into stirring powers of 0.0249, 0.0997, and 0.2243 W/t, respectively. The raw data from the previous experiments performed by the authors are summarized in Table 2.

**Table 2.** Raw data of the evaluation model.

| Serial Number | Bath Temperature | Stirring Energy Density | Specific Surface Area | Carbon Content | Mass Transfer Coefficient |
|---------------|------------------|-------------------------|-----------------------|----------------|---------------------------|
| -             | K                | W/t                     | m <sup>2</sup> /t     | wt%            | ×10 <sup>-4</sup> m/s     |
| 1             | 1573             | 0                       | 56.03                 | 0.168          | 0.314                     |
| 2             | 1623             | 0                       | 56.03                 | 0.168          | 0.333                     |
| 3             | 1673             | 0                       | 56.03                 | 0.168          | 0.501                     |
| 4             | 1723             | 0                       | 56.03                 | 0.168          | 0.531                     |
| 5             | 1573             | 0.0249                  | 56.03                 | 0.168          | 0.750                     |
| 6             | 1573             | 0.0997                  | 56.03                 | 0.168          | 1.097                     |
| 7             | 1573             | 0.2243                  | 56.03                 | 0.168          | 1.540                     |
| 8             | 1673             | 0.0249                  | 56.03                 | 0.168          | 0.842                     |
| 9             | 1673             | 0.0997                  | 56.03                 | 0.168          | 1.159                     |
| 10            | 1673             | 0.2243                  | 56.03                 | 0.168          | 1.720                     |
| 11            | 1573             | 0                       | 56.03                 | 0.491          | 0.379                     |
| 12            | 1573             | 0                       | 52.7                  | 0.180          | 0.131                     |
| 13            | 1573             | 0                       | 95.02                 | 0.180          | 0.452                     |
| 14            | 1573             | 0                       | 121.2                 | 0.180          | 0.569                     |
| 15            | 1573             | 0                       | 85.87                 | 0.180          | 0.375                     |

Correlation analysis is used to test the significance of the relationship between two variables. The Pearson's correlation coefficient ( $r$ ) can be expressed as follows:

$$r = \frac{S_{xy}}{S_x S_y}, S_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n - 1}, S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}, \text{ and } S_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n - 1}} \quad (4)$$

where  $S_{xy}$  is the sample covariance,  $S_x$  and  $S_y$  are the sample standard deviations of  $x$  and  $y$ , respectively,  $N$  is the number of samples, and  $\bar{x}$ ,  $\bar{y}$  are the average values of  $x$  and  $y$ , respectively. Positive and negative correlations are indicated by  $r > 0$  and  $r < 0$ , respectively. The magnitude of the absolute value indicates the strength of the correlation; that is, the larger the absolute value, the stronger the correlation.

Correlation analysis of the original data was performed using the Statistical Product and Service Solutions software to ensure that the independent variables were correlated with the evaluation variables, and the results are presented in Table 3.

**Table 3.** Correlation analysis results.

|                           | Bath Temperature | Stirring Energy Density | Scrap Specific Surface Area | Scrap Carbon Content |
|---------------------------|------------------|-------------------------|-----------------------------|----------------------|
| Mass transfer coefficient | 0.533            | 0.960                   | 0.362                       | 0.016                |

The analyzed factors ( $\epsilon$ ,  $T$ ,  $S_{sp}$ , and  $C_s$ ) were positively correlated to  $k$  (Table 3). The  $r$ -value of  $\epsilon$  and  $k$  was the largest, followed by that of  $T$  and  $k$ . According to previous studies, the mass transfer of carbon from the hot metal pool to the scrap surface was the limiting step of the melting process. Stirring promoted mass transfer and convective heat transfer in the bath. The melting rate increased with increasing stirring rate [2,3,9,16,17]. Liu et al. [18] analyzed the effects of different bottom-blowing systems on ice melting and concluded that the stirring effect of the bottom-blowing on the molten pool accelerated the mass transfer of carbon from the hot metal to the scrap surface. Furthermore, the convective heat transfer between the molten pool and scrap was enhanced to promote scrap melting. The aforementioned results were consistent with those of our correlation analysis. Therefore, the qualitative results of the correlation analysis provided a direction for further exploration of the quantitative relationship between the melting process parameters and several factors.

### 3.2. Qualitative Evaluation of Mass Transfer Coefficient Using the Entropy Weight Method

It is beneficial to promote the scrap to melt to determine the degree of influence of each factor on the mass transfer coefficient [19]. The entropy weight method, an objective

assignment method, can utilize the information provided by the data to determine the objective weights and to remove the subjective influence. It is more advantageous in comparison with other subjective assignment methods. The main steps of the entropy weight method are as follows [20–22]:

The evaluation matrix composed of  $m$  evaluation schemes and  $n$  indicators is defined as  $X = (x_{ij})_{m \times n}$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n$ . The index standardization method is defined in Equation (5):

$$P_{ij} = x_{ij} / \sum_{i=1}^m x_{ij} \quad (5)$$

where  $P_{ij}$  is the standardized index data. This indicates the proportion of the  $i$ -th scheme under the  $j$ -th indicator in this index. The incommensurability among indicators is effectively eliminated through standardization. The entropy of each evaluation index is expressed as Equation (6):

$$E_j = \left( \sum_{i=1}^m P_{ij} \ln P_{ij} \right) / \ln m \quad (6)$$

In particular,  $P_{ij} \ln P_{ij} = 0$  when  $P_{ij} = 0$ .  $\omega_j$  is the weight of each indicator which can be expressed as Equation (7):

$$\begin{cases} \omega_j = (1 - E_j) / \sum_{j=1}^n (1 - E_j) \\ \sum_{j=1}^n \omega_j = 1 \end{cases} \quad (7)$$

$r_i$  is defined as the comprehensive evaluation score of the  $i$ -th system. As shown in Equation (8), it is obtained using the linear weighted comprehensive evaluation formula.

$$r_i = \sum_{j=1}^n p_{ij} \omega_j \quad (8)$$

Finally, the influence of each factor on scrap melting is determined by comparing the comprehensive scores. As shown in Table 4, the comprehensive evaluation scores of  $r_1$ – $r_{15}$  can be obtained using Equations (5)–(8).

**Table 4.** Comprehensive evaluation scores.

| Influencing Factors         | Factors                     | Serial Number | Value  | Rank |
|-----------------------------|-----------------------------|---------------|--------|------|
| Bath temperature            | Bath temperature            | $r_1$         | 0.0453 | 2    |
|                             |                             | $r_2$         | 0.0359 |      |
|                             |                             | $r_3$         | 0.0541 |      |
|                             |                             | $r_4$         | 0.0573 |      |
|                             |                             | $r_5$         | 0.1279 |      |
| Stirring of the molten pool | Rotating speed              | $r_6$         | 0.1184 | 1    |
|                             |                             | $r_7$         | 0.1663 |      |
|                             |                             | $r_8$         | 0.0909 |      |
|                             |                             | $r_9$         | 0.1251 |      |
|                             |                             | $r_{10}$      | 0.1857 |      |
|                             |                             | $r_{11}$      | 0.0411 |      |
| Scrap type                  | Scrap specific surface area | $r_{12}$      | 0.0141 | 3    |
|                             |                             | $r_{13}$      | 0.0488 |      |
|                             |                             | $r_{14}$      | 0.0614 |      |
|                             |                             | $r_{15}$      | 0.0405 |      |

The comprehensive evaluation scores of the affecting factors were ranked as follows: molten pool stirring > bath temperature > scrap type (Table 4). Therefore, the molten pool kinetics affected scrap melting the most, followed by the thermodynamics of the molten pool and scrap selection. Consequently, the following suggestions were evaluated in this study. The dynamics and thermodynamics of the molten pool can be improved

to accelerate scrap melting and promote the steelmaking process. Similar processes are used in conventional steelmaking to promote scrap melting. Top/bottom-blowing was used to improve the kinetic conditions. Procedures such as injecting pulverized coal or top-blowing oxygen were implemented to accelerate the exothermic reaction of carbon and oxygen.  $T$  was increased to improve the thermodynamic conditions.

#### 4. Quantitative Evaluation of Mass Transfer Coefficient Using Explicit Functions

##### 4.1. Explicit Function under Natural Convection Conditions

The schematic diagram of cylindrical scrap melting is presented in Figure 3.

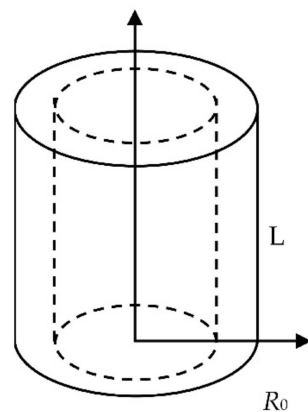


Figure 3. Schematic diagram of cylindrical scrap melting.

For a stationary specimen, the flow of liquid conformed to natural convection conditions. According to the dimensionless correlations of mass transfer under natural convection conditions ( $Sh = A(GrSc)^B$ ) [23],  $k$  can be calculated as follows:

$$k = A \left( \frac{g\Delta\rho}{\mu} \right)^B L^{3B-1} (D_C)^{1-B} \quad (9)$$

where  $A$  and  $B$  are constants obtained by fitting the experimental results.  $g$  is the gravitational acceleration in  $m/s^2$ .  $\Delta\rho$  is the density difference in  $kg/m^3$ .  $\mu$  is the hydrodynamic viscosity in  $N\cdot s/m^2$ .  $L$ , the specific length of the scrap, is the immersion depth of the cylindrical scrap under stationary conditions. And  $D_C$  is the diffusion coefficient in  $m^2/s$ .

According to previous studies [7,8,13,23–26],  $D_C$  is a function of the temperature and carbon concentration. For example, Shukla et al. [13] indicated that  $D_C$  can be calculated as follows:  $D_C = 10^{-4} \times \exp(-(12,100/T + 2.568) + (1320/T - 0.554) \times C_b)$ . Therefore,  $D_C$  can be expressed as  $D_C = f(T, C_b)$ , where  $T$  is the diffusion temperature (K), which is equal to the bath temperature.  $D_C$  increased upon increasing  $T$  and  $C_b$ .

$S_{sp}$  for a cylindrical scrap specimen can be calculated as follows:

$$S_{sp} = \frac{2}{\rho_s} \left( \frac{1}{L} + \frac{1}{R_0} \right) \quad (10)$$

Therefore, by combining Equations (9)–(11),  $k$  can be expressed as follows:

$$k = A \left( \frac{g\Delta\rho}{\mu} \right)^B \left( \frac{S_{sp}\rho_s}{2} - \frac{1}{R_0} \right)^{1-3B} (D_C)^{1-B} \quad (11)$$

Equation (11) gives an explicit relationship between  $k$ ,  $T$ ,  $C_s$ , and  $S_{sp}$  under natural convection conditions, providing a basis for quantitative discussion. Under the experimental conditions in this study,  $A = 0.144$  and  $B = 0.325$  [15].

#### 4.2. Explicit Function under Forced Convection Conditions

The dimensionless correlation of  $k$  under forced convection conditions is the Chilton–Colburn analogy:  $(J_D = \left(\frac{k}{U}\right)(Sc)^{0.644} = ARe^B)$  [8]. Therefore,  $k$  can be expressed as follows:

$$k = A(\pi)^{B+1} \left(\frac{\rho_b}{\mu}\right)^{B+0.644} (D_0 e^{-\frac{Q}{Rt}})^{0.644} L^{2B+1} n^{B+1} \quad (12)$$

Under forced convection conditions,  $S_{sp}$  can be expressed as follows:

$$S_{sp} = \frac{2(L + 2h)}{Lh\rho_s} \quad (13)$$

where  $h$  is the height of the cylindrical scrap (m) and  $L$  is the average diameter of the cylindrical scrap ( $L = 2R_0$ )

In addition, Equation (14) was derived from the relationship between  $\varepsilon$  and the rotation speed.

$$n = \frac{2\left(\frac{m}{m_s}\varepsilon\right)^{1/2}}{\pi L} \quad (14)$$

Combining Equations (12)–(14),  $k$  can be expressed as follows:

$$k = A(2)^{B+1} \left(\frac{\rho_b}{\mu}\right)^{B+0.644} (D_0 e^{-\frac{Q}{Rt}})^{0.644} \left(\frac{4h}{S_{sp}h\rho_s - 2}\right)^B \left(\frac{m}{m_s}\varepsilon\right)^{\frac{B+1}{2}} \quad (15)$$

Equation (15) gives an explicit relationship between  $k$ ,  $T$ ,  $\varepsilon$ ,  $C_s$ , and  $S_{sp}$  under forced convection conditions, providing a basis for quantitative discussion. Under the experimental conditions in this study,  $A = 0.133$  and  $B = -0.356$  [15].

#### 4.3. Validation of Explicit Functions under Natural and Forced Convection Conditions

The dependence of  $k$  on  $T$  under natural convection conditions is illustrated in Figure 4. The  $k$  values determined using the evaluation model were consistent with the experimental values;  $k$  increased with increasing  $T$ , confirming the accuracy of the explicit function under natural convection conditions.

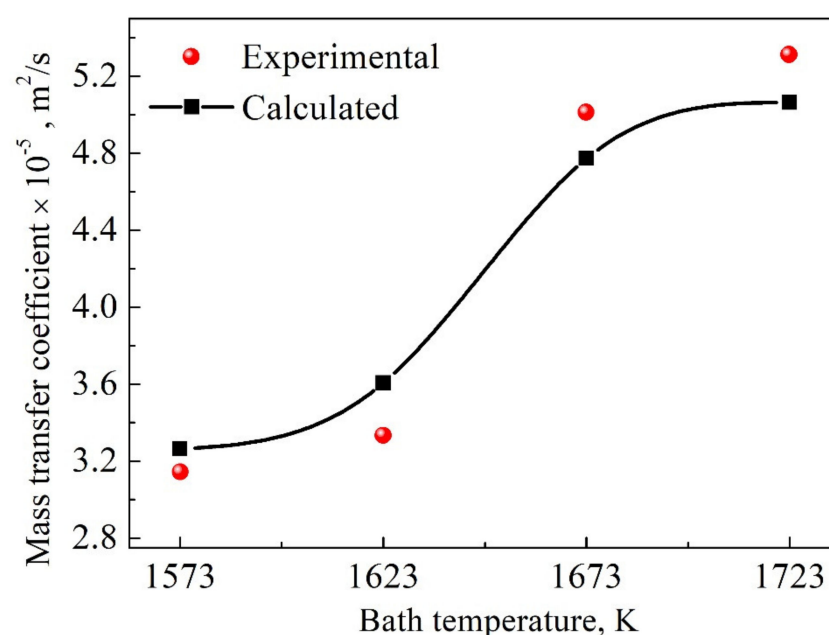


Figure 4. Validation of explicit function under natural convection conditions.

The dependence of  $k$  on  $\varepsilon$  under forced convection conditions is illustrated in Figure 5. The  $k$  values determined using the evaluation model were consistent with the experimental values;  $k$  increased with increasing  $\varepsilon$ , confirming the accuracy of the explicit function under forced convection conditions.

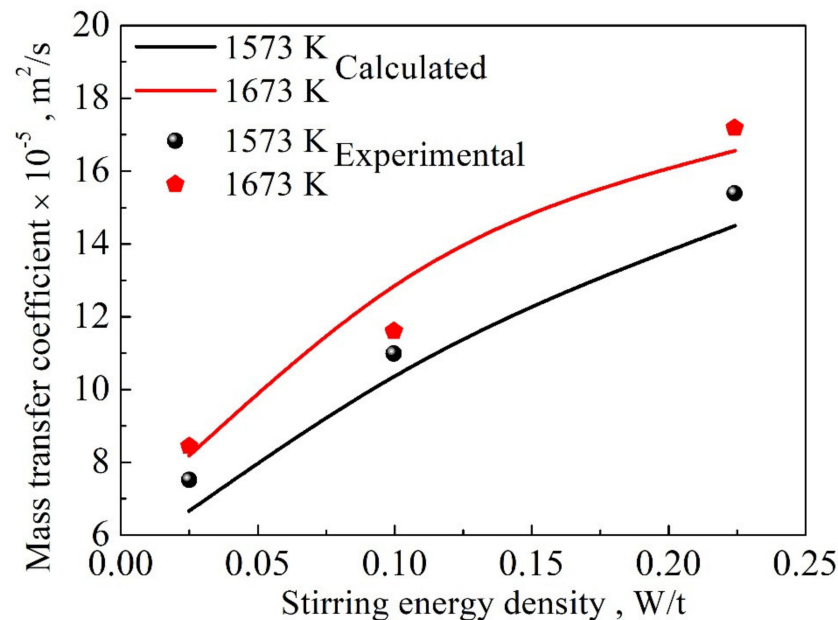


Figure 5. Validation of the explicit function under forced convection conditions.

## 5. Conclusions

The  $k$  values under different scrap melting conditions were obtained by laboratory-scale experiments. At  $T$  of 1573 and 1673 K, when the rotation speed was increased from 141 to 423 r/min,  $k$  increased from  $7.50 \times 10^{-5}$  to  $1.54 \times 10^{-4}$  m/s and from  $8.42 \times 10^{-5}$  to  $1.72 \times 10^{-4}$  m/s, respectively. For the stationary specimen,  $k$  increased from  $3.14 \times 10^{-5}$  to  $5.31 \times 10^{-5}$  m/s as  $T$  was increased from 1573 to 1723 K, respectively, from  $3.14 \times 10^{-5}$  to  $3.799 \times 10^{-5}$  m/s when  $C_s$  increased from 0.1675 to 0.49 wt%, respectively, and from  $1.19 \times 10^{-5}$  to  $5.99 \times 10^{-5}$  m/s when  $S_{sp}$  increased from 52.7 to 121.23 m²/t, respectively.

The effects of various factors on  $k$  were evaluated using correlation analysis. The stirring energy density had the greatest correlation with the mass transfer coefficient, followed by the bath temperature.

The results of the entropy weight method indicated that the effects of each factor on  $k$  decreased as follows: molten pool stirring rate > bath temperature > scrap type.

The explicit functions between  $k$  and the factors affecting  $k$  under natural and forced convection conditions were described as follows: under natural convection,

$$k = A \left( \frac{g \Delta \rho}{\mu} \right)^B \left( \frac{S_{sp} \rho_s}{2} - \frac{1}{R_0} \right)^{1-3B} (D_C)^{1-B}$$

Under forced convection, the explicit function was as follows:

$$k = A(2)^{B+1} \left( \frac{\rho_b}{\mu} \right)^{B+0.644} (D_0 e^{-\frac{Q}{RT}})^{0.644} \left( \frac{4h}{S_{sp} h \rho_s - 2} \right)^B \left( \frac{m}{m_s} \varepsilon \right)^{\frac{B+1}{2}}$$

**Author Contributions:** M.G., J.-T.G., and Y.-L.Z., conceived and designed the experiments; M.G. performed the experiments; M.G. analyzed the data; M.G. and Y.-L.Z. contributed to the writing and editing of the manuscript. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Key R&D Program of China, grant number 2019YFC1905701. This research was funded by Key projects of NSFC, grant number U1960201.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare that they have no conflict of interest.

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