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# Cosmological Constant and Renormalization of Gravity

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**Abstract:** In arXiv:1601.02203 and arXiv:1702.07063, we have proposed a topological model with a simple Lagrangian density and have tried to solve one of the cosmological constant problems. The Lagrangian density is the BRS exact and therefore the model can be regarded as a topological theory. In this model, the divergence of the vacuum energy coming from the quantum corrections from matters can be absorbed into the redefinition of the scalar field. In this paper, we consider the extension of the model in order to apply the mechanism to other kinds of divergences coming from the quantum correction and consider the cosmology in an extended model.

**Keywords:** cosmological constant; quantum gravity; renormalization

By the recent cosmological observations, we now believe the accelerating expansion of the present universe, whose simplest model may be given by a cosmological term with a small cosmological constant. We also know that the quantum correction coming from the contributions from matters to the vacuum energy, which may be identified with the cosmological constant, diverges strongly and we need the very finely tuned counterterm to cancel the divergence. For a discussion about the small but non-vanishing vacuum energy, see, for example, [1]. In order to tackle the problem of the large quantum corrections to the vacuum energy, the unimodular gravity theories [2–28] has been proposed and discussed. Other scenarios like the sequestering mechanism have been also proposed [29–35]. In [36], we have proposed a new model which could be regarded with a topological field theory and discussed the cosmology in the model [37]. We should note that by the quantum corrections from the matter, the following terms, besides the cosmological constant, are generated,

$$\mathcal{L}_{\text{qc}} = \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu} + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \quad (1)$$

Here, the coefficient  $\alpha$  diverges quadratically and  $\beta$ ,  $\gamma$ , and  $\delta$  diverge logarithmically without the cut-off scale. If we further include the quantum corrections from the graviton, infinite numbers of diverging quantum corrections appear. In order to solve these problems of the divergences and the renormalizations, we extend the model in [36] and discuss the cosmology given by the extended models.

The action of the model in [36] has the following form,

$$S' = \int d^4x \sqrt{-g} \{ \mathcal{L}_{\text{gravity}} - \lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c \} + S_{\text{matter}}. \quad (2)$$

Although  $\lambda$  and  $\varphi$  are ordinary scalar fields,  $b$  and  $c$  are fermionic (Grassmann odd) scalar fields and we regard that  $b$  is an anti-ghost field and  $c$  with a ghost field.<sup>1,2</sup> In (2),  $S_{\text{matter}}$  is the action of matters and  $\mathcal{L}_{\text{gravity}}$  is the Lagrangian density of arbitrary gravity. There does not appear any parameter in the action (2) except in the parts of  $S_{\text{matter}}$  and  $\mathcal{L}_{\text{gravity}}$ .

By separating the gravity Lagrangian density  $\mathcal{L}_{\text{gravity}}$  into the sum of some constant  $\Lambda$ , which may include the large quantum corrections from matters, and other part  $\mathcal{L}_{\text{gravity}}^{(0)}$ ,  $\mathcal{L}_{\text{gravity}} = \mathcal{L}_{\text{gravity}}^{(0)} - \Lambda$ , we redefine the scalar field  $\lambda$  by  $\lambda \rightarrow \lambda - \Lambda$ . The obtained action has the following form,

$$S' = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\text{gravity}}^{(0)} - \lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c \right\} + S_{\text{matter}}. \quad (3)$$

Note that the action (3) does not include the cosmological constant  $\Lambda$  and therefore the constant  $\Lambda$  never affects the dynamics in the model. This tells that the large quantum corrections from the matters can be tuned to vanish.

As shown in [36], ghosts appear, which generates the negative norm states in the quantum theory, in the model (2). The existence of the negative norm states makes the so-called Copenhagen interpretation invalid and therefore the model becomes inconsistent. The negative norm states, however, can be eliminated by defining the physical states which are annihilated by the BRS charge [40]. We can find that the action (2) is invariant under the infinite numbers of the BRS transformation,

$$\delta\lambda = \delta c = 0, \quad \delta\varphi = \epsilon c, \quad \delta b = \epsilon(\lambda - \lambda_0). \quad (4)$$

Here,  $\epsilon$  is a fermionic (Grassmann odd) parameter and  $\lambda_0$  should satisfy the following equation by putting  $\lambda = \lambda_0$ ,

$$0 = \nabla^\mu \partial_\mu \lambda, \quad (5)$$

which is obtained by the variation of the action (2) with respect to  $\varphi$ .<sup>3</sup> If the physical states are defined as the states invariant under the BRS transformation in (4), the negative norm states can be eliminated by the Kugo-Ojima mechanism in the gauge theory [41,42].<sup>4</sup>

Equation (4) tells us that  $\lambda - \lambda_0$  is given by the BRS transformation of the anti-ghost  $b$ , therefore the vacuum expectation value of  $\lambda - \lambda_0$  must vanish in the physical states. Therefore, there occurs the spontaneous breakdown of the corresponding BRS symmetry in case that the vacuum expectation value of  $\lambda - \lambda_0$  does not vanish. For the broken BRS symmetry, it is impossible to impose the physical state condition. It should be noted, however, there is one and only one unbroken BRS symmetry in the infinite numbers of the BRS symmetries in (4). The point is that Equation (5) is nothing but the field equation for  $\lambda$ . Because the real world is realized by one and only one solution of (5) for  $\lambda$ , one and only one  $\lambda_0$  is chosen so that  $\lambda = \lambda_0$  and therefore the corresponding BRS symmetry is not broken, which eliminates the negative norm states, which are the ghost states, and the unitarity is guaranteed. Although the quantum fluctuations are prohibited by the BRS symmetry,  $\lambda_0$  can include the classical fluctuation as long as  $\lambda_0$  satisfies the classical Equation (5).

We can regard the Lagrangian density in the action (2),

$$\mathcal{L} = -\lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c, \quad (6)$$

as the Lagrangian density of a topological field theory [43], where the Lagrangian density is given by the BRS transformation of some quantity. If we consider the model which only includes the scalar

<sup>1</sup> The action without  $c$  and  $b$  has been proposed in [38] in order to solve the problem of time.

<sup>2</sup> The cosmological perturbation in the model motivated in the model (2) has been investigated in [39].

<sup>3</sup> The existence of the BRS transformation where  $\lambda_0$  satisfies Equation (5) was pointed out by R. Saitou.

<sup>4</sup> We can assign the ghost number, which is conserved, 1 for  $c$  and  $-1$  for  $b$  and  $\epsilon$ . The four scalar fields  $\lambda$ ,  $\varphi$ ,  $b$ , and  $c$  are called a quartet [41,42]

field  $\varphi$  but whose Lagrangian density identically vanishes, which tells us that the action is trivially invariant under any transformation of  $\varphi$ . We may fix the gauge symmetry by imposing the following gauge condition,

$$1 + \nabla_\mu \partial^\mu \varphi = 0. \quad (7)$$

By following the paper [44], we find the gauge-fixed Lagrangian is given by the BRS transformation (4) of  $-b(1 + \nabla_\mu \partial^\mu \varphi)$  and we obtain

$$\delta(-b(1 + \nabla_\mu \partial^\mu \varphi)) = \epsilon(-(\lambda - \lambda_0)(1 + \nabla_\mu \partial^\mu \varphi) + b\nabla_\mu \partial^\mu c) = \epsilon(\mathcal{L} + \lambda_0 + (\text{total derivative terms})). \quad (8)$$

Then we find that the Lagrangian density (6) is given by the BRS transformation of the quantity  $-b(1 + \nabla_\mu \partial^\mu \varphi)$  up to the total derivative terms if  $\lambda_0 = 0$ . The action is not given by the BRS transformation (4) with the non-vanishing  $\lambda_0$ , which could be a reason why the Lagrangian density (6) gives non-trivial and physically relevant contributions.

The above mechanism can work for the divergences in (1) or more general divergences [37]. If we include the divergences in (1), we may generalize the model in (6) as follows,

$$\begin{aligned} \mathcal{L} = & -\Lambda - \lambda_{(\Lambda)} + (\alpha + \lambda_{(\alpha)})R + (\beta + \lambda_{(\beta)})R^2 + (\gamma + \lambda_{(\gamma)})R_{\mu\nu}R^{\mu\nu} + (\delta + \lambda_{(\delta)})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ & + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\ & + \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} + \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)}. \end{aligned} \quad (9)$$

As in the case of the vacuum energy, the divergences are included in the coefficients  $\Lambda$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  but if we shift the parameters  $\lambda_{(\Lambda)}$ ,  $\lambda_{(\alpha)}$ ,  $\lambda_{(\beta)}$ ,  $\lambda_{(\gamma)}$ , and  $\lambda_{(\delta)}$  as follows,

$$\lambda_{(\Lambda)} \rightarrow \lambda_{(\Lambda)} - \Lambda, \quad \lambda_{(\alpha)} \rightarrow \lambda_{(\alpha)} - \alpha, \quad \lambda_{(\beta)} \rightarrow \lambda_{(\beta)} - \beta, \quad \lambda_{(\gamma)} \rightarrow \lambda_{(\gamma)} - \gamma, \quad \lambda_{(\delta)} \rightarrow \lambda_{(\delta)} - \delta, \quad (10)$$

we can rewrite the Lagrangian density (9),

$$\begin{aligned} \mathcal{L} = & -\lambda_{(\Lambda)} + \lambda_{(\alpha)}R + \lambda_{(\beta)}R^2 + \lambda_{(\gamma)}R_{\mu\nu}R^{\mu\nu} + \lambda_{(\delta)}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ & + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\ & + \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} + \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)}, \end{aligned} \quad (11)$$

which tells that we can absorb the divergences into the redefinition of  $\lambda_{(i)}$ , ( $i = \Lambda, \alpha, \beta, \gamma, \delta$ ) and the divergences becomes irrelevant for the dynamics. The Lagrangian density (11) is also invariant under the following BRS transformations

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0, \quad \delta\varphi_{(i)} = \epsilon c, \quad \delta b_{(i)} = \epsilon(\lambda_{(i)} - \lambda_{(i)0}), \quad (i = \Lambda, \alpha, \beta, \gamma, \delta), \quad (12)$$

where  $\lambda_{(i)0}$ 's satisfy the equation,

$$0 = \nabla^\mu \partial_\mu \lambda_{(i)0}, \quad (13)$$

as in (5). The Lagrangian density (11) is also given by the BRS transformation (12) with  $\lambda_{(i)0} = 0$ ,

$$\delta \left( \sum_{i=\Lambda, \alpha, \beta, \gamma, \delta} (-b_{(i)} (1 + \nabla_\mu \partial^\mu \varphi_{(i)})) \right) = \epsilon(\mathcal{L} + (\text{total derivative terms})). \quad (14)$$

As mentioned, due to the quantum correction from the graviton, the divergences in infinite numbers of quantum corrections appear. Let  $\mathcal{O}_i$  be possible gravitational operators then a further generalization of the Lagrangian density (11) is given by

$$\mathcal{L} = \sum_i \left( \lambda_{(i)} \mathcal{O}_{(i)} + \partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)} \right). \quad (15)$$

Then all the divergences are absorbed into the redefinition of  $\lambda_i$ . The Lagrangian density (15) is invariant under the BRS transformation and given by the the BRS transformation of some quantity and therefore the model can be regarded as a topological field theory, again.

By the above arguments, the problems of the divergences in the quantum theory might be solved but there is not any principle to determine the values of the observed cosmological constant and other coupling constants. The values could be determined by the initial conditions or the boundary conditions in the classical theory. Therefore, it could be interesting to investigate the cosmology and specify the region of the initial conditions which could be consistent with the evolution of the observed universe. For the model (2), in [37], it has been shown that we need the fine-tuning for the initial conditions although the constraints on the conditions are relaxed a little bit.

For simplicity, we consider the following reduced model,

$$\mathcal{L} = -\lambda_{(\Lambda)} + \lambda_{(\alpha)}R + \partial_\mu\lambda_{(\Lambda)}\partial^\mu\varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)}\partial^\mu c_{(\Lambda)} + \partial_\mu\lambda_{(\alpha)}\partial^\mu\varphi_{(\alpha)} - \partial_\mu b_{(\alpha)}\partial^\mu c_{(\alpha)}. \quad (16)$$

In order to consider the cosmology, we assume  $b_{(\Lambda)} = c_{(\Lambda)} = b_{(\alpha)} = c_{(\alpha)} = 0$  because the ghost number should be conserved and superselection rule should hold. We assume that the space-time is given by the FRW universe with flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^3 (dx^i)^2, \quad (17)$$

and we assume that all the scalar fields  $\lambda_{(\Lambda)}$ ,  $\varphi_{(\Lambda)}$ ,  $\lambda_{(\alpha)}$ , and  $\varphi_{(\alpha)}$  only depend on the cosmological time  $t$ . Then the variation of the action with respect to the scalar fields and metric gives the following equations.

$$0 = 1 + \left( \frac{d^2\varphi_{(\Lambda)}}{dt^2} + 3H\frac{d\varphi_{(\Lambda)}}{dt} \right), \quad 0 = \frac{d^2\lambda_{(\Lambda)}}{dt^2} + 3H\frac{d\lambda_{(\Lambda)}}{dt}, \quad (18)$$

$$0 = 12H^2 + 6\frac{dH}{dt} + \left( \frac{d^2\varphi_{(\alpha)}}{dt^2} + 3H\frac{d\varphi_{(\alpha)}}{dt} \right), \quad 0 = \frac{d^2\lambda_{(\alpha)}}{dt^2} + 3H\frac{d\lambda_{(\alpha)}}{dt}, \quad (19)$$

$$3\lambda_{(\alpha)}H^2 = -3H\frac{d\lambda_{(\alpha)}}{dt} + \lambda_{(\Lambda)} - \frac{d\lambda_{(\Lambda)}}{dt}\frac{d\varphi_{(\Lambda)}}{dt} - \frac{d\lambda_{(\alpha)}}{dt}\frac{d\varphi_{(\alpha)}}{dt}, \quad (20)$$

$$-\lambda_{(\alpha)}\left(3H^2 + 2\frac{dH}{dt}\right) = 2\frac{d^2\lambda_{(\alpha)}}{dt^2} + 3H\frac{d\lambda_{(\alpha)}}{dt} - \lambda_{(\Lambda)} - \frac{d\lambda_{(\Lambda)}}{dt}\frac{d\varphi_{(\Lambda)}}{dt} - \frac{d\lambda_{(\alpha)}}{dt}\frac{d\varphi_{(\alpha)}}{dt}. \quad (21)$$

If  $\lambda_{(\Lambda)}$  and  $\lambda_{(\alpha)}$  are constant,

$$\lambda_{(\Lambda)} = \lambda_{(\Lambda)0}, \quad \lambda_{(\alpha)} = \lambda_{(\alpha)0}, \quad (22)$$

the second equations in (18) and (19) are satisfied. Then Equation (20) or Equation (21) gives  $H$  is also a constant,

$$H = H_0 \equiv \sqrt{\frac{\lambda_{(\Lambda)0}}{3\lambda_{(\alpha)0}}}. \quad (23)$$

Then a solution for the first equations in (18) and (19) is given by

$$\varphi_{(\Lambda)} = -\frac{t}{3H_0}, \quad \varphi_{(\alpha)} = -4H_0t. \quad (24)$$

Because  $H$  is a constant, we find that the de Sitter space-time is a solution of this model.

We now consider the stability of the obtained solution describing the de Sitter space-time by considering the perturbation,

$$H = H_0 + \delta H, \quad \lambda_{(\Lambda)} = \lambda_{(\Lambda)0} + \delta\lambda_{(\Lambda)}, \quad \varphi_{(\Lambda)} = -\frac{t}{3H_0} + \delta\varphi_{(\Lambda)}, \quad \lambda_{(\alpha)} = \lambda_{(\alpha)0} + \delta\lambda_{(\alpha)}, \quad \varphi_{(\alpha)} = -4H_0t + \delta\varphi_{(\alpha)}. \quad (25)$$

Then we obtain the following perturbed equations,

$$0 = \delta\ddot{\phi}_{(\Lambda)} + 3H_0\delta\dot{\phi}_{(\Lambda)} - \frac{1}{3H_0}\delta H, \quad (26)$$

$$0 = \delta\ddot{\lambda}_{(\Lambda)} + 3H_0\delta\dot{\lambda}_{(\Lambda)}, \quad (27)$$

$$0 = \delta\ddot{\phi}_{(\alpha)} + 3H_0\delta\dot{\phi}_{(\alpha)} + 20H_0\delta H + 6\delta\dot{H}, \quad (28)$$

$$0 = \delta\ddot{\lambda}_{(\alpha)} + 3H_0\delta\dot{\lambda}_{(\alpha)}, \quad (29)$$

$$6\lambda_{(\alpha)0}H_0\delta H + 3H_0^2\delta\lambda_{(\alpha)} = -3H_0\delta\dot{\lambda}_{(\alpha)} + \delta\lambda_{(\Lambda)} + \frac{1}{3H_0}\delta\dot{\lambda}_{(\Lambda)}. \quad (30)$$

By using (27), (29) and (30), we obtain

$$\lambda_{(\alpha)0}\delta\dot{H} = H_0\delta\dot{\lambda}_{(\alpha)}. \quad (31)$$

By using (30), we delete  $\delta H$  in (26) and (28) and obtain

$$0 = \delta\ddot{\phi}_{(\Lambda)} + 3H_0\delta\dot{\phi}_{(\Lambda)} - \frac{1}{18H_0^2\lambda_{(\alpha)0}} \left( -3H_0^2\delta\lambda_{(\alpha)} - 3H_0\delta\dot{\lambda}_{(\alpha)} + \delta\lambda_{(\Lambda)} + \frac{1}{3H_0}\delta\dot{\lambda}_{(\Lambda)} \right), \quad (32)$$

$$0 = \delta\ddot{\phi}_{(\alpha)} + 3H_0\delta\dot{\phi}_{(\alpha)} + \frac{10}{3\lambda_{(\alpha)0}} \left( -3H_0^2\delta\lambda_{(\alpha)} - \frac{6}{5}H_0\delta\dot{\lambda}_{(\alpha)} + \delta\lambda_{(\Lambda)} + \frac{1}{3H_0}\delta\dot{\lambda}_{(\Lambda)} \right). \quad (33)$$

We now define new variables  $\delta\eta_{(\Lambda)}$  and  $\delta\eta_{(\alpha)}$  by

$$\delta\eta_{(\Lambda)} \equiv \delta\dot{\lambda}_{(\Lambda)}, \quad \delta\eta_{(\alpha)} \equiv \delta\dot{\lambda}_{(\alpha)}, \quad (34)$$

and we rewrite (27), (29), (32) and (33) as follows,

$$0 = \delta\dot{\eta}_{(\Lambda)} + 3H_0\delta\eta_{(\Lambda)}, \quad (35)$$

$$0 = \delta\dot{\eta}_{(\alpha)} + 3H_0\delta\eta_{(\alpha)}, \quad (36)$$

$$0 = \delta\ddot{\phi}_{(\Lambda)} + 3H_0\delta\dot{\phi}_{(\Lambda)} - \frac{1}{18H_0^2\lambda_{(\alpha)0}} \left( -3H_0^2\delta\lambda_{(\alpha)} - 3H_0\delta\eta_{(\alpha)} + \delta\lambda_{(\Lambda)} + \frac{1}{3H_0}\delta\eta_{(\Lambda)} \right), \quad (37)$$

$$0 = \delta\ddot{\phi}_{(\alpha)} + 3H_0\delta\dot{\phi}_{(\alpha)} + \frac{10}{3\lambda_{(\alpha)0}} \left( -3H_0^2\delta\lambda_{(\alpha)} - \frac{6}{5}H_0\delta\eta_{(\alpha)} + \delta\lambda_{(\Lambda)} + \frac{1}{3H_0}\delta\eta_{(\Lambda)} \right). \quad (38)$$

Furthermore, we define

$$\delta\varphi \equiv \delta\dot{\phi}_{(\Lambda)} + \frac{1}{60h_0^2}\delta\dot{\phi}_{(\alpha)}, \quad (39)$$

Equations (37) and (38) give

$$0 = \delta\dot{\varphi} + 3H_0\delta\varphi - \frac{\delta\eta_{(\alpha)}}{10H_0\lambda_{(\alpha)0}}. \quad (40)$$

We now write Equations (34)–(37) and (40) in a matrix form,

$$0 = \left( \frac{d}{dt} + A \right) \begin{pmatrix} \delta\eta_{(\Lambda)} \\ \delta\eta_{(\alpha)} \\ \delta\lambda_{(\Lambda)} \\ \delta\lambda_{(\alpha)} \\ \delta\varphi \\ \delta\dot{\phi}_{(\Lambda)} \end{pmatrix}, \quad A \equiv \begin{pmatrix} 3H_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3H_0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3H_0 & 0 \\ -\frac{1}{54H_0^3\lambda_{(\alpha)0}} & 16H_0\lambda_{(\alpha)0} & -\frac{1}{18H_0^2\lambda_{(\alpha)0}} & \frac{1}{6\lambda_{(\alpha)0}} & 0 & 3H_0 \end{pmatrix}. \quad (41)$$

If there is negative eigenvalue in the matrix  $A$ , the solution describing the de Sitter space-time is unstable but as clear from the form of the matrix  $A$ , which is triangular, the eigenvalues are given

by four  $3H_0$ 's and two  $0$ 's. Therefore, the solution describing the de Sitter space-time is stable or at least quasi-stable as in the model (3) with  $\mathcal{L}_{\text{gravity}}^{(0)} = \frac{R}{2\kappa^2}$ , that is, the case of the Einstein gravity [37]. The stability tells us that the solution might describe the acceleratingly expanding universe at present.

In the model (16), the divergences in the cosmological constant and the gravitational constant coming from the quantum corrections may not affect the dynamics but there is no principle to determine the constants, which may correspond to  $\lambda_{(\Lambda)0}$  and  $\lambda_{(\alpha)0}$  in (22). These constants could be determined by the initial conditions or something else and therefore it could be interesting to investigate the cosmology by including the matters as in the model of [37]. In the model (15), however, we need infinite numbers of the initial conditions, which might be unphysical but this problem might give a clue to the quantum gravity. In fact, the second equations in (18) and (19) describe the evolutions of the effective coupling constants  $\lambda_{(\Lambda)}$  and  $\lambda_{(\alpha)}$  with respect to the scale  $a(t)$  as in the renormalization group equations.

Finally, we would like to mention the relation with the Weinberg no-go theorem [49]. In the paper, it was assumed that the system has translational invariance and  $GL(4)$  invariance. Then it has been shown that we need to fine-tune the parameters in order to obtain the vanishing or small cosmological constant. In the paper, the translational invariance was assumed even for the fields and therefore all the fields are constant. As is clear from Equation (24), some of the scalar fields must depend on the time and not constants. Therefore, the Weinberg no-go theorem cannot apply to the model in this paper, although there might be a problem of fine-tuning for the initial conditions.

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