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Article

A Toy Cosmology Using a Hubble-Scale Casimir Effect

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Abstract: The visible mass of the observable universe agrees with that needed for a flat cosmos, and the reason for this is not known. It is shown that this can be explained by modelling the Hubble volume as a black hole that emits Hawking radiation inwards, disallowing wavelengths that do not fit exactly into the Hubble diameter, since partial waves would allow an inference of what lies outside the horizon. This model of "horizon wave censorship" is equivalent to a Hubble-scale Casimir effect. This incomplete toy model is presented to stimulate discussion. It predicts a minimum mass and acceleration for the observable universe which are in agreement with the observed mass and acceleration, and predicts that the observable universe gains mass as it expands and was hotter in the past. It also predicts a suppression of variation on the largest cosmic scales that agrees with the low-l cosmic microwave background anomaly seen by the Planck satellite.

Keywords: cosmology; Hawking radiation; Hubble-scale Casimir effect; mass of the observable universe; large-scale CMB anomaly

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1. Introduction

Using the Hubble space telescope it has been determined that there are about 9×10^{21} stars in the observable universe. Assuming an average stellar mass based on the Sun, of 2×10^{30} kg, the universe's *visible* mass can be calculated to be about $1.8 \times 10^{52\pm1}$ kg (note the error bars on the exponent). Another similar estimate obtained by [1] was 2.4×10^{52} kg. A recent study has tripled the number of estimated red dwarf stars in elliptical galaxies so this may be an underestimate [2].

Given its error bars this mass is indistinguishable from a critical value that determines whether the universe is gravitationally closed or open. This is the so-called flatness problem pointed out by [3] and the most popular explanation for it is the theory of inflation. Inflation was first proposed by [4,5] who showed that an early, small universe would be so curved that quantum effects would produce an effective cosmological constant. They proposed that the universe was hugely inflated early on so that now it is many times larger than our observable universe. Some early problems with inflation were solved by [6]. Inflation explains the flatness problem, since we can see only a small proportion of the cosmos. However, inflation is difficult to test directly.

Another model is suggested here using a Hubble-scale Casimir effect (HsCe) that has been applied to Unruh radiation to explain inertial mass, but we apply it here to Hawking radiation to model gravitational mass.

2. Inertia from a HsCe

Work by [7,8] showed that a body with a linear acceleration of a sees thermal radiation of temperature T where,

$$T = \frac{\hbar a}{2\pi c k} \tag{1}$$

where $\hbar$ is the reduced Planck's constant, c is the speed of light and k is Boltzmann's constant. The dominant wavelength of this radiation (λ) is given by Wien's displacement law:

$$\lambda = \frac{\beta h c}{kT} \tag{2}$$

where $\beta = 0.2$ (determined theoretically by Wien). Replacing T using Equation (1) gives:

$$\lambda = \frac{4\pi^2 \beta c^2}{a} \tag{3}$$

In galaxies, the orbital acceleration of stars decreases as the galactic radius (r) increases ($a = v^2/r$), so the Unruh wavelength seen by stars (λ) should lengthen. Milgrom [9,10] noted that at the radius where the galaxy rotation problem begins, λ becomes equivalent to the Hubble distance ($\Theta = 2.7 \times 10^{26}$ m $= 2c/H$) where H is the Hubble constant. Assuming that inertia is caused by a form of Unruh radiation [9,10] speculated that at this point there might be a "break in the response of the vacuum" and that inertial mass might collapse for low accelerations. This suggested a link with the modified inertia version of empirical Modified Newtonian Dynamics (MoND) [11] but the *abrupt* break in inertia implied by this model did not fit the observed behaviour of galaxies.

McCulloch [12] proposed that inertia was due to Unruh radiation and that, instead of an abrupt break, that a Hubble-scale Casimir effect (HsCe) was acting on the Unruh radiation. This model assumes that only Unruh wavelengths that fit exactly into twice the Hubble diameter are allowed, and this predicts a far more gradual reduction in the inertial mass m_i below the gravitational mass m_g as accelerations become tiny. This model is analogous to a seiche in physical oceanography, in which only certain-sized waves are allowed within a bounded body of water such as a lake or harbour. This model can also be interpreted as "horizon wave censorship" in that any patterns or waves that do not fit exactly within the

Hubble scale immediately imply that the pattern extends outside it and that would give an inside observer the ability to determine information from outside the Hubble horizon and this cannot be allowed. In this model (for which either the HsCe or horizon censorship interpretation can be used) the modified inertial mass (m_i) is:

$$m_i = m_g \left(1 - \frac{\lambda}{4\Theta}\right) \tag{4}$$

Replacing the Unruh wavelength λ using Equation (3):

$$m_i = m_g \left(1 - \frac{\beta\pi^2 c^2}{|a|\,\Theta}\right) \sim m_g \left(1 - \frac{2c^2}{|a|\,\Theta}\right) \tag{5}$$

The derivation of Equation (5) can be seen in [12]. It should be pointed out that although Equation (5) suggests that a tiny acceleration would produce a negative inertial mass, this would never occur, since if the acceleration becomes tiny and the inertial mass approaches zero, then the acceleration due to external forces would increase again. This model then predicts a minimum acceleration. This model could be called Modified inertia by a Hubble-scale Casimir effect (MiHsC) or quantised inertia. For terrestrial accelerations ($a = 9.8$ m/s^2) the modification of inertia is negligible, but for the tiny accelerations seen in deep space the second term in Equation (5) can become important. Although MiHsC makes some bold assumptions (e.g., that Wien's law holds at these huge scales) these are somewhat justified by the fact that the minimum acceleration predicted by MiHsC agrees well with the cosmic acceleration attributed to dark energy [12,13], and MiHsC also predicts the anomalous Tajmar effect seen for supercooled spinning rings [14] and galaxy and galaxy cluster rotation without the need for dark matter [15]. MiHsC violates the equivalence principle, but not in a way that could have been detected in the usual torsion balance experiments [14]. Further, standard inertia has been shown to be explained to within 26% by this model [16,17].

3. Gravity from the HsCe

For an observer in an expanding universe there is a maximum volume that can be observed, since beyond the Hubble distance the velocity of recession is greater than the speed of light and the redshift is infinite: this is the Hubble volume. Its boundary is similar to the event horizon of a black hole [18] because it marks a boundary to what can be observed. This means that it is reasonable to assume that Hawking radiation is emitted at this boundary both outwards and inwards to conserve energy, and any wavelength that does not fit exactly within this size cannot be allowed for the inwards radiation, and therefore also for the outwards radiation. The same principle was applied to Unruh radiation above. Hawking's result for the temperature of a black hole was:

$$T \sim \frac{\hbar c^3}{8\pi G M k} \tag{6}$$

where M is the gravitational mass and G is the gravitational constant. McCulloch [12] used Wien's law and Equation (6) to give an expression for the wavelength of emission of a black hole:

$$\lambda \sim \frac{16\pi^2 GM\beta}{c^2} \tag{7}$$

and then calculated the black hole mass implied by the maximum wavelength allowed with the Hubble-scale Casimir effect (setting $\lambda = \Theta$, in fact it should have been $\lambda = 2\Theta$). The model assumed that the wavelength of the Hawking radiation emitted must fit exactly within the Hubble diameter, a Hubble-scale Casimir effect (HsCe). The model predicted a maximum black hole mass of $M = 1.165 \times 10^{52}$ kg and this was intriguingly close to the observed baryonic mass of the observable universe: $M = 1.8 \times 10^{52\pm1}$ kg. It is this agreement that this paper is designed to explore.

Here, the black hole model used above is turned inside out. The observable universe is now taken to be a black hole, and it is assumed that the wavelengths of Hawking radiation it emits inward from its boundary must fit exactly into twice its own diameter. If they do not, they are disallowed for the same reasons mentioned above.

Equation (7) predicts that if the mass of the universe (M) increases, so does the Hawking wavelength λ emitted by its boundary, but because of the HsCe there is a limitation on λ, so the observable universe's mass and temperature are determined by its diameter.

In the calculation above, this mass was derived crudely using an abrupt radiation cutoff at the Hubble scale. A similar result is derived here using a more complete model of the Hubble-scale Casimir effect (though still a parameterisation of it) by applying the Stefan-Boltzmann law to the inwards Hawking radiation:

$$E = \sigma T^4 \tag{8}$$

where E is the radiated energy, σ is the Stefan-Boltzmann constant and T is the Hawking temperature of the event horizon. A positive value of E implies that the edge of the Hubble volume will radiate energy E inwards and the Hubble volume will gain mass, just as black holes are purported to lose mass by evaporating Hawking radiation outwards (this analogy was used by [18]). The energy at longer wavelengths is now increasingly disallowed because of the HsCe as in [12], or the equivalent model of horizon censorship, to give a modified emitted energy E':

$$E' = \sigma T^4 \left(1 - \frac{\lambda}{4\Theta}\right) \tag{9}$$

replacing the wavelength (λ) using Equation (7) we get:

$$E' = \sigma T^4 \left(1 - \frac{4\pi^2 GM\beta}{\Theta c^2}\right) \tag{10}$$

If M is below a certain value then $E' > 0$, the edge of the Hubble volume radiates inwards and the observable universe (OU) gains mass. If M is less than this value then $E' < 0$, the particle horizon absorbs radiation and the OU looses mass. The equilibrium value of M, called M_{eq}, can be found by equating the two terms in brackets and rearranging:

$$M_{eq} = \frac{\Theta c^2}{4\pi^2 G\beta} \sim \frac{\Theta c^2}{8G} \sim \frac{c^3}{4GH} \sim 4.6 \pm 0.4 \times 10^{52}\ kg \tag{11}$$

where the error bar arises from an assumed 9% uncertainty in the Hubble constant H [19]. This predicted mass is within the error bars of the observed baryonic mass of the Hubble volume: $1.8 \times 10^{52\pm1}$ kg. Also, rearranging Equation (11) we find that:

$$c = \sqrt{\frac{8GM_{eq}}{\Theta}} = 2 \times \sqrt{\frac{2GM_{eq}}{\Theta}} \tag{12}$$

This formula for the speed of light c is twice the escape velocity for a mass M_{eq}. Therefore, for this cosmology the speed of light behaves rather like a cosmic escape velocity.

4. Discussion

This model can be explained more intuitively as follows. The edge of the observable universe is an event horizon, so, radiation, including Hawking radiation, with a wavelength bigger than this cannot exist since it cannot, even in principle, be seen (following Ernst Mach). Also, wavelengths that do not fit exactly into this scale cannot exist either because of a Hubble-scale Casimir effect or because they would give us information from beyond the horizon (horizon wave censorship).

According to Hawking, the mass of a black hole is linearly related to its temperature or inversely-linearly related to the wavelength of the Hawking radiation it emits. Therefore, for a given size of the universe there is a maximum Hawking wavelength it can have and a minimum allowed gravitational mass it can have. If its mass was less than this then the Hawking radiation would have a wavelength that is bigger than the size of the observed universe and would be disallowed. The minimum mass it predicts is encouragingly close to the observed mass of the Hubble volume.

Equation (11) implies that the baryonic mass of the observable universe is linearly related to its diameter, so it increases with time. This is similar to the behaviour of the Steady State Theory [20] and also the mass predicted here is half of the mass of the observable universe derived from that theory:

$$M_{SST} = \frac{c^3}{2GH} \sim \frac{\Theta c^2}{4G} \tag{13}$$

The Steady State Theory (SST) assumed that, as the universe expands, matter is created to maintain a constant density, but it was discredited because it was unable to explain the past hot universe suggested by the observed Cosmic Microwave Background (CMB).

In the HsCe model the Hubble-mass also increases in time as the universe expands (Equation (11)), but the HsCe also predicts a hotter early universe, since by combining Equation (2) (Wien's law) and taking the maximum wavelength allowed by the HsCe, $\lambda = 2\Theta$:

$$T \geqq \frac{hc\beta}{2k\Theta} \tag{14}$$

Therefore, when the universe was younger, and Θ was smaller, the Hawking temperature emitted from the universe's edge was higher.

Evidence for the HsCe model may already have been seen. Data sets from the Cosmic Background Explorer (COBE), Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck satellite [21] have shown that the angular two-point correlation of the Cosmic Microwave Background (CMB) on the largest

angular scales (multipole $l < 40$) is 5%–10% lower than would be expected from the Λ-CDM model. In the Planck data this anomaly is significant to 2.5–3 sigma.

Using the Hubble-scale Casimir effect (or horizon wave censorship) to suppress patterns of variation, instead of Unruh waves, the energy of the CMB blackbody radiation spectrum E would be modified to E' in the same way as for Equations (4) and (9) above, as follows:

$$E' = E\left(1 - \frac{\lambda_m}{4\Theta}\right) \tag{15}$$

where λ_m is the peak wavelength of the variation and Θ is a Hubble diameter. The CMB data is presented with respect to the monopole moment, and the first monopole moment ($l = 1$, or here: $L = 1$) indicates the longest wave observable in the sky, which has a wavelength equal to the width of the sky: 2Θ. Therefore, the monopole moment can be written in terms of the Hubble diameter, and assuming a flat space, as $L = 2\Theta/\lambda_m$ so that:

$$E' = E\left(1 - \frac{1}{2L}\right) \tag{16}$$

Equation (16) predicts that the Hubble-scale Casimir effect or horizon censorship model, when applied to patterns of variation, predicts a decrease in variation for $l < 40$ of 5.5%, which agrees with the decrease observed by the Planck satellite (which was 5%–10%).

5. Conclusions

The Hubble volume is modelled here by assuming it behaves like a black hole and emits Hawking radiation inwards from its edge whose wavelengths are subject to a Hubble-scale Casimir effect (HsCe) or an equivalent horizon wave censorship model. This model predicts a Hubble-mass of $4.6 \pm 0.4 \times 10^{52}$ kg in agreement with the observed mass of $1.8 \times 10^{52\pm1}$ kg and therefore provides an alternative explanation for the flatness problem.

The HsCe model predicts an increase in mass as the universe expands similar to the behaviour of the steady state theory. Unlike that theory, the HsCe predicts that the universe could have been hotter in the past.

The HsCe model is presented here as a toy model to stimulate discussion. However, it is supported to some extent by the recent anomalous results of the Planck satellite which show a suppression of variation at the largest cosmic scales that agree with those proposed here for Unruh-Hawking radiation.

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Conflicts of Interest

The author declares no conflict of interest.

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