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On the Maximum Energy Release from Formation of Static Compact Objects

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Abstract: Type II Supernova 1987A (SN 1987A), observed in 1987, released an energy of $Q \approx 3 \times 10^{53}$ erg. This huge energy is essentially the magnitude of gravitational potential or self-gravitational energy (PE) of a new born cold neutron star having a gravitational compactness or redshift $z_b \approx 0.15$. One may wonder what could be the upper limit on the amount of energy that might be released with the formation of a cold Ultra Compact Object (UCO) with an arbitrary high z_b. Accordingly, here, for the first time, we obtain an analytical expression for the PE of a homogeneous general relativistic UCO assuming it to be cold and static. It is found that the PE of a homogeneous UCO of mass M may exceed Mc^2 and be as large as 1.34 Mc^2. This result, though surprising, follows from an *exact and correct* analytical calculation based on the standard General Theory of Relativity (GTR). Further, UCOs supported by tangential stresses may be inhomogeneous and much more massive than neutron stars with PE $\sim$ 2.1 Mc^2 Thus, in principle, formation of an UCO of a few solar masses ($M_\odot$) might release an energy $Q \sim 10^{55}$ erg.

Keywords: Supernova SN 1987A; general relativity; compact objects; anisotropic stars

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1. Introduction

Compact objects are mostly formed through the gravitational collapse of normal stars at the end of their lives. White Dwarfs, Neutron Stars, and Black Holes (BHs) belong to the family of relativistic compact objects in astrophysics. Static or non-rotating white dwarfs, with a radius comparable to that of Earth, have masses lower than the well-known Chandrashekhar mass-limit of 1.4 $M_\odot$ [1]. Neutron stars, with a typical radius of 10 km, have masses up to the Oppenheimer-Volkoff limit of 2–3 $M_\odot$ [2]. Above this limit, stars may collapse into black holes, whose radius is given by the radius of event horizon $R_g = 2M$, in natural units with $G = c = 1$. Studies on radiative spherical general relativistic collapse suggest that the fluid becomes hotter during the collapse [3]. Two equations of states of the perfect fluid, based on the formulations using baryonic number density and mass density, are used in the literature to study the binding energy of static fluids [4]. The latter approach is preferred in the problems of General Relativity (GR). Polytropic equations of state are also used to calculate the potential energy, (Ω), of the fluids [4]. Emission of electromagnetic photons, neutrinos, and gravitational waves is believed to take place during the general relativistic collapse and the radiative energy increases steadily [3,5].

The enormous amounts of energy, observed in various astrophysical energetic phenomena are believed to be powered by the compact objects from the interactions with strong and large-scale magnetic fields on their surrounding magnetospheres. More than three decades ago, the titanic Supernova 1987A (SN 1987A) was discovered on 23 February, 1987 in the nearby Large Magellanic Cloud, which is a satellite galaxy of the Milky-Way at a distance of 160 kilo-light years. This was the first time a burst of cosmic neutrinos (ν), accompanying the stellar collapse, was detected on Earth from beyond our solar system [6]. Even after more than three decades, this remains the only case to support the idea that general relativistic gravitational collapses are accompanied by the emission of electromagnetic radiation and neutrinos. Multi-messenger observations including gravitational waves are expected to provide more insights into the modern supernova models [7]. Long duration Gamma Ray Bursts (GRBs) are the most energetic explosions in the Universe involving isotropic equivalent gamma ray energy release of 10^{51-55} erg over a duration of $T_{90} > 2s$. In general, more than 15 such GRBs have been observed so far [8]. It is widely believed that long duration GRBs are produced in the collapse of a massive star and this may lead to the black holes or neutron stars/magnetars having a physical surface. The gravitational or magnetic energy release in these cosmic explosions is speculated to power the bright γ-ray emission as well as the cosmic ray acceleration [9]. They are considered as a possible source of high energy neutrinos. Neutrino bursts could be the primary source of energy emission in other astrophysical scenarios as well [10,11].

SN 1987A also confirmed the prediction on the occurrence of neutrino-antineutrino ($\nu - \bar{\nu}$) bursts by the theories of supernova involving stellar core collapse [12–14]. The measured energy of the ν-burst, $(2.9 \pm 1.2) \times 10^{53}$ erg, can be understood as the *magnitude* of the potential energy ($Q = |\Omega|$), as the positive energy is liberated to compensate for the increase of the (negative) self-gravitational energy. By using the equation given below, such an equality ($Q = |\Omega|$) suggests that the gravitational mass of the new born neutron star to be (1.38 ± 0.43) $\mathrm{M}_\odot$ where $\mathrm{M}_\odot$ is the solar mass:

$$Q = |\Omega| \approx \frac{\mathrm{GM}^2}{R} = 3 \times 10^{53} \left(\frac{M}{1.4\, M_\odot} \right)^2 \left(\frac{10\ \mathrm{km}}{R} \right) \mathrm{erg} \tag{1}$$

For proper appreciation of the foregoing equation, we need to briefly revisit the idea of polytropes, self-gravitating fluid spheres. The equation of state connecting the (isotropic) pressure (P) and density (ρ) satisfies

$$P = K\rho^{\gamma}, \tag{2}$$

where K is a constant. If the fluid sphere is an inhomogeneous polytrope of index n, then $\gamma = (1 + 1/n)$ [15]. It turns out that the case of a homogeneous sphere corresponds to $n = 0,\ \gamma = \infty$. For a polytrope having index n, it is found that

$$|\Omega| = \frac{3}{5-n} \frac{GM^2}{R} \tag{3}$$

Clearly, the uniform density case of $|\Omega| = (3/5)GM^2/R$ corresponds to $n = 0$ case. The internal energy density (u) of the polytropic fluid is [15]

$$u = \frac{P}{\gamma - 1} \tag{4}$$

Then it is seen that, for uniform density case with $\gamma = \infty$, $u = 0$ and so is the thermal internal energy $U = 0$ despite arbitrary high pressure. A related concept here is the Newtonian total energy of the system (E^N) which is the sum of the negative self-gravitational energy (Ω) and positive internal/thermal energy (U). E^N is given as [16]

$$E^N = U + \Omega \tag{5}$$

For a Newtonian polytrope, one finds that [16]

$$E^N = \frac{n-3}{5-n} \frac{GM^2}{R} \tag{6}$$

When $u = U = 0$, one has $E^N = \Omega$, and from energy conservation, the energy liberated in the formation of the compact object

$$Q = |E^N| = |\Omega| = \frac{3}{5}\frac{GM^2}{R} \tag{7}$$

The key idea here is that for the sake of the principle of energy conservation, an amount of energy equal to the negative Ω must be radiated away in some form or other. This point will become clearer when we will move to a general relativistic viewpoint of this issue. However, here we run into a small problem because the neutron star mass in SN 1987A was determined from Equation (1) : $\Omega = GM^2/R$. The only way to resolve this contradiction is that the cold compact object is not homogeneous, but has a structure similar to a $n = 3/2$ polytrope.

$$|\Omega| = \frac{3}{3.5}\frac{GM^2}{R} \approx \frac{GM^2}{R} \tag{8}$$

And we shall make this revision for the case of General Relativity too. Now let us dig deeper into this idea.

By comparing Equations (7) and (8), one can find that

$$|\Omega|^N(Observation) = 1.6|\Omega|^N(Hom) \tag{9}$$

Essentially, $|\Omega|$ increases significantly with extreme inhomogeneity because more and more mass lie in the deeper potential well as compared to the homogeneous case. And this effect of enhancement in PE with inhomogeneity is expected to be more pronounced for general relativistic case for the extreme non-linearity of General Relativity.

Having made this discussion in the framework of simple Newtonian gravitation, we shall now explore the question of highest possible value of $|\Omega|$ by using the GTR. Accordingly, for the first time, we shall work out an exact expression for general relativistic $|\Omega|$ for a homogeneous fluid sphere in Section 2.

2. Potential Energy in General Relativity

While in Newtonian physics, ρ represents the baryonic mass alone, in GTR, ρ includes contribution from internal energy as well. It is well known that, the gravitational or ADM (Arnowitt, Deser and Misner) mass of the fluid sphere is given as [16–19]

$$M = \int_0^R \rho\, 4\pi r^2\, dr \tag{10}$$

where r is the areal radius or Schwarzschild radial coordinate for a static case. Unlike the Newtonian case, in GTR case, ρ includes all internal and radiation energies along with the baryonic energy density. Also, the total mass energy is $E = Mc^2$ in contrast to the Newtonian total energy E^N. However M is not the sum of the locally added mass-energies because the proper radial distance $dl \neq dr$:

$$dl = \sqrt{-g_{rr}}\, dr = \frac{dr}{\sqrt{1 - 2m/r}}, \tag{11}$$

where

$$m(r) = \int_0^r \rho\, 4\pi r^2 dr, \tag{12}$$

is the gravitational mass of an interior spherical section. Accordingly, the proper mass or the sum of locally added mass-energies in the curved space-time is [16–19]

$$M_p = \int_0^R \rho\, 4\pi r^2 \sqrt{-g_{rr}}\, dr = \int_0^R \frac{4\pi\rho r^2}{\sqrt{1 - 2m/r}}\, dr \tag{13}$$

Since for the asymptotically flat space-time, $\sqrt{-g_{rr}} \to 1$ as $r \to \infty$, proper mass is the initial or original gravitational mass at $r \to \infty$. In GTR, the fundamental definition of PE is given by (see Equation 3.31 in [17]):

$$\Omega = M \; - \; M_p \tag{14}$$

Clearly, $M < M_p$ because self-gravitational energy is negative. And this shows the energy released in the process of gravitational collapse. We shall assume here that the compact object is born in a catastrophic collapse and the new born object is static and cold, i.e., its internal energy is negligible compared to its proper mass energy $U \ll M_p$ (this is the case for white dwarfs and neutron stars). In such a case the energy released in the collapse process will be:

$$Q \; = \; M_p - M = |\Omega| = \int_0^R 4\pi\rho r^2 \frac{dr}{\sqrt{1 - 2m/r}} - M \tag{15}$$

In the Newtonian limit of $2m/r \ll 1$, one finds $\sqrt{-g_{rr}} \approx (1 + m/r)$, so that

$$Q = \; |\Omega| \; = \int_0^M \frac{mdm}{r} \tag{16}$$

which for $\rho = \textit{constant}$ case reduces to $|\Omega| = (3/5)M^2/R$.

Equation (15) cannot be integrated for an inhomogeneous sphere having arbitrary form of $\rho(r)$. However, for the first time, we point out that, even for a general relativistic case, it is possible to obtain an exact analytical expression for M_p and hence $|\Omega|$ for an uniform density case. In such a case, we have

$$M_p = \int_0^R \frac{4\pi\rho r^2}{\sqrt{1 - (8\pi/3)\rho r^2}} \, dr \tag{17}$$

Writing $x = 2m/r = (8\pi/3)\rho r^2$ and $X = 2M/R = (8\pi/3)\rho R^2$, one can integrate the foregoing equation:

$$M_p = \frac{3}{4}\sqrt{\frac{3}{8\pi\rho}}\left[\sin^{-1}\sqrt{X} - \sqrt{X}\sqrt{1 - X}\right], \tag{18}$$

which may be rewritten as

$$M_p = \frac{3M}{2X^{3/2}}\left[\sin^{-1}\sqrt{X} - \sqrt{X}\sqrt{1 - X}\right] \tag{19}$$

Thus we obtain the *maiden analytical expression* for the PE of a homogeneous fluid sphere in general relativistic case as:

$$|\Omega| = \frac{3M}{2X^{3/2}}\left[\sin^{-1}\sqrt{X} - \sqrt{X}\sqrt{1 - X}\right] - M = yM \tag{20}$$

where

$$y = \frac{|\Omega|}{M} = \frac{3}{2X^{3/2}}\left[\sin^{-1}\sqrt{X} - \sqrt{X}\sqrt{1 - X}\right] - 1 \tag{21}$$

For sufficiently small values of $X = 2M/R$ one can expand

$$\sin^{-1}\sqrt{X} = \sqrt{X} + \frac{X^{3/2}}{6} + \frac{3X^{5/2}}{40} \tag{22}$$

and

$$\sqrt{X}\sqrt{1 - X} = \sqrt{X} - \frac{X^{3/2}}{2} - \frac{X^{5/2}}{8} \tag{23}$$

Then one recovers

$$y = \frac{3}{10}X = \frac{3}{5}\frac{M}{R} \tag{24}$$

and

$$|\Omega| = yM = \frac{3}{5}\frac{M^2}{R} \tag{25}$$

This shows the correctness of Equation (20) derived above.

3. Binding Energy of Ultra-Compact Objects

The real measure of the gravitational potential well of ultra-compact objects may be gauged from their surface gravitational redshifts

$$z_b = (1 - 2M/R)^{-1/2} - 1 = (1 - X)^{-1/2} - 1 \tag{26}$$

Note, for a typical neutron star, $z_b \approx 0.15$.

For a compact object supported by isotropic pressure alone, the Buchdahl upper limit of $X = 2M/R = 8/9$ or $z_b = 2.0$ [20]. But for an anisotropic compact object supported partially or fully by tangential pressure too, this upper limit on X increases, and for the case of extreme pressure anisotropy, this *upper limit* is $X \to 1.0$, i.e., $z_b \to \infty$ [21–23].

It may be borne in mind that an exotic compact object (ECO) having $X \to 1$ or $R \to 2M$ is still a *non-singular* object filled with matter and having a physical boundary. This is in contrast to the limiting case of $X = 1$ or $R = 2M$, when one would obtain a singular, vacuum BH except for the central singularity.

In fact, it has been claimed that such ECO could be as massive as BHs [23]. For a homogeneous ECO, for the extreme case of $X = 2M/R \to 1$, one finds that

$$|\Omega|(Hom) = \left(\frac{3\pi}{4} - 1\right)M \approx 1.34M \tag{27}$$

In Figure 1, we plot the values of $|\Omega|/M$ against $X = 2M/R$ from Equation (20).

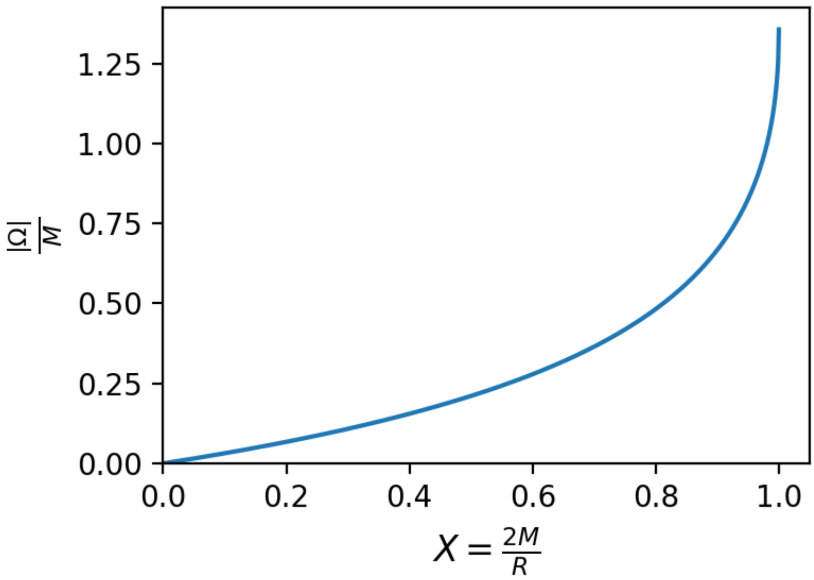

Figure 1. Gravitational Potential Energy Per Unit Mass ($|\Omega|/M$) as a function of Compactness ($X = 2M/R$) from Equation (20).

If we take the new born neutron star in SN1987A to be of uniform density, then we would obtain a value of $\Omega = 0.6M$. For $M = 1.4M$, we would get a value of $Q \approx 1.6 \times 10^{53}$ erg. Thus the assumption of uniform density contradicts with the observation that $Q(SN1987A) \approx 3 \times 10^{53}$ erg. For the GR case too we expect a similar, if not higher, value of $|\Omega|$ compared to the homogeneous case. This is so because relativistic polytropes have stronger mass concentration towards their centers compared to the corresponding Newtonian cases because general relativistic case is much more non-linear than Newtonian gravity. Then for the GR UCO too, we may tentatively write

$$|\Omega|(Observation) = 1.6\,|\Omega|(Hom) \approx 2.1M \tag{28}$$

This implies that, in the extreme case, a cold ECO of mass $1M_\odot$ and extreme pressure anisotropy ($X \to 1$) might be born from an original core of mass $M_p^i \approx 3.1\, M_\odot$ with an attendant release of Q = 2.1 $M_\odot c^2$ of energy.

While for an extreme pressure anisotropy, it is possible to have $z_b \to \infty$ [21–23], a realistic upper limit could be $z_b \approx 5.0$ [24–26]. Table 1 gives the values of $|\Omega|$ for (i) $z_b = 0.15$, the neutron star case, (ii) $z_b = 2.0$, the Buchdahl upper limit for isotropic pressure case, (iii) $z_b = 5.0$, the realistic upper limit for anisotropic pressure case [24–26], and (iv) $z_b = \infty$, the theoretical upper limit of compactness for extremely anisotropic pressure case [21–23].

Table 1. Magnitude of Gravitational Potential Energy of Highly Inhomogeneous Ultra-Compact Objects.

z_b	X = 2M/R	$\|\Omega\|\ (Inhom)$
0.15	≈0.15	0.15 M
2.0	≈0.89	1.25 M
5.0	≈0.97	2.0 M
∞	1.0	2.1 M

4. Discussion and Outlook

General Relativistic astrophysics is almost hundred years old field of research. Even though numerical studies of polytropes in General Relativity were first carried out by Tooper long ago [17], here for the first time, we have obtained an exact expression for the gravitational potential energy (Ω) of a homogeneous ultra-compact object. Our exact result shows that a homogeneous UCO has an *upper limit* of $|\Omega|(Hom) \approx 1.34$ Mc^2. Here it must be borne in mind that though it is possible that $|\Omega| > M$, *we always have* $|\Omega| < M_p$, *the proper mass-energy of the fluid sphere*. For highly inhomogeneous case, the value of $|\Omega|$ increases as more and more mass resides in deep gravitational potential well [17]. Observation of SN 1987A suggests that in Newtonian case, $|\Omega^N|(Observation) = 1.6|\Omega^N|(Hom)$. For the general relativistic case, this enhancement factor is expected to be larger. Yet if we adopt the enhancement factor of 1.6, the highest value of $|\Omega|(Observation) \approx 2.1Mc^2$. Indeed an in-depth study of GR gravitational collapse shows that the energy emission in the GR case should be higher than the corresponding Newtonian case [5].

It is important here to mention that neutron stars may be better represented by a polytrope of $n \approx 1.0$ instead of $n = 1.5$, appropriate for mildly reletivistic degenerate fluid. In this case, the enhancement factor will become $1.6 \times (3.5/4) \approx 1.3$. However, as before, because of extreme nonlinearity of GR, this enhancement factor is expected to be larger than 1.3. Yet, if we adopt the lower value of 1.3, we will have

$$|\Omega|(n = 1.0) = 1.3\,|\Omega|(Hom) \approx 1.8M \tag{29}$$

and corresponding $Q = 1.8Mc^2$.

Here we may offer yet another clarification. The exact upper limit $X = 1;\ R = 2M$ corresponds to the Schwarzschild black holes which are vacuum except for their central singularity. However, the limit $X \to 1$ and yet $X < 1.0$, may correspond to anisotropic pressure supported *non-singular* compact objects [21–23] filled with matter and possessing physical boundaries. Technically, a singular black hole (point singularity) is homogeneous. But this does not imply that anisotropic non-singular ECOs are necessarily homogeneous. On the other hand, such configurations could to be highly inhomogeneous [21–23], and hence the inhomogeneity related modest boosting factors of 1.6 or 1.3 adopted here is justified.

Further, such anisotropic pressure supported compact objects have mass upper limits higher than that of the neutron stars [21,22], and it has even been claimed that they could be as massive as black holes [23]. Even if we consider the Buchdahl upper limit of compactness for compact objects supported by isotropic pressure alone ($z_b = 2.0$), it is possible that $|\Omega|(Inhom) \approx 1.25$ Mc^2. While for an extreme pressure anisotropy, it is possible to have $z_b \to \infty$ [21–23], the realistic upper limit could be $z_b \approx 5.0$ [24–26]. After accepting such a realistic upper limit on compactness, it is seen that the birth of a $M = 2.5 - 3.0 M_\odot$ ultra-compact object might be accompanied by the emission of an energy $Q \sim 10^{55}$ erg. This novel result has relevance for most powerful cosmic explosions. In fact the isotropic energy release in the brightest Gamma Ray Burst GRB 2210009A is $Q \sim 1.5 \times 10^{55}$ erg [8]. This enormous isotropic energy and close proximity (redshift ~ 0.1505) of the source push the limits of modern theories of GRBs and event rate. Multi-wavelength observations suggest that structured jets launched by a common central engine may power the most extreme explosions like GRB 2210009A [27]. One of the important highlights of GRB 221009A is the first detection of very high energy γ-rays above an energy of 10 TeV from a GRB [28]. This provides a unique opportunity to explore for possible detection of high-energy neutrinos from GRBs. However, no statistically significant neutrino emission in the energy range 10^6 eV to 10^{15} eV has been reported so far from GRB 221009A during or after the high energy γ-ray emission [29]. Only upper limits on the time-averaged integral flux of neutrinos are estimated for GRB 221009A. Taking into account the contribution of TeV emission as well as the blast wave kinetic energy involved in the afterglow emission, the isotropic equivalent energy budget of GRB 2210009A corresponds to an energy release of more than 10^{55} erg [27] which is 30 times higher than the energy released in SN1987A. We remind the reader that we only explored theoretical *upper limits* of Q under most favourable cases, and actual value of Q will be lower in real life (unless M will be higher).

Finally, whether $Q = 1.34M$ or $Q = 2.1M$ or $Q = 1.8M$, it is always possible that $Q \approx 10^{55}$ erg, because ECOs supported by extreme anisotropic pressure might be arbitrarily massive unlike neutron stars [23]. Thus, in principle, it is possible to understand the energy budget of GRB 2210009A, $Q \sim 1.5 \times 10^{55}$ erg [8]. Accordingly, it seems that, in future, we might hopefully detect cosmic $\nu - \bar{\nu}$ burst having luminosity much higher than that of SN 1987A.

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