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Weyl Conformal Symmetry Model of the Dark Galactic Halo

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Abstract: The postulate of universal conformal (local Weyl scaling) symmetry modifies both general relativity and the Higgs scalar field model. The conformal Higgs model (CHM) generates an effective cosmological constant that fits the observed accelerating Hubble expansion for redshifts $z \leq 1$ (7.33 Gyr) accurately with only one free parameter. Growth of a galaxy is modeled by the central accumulation of matter from an enclosing empty spherical halo whose radius expands with depletion. Details of this process account for the nonclassical, radial centripetal acceleration observed at excessive orbital velocities in galactic haloes. There is no need for dark matter.

Keywords: galactic dark halo; conformal Higgs model; conformal gravity

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1. Introduction

Universal conformal symmetry, requiring local Weyl scaling covariance [1–4] of all elementary physical fields [5], offers a paradigm alternative to consensus ΛCDM for cosmology, motivated by the absence of experimental confirmation of conjectured dark matter and need for an explanation of currently accelerating Hubble expansion. The conformal Higgs model (CHM) [6–9] retains the Higgs mechanism for gauge boson mass, but acquires a gravitational term in the scalar field Lagrangian density. The CHM determines centrifugal cosmic acceleration accurately for redshifts $z \leq 1$ (7.33 Gyr) [6,7]. Conformal gravity (CG) replaces the Einstein–Lagrangian density by a quadratic contraction of the conformal Weyl tensor [3,10–14]. Substantial empirical support for this proposed break with convention is provided by applications of CG to anomalous galactic rotation velocities. CG has recently been fitted to rotation data for 138 galaxies [15–19]. The CHM precludes the existence of a massive Higgs particle, but conformal theory is found to be compatible with a compound gauge diboson, W_2, of mass 125 GeV [20], consistent with the observed LHC resonance [21,22]. Fits of CG and the CHM to observed galactic and cosmological data do not require dark matter [9].

2. Dark Matter

When it became possible to measure orbital velocities in the outer reaches of galaxies, they were found to systematically exceed the uniformly decreasing value implied by standard Einstein/Newton gravity. The general functional form of $v(r)$ was observed to level off at a characteristic radial acceleration of $a_0 \simeq 10^{-10}$ m/s^2. This led to the conjecture that standard gravity, due to observed galactic mass, was augmented by some additional gravitational source. Since this source was not directly observed, it was called dark matter.

Alternatively, general relativity might be modified to account for this excess centripetal acceleration. The most successful model assumes modified Newtonian dynamics (MOND) [23,24]. The basic postulate for radial acceleration a, given Newtonian a_N, is that $a^2 \to a_N a_0$ for $a_N \ll a_0$.

When conformal gravity (CG) is applied to a Schwarzschild model (a central gravitational source with spherical symmetry), it has an exact solution in the form of the

Schwarzschild radial potential, $B(r)$ [10,13], for constants related by $\alpha^2 = 1 - 6\beta\gamma$ [13]. Outside a source of a finite radius [10],

$$B(r) = -2\beta/r + \alpha + \gamma r - \kappa r^2. \tag{1}$$

$B(r)$ determines circular geodesics with orbital velocity, v, such that $v^2/c^2 = ra/c^2 = \frac{1}{2}rB'(r) = \beta/r + \frac{1}{2}\gamma r - \kappa r^2$. The Kepler formula is $ra_N/c^2 = \beta/r$, from a 2nd-order equation. The 4th-order conformal equation introduces two additional constants of motion, radial acceleration, γ, and a cutoff parameter, κ, which determines the radius of a galactic halo [8]. Classical gravitation is retained at subgalactic distances by setting $\beta = Gm/c^2$ for a spherical source of mass m [3].

The depleted halo model [8], described below, treats all matter outside a defined galactic radius as uniform and isotropic. Only spherical symmetry is considered. Following Mannheim and Kazanas [11], galactic mass within this radius is essentially treated by classical gravitation, describing a detailed, nonspherical geometric structure. Dark matter is replaced by the anomalous acceleration parameter γ [10].

An alternative to the multiplicative postulate of MOND is provided by the additive acceleration parameter γ, which has the conceptual advantage of arising from a well-defined variational field theory. Fits to anomalous galactic rotational velocities by CG and MOND are of comparable accuracy in the flat velocity range. However, the halo cutoff parameter κ, unique to CG, is found to be relevant at very large galactic radii [16,17,25].

3. Dark Energy and Hubble Expansion

The Higgs scalar field [26,27] is an essential element of electroweak physics. It has a spontaneously generated finite amplitude, constant in spacetime, responsible for the finite mass of gauge bosons and fermions. Retaining Higgs $V(\Phi^\dagger\Phi) = -(w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$, which depends on the two assumed constants w^2 and λ [26,27], the postulate of universal conformal symmetry [5] requires the CHM Higgs–Lagrangian density to acquire a gravitational term, $-\frac{1}{6}R\Phi^\dagger\Phi$ [3], where $R = g_{\mu\nu}R^{\mu\nu}$, a trace of the Ricci tensor. The variation of Ricci R on a cosmic time scale implies a very small, but universal, source density for the Z_μ neutral gauge field. Dressing of the scalar field by Z_μ determines the Higgs parameter w^2, and dressing by diboson W_2 determines λ [20]. These two parameters and Ricci scalar R imply finite Φ amplitude and broken gauge and conformal symmetry.

In the uniform, isotropic cosmic geometry assumed for cosmology, the CHM implies a Friedmann cosmic evolution equation [5,7] with parameters determined by the scalar Higgs field. This modified Friedmann equation contains an effective cosmological constant, defining dark energy density. The integrated luminosity distance, computed as a function of redshift, fits observed data back to the CMB (cosmic microwave background) [6]. Omitting cosmic mass and curvature, the fit to observed Hubble expansion data, with centrifugal acceleration, is accurate back to redshift $z = 1$(7.33Gyr) [7].

4. Depleted Halo Model

CG and the CHM are consistent, but interdependent [9], in the context of a depleted dark halo model [8] for an isolated galaxy. A galaxy of mass M is modeled by a spherically averaged mass density, ρ_G/c^2, within an effective galactic radius r_G, formed by the condensation of primordial uniform, isotropic matter of uniform mass density ρ_m/c^2. A model valid for nonclassical gravitation can take advantage of spherical symmetry at large galactic radii, assuming classical gravitation within r_G. Nonspherical gravitation is neglected outside r_G. The dark halo inferred from gravitational lensing and centripetal acceleration is identified with the resulting depleted sphere of a large radius, r_H [8]. Given mean mass density, $\bar{\rho}_G/c^2$, within r_G, this implies an empty halo radius of $r_H = r_G(\bar{\rho}_G/\rho_m)^{\frac{1}{3}}$.

CG determines source-free Schwarzschild potential, $B(r)$, as Equation (1) outside a galactic radius r_G [3,10]. As shown in detail below, the physically relevant particular solution for $B(r)$ [9] incorporates nonclassical radial acceleration, γ, as a free parameter. Its value is determined by the halo model. Gravitational lensing by a spherical halo is observed as the centripetal deflection of a photon geodesic passing from the external intergalactic space with a postulated universal isotropic mass-energy density of ρ_m into the empty halo sphere. The conformal Friedmann cosmic evolution equation implies dimensionless cosmic acceleration parameters $\Omega_q(\rho)$ [8], which are locally constant but differ across the halo boundary r_H. Smooth evolution of the cosmos implies an observable centripetal particle acceleration γ within r_H proportional to $\Omega_q(in) - \Omega_q(out) = \Omega_q(0) - \Omega_q(\rho_m)$. Uniform cosmological ρ_m implies a constant γ for $r \leq r_H$, independent of galactic mass [9]. This surprising result is consistent with recent observations of galactic rotational velocities for galaxies with directly measured mass [25,28], implying that radial acceleration, a, observed as orbital velocity, is a function of Newtonian a_N, independent of orbital radius and galactic mass.

In the CHM, observed nonclassical gravitational acceleration, $\frac{1}{2}\gamma c^2$, in the halo is proportional to [8] $\Delta\Omega_q = \Omega_q(0) - \Omega_q(\rho_m) = \Omega_m(\rho_m)$, where, given ρ_m and H_0, $\Omega_m(\rho_m) = \frac{2}{3}\frac{\bar{\tau}c^2\rho_m}{H_0^2}$ [6], for the Hubble constant H_0 and $\bar{\tau} < 0$. Thus, the halo model determines constant γ from uniform universal cosmic baryonic mass density ρ_m/c^2, which includes radiation energy density here.

5. Consistency of CG and CHM

CG and CHM must be consistent for an isolated galaxy and its dark halo, observed by gravitational lensing. CG is valid for anomalous outer galactic rotation velocities in the static spherical Schwarzschild metric, solving a differential equation for Schwarzschild gravitational potential, $B(r)$ [3,10]. The CHM is valid for cosmic Hubble expansion in the uniform, isotropic FLRW metric, solving a differential equation for the Friedmann scale factor, $a(t)$ [6]. Concurrent validity is achieved by introducing a common hybrid metric:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -B(r)dt^2 + a^2(t)(\frac{dr^2}{B(r)} + r^2 d\omega^2). \quad (2)$$

Metric tensor $g_{\mu\nu}$ is determined by conformal field equations derived from $\mathcal{L}_g + \mathcal{L}_\Phi$ [8], driven by the energy–momentum tensor $\Theta_m^{\mu\nu}$, where subscript m refers to conventional matter and radiation. The gravitational field equation within the halo radius r_H is:

$$X_g^{\mu\nu} + X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}, \quad (3)$$

where $X^{\mu\nu}$ is a metric functional derivative [3,9]. The gravitational equations are decoupled by separating mass/energy source density, ρ, into uniform isotropic mean density, $\bar{\rho}$, and residual $\hat{\rho} = \rho - \bar{\rho}$, which extends only to the galactic radius r_G and integrates to zero over the defining volume. Defining mean density, $\bar{\rho}_G$, and residual density, $\hat{\rho}_G = \rho_G - \bar{\rho}_G$, and assuming $\Theta_m^{\mu\nu}(\rho) \simeq \Theta_m^{\mu\nu}(\bar{\rho}) + \Theta_m^{\mu\nu}(\hat{\rho})$, solutions for $r \leq r_G$ of the two equations

$$X_g^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}(\hat{\rho}_G), X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}(\bar{\rho}_G) \quad (4)$$

decouple, implying a solution of the full equation.

6. Computed Parameters of the Schwarzschild Potential, *B*(*r*)

Given the mass/energy source density, $f(r)$, enclosed within $\bar{r}$, the Schwarzschild field equation is [10,13]:

$$\partial_r{}^4(rB(r)) = rf(r), \quad (5)$$

for $f(r) \sim (\Theta_0^0 - \Theta_r^r)_m$, as determined by the source– energy–momentum tensor $\Theta_m^{\mu\nu}$ [3].

Derivative functions $y_i(r) = \partial_r^i(rB(r))$ for $0 \leq i \leq 3$ satisfy differential equations [3,9]

$$\partial_r y_i = y_{i+1}, 0 \leq i \leq 2,$$
$$\partial_r y_3 = rf(r). \tag{6}$$

The general solution, for independent constants $c_i = y_i(0)$, determines the coefficients β, α, γ, and κ such that at endpoint $\bar{r}$

$$y_0(\bar{r}) = -2\beta + \alpha\bar{r} + \gamma\bar{r}^2 - \kappa\bar{r}^3,$$
$$y_1(\bar{r}) = \alpha + 2\gamma\bar{r} - 3\kappa\bar{r}^2,$$
$$y_2(\bar{r}) = 2\gamma - 6\kappa\bar{r},$$
$$y_3(\bar{r}) = -6\kappa. \tag{7}$$

Gravitational potential $B(r)$ is required to be differentiable and free of singularities. $c_0 = 0$ prevents a singularity at the origin. Specific values of γ and κ, consistent with Hubble expansion and the observed galactic dark halo [6,8], can be fitted by adjusting c_1, c_2, and c_3, subject to $c_0 = 0, \alpha^2 = 1 - 6\beta\gamma$ [13].

A particular solution for $B(r)$ [10,13], assumed by previous authors, derives an integral for γ that vanishes for residual source density $\hat{\rho}$. This is replaced here by an alternative solution for which γ is a free parameter [9]. Since the Weyl tensor vanishes identically in uniform geometry, CG applies only to the residual density $\hat{\rho}$.

The proposed particular solution, given γ and κ is:

$$rB(r) = y_0(r) = -\frac{1}{6}\int_0^r q^4 f dq + \alpha r - \frac{1}{2}r\int_r^{\bar{r}} q^3 f dq$$
$$+\gamma r^2 + \frac{1}{2}r^2\int_r^{\bar{r}} q^2 f dq - \kappa r^3 - \frac{1}{6}r^3\int_r^{\bar{r}} q f dq. \tag{8}$$

The integrated parameters $c_i = y_i(0)$ are $c_1 = \alpha - \frac{1}{2}\int_0^{\bar{r}} q^3 f dq$, $c_2 = 2\gamma + \int_0^{\bar{r}} q^2 f dq$, $c_3 = -6\kappa - \int_0^{\bar{r}} q f dq$, and at $r = \bar{r}$, $2\beta = \frac{1}{6}\int_0^{\bar{r}} q^4 f dq$. Term $\gamma r^2 + \frac{1}{2}r^2\int_r^{\bar{r}} q^2 f dq$ in this solution differs from prior reference [10]. γ here is a free parameter that determines generally nonzero c_2.

For an isolated single spherical solar mass in a galactic halo, mean internal mass density, $\bar{\rho}_\odot$, within $r_\odot$ determines an exact solution of the conformal Higgs gravitational equation, giving the internal acceleration $\Omega_q(\bar{\rho}_\odot)$. Given γ outside $r_\odot$, continuous acceleration across boundary $r_\odot$,

$$\frac{1}{2}\gamma_{\odot,in}c^2 - cH_0\Omega_q(\bar{\rho}_\odot) = \frac{1}{2}\gamma c^2 - cH_0\Omega_q(0), \tag{9}$$

determines constant $\gamma_{\odot,in}$, which is valid inside $r_\odot$. $\gamma_{\odot,in}$ is determined by local mean source density $\bar{\rho}_\odot$. γ in the halo is not changed. Its value is a constant of integration that cannot vary in the source-free halo [8,9]. Hence, there is no way to determine a mass-dependent increment to γ. This replaces the usually assumed $\gamma = \gamma_0 + N^*\gamma^*$ by $\gamma = \gamma_H$, determined at the halo boundary r_H.

7. Implications for Cosmology

The common assumption for galactic growth is that a primordially accumulated dark matter halo subsequently attracts baryonic matter to form an observable galaxy. Conformal theory, as well as MOND, reverse this sequence, while eliminating the need for dark matter. The nonclassical CG gravitational acceleration is a byproduct of the gravitational accumulation of baryonic matter attracted to a growing galaxy. The CHM generates a uniform, constant, nonclassical centripetal acceleration within a halo of large expanding radius. The rate of galactic growth must depend on the net incoming flux of matter diffusing across the halo boundary, where the net gravitational radial acceleration vanishes.

Galactic collision is initiated by halo contact. Since halo volume is determined by galactic mass, it must remain constant, implying the distortion of colliding halo boundaries analogous to the collision of two spherical balloons. A dynamical model must consider the diffusion of matter across a changing halo boundary. A new, observable phenomenon affects neighboring galaxies in a galactic cluster. Once halos are in contact, the accessible source of primordial matter is restricted, thus reducing the rate of growth for both colliding galaxies. This should be observed as a cessation of growth from the primordial background in the extreme case of a galaxy completely surrounded by a cluster of contiguous halos.

The empirical correlation relation of McGaugh et al. [28] establishes total radial acceleration as a function of its baryonic Newtonian value. This implies CG γ independent of galactic mass [25], which places a strong constraint on galactic rotation curves. The selected particular solution, Equation (8) for $B(r)$, which differs from [10], depends on an independently determined γ. Resulting nonzero c_2 implies a singular Ricci scalar at the galactic center [9], relevant to the formation of a supermassive black hole.

Since the coefficient of the source term in the conformal Friedman equation is negative, primordial energy density must cause centrifugal acceleration. This may create a dynamical Big Bang in the CHM without requiring a separate field. The relevancy of CHM should be explored. A weak time-dependence of scalar field Φ is implied [6,29]. For large redshifts, the Friedmann equation for scale factor $a(t)$ and the CHM equation for $\Phi(t)$ must be integrated together. A changed Higgs amplitude ϕ_0 affects initial atomic abundances.

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