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Almost Ćirić Type Contractions and Their Applications in Complex Valued b-Metric Spaces

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Abstract: In this article, we present the use of a unique and common fixed point for a pair of mappings that satisfy certain rational-type inequalities in complex-valued b-metric spaces. We also provide applications related to authenticity concerns in integral equations. Our results combine well-known contractions, such as the Ćirić contraction and almost contractions.

Keywords: complex valued b-metric space; common fixed point; continuous mappings; Urysohn integral equations; integral contractions

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1. Introduction and Preliminaries

One of the most significant and vital tools utilized by authors in the fields of nonlinear analysis, quantum physics, hydrodynamics, number theory, and economics is the Banach contraction principle [1]. This contraction has been generalized by weakening the contraction principle and enhancing the working spaces in different structures and generalized metrics, such as quasi-metric, b-metric, cone-metric, etc. For examples, see [2–11].

One of the remarkable and interesting generalizations of contraction mappings is Ćirić-type contractions (see [12]). For analyzing fixed points of self-mappings in different metrics spaces, Ćirić-type contractions offer a broader framework. A variety of results, such as the existence and uniqueness of fixed points, their stability, and the convergence of iterative procedures are investigated in the study of Ćirić-type contractions.

Similarly, a weakened form of contraction mapping, the "almost contraction", was introduced in 2004 by Berinde [13]. This contraction comprises the class of many mappings, notably Banach [14], Chatterjea [15], and Kannan [16]. However, it must be noted that unlike traditional contractions, almost contractions do not guarantee a unique fixed point.

A new concept called b-metric space was introduced in 1989 by Bakhtin [17]. Several important studies have been conducted by researchers in the field of b-metric space, including refs. [18–21]. In 2011, the metric space in complex version was firstly presented by Azam et al. [22]. Similarly, b-metric space in complex plane has been introduced in 2013 by Rao et al. [23].

Let us now recall the mentioned notions.

Definition 1 ([24])**.** *For complex numbers set $\mathbb{C}$, relation of partial order $\preceq$ on $\mathbb{C}$ is defined by*

$$\wp_1 \precsim \wp_2 \text{ if and only if } \mathfrak{Real}(\wp_1) \leq \mathfrak{Real}(\wp_2) \text{ and } \mathfrak{Img}(\wp_1) \leq \mathfrak{Img}(\wp_2).$$

Therefore, we can say that $\wp_1 \precsim \wp_2$ if one of the below condition is fulfilled:
(I) $\mathfrak{Real}(\wp_1) = \mathfrak{Real}(\wp_2)$, $\mathfrak{Img}(\wp_1) < \mathfrak{Img}(\wp_2)$,

(II) $\mathfrak{Real}(\wp_1) < \mathfrak{Real}(\wp_2)$, $\mathfrak{Img}(\wp_1) = \mathfrak{Img}(\wp_2)$,
(III) $\mathfrak{Real}(\wp_1) < \mathfrak{Real}(\wp_2)$, $\mathfrak{Img}(\wp_1) < \mathfrak{Img}(\wp_2)$,
(IV) $\mathfrak{Real}(\wp_1) = \mathfrak{Real}(\wp_2)$, $\mathfrak{Img}(\wp_1) = \mathfrak{Img}(\wp_2)$.
We can say that $\wp_1 \precnsim \wp_2$ *if* $\wp_1 \neq \wp_2$ *and one of the mentioned necessities is fulfilled and we can say that* $\wp_1 \prec \wp_2$ *only if condition (III) is satisfied.*

Definition 2 ([25])**.** *Let* $\wp_1, \wp_2 \in \mathbb{C}$. *The max function for the partial order* $\preceq$, *defined on* $\mathbb{C}$ *as:*
(a) $\max\{\wp_1, \wp_2\} \preceq \wp_2 \Leftrightarrow \wp_1 \preceq \wp_2$;
(b) $\wp_1 \preceq \max\{\wp_2, \wp_3\} \Rightarrow \wp_1 \preceq \wp_2$ *or* $\wp_1 \preceq \wp_3$;
(c) $\max\{\wp_1, \wp_2\} = \wp_2 \Leftrightarrow \wp_1 \preceq \wp_2$ *or* $|\wp_1| \leq |\wp_2|$.

Another important lemma that is helpful in justifying our new results is the following.

Lemma 1 ([25])**.** *Let* $\wp_i \in \mathbb{C}, i \in \{1,2,3,4,5\}$ *and partial order relation* $\preceq$ *defined on* $\mathbb{C}$. *Then, these statements fulfil:*
(a) If $\wp_1 \preceq \max\{\wp_2, \wp_3\}$ *then,* $\wp_1 \preceq \wp_2$ *if* $\wp_3 \preceq \wp_2$;
(b) If $\wp_1 \preceq \max\{\wp_2, \wp_3, \wp_4\}$ *then,* $\wp_1 \preceq \wp_2$ *if* $\max\{\wp_3, \wp_4\} \preceq \wp_2$;
(c) If $\wp_1 \preceq \max\{\wp_2, \wp_3, \wp_4, \wp_5\}$ *then,* $\wp_1 \preceq \wp_2$ *if* $\max\{\wp_3, \wp_4, \wp_5\} \preceq \wp_2$.

Definition 3 ([24])**.** *For the provided real number* $b \geq 1$ *and a nonempty set* $\mathcal{Z}$, *a functional* $\Lambda : \mathcal{Z} \times \mathcal{Z} \to \mathbb{C}$ *is termed as a complex valued b-metric (CVbM), if for all* $\aleph, \varsigma, £ \in \mathcal{Z}$ *the necessities below fulfil:*
(1) $\Lambda(\aleph, £) = 0$ *if and only if* $\aleph = £$,
(2) $\Lambda(\aleph, £) \succeq 0$,
(3) $\Lambda(\aleph, £) = \Lambda(\aleph, £)$,
(4) $\Lambda(\aleph, £) \preceq b[\Lambda(\aleph, \varsigma) + \Lambda(\varsigma, £)]$.
Then $(\mathcal{Z}, \Lambda)$ *is a complex valued b-metric space (CVbM space).*

Example 1 ([24])**.** *Let* $\mathcal{Z} = \mathbb{C}$, *define* $\Lambda : \mathcal{Z} \times \mathcal{Z} \to \mathbb{C}$ *by*

$$\Lambda(\nu_1, \nu_2) = \mid \nu_1 - \nu_2 \mid^2 + i \mid \nu_1 - \nu_2 \mid^2 \text{ for all } \nu_1, \nu_2 \in \mathcal{Z}.$$

Then $(\mathcal{Z}, \Lambda)$ *is a CVbM space with* $b = 2$.

Definition 4 ([23])**.** *Let* $(\mathcal{Z}, \Lambda)$ *be a CVbM space and* $\{\aleph_n\}$ *a sequence in* $\mathcal{Z}$ *and* $\aleph \in \mathcal{Z}$.
(i) A sequence $\aleph_n$ *in* $\mathcal{Z}$ *is convergent to* $\aleph \in \mathcal{Z}$ *if for every* $0 \prec c \in \mathbb{C}$ *there exists* $n_0 \in \mathbb{N}$, *such that* $\Lambda(\aleph_n, \aleph) \prec c$ *for every* $n > n_0$. *In that case, we use the notation* $\lim_{n \to +\infty} \aleph_n = \aleph$ *or* $\aleph_n \to \aleph$ *as* $n \to +\infty$.
(ii) If for every $0 \prec c \in \mathbb{C}$ *there exists* $n_0 \in \mathbb{N}$, *such that* $\Lambda(\aleph_n, \aleph_{n+m}) \prec c$ *for every* $n > n_0$ *and* $m \in \mathbb{N}$. *Then,* $\{\aleph_n\}$ *is called a Cauchy sequence in* $(\mathcal{Z}, \Lambda)$.
(iii) If every Cauchy sequence in $\mathcal{Z}$ *is convergent in* $\mathcal{Z}$, *then* $(\mathcal{Z}, \Lambda)$ *is called a complete CVbM space.*

Lemma 2 ([23])**.** *Let* $(\mathcal{Z}, \Lambda)$ *be a CVbM space and* $\{\aleph_n\}$ *be a sequence in* $\mathcal{Z}$.
(i) Then, a sequence $\{\aleph_n\}$ *converges to* $\aleph$ *if and only if* $|\Lambda(\aleph_n, \aleph)| \to 0$ *as* $n \to +\infty$.
(ii) Then, a sequence $\{\aleph_n\}$ *is a Cauchy sequence if and only if* $|\Lambda(\aleph_n, \aleph_{n+m})| \to 0$ *as* $n \to +\infty$, *where* $m \in \mathbb{N}$.

Next, the contraction principle in [12] is to be recalled, which is the generalization of Lj Ćirić.

Theorem 1 ([12])**.** *Let* $(\daleth, \Lambda)$ *be a metric space and for a mapping* $Q : \daleth \to \daleth$ *there exists* $\varsigma \in (0,1)$, *such that for all* $z_1, z_2 \in \daleth$, *we have*

$$\Lambda(Qz_1, Qz_2) \leq \varsigma \max\left\{\Lambda(z_1, z_2), \Lambda(z_1, Qz_1), \Lambda(z_2, Tz_2), \frac{1}{2}(\Lambda(z_1, Qz_2) + \Lambda(z_2, Qz_1))\right\}.$$

If $\urcorner$ is complete Q-orbitally then:
(1) Fix(Q)=z;*
(2) For all $z* \in \urcorner$ sequence $(Q^i z)_{i \in N}$ converges to z;*
(3) $\Lambda(Q^i z, z*) \leq \frac{\varsigma^i}{1-\varsigma}\Lambda(z, Qz)$, for all $z \in \urcorner, i = 1, 2, \ldots$

Similarly, the generalisation of the fixed-point theorem of Zamfirescu [26] has been further elongated in [13] to an almost contraction.

Theorem 2 ([13])**.** *Let $(\mathcal{C}, \digamma)$ be a complete metric space and $G : \mathcal{C} \to \mathcal{C}$ be an almost contraction, that is a mapping for which exists a constant $\kappa \in [0, 1)$ and for some $\varrho \geq 0$, such that*

$$\digamma(Gu, Gw) \leq \kappa \digamma(u, w) + \varrho \digamma(w, Gu),$$

for all $u, w \in \mathcal{C}$. Then
(1) $Fix(G) = u \in \mathcal{C} : Gu = u \neq 0$;
(2) For any $u_0 \in \mathcal{C}$, the Picard iteration u_n converges to $u* \in Fix(G)$;
(3) The following estimate holds $\digamma(u_{n+i-1}, u*) \leq \frac{\delta^i}{1-\delta}\digamma(u_n, u_{n-1}), n = 1, 2, \ldots, i = 1, 2, \ldots$

In this manuscript our aim is to combine and extend the Ćirić and almost contraction conditions in the context of CVbM spaces. In addition, some examples and applications have been provided for the authenticity of our new generalization results.

We will use the following variant of the results from Miculescu and Mihail [27] (see also [28]).

Lemma 3 ([29])**.** *Let $\{\omega_n\}$ be a sequence in CVbM space $(\mathcal{Z}, \Lambda)$ and exists $\hbar \in [0, 1)$, such that*

$$\Lambda(\omega_{n+1}, \omega_n) \preceq \hbar \Lambda(\omega_n, \omega_{n-1}),$$

for all $n \in \mathbb{N}$. Then $\{\omega_n\}$ is a Cauchy sequence.

2. Main Results

Here we present our first new result in the case of a CVbM space for a unique and common fixed point of almost Ćirić-type contractions.

Theorem 3. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space $W, T : \mathcal{C} \to \mathcal{C}$ be two continuous mappings, such that:*

$$\begin{aligned} d_b(Wz_1, Tz_2) \quad \preceq \quad & q \max\left\{ d_b(z_1, z_2), d_b(z_1, Wz_1), \frac{d_b(z_2, Tz_2)}{1 + d_b(z_2, Wz_2)}, \right. \\ & \left. \frac{1}{2}(d_b(z_1, Tz_2) + d_b(z_2, W_1)) \right\} \\ + \quad & qLmin\left\{ d_b(z_1, z_2), d_b(z_1, Wz_1), d_b(z_2, Tz_2), \frac{d_b(z_1, Tz_2) d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)} \right\}, \end{aligned} \tag{1}$$

for all $z_1, z_2 \in \mathcal{C}$, where $0 \leq q < \frac{1}{s}$, $L \geq 0$ and all elements on the right side can be compared to one another with partial order $\preceq$. Then, the pairs (W,T) has a unique common fixed point.

Proof. Let μ_0 be an arbitrary point in $\mathcal{C}$ that defines a sequence μ_η, as follows:

$$\mu_{2\eta+1} = W\mu_{2\eta} \text{ and } \mu_{2\eta+2} = T\mu_{2\eta+1}, n = 0, 1, \ldots \tag{2}$$

Then, by (1) and (2) we obtain

$$\begin{aligned}
d_b(\mu_{2\eta+1},\mu_{2\eta+2}) &= d_b(W\mu_{2\eta},T\mu_{2\eta+1}) \\
&\preceq q\max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta},W\mu_{2\eta}),\frac{d_b(\mu_{2\eta+1},T\mu_{2\eta+1})}{1+d_b(\mu_{2\eta+1},W\mu_{2\eta})},\right. \\
&\left.\frac{1}{2}(d_b(\mu_{2\eta},T\mu_{2\eta+1})+d_b(\mu_{2\eta+1},W\mu_{2\eta}))\right\} \\
&+ qL\min\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta},W\mu_{2\eta}),d_b(\mu_{2\eta+1},T\mu_{2\eta+1}), \\
&\left.\frac{d_b((\mu_{2\eta},T\mu_{2\eta+1}))d_b(\mu_{2\eta+1},W\mu_{2\eta})}{1+d_b(\mu_{2\eta},\mu_{2\eta+1})}\right\} \\
&\preceq q\max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta},\mu_{2\eta+1}),\frac{d_b(\mu_{2\eta+1},\mu_{2\eta+2})}{1+d_b(\mu_{2\eta+1},\mu_{2\eta+1})},\right. \\
&\left.\frac{1}{2}(d_b(\mu_{2\eta},\mu_{2\eta+2})+d_b(\mu_{2\eta+1},\mu_{2\eta+1}))\right\} \\
&+ qL\min\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta+1},\mu_{2\eta+2}), \\
&\left.\frac{d_b((\mu_{2\eta},\mu_{2\eta+2}))d_b(\mu_{2\eta+1},\mu_{2\eta+1})}{1+d_b(\mu_{2\eta},\mu_{2\eta+1})}\right\},
\end{aligned}$$

so,

$$d_b(\mu_{2\eta+1},\mu_{2\eta+2})\preceq q\max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta+1},\mu_{2\eta+2}),\frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2})\right\}.$$

We have three possible maximums.

If

Case I.

$$\max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta+1},\mu_{2\eta+2}),\frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2})\right\}=d_b(\mu_{2\eta+1},\mu_{2\eta+2}),$$

we have

$$d_b(\mu_{2\eta+1},\mu_{2\eta+2})\preceq qd_b(\mu_{2\eta+1},\mu_{2\eta+2}).$$

This implies that $q\geq 1$, which is a contradiction.

Case II.

If

$$\max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}),d_b(\mu_{2\eta+1},\mu_{2\eta+2}),\frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2})\right\}=d_b(\mu_{2\eta},\mu_{2\eta+1}),$$

we have

$$d_b(\mu_{2\eta+1},\mu_{2+\eta 2})\preceq qd_b(\mu_{2\eta},\mu_{2\eta+1}). \tag{3}$$

Next, we have

$$d_b(\mu_{2\eta+2},\mu_{2\eta+3})\preceq q\max\left\{d_b(\mu_{2\eta+1},\mu_{2\eta+2}),d_b(\mu_{2\eta+2},\mu_{2\eta+3}),\frac{1}{2}d_b(\mu_{2\eta+1},\mu_{2\eta+3})\right\}.$$

Then we find to have these three cases as below.

Case IIa.

$$d_b(\mu_{2\eta+2},\mu_{2\eta+3})\preceq qd_b(\mu_{2\eta+2},\mu_{2\eta+3}),$$

which is again the same contradiction.

Case IIb.

$$d_b(\mu_{2\eta+2},\mu_{2\eta+3})\preceq qd_b(\mu_{2\eta+1},\mu_{2\eta+2}). \tag{4}$$

From (3) and (4), for all $n = 0, 1, 2, \ldots$ we obtain

$$d_b(\mu_{\eta+1}, \mu_{\eta+2}) \preceq q d_b(\mu_\eta, \mu_{\eta+1}) \preceq \cdots \preceq q^{\eta+1} d_b(\mu_0, \mu_1). \tag{5}$$

For $m, \eta \in \mathbb{N}$ and $m > \eta$, we have

$$\begin{aligned} d_b(\mu_\eta, \mu_m) \quad & \preceq s[d_b(\mu_\eta, \mu_{\eta+1}) + d_b(\mu_{\eta+1}, \mu_m)] \\ & \preceq s(d_b(\mu_\eta, \mu_{\eta+1})) + s^2[d_b(\mu_{\eta+1}, \mu_{\eta+2}) + d_b(\mu_{\eta+2}, \mu_m)] \\ & \preceq s(d_b(\mu_\eta, \mu_{\eta+1})) + s^2(d_b(\mu_{\eta+1}, \mu_{\eta+2})) + s^3(d_b(\mu_{\eta+2}, \mu_{\eta+3}) \\ & + \cdots + s^{m-\eta-1}(d_b(\mu_{m-2}, \mu_{m-1})) + s^{m-\eta}(d_b(\mu_{m-1}, \mu_m)). \end{aligned}$$

Moreover, using (5) we have

$$\begin{aligned} d_b(\mu_\eta, \mu_m) \quad \preceq & \quad s q^\eta (d_b(\mu_0, \mu_1)) + s^2 q^{\eta+1}(d_b(\mu_0, \mu_1)) + s^3 q^{\eta+2}(d_b(\mu_0, \mu_1)) + \cdots \\ + & \quad s^{m-\eta-1} q^{m-2}(d_b(\mu_0, \mu_1)) + s^{m-\eta} q^{m-1}(d_b(\mu_0, \mu_1)). \end{aligned}$$

This implies that

$$d_b(\mu_\eta, \mu_m) \preceq \sum_{i=\eta}^{m-\eta} s^i q^{i+\eta-1}(d_b(\mu_0, \mu_1)).$$

Therefore,

$$\begin{aligned} \left| d_b(\mu_\eta, \mu_m) \right| \quad & \preceq \quad \sum_{i=\eta}^{m-\eta} s^i q^{i+\eta-1} \left| (d_b(\mu_0, \mu_1)) \right| \\ & \preceq \quad \sum_{i=\eta}^{\infty} (sq)^i \left| (d_b(\mu_0, \mu_1)) \right| \\ & = \quad \frac{(sq)^\eta}{1 - sq} \left| (d_b(\mu_0, \mu_1)) \right|. \end{aligned}$$

As a result, we have

$$\left| d_b(\mu_\eta, \mu_m) \right| \preceq \frac{(sq)^\eta}{1 - sq} \left| (d_b(\mu_0, \mu_1)) \right| \to 0 \text{ as } \eta \to \infty.$$

Thus, $\{\mu_\eta\}$ has been proven to be a Cauchy sequence in $\mathcal{C}$.
Case IIc.

$$\begin{aligned} d_b(\mu_{2\eta+2}, \mu_{2\eta+3}) \quad & \preceq \quad q\frac{1}{2} d_b(\mu_{2\eta+1}, \mu_{2\eta+3}) \\ & \preceq \quad \frac{qs}{2}(d_b(\mu_{2\eta+1}, \mu_{2\eta+2}) + d_b(\mu_{2\eta+2}, \mu_{2\eta+3})), \end{aligned}$$

this implies that

$$(1 - \frac{qs}{2}) d_b(\mu_{2\eta+2}, \mu_{2\eta+3}) \preceq \frac{qs}{2} d_b(\mu_{2\eta+1}, \mu_{2\eta+2}).$$

In addition,

$$d_b(\mu_{2\eta+1}, \mu_{2\eta+2}) \preceq \frac{qs}{2}(d_b(\mu_{2\eta}, \mu_{2\eta+1}) + d_b(\mu_{2\eta+1}, \mu_{2\eta+2})),$$

which implies that

$$(1 - \frac{qs}{2}) d_b(\mu_{2\eta+2}, \mu_{2\eta+3}) \preceq \frac{qs}{2} d_b(\mu_{2\eta+1}, \mu_{2\eta+2}).$$

Thus we obtain
$$d_b(\mu_{2n+2},\mu_{2n+3}) \preceq \frac{qs}{2-qs} d_b(\mu_{2n+1},\mu_{2n+2}). \tag{6}$$
From (3) and (6) we obtain
$$d_b(\mu_{\eta+1},\mu_{\eta+2}) \preceq \varsigma d_b(\mu_\eta,\mu_{\eta+1}),$$
where $\varsigma = \max\left\{\frac{qs}{2-qs}, q\right\} < 1$, by Lemma 3, we conclude that $\{\mu_n\}$ is a Cauchy sequence.

Case III

If
$$If \max\left\{d_b(\mu_{2\eta},\mu_{2\eta+1}), d_b(\mu_{2\eta+1},\mu_{2\eta+2}), \frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2})\right\} = \frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2}),$$
we have
$$\begin{aligned} d_b(\mu_{2\eta+1},\mu_{2\eta+2}) &\preceq \frac{1}{2}d_b(\mu_{2\eta},\mu_{2\eta+2}) \\ &\preceq \frac{qs}{2}(d_b(\mu_{2\eta},\mu_{2\eta+1}) + d_b(\mu_{2\eta+1},\mu_{2\eta+2})). \end{aligned}$$
Thus,
$$(1-\frac{qs}{2})d_b(\mu_{2\eta+1},\mu_{2\eta+2}) \preceq \frac{qs}{2}d_b(\mu_{2\eta},\mu_{2\eta+1}).$$
Then, we obtain
$$d_b(\mu_{2n+1},\mu_{2n+2}) \preceq \frac{qs}{2-qs} d_b(\mu_{2n},\mu_{2n+1}). \tag{7}$$
Further, for the next step we obtain
$$d_b(\mu_{2\eta+2},\mu_{2\eta+3}) \preceq q \max[d_b(\mu_{2\eta+1},\mu_{2\eta+2}), d_b(\mu_{2\eta+2},\mu_{2\eta+3}), \frac{1}{2}d_b(\mu_{2\eta+1},\mu_{2\eta+3})].$$
Then, once again, we have three cases:

Case IIIa
$$d_b(\mu_{2\eta+2},\mu_{2\eta+3}) \preceq q d_b(\mu_{2\eta+2},\mu_{2\eta+3}),$$
which is a contradiction, because we have $q \geq 1$ here.

Case IIIb
$$d_b(\mu_{2\eta+2},\mu_{2\eta+3}) \preceq q d_b(\mu_{2\eta+1},\mu_{2\eta+2}). \tag{8}$$
It follows from (7) and (8) that
$$d_b(\mu_{\eta+1},\mu_{\eta+2}) \preceq \varsigma d_b(\mu_\eta,\mu_{\eta+1}),$$
where $\varsigma = \max\left\{\frac{qs}{2-qs}, q\right\} < 1$; by Lemma 3, we obtain that $\{\mu_\eta\}$ is a Cauchy sequence.

Case IIIc
$$d_b(\mu_{2\eta+2},\mu_{2\eta+3}) \preceq \frac{1}{2}d_b(\mu_{2\eta+1},\mu_{2\eta+3}).$$
After some calculation, as completed before, we obtain
$$d_b(\mu_{2\eta+2},\mu_{2\eta+3}) \preceq \frac{qs}{2-qs} d_b(\mu_{2\eta+1},\mu_{2\eta+2}). \tag{9}$$
Then, by (7) and (9) we obtain
$$d_b(\mu_{\eta+1},\mu_{\eta+2}) \preceq \hbar d_b(\mu_\eta,\mu_{\eta+1}), \tag{10}$$

where $0 \leq \hbar = \frac{qs}{2-qs} < 1$. Then, for all η = 0,1,2, . . . , we obtain

$$d_b(\mu_{\eta+1}, \mu_{\eta+2}) \preceq \hbar d_b(\mu_\eta, \mu_{\eta+1}) \preceq \cdots \preceq \hbar^{\eta+1} d_b(\mu_0, \mu_1). \tag{11}$$

This will implies

$$\begin{aligned} d_b(\mu_\eta, \mu_m) &\preceq s[d_b(\mu_\eta, \mu_{\eta+1}) + d_b(\mu_{\eta+1}, \mu_m)] \\ &\preceq s(d_b(\mu_\eta, \mu_{\eta+1})) + s^2[d_b(\mu_{\eta+1}, \mu_{\eta+2}) + d_b(\mu_{\eta+2}, \mu_m)] \\ &\preceq s(d_b(\mu_\eta, \mu_{\eta+1})) + s^2(d_b(\mu_{\eta+1}, \mu_{\eta+2})) + s^3(d_b(\mu_{\eta+2}, \mu_{\eta+3})) \\ &+ \cdots + s^{m-\eta-1}(d_b(\mu_{m-2}, \mu_{m-1})) + s^{m-n}(d_b(\mu_{m-1}, \mu_m)). \end{aligned}$$

Using (11), we obtained

$$\begin{aligned} d_b(\mu_\eta, \mu_m) &\preceq s\hbar^\eta(d_b(\mu_0, \mu_1)) + s^2\hbar^{\eta+1}(d_b(\mu_0, \mu_1)) + s^3\hbar^{\eta+2}(d_b(\mu_0, \mu_1)) + \cdots \\ &+ s^{m-\eta-1}\hbar^{m-2}(d_b(\mu_0, \mu_1)) + s^{m-\eta}\hbar^{m-1}(d_b(\mu_0, \mu_1)). \end{aligned}$$

This implies that

$$d_b(\mu_\eta, \mu_m) \preceq \sum_{i=\eta}^{m-\eta} s^i \hbar^{i+\eta-1}(d_b(\mu_0, \mu_1)).$$

Therefore,

$$\begin{aligned} \left| d_b(\mu_\eta, \mu_m) \right| &\preceq \sum_{i=\eta}^{m-\eta} s^i \hbar^{i+\eta-1} \left| (d_b(\mu_0, \mu_1)) \right| \\ &\preceq \sum_{i=\eta}^{\infty} (s\hbar)^i \left| (d_b(\mu_0, \mu_1)) \right| \\ &= \frac{(s\hbar)^\eta}{1 - s\hbar} \left| (d_b(\mu_0, \mu_1)) \right|. \end{aligned}$$

As a result, we have

$$\left| d_b(\mu_\eta, \mu_m) \right| \preceq \frac{(s\hbar)^\eta}{1 - s\hbar} \left| (d_b(\mu_0, \mu_1)) \right| \to 0 \text{ as } \eta \text{ goes to } \infty.$$

Thus, μ_η is a Cauchy sequence in $\mathcal{C}$. We obtain μ_η in all the above discussed cases as a Cauchy sequence. Because $\mathcal{C}$ is a complete space there, we have $\bar{g} \in \mathcal{C}$, such that $d_b(\mu_\eta, \bar{g}) \to 0$ as $\eta \to \infty$. This yields $d_b(\mu_{2\eta}, \bar{g}) \to 0$ as $\eta \to \infty$. Because we have W continuous, this implies that $\mu_{2\eta+1} = W\mu_{2\eta} \to W\bar{g}$ as $\eta \to \infty$. In the same way, $d_b(\mu_{2\eta+1}, \bar{g}) \to 0$ as $\eta \to \infty$. As we have T continuous, this implies that $\mu_{2\eta+2} = T\mu_{2\eta+1} \to T\bar{g}$ as $\eta \to \infty$. Since the limit is unique, we obtain $\bar{g} = T\bar{g}$. Thus, $\bar{g}$ is a common fixed point of the pair (W,T).

Uniqueness

To justify that $\bar{g}$ is unique, let $\ell \in \mathcal{C}$ be considered as another common fixed point of (W,T). Therefore, we have

$$\begin{aligned} d_b(\ell, \bar{g}) &= d_b(W\ell, T\bar{g}) \\ &\preceq q \max\left\{ d_b(\ell, \bar{g}), d_b(\ell, W\ell), \frac{d_b(\bar{g}, T\bar{g})}{1 + d_b(\bar{g}, T\bar{g})}, \frac{1}{2}(d_b(\ell, T\bar{g}) + d_b(\bar{g}, W\ell)) \right\} \\ &+ qL \min\left\{ d_b(\ell, \bar{g}), d_b(\ell, W\ell), d_b(\bar{g}, T\bar{g}), \frac{d_b(\ell, T\bar{g}) d_b(\bar{g}, W\ell)}{1 + d_b(\ell, \bar{g})} \right\} \end{aligned}$$

This implies that

$$\begin{aligned}
d_b(\ell,\bar{g}) &= d_b(W\ell,T\bar{g})\\
&\preceq q\max\left\{d_b(\ell,\bar{g}),d_b(\ell,\ell),\frac{d_b(\bar{g},\bar{g})}{1+d_b(\bar{g},\bar{g})},\frac{1}{2}(d_b(\ell,\bar{g})+d_b(\bar{g},\ell))\right\}\\
&+ qL\min\left\{d_b(\ell,\bar{g}),d_b(\ell,\bar{g}),d_b(\bar{g},\bar{g}),\frac{d_b(\ell,\bar{g})d_b(\bar{g},\ell)}{1+d_b(\ell,\bar{g})}\right\},
\end{aligned}$$

so $d_b(\ell,\bar{g}) \preceq qd_b(\ell,\bar{g})$. This means that $q \geq 1$, which causes a contradiction. Thus, $\ell = \bar{g}$. Thus, $\bar{b}$ is unique. □

Theorem 4. *Let $(\mathcal{C},d_b)$ be a complete CVbM space with $s \geq 1$, a provided real number, and $W,T:\mathcal{C}\to\mathcal{C}$ be two mappings such that:*

$$\begin{aligned}
d_b(Wz_1,Tz_2) &\preceq q\max\left\{d_b(z_1,z_2),d_b(z_1,Wz_1),\frac{d_b(z_2,Tz_2)}{1+d_b(z_2,Wz_1)},\frac{1}{2}(d_b(z_1,Tz_2)+d_b(z_2,Wz_1))\right\}\\
&+ L\min\left\{d_b(z_1,z_2),d_b(z_1,Wz_1),d_b(z_2,Tz_2),\frac{d_b(z_1,Tz_2)d_b(z_2,Wz_1)}{1+d_b(z_1,z_2)}\right\},
\end{aligned}$$

for all $z_1,z_2 \in \mathcal{C}$, where $0 \leq q \leq \frac{1}{s}$ and $L \geq 0$ and all the element on the right side can be compared to one another with partial order $\preceq$. Then, W and T possess a unique common fixed point.

Proof. The sequence $\{u_\eta\}$ could be obtained as a Cauchy sequence using the same procedure used in Theorem 3. Because $\mathcal{C}$ is complete, there exists $\bar{g}\in\mathcal{C}$, such that $d_b(u_\eta,\bar{g})\to 0$ as $\eta\to\infty$. Because W and T omitted to have continuity, we have $d_b(\bar{g},W\bar{g}) = k > 0$. Then, we can estimate that

$$\begin{aligned}
k &= d_b(\bar{g},W\bar{g}) \preceq s[d_b(\bar{g},u_{2\eta+2})+d_b(u_{2\eta+2},W\bar{g})]\\
&\preceq sd_b(\bar{g},u_{2\eta+2})+sd_b(Tu_{2\eta+1},W\bar{g})\\
&\preceq sd_b(\bar{g},u_{2\eta+2})+sq\max\{d_b(\bar{g},u_{2\eta+1}),d_b(\bar{g},W\bar{g}),\frac{d_b(u_{2\eta+1},Tu_{2\eta+1})}{1+d_b(u_{2\eta+1},Wu_{2\eta+1})},\\
&\quad \frac{1}{2}(d_b(\bar{g},Tu_{2\eta+1})+d_b(u_{2\eta+1},W\bar{g}))\}+L\min\{d_b(\bar{g},u_{2\eta+1}),d_b(\bar{g},W\bar{g})\},\\
&\quad d_b(u_{2\eta+1},Tu_{2\eta+1})\frac{d_b(\bar{g},Tu_{2\eta+1})d_b(u_{2\eta+1},W\bar{g})}{1+d_b(\bar{g},u_{2\eta+1})}\\
&\preceq sd_b(\bar{g},\bar{g})+sq\max\{d(\bar{g},\bar{g}),d(\bar{g},W\bar{g}),\frac{d_b(\bar{g},\bar{g})}{1+d_b(\bar{g},W\bar{g})}\frac{1}{2}(d_b(\bar{g},\bar{g})+d_b(\bar{g},W\bar{g}))\}\\
&+ L\min\{d_b(\bar{g},\bar{g}),d_b(\bar{g},W\bar{g}),d_b(\bar{g},\bar{g}),\frac{d_b(\bar{g},\bar{g})d_b(\bar{g},W\bar{g})}{1+d_b(\bar{g},\bar{g})}\}\\
&\preceq sqd_b(\bar{g},W\bar{g}),
\end{aligned}$$

so, $k \preceq sqk$. This implies that $|k| \leq sq|k|$, which causes a contradiction. Consequently, $\bar{g} = W\bar{g}$. In the same way, one can obtain $\bar{g} = T\bar{g}$. Hence, $\bar{g}$ is a common fixed point of (W,T). To justify the uniqueness of $\bar{g}$, one can use the similar approach as followed in Theorem 3. □

Taking $W = T$ we achieve the results below for, almost Ćirić, type operators on CVbM spaces.

Theorem 5. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with $s \geq 1$, a real number and $W : \mathcal{C} \to \mathcal{C}$ be a continuous mapping that fulfils:*

$$\begin{aligned} d_b(Wz_1, Wz_2) &\preceq q \max\Big\{ d_b(z_1, z_2), d_b(z_1, Wz_1), \frac{d_b(z_2, Wz_2)}{1 + d_b(z_2, Wz_1)}, \\ &\quad \frac{1}{2}(d_b(z_1, Wz_2) + d_b(z_2, Wz_1)) \Big\} \\ &+ qL \min\Big\{ d_b(z_1, z_2), d_b(z_1, Wz_1), d_b(z_2, Wz_2), \frac{d_b(z_1, Wz_2) d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)} \Big\} \end{aligned}$$

for all $z_1, z_2 \in \mathcal{C}$, where $0 \leq q \leq \frac{1}{s}$ and $L \geq 0$, and all the element on the right side can be compared to one another with partial order $\precsim$. Then, W possesses a unique fixed point.

Remark 1. *If operator W is omitted to be continuous, we would have a similar fixed point result.*

Corollary 1. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with $s \geq 1$, coefficient, and $W : \mathcal{C} \to \mathcal{C}$ be a continuous mapping that fulfils:*

$$\begin{aligned} & d_b(W^n z_1, W^n z_2) \preceq q \max\Big\{ d_b(z_1, z_2), d_b(z_1, W^n z_1), \frac{d_b(z_2, W^n z_2)}{1 + d_b(z_2, W^n z_1)}, \\ & \frac{1}{2}(d_b(z_1, W^n z_2) + d_b(z_2, W^n z_1)) \Big\} \\ + \quad & L \min\Big\{ d_b(z_1, z_2), d_b(z_1, W^n z_1), d_b(z_2, W^n z_2), \frac{d_b(z_1, W^n z_2) d_b(z_2, W^n z_1)}{1 + d_b(z_1, z_2)} \Big\} \end{aligned}$$

for all $z_1, z_2 \in \mathcal{C}$, where $0 \leq q \leq \frac{1}{s}, L \geq 0$ $n \in N$ and all the elements of the right side can be compared to one another's partial order $\preceq$. Then W possesses a unique fixed point.

Proof. Considering Theorem 3, one can obtain $\bar{g} \in \mathcal{C}$, such that $W^\eta \bar{g} = \bar{g}$. Therefore, we can obtain

$$\begin{aligned} d_b(W\bar{g}, \bar{g}) &= d(WW^\eta \bar{g}, W^\eta \bar{g}) = d_b(W^\eta W\bar{b}, W^\eta \bar{b}). \\ &\preceq q d_b(W\bar{g}, \bar{g}). \end{aligned}$$

Then $W^\eta \bar{g} = W\bar{g} = \bar{g}$ and fixed point $\bar{g}$ is unique. □

Remark 2. *From Corollary 1, if one omits and does not consider the continuity of T, a similar result can be achieved.*

Next, for almost Ćirić type operators in CVbM spaces, we extend another generalization of a common fixed-point theorem.

Theorem 6. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with $s \geq 1$, a provided real number, and $W, T : \mathcal{C} \to \mathcal{C}$ be two continuous mappings, such that:*

$$\begin{aligned} d_b(Wb_1, Tb_2) &\preceq q \max\Big\{ d_b(b_1, b_2), \frac{d_b(b_1, Wb_1) d_b(b_2, Tb_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, Tb_2) d_b(b_2, Wb_1)}{1 + d_b(b_1, b_2)} \Big\} \\ &+ qL \min\Big\{ d_b(b_1, b_2), \frac{d_b(b_1, Wb_1) d_b(b_2, Tb_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, Tb_2) d_b(b_2, Wb_1)}{1 + d_b(b_1, b_2)} \Big\} \end{aligned} \tag{12}$$

for all $b_1, b_2 \in \mathcal{C}$, where $0 \leq q < \frac{1}{s}$, $L \geq 0$ and all the elements of the right side can be compared to one another with partial order $\preceq$. Then, the pairs W and T possess a unique common fixed point.

Proof. Let b_0 be an arbitrary point in $\mathcal{C}$ and define a sequence b_η as follows:

$$b_{2\eta+1} = Wb_{2\eta} \text{ and } b_{2\eta+2} = Tb_{2\eta+1}, n = 0, 1, \ldots \tag{13}$$

Then by (12) and (13) we obtain

$$\begin{aligned}
& d_b(b_{2\eta+1}, b_{2\eta+2}) = d_b(Wb_{2\eta}, Tb_{2\eta+1}) \\
\preceq\ & q\max\left\{d_b(b_{2\eta}, b_{2\eta+1}), \frac{d_b(b_{2\eta}, Wb_{2\eta}d_b(b_{2\eta+1}Tb_{2\eta+1}))}{1+d_b(b_{2\eta}, b_{2\eta+1})}, \frac{d_b(b_{2\eta}, Tb_{2\eta+1})d_b(b_{2\eta+1}, Wb_{2\eta})}{1+d_b(b_{2\eta}, b_{2\eta+1})}\right\} \\
+\ & L\min\left\{d_b(b_{2\eta}, b_{2\eta+1}), \frac{d_b(b_{2\eta}, Wb_{2\eta})d_b(b_{2\eta+1}, Tb_{2\eta+1})}{1+d_b(b_{2\eta+1}, Wb_{2\eta})}, \frac{d_b((b_{2\eta}, Tb_{2\eta+1}))d_b(b_{2\eta+1}, Wb_{2\eta})}{1+d_b(b_{2\eta}, b_{2\eta+1})}\right\} \\
\preceq\ & q\max\left\{d_b(b_{2\eta}, b_{2\eta+1}), \frac{d_b(b_{2\eta}, b_{2\eta+1}d_b(b_{2\eta+1}b_{2\eta+2}))}{1+d_b(b_{2\eta}, b_{2\eta+1})}, \frac{d_b(b_{2\eta}, b_{2\eta+2})d_b(b_{2\eta+1}, b_{2\eta+1})}{1+d_b(b_{2\eta}, b_{2\eta+1})}\right\} \\
+\ & L\min\left\{d_b(b_{2\eta}, b_{2\eta+1}), \frac{d_b(b_{2\eta}, b_{2\eta+1})d_b(b_{2\eta+1}, b_{2\eta+2})}{1+d_b(b_{2\eta+1}, b_{2\eta+1})}, \frac{d_b((b_{2\eta}, b_{2\eta+2}))d_b(b_{2\eta+1}, b_{2\eta+1})}{1+d_b(b_{2\eta}, b_{2\eta+1})}\right\} \\
\preceq\ & q\max\{d_b(b_{2\eta}, b_{2\eta+1}), d_b(b_{2\eta+1}, b_{2\eta+2})\}.
\end{aligned}$$

If

$$\max\{d_b(b_{2\eta}, b_{2\eta+1}), d_b(b_{2\eta+1}, b_{2\eta+2})\} = d_b(b_{2\eta+1}, b_{2\eta+2})$$

then

$$d_b(b_{2\eta+1}, b_{2\eta+2}) \preceq qd_b(b_{2\eta+1}, b_{2\eta+2}).$$

This yields $q \geq 1$, which is a contradiction. Therefore,

$$d_b(b_{2\eta+1}, b_{2\eta+2}) \preceq qd_b(b_{2\eta}, b_{2\eta+1}). \tag{14}$$

In the same way, we can obtain

$$d_b(b_{2\eta+2}, b_{2\eta+3}) \preceq qd_b(b_{2\eta+1}, b_{2\eta+2}). \tag{15}$$

From (14) and (15) for all $\eta = 0, 1, 2, 3\ldots$, we obtain

$$d_b(b_{\eta+1}, b_{\eta+2}) \preceq qd_b(b_\eta, b_{\eta+1}) \preceq q^{\eta+1}d_b(b_0, b_1).$$

For $m, \eta \in N$, and $m > \eta$, we obtain

$$\begin{aligned}
d_b(b_\eta, b_m) & \preceq s[d_b(b_\eta, b_{\eta+1}) + d_b(b_{\eta+1}, b_m)] \\
& \preceq s(d_b(b_\eta, b_{\eta+1})) + s^2[d_b(b_{\eta+1}, b_{\eta+2}) + d_b(b_{\eta+2}, b_m)] \\
& \preceq s(d_b(b_\eta, b_{\eta+1})) + s^2(d_b(b_{\eta+1}, b_{\eta+2})) + s^3(d_b(b_{\eta+2}, b_{\eta+3}) \\
& + \cdots + s^{m-\eta-1}(d_b(b_{m-2}, b_{m-1})) + s^{m-\eta}(d_b(b_{m-1}, b_m)).
\end{aligned}$$

This implies that

$$d_b(b_\eta, b_m) \preceq \textstyle\sum_{i=\eta}^{m-\eta} s^i q^{i+\eta-1}(d_b(b_0, b_1)).$$

Therefore,

$$\begin{aligned}\left|d_b(b_\eta, b_m)\right| &\preceq \sum_{i=\eta}^{m-\eta} s^i q^{i+\eta-1}\left|(d_b(b_0, b_1))\right| \\ &\preceq \sum_{i=\eta}^{\infty}(sq)^i\left|(d_b(b_0, b_1))\right| \\ &= \frac{(sq)^\eta}{1-sq}\left|(d_b(b_0, b_1))\right|.\end{aligned}$$

Thus, we have

$$\left|d_b(b_\eta, b_m)\right| \preceq \frac{(sq)^\eta}{1-sq}\left|(d_b(b_0, b_1))\right| \to 0 \text{ as } \eta \to \infty.$$

Consequently, b_η is referred to as a Cauchy sequence in $\mathcal{C}$. Because $\mathcal{C}$ is complete, there exists $\bar{g} \in \mathcal{C}$, such that $d_b(b_\eta, \bar{g}) \to 0$ as $\eta \to \infty$. This results in $d_b(b_{2\eta}, \bar{g}) \to 0$ as $\eta \to \infty$. Because W is continuous, this implies that $b_{2\eta+1} = Wb_{2\eta} \to W\bar{g}$ as $\eta \to \infty$. In the same way, $d(b_{2\eta+1}, \bar{g}) \to 0$ as $\eta \to \infty$. Similarly, T is continuou, so $b_{2\eta+2} = Tb_{2\eta+1} \to T\bar{g}$ as $\eta \to \infty$. Because the limit is unique, we obtain $\bar{g} = T\bar{g}$. Thus, $\bar{g}$ is a common fixed point of the pair (W,T).

To justify the uniqueness, $\bar{l} \in \mathcal{C}$ is supposed to be another common fixed point of (W,T). Therefore,

$$\begin{aligned}d_b(\bar{l}, \bar{g}) &= d_b(W\bar{l}, T\bar{g}) \\ &\preceq q\max\left\{d_b(\bar{l}, \bar{g}), \frac{d_b(\bar{l}, W\bar{l})d_b(\bar{g}, T\bar{g})}{1+d_b(\bar{l}, \bar{g})}, \frac{d_b(\bar{l}, T\bar{g})d_b(\bar{g}, W\bar{l})}{1+d_b(\bar{l}, \bar{g})}\right\} \\ &+ L\min\left\{d_b(\bar{l}, \bar{g}), \frac{d_b(\bar{l}, W\bar{l})d_b(\bar{g}, T\bar{g})}{1+d_b(\bar{l}, \bar{g})}, \frac{d_b(\bar{l}, T\bar{g})d_b(\bar{g}, W\bar{l})}{1+d_b(\bar{l}, \bar{g})}\right\}\end{aligned}$$

This implies that $d_b(\bar{l}, \bar{g}) \preceq d_b(\bar{l}, \bar{g})$, which causes a contradiction. Consequently, $\bar{g}$ is a unique fixed point. □

If the continuity of T and W is omitted in the above theorem, the below common fixed point result would be obtained.

Theorem 7. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with $s \geq 1$, a provided real number, and $W, T : \mathcal{C} \to \mathcal{C}$ be two mappings such that*

$$\begin{aligned}d_b(Wb_1, Tb_2) &\preceq q\max\left\{d_b(b_1, b_2), \frac{d_b(b_1, Wb_1)d_b(b_2, Tb_2)}{1+d_b(b_1, b_2)}, \frac{d_b(b_1, Tb_2)d_b(b_2, Wb_1)}{1+d_b(b_1, b_2)}\right\} \\ &+ L\min\left\{d_b(b_1, b_2), \frac{d_b(b_1, Wb_2)d_b(b_2, Tb_2)}{1+d_b(b_1, B_2)}, \frac{d_b(b_1, Tb_2)d_b(b_2, Wb_1)}{1+d_b(b_1, b_2)}\right\},\end{aligned}$$

for all $b_1, b_2 \in \mathcal{C}$ where $0 \leq q < \frac{1}{s}, L \geq 0$ and all the elements of the right side can be compared to one another with partial order $\preceq$. Then, the pair (W,T) possesses a unique common fixed point.

Proof. It could be obtained that b_η is a Cauchy sequence, using the same procedure used in Theorem 6. Because $\mathcal{C}$ is a complete space, there exists $b* \in \mathcal{C}$, such that $d_b(b_\eta, b*) \to 0$ as $n \to \infty$. Because we cannot consider the continuity of W and T, we obtain $d_b(b*, Wb*) = k > 0$. Then, we can estimate that

$$
\begin{aligned}
k = d_b(b*, Wb*) &\preceq s[d_b(b*, b_{2\eta+2}) + d_b(b_{2\eta+2}, Wb*)] \\
&\preceq sd_b(b*, b_{2\eta+2}) + sd_b(Tb_{2\eta+1}, Wb*) \\
&\preceq sd_b(b*, b_{2\eta+2}) + sq\max\{d_b(b*, b_{2\eta+1}), \\
&\quad \frac{d_b(b*, Wb*)d_b(b_{2\eta+1}Tb_{2\eta+1})}{(1 + d_b(b*, b_{2\eta+1}))}, \frac{d_b(b*, Tb_{2\eta+1})d_b(b_{2\eta+1}, Wb*)}{(1 + d_b(b*, b_{2\eta+1}))}\} + \\
&\quad L\min\{d_b(b*, b_{2\eta+1}), \frac{d_b(b*, Wb*)d_b(b_{2\eta+1}, Tb_{2\eta+1})}{(1 + d_b(b*, b_{2\eta+1}))}, \\
&\quad \frac{d_b(b*, Tb_{2\eta+1})d_b(b_{2\eta+1}, Wb*)}{(1 + d_b(b*, b_{2\eta+1}))}\} \\
&\to sqd(b*, Wb*) \text{ as } \eta \to \infty.
\end{aligned}
$$

This implies that $|k| \leq sq|k|$, which causes the contradiction. Thus, $b* = Wb*$. Similarly, one can obtain $b* = Tb*$. Hence, b* is common fixed point of (W,T). To justify the uniqueness of b*, we can use the similar approach as followed proving Theorem 6. □

For $W = T$ in the previous result, we have the following result.

Theorem 8. *Let $(\mathcal{C}, d_b)$, a complete CVbM space with coefficient $s \geq 1$, and $W : \mathcal{C} \to \mathcal{C}$ be a continuous mapping such that*

$$
\begin{aligned}
d_b(Wb_1, Wb_2) &\preceq q\max\left\{d_b(b_1, b_2), \frac{d_b(b_1, Wb_2)d_b(b_2, Wb_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, Wb_2)d_b(b_2, Wb_1)}{1 + d_b(b_1, b_2)}\right\} \\
&+ qL\min\left\{d_b(b_1, b_2), \frac{d_b(b_1, Wb_1)d_b(b_2, Wb_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, Wb_2)d_b(b_2, Wb_1)}{1 + d_b(b_1, b_2)}\right\},
\end{aligned}
$$

for all $b_1, b_2 \in \mathcal{C}$ where $0 \leq q < \frac{1}{s}$, $L \geq 0$ and all the elements of the right side can be compared to one another with partial order $\preceq$. Then, W has a unique fixed point.

Remark 3. *If continuity of W is to be excluded, we can obtain the similar result.*

Corollary 2. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with coefficient $s \geq 1$, and $W : \mathcal{C} \to \mathcal{C}$ be a continuous mapping fulfilling*

$$
\begin{aligned}
d_b(W^\eta b_1, W_2^\eta) &\preceq q\max\left\{d_b(b_1, b_2), \frac{d_b(b_1, W^\eta b_1)d_b(b_2, W^\eta b_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, W^\eta b_2)d_b(b_2, W^\eta b_1)}{1 + d_b(b_1, b_2)}\right\} \\
&+ L\min\left\{d_b(b_1, b_2), \frac{d_b(b_1, W^\eta b_1)d_b(b_2, W^\eta b_2)}{1 + d_b(b_1, b_2)}, \frac{d_b(b_1, W^\eta b_2)d_b(b_2, W^\eta b_1)}{1 + d_b(b_1, b_2)}\right\}
\end{aligned}
$$

for all $b_1, b_2 \in \mathcal{C}$, where $0 \leq q \leq \frac{1}{s}$, $L \geq 0$, and all the element at the right side can be compared to one another with partial order $\preceq$. Then W possess a unique fixed point.

Proof. Considering Theorem 8, one can obtain $b* \in \mathcal{C}$, in such a way that $W^\eta b* = b*$. Then, one could obtain

$$
\begin{aligned}
d_b(Wb*, b*) &= d_b(WW^\eta b*, W^\eta b*) = d_b(W^\eta Wb*, W^\eta b*). \\
&\preceq qd_b(Wb*, b*) + L(0). \\
&\preceq qd_b(Wb*, b*).
\end{aligned}
$$

Then, $W^\eta b* = Wb* = b*$. Therefore, the fixed point of W is unique. □

Remark 4. *(i) Omitting continuity of W, we can obtain similar result from Corollary 2.*
(ii) Plugging L = 0 into all the above results, one can obtain the results of [29].

Example 2. *Let* $Y = \mathbb{C}$ *be a complex numbers set. Define* $d_b : Y \times Y \to \mathbb{C}$ *by*

$$d_b(\wp_1, \wp_2) = |\vartheta_1 - \vartheta_2|^2 + i|\eta_1 - \eta_2|^2,$$

for all $\wp_1, \wp_2 \in Y$, *where* $\wp_1 = \vartheta_1 + i\eta_1 = (\vartheta_1, \eta_1)$ *and* $\wp_2 = \vartheta_2 + i\eta_2 = (\vartheta_2, \eta_2)$.
Certainly, Y *is a complete CVbM space having coefficient* $s \geq 1$.
Let us define two mappings

$$W(\wp) = W(\vartheta + \eta) = \begin{cases} 0, & \text{if } \vartheta, \eta \in Q \\ i, & \text{if } \vartheta, \eta \in \acute{Q} \\ 3 - 2i, & \text{if } \vartheta \in \acute{Q}, \eta \in Q \\ 1, & \text{if } \vartheta \in Q, \eta \in \acute{Q} \end{cases}$$

and

$$T(\wp) = T(\vartheta + \eta) = \begin{cases} 0, & \text{if } \vartheta, \eta \in Q \\ 2 - 2i, & \text{if } \vartheta, \eta \in \acute{Q} \\ 2, & \text{if } \vartheta \in \acute{Q}, \eta \in Q \\ 2i, & \text{if } \vartheta \in Q, \eta \in \acute{Q} \end{cases}$$

where Q *is a set of rational numbers and* $\acute{Q}$ *a set of irrational numbers.*
(i) if $\vartheta, \eta \in Q$ *let* $\vartheta = \frac{1}{2}$ *and* $\eta = 0$ *then*

$$d(W\vartheta, T\eta) = d(W(\frac{1}{2}), T(0)) = d(0, 0) = 0$$

$$d(\vartheta, \eta) = d(\frac{1}{2}, 0) = \frac{1}{4}.$$

There is no need to check the other conditions, because they fulfil the inequality (1) in Theorem 3.
(ii) If $\vartheta, \eta \in \acute{Q}$, *let* $\vartheta = \frac{1}{\sqrt{2}}$ *and* $\eta = \pi$ *then*

$$d(W\vartheta, T\eta) = d(W(\frac{1}{\sqrt{2}}), T(\pi)) = d(\iota, 2 - 2\iota) = \iota^2 + \iota(2 - 2\iota)^2 = 3,$$

$$d(\vartheta, \eta) = d(\frac{1}{\sqrt{2}}, 2 - 2\iota) = (\frac{1}{\sqrt{2}})^2 + \iota(2 - 2\iota)^2 = \frac{9}{5}.$$

Similarly, one can check (iii) and (iv). Thus, the fixed point of W and T is unique and common.

3. Applications

3.1. Applications to Integral-Type Contractions

In the present section, the fixed point results, derived in the above section, are implemented to prove common fixed points of some integral-type contractions. Initially, let us define altering distance function.

Definition 5. *A function* $\Gamma : [0, \infty) \to [0, \infty)$ *is referred to as an altering distance function if it fulfils these necessities:*
(a) Γ *is continuous and nondecreasing.*
(b) $\Gamma(\nu) = 0$ *iff* ν*=0.*

Now, let us provide the following definition.

Definition 6. *Let* $\aleph$ *be the set of the functions* $\hbar : [0, \infty) \to [0, \infty)$ *that fulfills these requirements:*
(i) $\hbar$ *for each subset of* $[0, \infty)$, *such that the subset is compact, is Lebesgue integrable.*
(ii) $\int_0^\beta \hbar(\nu) d\nu > 0$, *for all* $\beta > 0$.

Remark 5. *It is quite simple to demonstrate whether the mapping* $\tau; [0,\infty) \to [0,\infty)$ *defined as*

$$\tau(t) = \int_0^{\zeta} \hbar(t)dt > 0$$

is an altering distance function.

Further, the first new result of this section is presented.

Theorem 9. *Let* $(\mathcal{C}, d_b)$ *be a complete CVbM space having* $s \geq 1$*, a given real number, and* $W, T : \mathcal{C} \to \mathcal{C}$ *are two continuous mappings holding*

$$\int_0^{d_b(Wz_1,Tz_2)} \hbar(\nu)d\nu \preceq q \int_0^{M(z_1,z_2)+Lm(z_1,z_2)} \hbar(\nu)d\nu,$$

for all $z_1, z_2 \in \mathcal{C}, 0 \leq q \leq \frac{1}{s}, L \geq 0$ *and* $\hbar \in \aleph$ *with*

$$M(z_1,z_2) = \max\left[d_b(z_1,z_2), d_b(z_1,Wz_1), \frac{d_b(z_2,Tz_2)}{1+d_b(z_2,Wz_2)}, \frac{1}{2}(d_b(z_1,Tz_2) + d_b(z_2,Wz_1))\right]$$

and

$$m(z_1,z_2) = \min\left[d_b(z_1,z_2), d_b(z_1,Wz_1), d_b(z_2,Tz_2), \frac{d_b(z_1,Tz_2)d_b(z_2,Wz_1)}{1+d_b(z_1,z_2)}\right],$$

where all the element of $M(z_1,z_2)$ *and* $m(z_1,z_2)$ *can be compared to one another w.r.t* $\preceq$*. Then (W,T) possess a unique common fixed point.*

Proof. Considering Theorem 3, such that $\tau(w) = \int_0^w \hbar(v)dv$, one can achieve the required solution. $\square$

Remark 6. *The same result can be achieved, if one omit continuity of the mappings.*

We deduce two fixed points theorems of integral-type results, if we take W = T, with and without continuity of W.

Theorem 10. *Let* $(\mathcal{C}, d_b)$ *be a complete CVbM space having* $s \geq 1$*, a provided real number, and* $W : \mathcal{C} \to \mathcal{C}$ *be a continuous mappings that fulfil*

$$\int_0^{d_b(Wz_1,Wz_2)} \hbar(t)dt \preceq q \int_0^{M(z_1,z_2)+Lm(z_1,z_2)} \hbar(t)dt,$$

for all $z_1, z_2 \in \mathcal{C}, 0 \leq q \leq \frac{1}{s}, L \geq 0$ *and* $\hbar \in \aleph$ *with*

$$M(z_1,z_2) = \max\left\{d_b(z_1,z_2), d_b(z_1,Wz_1), \frac{d_b(z_2,Wz_2)}{1+d_b(z_2,Wz_2)}, \frac{1}{2}(d_b(z_1,Wz_2) + d_b(z_2,Wz_1))\right\}$$

and

$$m(z_1,z_2) = \min\left\{d_b(z_1,z_2), d_b(z_1,Wz_1), d_b(z_2,Wz_2), \frac{d_b(z_1,Wz_2)d_b(z_2,Wz_1)}{1+d_b(z_1,z_2)}\right\},$$

where all the elements of $M(z_1,z_2)$ *and* $m(z_1,z_2)$ *can be compared to one another w.r.t* $\preceq$*. Then, W posses a unique common fixed point.*

Proof. Considering Theorem 5, such that $\daleth(t) = \int_0^t \hbar(\nu)d\nu$, one can obtain the required. $\square$

We would have the following common fixed-point integral-type result for the extension and generalization of almost Ćirić-type contractions.

Theorem 11. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space with coefficient $s \geq 1$, and $W, T : \mathcal{C} \to \mathcal{C}$ be continuous mappings that fulfil*

$$\int_0^{d_b(Wz_1, Tz_2)} \hbar(t)dt \preceq q \int_0^{Q(z_1,z_2)+p(z_1,z_2)} \hbar(t)dt,$$

for all $z_1, z_2 \in \mathcal{C}, 0 \leq q \leq \frac{1}{s}$, $L \geq 0$ and $\hbar \in \aleph$ with

$$Q(z_1, z_2) = \max\left\{d_b(z_1, z_2), \frac{d_b(z_1, Wz_1)d_b(z_2, Tz_2)}{1 + d_b(z_1, z_2)}, \frac{d_b(z_1, Tz_2)d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)}\right\}$$

and

$$p(z_1, z_2) = \min\left\{d_b(z_1, z_2), \frac{d_b(z_1, Wz_1)d_b(z_2, Tz_2)}{1 + d_b(z_1, z_2)}, \frac{d_b(z_1, Tz_2)d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)}\right\},$$

where all the element of $M(z_1, z_2)$ and $m(z_1, z_2)$ can be compared to one another with $\preceq$. Then, (W,T) possesses a unique common fixed point.

Proof. Utilizing Theorem 6, such that taking $\daleth(t) = \int_0^t \hbar(v)dv$ one can achieve the required result. □

Remark 7. *One can reach a similar conclusion if one excludes continuity and take the mappings as non-continuous.*

Taking W = T, one can deduce two fixed-point theorems of integral-type results for the almost Ćirić-type contractions, with and without continuous W.

Theorem 12. *Let $(\mathcal{C}, d_b)$ be a complete CVbM space having $s \geq 1$, a provided real number, and $W : \mathcal{C} \to \mathcal{C}$ be continuous mapping which fulfils*

$$\int_0^{d(Wz_1, Wz_2)} \hbar(t)dt \preceq q \int_0^{Q(z_1,z_2)+p(z_1,z_2)} \hbar(t)dt,$$

for all $z_1, z_2 \in \mathcal{C}, 0 \leq q \leq \frac{1}{s}, L \geq 0$ and $\hbar \in \aleph$, with

$$Q(z_1, z_2) = \max\left\{d_b(z_1, z_2), \frac{d_b(z_1, Wz_1)d_b(z_2, Wz_2)}{1 + d_b(z_1, z_2)}, \frac{d_b(z_1, Wz_2)d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)}\right\}$$

and

$$p(z_1, z_2) = \min\left\{d_b(z_1, z_2), \frac{d_b(z_1, Wz_1)d_b(z_2, Wz_2)}{1 + d_b(z_1, z_2)}, \frac{d_b(z_1, Wz_2)d_b(z_2, Wz_1)}{1 + d_b(z_1, z_2)}\right\},$$

where all the element of $M(z_1, z_2)$ and $m(z_1, z_2)$ can be compared to one another w.r.t $\preceq$. Then, W possesses a unique fixed point.

Proof. Utilizing Theorem 8, such that taking $\Upsilon(t) = \int_0^t \hbar(v)dv$, we would achieve the result. □

3.2. Application to the System of Urysohn-Type Integral Equations

In the last decades, the Banach contraction principle has troubled many researchers as it was considered to be one of the most prominent tools in the formulation of the existence and uniqueness of a common solution to integral-type equations in many disciplines, notably non-linear analysis. In this section, for the authenticity of our results, we implement the results we achieved in previous sections, to establish the existence of a unique and common solution to system of integral-type equations. The motivation we had to consider these applications is from the publication of W. Sintunavarat et al. [30]. Let us take the system of Urysohn integral equations under consideration.

$$\begin{cases} \gamma_1(\omega) = g(\omega) + \int_{p_1}^{p_2} K_1(\omega, s, \gamma_1(s))ds \\ \gamma_2(\omega) = g(\omega) + \int_{p_1}^{p_2} K_2(\omega, s, \gamma_2(s))ds \end{cases} \tag{16}$$

where
(i) $\gamma_1(\omega)$ and $\gamma_2(\omega)$ are variables which are unknown for all $\omega \in [p_1, p_2]$, $p_1 > 0$,
(ii) g(ω) is the term which is deterministic free, defined for $\omega \in [p_1, p_2]$,
(iii) $K_1(\omega, s)$ and $K_2(\omega, s)$ are deterministic kernels defined for $\omega, s \in [p_1, p_2]$.
Let $\digamma = (C[p_1, p_2], R^n), \ell > 0$ and $d_b : \digamma \times \digamma \to R^n$ defined by

$$d_b(\gamma_1, \gamma_2) = ||\gamma_1(\omega) - \gamma_2(\omega)||_\infty = \sup |\gamma_1(\omega) - \gamma_2(\omega)|^2 \sqrt{1+\ell^2} e^{\iota \tan^{-1} \ell},$$

for all $\gamma_1, \gamma_2 \in \digamma, \iota = \sqrt{-1} \in C$. Certainly, $(C[p_1, p_2], R^n, ||.||_\infty)$ is a complete CVbM with s = 2. Moreover, let us take the Urysohn integral equations system (16) under the following requirements;
$(Q_1); g(\omega) \in \digamma$,
$(Q_2); K_1, K_2$:$[p_1, p_2] \times [p_1, p_2] \times R^n \to R^n$ are continuous functions satisfying

$$|K_1(\omega, s, u(s)) - K_2(\omega, s, v(s))| \preceq \frac{1}{\sqrt{(p_2 - p_1)e^\ell}} M(\nu_1, \nu_2),$$

where

$$\begin{aligned} M(\nu_1, \nu_1) &= \max\left\{ d_b(\nu_1, \nu_2), d_b(\nu_1, W\nu_1), \frac{d_b(\nu_2, T\nu_2)}{1 + d_b(\nu_2, W\nu_2)}, \frac{1}{2}(d_b(\nu_1, T\nu_2) + d_b(\nu_2, W\nu_1)) \right\} \\ &+ L \min\left\{ d_b(\nu_1, \nu_2), d_b(\nu_1, W\nu_1), d_b(\nu_2, T\nu_2), \frac{d_b(\nu_1, T\nu_2) d_b(\nu_2, W\nu_1)}{1 + d_b(\nu_1, \nu_2)} \right\} \end{aligned}$$

In this portion, with the help of the result from the previous section, Theorem 3 we attempt to prove the existence of a unique solution of system (16).

Theorem 13. *If $(C[p_1, p_2], R^n, ||.||_\infty)$ is a complete CVbM space, then the above system (16) under the assumptions (Q_1) and (Q_2) has a unique common solution.*

Proof. Define two continuous mappings, T,W: $\digamma \to \digamma$, for $\gamma_1, \gamma_2 \in (C[p_1, p_2], R^n)$ and $\omega \in [p_1, p_2]$ as

$$W\gamma_1(\omega) = g(\omega) + \int_{p_2}^{p_1} K_1(\omega, s, \gamma_1(s))ds$$

$$T\gamma_2(\omega) = g(\omega) + \int_{p_2}^{p_1} K_2(\omega, s, \gamma_2(s))ds.$$

Then,

$$\begin{aligned}
|W\gamma_1(\omega) - T\gamma_2(\omega)|^2 &= \int_{p_1}^{p_2} |K_1(\omega, s, \gamma_1(s)) - K_2(\omega, s, \gamma_2(s))|^2 ds \\
&\preceq \int_{p_1}^{p_2} \frac{1}{(p_2 - p_1)e^{\ell}} M(\gamma_1, \gamma_2) \\
&= \frac{1}{(p_2 - p_1)e^{\ell}} \int_{p_1}^{p_2} \frac{e^{-i\tan^{-1}\ell}}{\sqrt{1+\ell^2}} |M(\gamma_1, \gamma_2)|^2 \sqrt{1+\ell^2} e^{i\tan^{-1}\ell} ds \\
&\preceq \frac{1}{(p_2 - p_1)e^{\ell}} \frac{e^{-i\tan^{-1}\ell}}{\sqrt{1+\ell^2}} ||M(\gamma_1, \gamma_2)||_\infty \int_{p_1}^{p_2} ds \\
&= \frac{1}{e^{\ell}} \frac{e^{-i\tan^{-1}\ell}}{\sqrt{1+\ell^2}} ||M(\gamma_1, \gamma_2)||_\infty.
\end{aligned}$$

Then, we obtain

$$|W\gamma_1(\omega) - T\gamma_2(\omega)|^2 \sqrt{1+\ell^2} e^{i} \tan^{-1}\ell \preceq \tfrac{1}{e^{\ell}} ||M(\gamma_1, \gamma_2)||_\infty.$$

This also implies that

$$||W\gamma_1(\omega) - T\gamma_2(\omega)|| \preceq \tfrac{1}{e^{i}} ||M(\gamma_1, \gamma_2)||_\infty.$$

Then,

$$d_b(W\gamma_1, T\gamma_2) \preceq \varrho M(\gamma_1, \gamma_2).$$

Therefore, the conditions of Theorem 3 are fulfilled for $0 < \varrho = \frac{1}{e^{\ell}} < 1$ and $\ell > 0$. Thus, the system (16) has a unique solution on $\digamma$. □

4. Conclusions

In the framework of CVbM spaces, the main goal of this publication is to combine and expand the Ćirić and almost contraction conditions. Numerous applications and examples support the validity of our proposed generalization. These findings have significance for future studies in this field and provide useful insights into the behavior of mappings in complex-valued b-metric spaces.

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