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Reformulated Silver-Meal and Similar Lot Sizing Techniques

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Abstract: Literature and most textbooks around the world describe Silver-Meal in such a way that periods with zero demand make Silver-Meal suggest a higher frequency of order replenishments than necessary and therefore higher total costs than necessary. Silver-Meal, still the best-known technique, is therefore inferior to other lesser-known techniques when the time interval in the calculations presently is days and not months. The purpose of this article is to show that another mathematical formulation of Silver-Meal avoids this trouble. We also point to characteristics such as Silver-Meal, Least Unit Cost, Part-Period Balancing, and lot-sizing techniques that are available in many textbooks for operations and supply chain management. We illustrate the techniques with different examples of periods without demand, declining demand, and varying demand. We point out possible problems with the different techniques. Literature mostly does not consider periods of zero demand, which was not so important before. Lot-sizing methods must cope with the important performance indicator "Days of inventory". Numerous practical situations with zero demand periods exist where a lot of sizing techniques help for efficient operations. It is necessary knowledge and a tool for students (future users, performers, and managers). "Lägsta periodkostnad" is a restored and reformulated Silver-Meal, with Silver-Meal's characteristics already presented in literature, except those difficulties with zero demand periods disappear.

Keywords: lot sizing; Silver-Meal; Least Unit Cost; Part-Period Balancing; education

MSC: 90B05; 90B30

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1. Introduction

Goggle searches for "Silver-Meal", "Least Unit Cost," and "Part-Period Balancing" present a huge surplus of hits for Silver-Meal. Silver-Meal, together with Material Requirement Planning and Economic Order Quantity, are probably the most cited and well-known concepts and techniques in operations and supply chain management.

Bitran et al. [1] wrote, "Due to their importance in production planning and inventory control, lot size problems have been widely studied (Peterson and Silver [2]). In particular, these problems play a key role in material requirements planning (Orlicky [3] and Smith [4]). And yet, despite all these efforts, the type of large-scale lot size problems that arise frequently in practice remain difficult to solve." Since 1984, many more articles have been published. While lot size problems are still difficult to solve, heuristics are a necessary tool to improve management's practical decisions.

With material requirements planning or similar implementation in mechanical/electrical manufacturing with mixed series production and a "make-to-stock" situation, varying order quantities forwarded upstream create unnecessary variations in the material flow. A fixed order quantity creates a "leaner" existence; managers can then generate a "takt" and a more efficient flow. Nevertheless, in many other practical situations, a fixed economic order quantity is unsuitable and the wrong solution. A construction company orders plasterboard, lagging, concrete, etc. for delivery on a special date (days). Between the delivery dates, there will be several days without demand. Other project-oriented productions contain demands for different quantities placed at uneven time intervals. Lot-sizing techniques are tools to improve practical operations. Historically, periods without demand have not been so important when the time scale was months, but now for a construction company or other project-oriented company, the time scale is days. Then there will be many periods without demand or requirements. Lot-sizing methods must cope with the important performance indicator "days of inventory".

From the literature, we have noticed that most investigations and evaluations of deterministic lot-sizing methods prefer Silver-Meal compared to Least Unit Cost, Part-Period Balancing, and other techniques, e.g., Blackburn and Millen [5], Saydam and Evans [6].

Bookbinder and Tan [7] note, and refer to other literature, that the Silver-Meal heuristic does not perform well when there are frequent periods with no demand and demand is sharply decreasing. Silver and Miltenburg [8] are aware of the difficulties for Silver-Meal to treat periods with zero demand and decreasing demand; therefore, they suggest two modifications to Silver-Meal. Bookbinder and Tan [7] also suggest two modified heuristics. These heuristics have several steps and rather complicated stop rules, which explain why these modifications have received limited attention and have not influenced textbooks.

Pujawan [9] tests Silver-Meal and Least Unit Cost in a supply chain receiving demand with stochastic variability from its downstream channel. Silver-Meal is shown to produce a series of orders with more stable intervals between orders but with more variable order quantities. Least Unit Cost results in more stable order quantities but more variable order intervals. However, the highest variability tested (Normal (200, 80)) presents a probability less than 0.01 for a period with zero demand. Govindan [10] also favors Silver-Meal in two-echelon supply chains (one vendor, multiple retailers), in tests with time-varying stochastic demand, and in some periods with zero demand.

Ho et al. [11] tested Silver-Meal with zero demand over more than one period. Instead of Silver-Meal, they suggest a heuristic they refer to as net least period cost (*nLPC*). The difference is that *nLPC* divides the total cost by the total number of non-zero demand periods, while Silver-Meal divides the total cost by the total number of periods between periods i and j. Ho et al. [11] argue that the justification for *nLPC* is that "zero demand does not require a setup and does not inflate holding costs when it is evaluated in isolation". If a quantity is carried over periods with zero demand to satisfy a demand in a future period, the carrying cost, or inventory-holding cost, increases.

In the following, we first present a small numerical deterministic lot-sizing example with a time scale in weeks. We use both the traditional and the new. The example is solved with different heuristics in steps to exemplify the techniques and make the reader certain about the different techniques. We compare the results for the entire known planning horizon.

Thereafter, we present another example with a time scale in days. The result of the day-example is presented in a table. Consequences for the whole horizon, with no new demand, are included. We also present two informative examples from Silver and Miltenburg [8]. Finally, we present conclusions, reflections, and suggested extensions.

2. A numerical Week-Example

Let us assume we have a demand in week 3 (an example from Segerstedt [12]) and must order at once. We also know future demand in the forthcoming weeks until week 10, according to Figure 1 and Table 1.

Should we also order more quantities to reduce costs? We have a setup/order cost of 200 Money Units (MU), the item costs 50 MU per unit, and we assume that capital holding in inventories costs 20% per year.

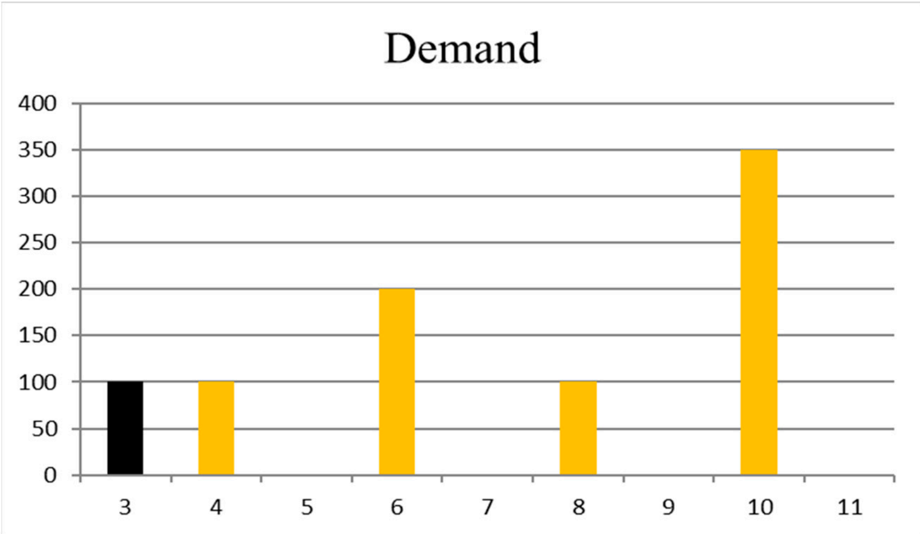

Figure 1. Future requirements, week-example.

We use the following notations:

Table 1. Notations.

A	ordering cost
r	inventory interest per year
p	the price or cost of one unit
h	the cost to keep one unit in stock for one period
X_t	the demand in period t can be zero or non-zero
$\hat{X}_0$	the demand that must be ordered now
$\hat{X}_i$	the i:th known future non-zero demand in period t_i
t_i	period when demand $\hat{X}_i$ must be available
M	A/h, a comparison number between different examples
$Q_{i,t}$	the i:th calculated and suggested order quantity necessary to be delivered in time period t (explanations $Q_{1,1} \geq \hat{X}_0$; $Q_{2,t}$ does not contain $\hat{X}_0$ and $t > 1$)

X_t is the common way, so far, to term demands that will be formed in lots. The stopping rules for Least Unit Cost and Part-Period Balancing are unaffected if X_t is zero or not, but not Silver-Meal. Therefore, $\hat{X}_i$ is proposed. The demand that must be ordered now is traditionally called X_1, here we introduce $\hat{X}_0 = X_1$ and $\hat{X}_1$ is the next forward demand that is non-zero.

We assume: $A = 200$ MU, $r = 20\%/\text{year}$, and $p = 50$ MU/unit; h then becomes 50 periods per year: $50 \cdot 0.2/50 = 0.2$ MU per unit and period. Our quota for comparison $M = A/h = 1000$.

To simplify, we "normalize" the calculations so that week 3 becomes period 1. Which is standard in most textbooks and other articles.

Then Figure 1 and Table 2 show: $\hat{X}_0 = X_1 = 100$, $t_0 = 1$; $\hat{X}_1 = X_2 = 100$, $t_1 = 2$; $\hat{X}_2 = X_4 = 200$, $t_2 = 4$; $\hat{X}_3 = X_6 = 100$, $t_3 = 6$; and $\hat{X}_4 = X_8 = 350$, $t_4 = 8$.

2.1. Solution with Silver-Meal (Silver and Meal [13])

The idea is to choose to have a new delivery when the average cost per period increases for the first time. Following the presentation and formulas in, e.g., Silver et al. [14], Axsäter [15], Nahmias [16], Olhager [17], Günter and Tempelmeier [18] and Domínguez-Machuca et al. [19], we should perform as follows:

$$\textbf{\textit{If}}\ \frac{A+h\sum_{t=1}^{k+1}(t-1)\cdot X_t}{k+1} > \frac{A+h\sum_{t=1}^{k}(t-1)\cdot X_t}{k}\ \textbf{\textit{then}}\ Q_{1,1} = \sum_{t=1}^{k} X_t.$$

Cost of only ordering the quantity in ***period 1***: $A/1$ = 200 MU/1 period = 200 MU/period.

Cost of also ordering the quantity in ***period 2***: $\left(A+h\sum_{t=1}^{2}(t-1)X_t\right)/2$ = $(200+0.2(0+1\cdot 100))/2$ = 110 MU/period.

Cost of also ordering the quantity in ***period 3***: $\left(A+h\sum_{t=1}^{3}(t-1)X_t\right)/3$ = $(200+0.2(1\cdot 100+2\cdot 0))/3$ = 220/3 = 73.33 MU/period.

Cost of also ordering the quantity in ***period 4***: $\left(A+h\sum_{t=1}^{4}(t-1)X_t\right)/4$ = $(200+0.2(1\cdot 100+2\cdot 0+3\cdot 200))/4$ = 85 MU/period.

85 > 77.33; therefore $Q_{1,1} = \sum_{t=1}^{3} X_t$ = 100 + 100 + 0 = 200 units.

Then, to calculate the next forthcoming order quantity, we start in period 4.

Cost of only ordering the quantity in ***period 4***: $A/1$ = 200 MU/1 period = 200 MU/period.

Cost of also ordering the quantity in ***period 5***: $\left(A+h\sum_{t=4}^{5}(t-4)X_t\right)/2$ = $(A+h\cdot 1\cdot t_5)/2$ = $(200+0.2\cdot 1\cdot 0)/2$ = 100 MU/period.

Cost of also ordering the quantity in ***period 6***: $\left(A+h\sum_{t=4}^{6}(t-4)X_t\right)/(6-4+1)$ = $(200+0.2(0+2\cdot 100))/3$ = 240/3 = 80 MU/period.

Cost of also ordering the quantity in ***period 7***: $\left(A+h\sum_{t=4}^{7}(t-4)X_t\right)/(7-4+1)$ = $(200+0.2(0+2\cdot 100+3\cdot 0))/4$ = 60 MU/period.

Cost of also ordering the quantity in ***period 8***: $\left(A+h\sum_{t=4}^{8}(t-4)X_t\right)/(8-4+1)$ = $(200+0.2(200+4\cdot 350))/5$ = 104 MU/period.

104 > 60; therefore $Q_{2,4} = \sum_{t=4}^{7} X_t$ = 200 + 0 + 100 + 0 = 300 units. Still to deliver are 350 units in period 8, therefore $Q_{3,8} = \sum_{t=8}^{9} X_t$ = 350 units. Total set-up and inventory holding cost: $200+0.2(1\cdot 100)+200+0.2(2\cdot 100)+200$ = 660 MU.

Table 2. Week-example.

Period	Quantity	t	i	t_i	$t_i - t_0$
3	100	1	0	1	0
4	100	2	1	2	1
6	200	4	2	4	3
8	100	6	3	6	5
10	350	8	4	8	7

2.2. Solution with Least Unit Cost (Gorham [20])

Furthermore, following the presentation and formulas in, e.g., Silver et al. [14] and Nahmias [16], we should perform as follows:

$$\textbf{\textit{If}}\ \frac{A+h\sum_{t=1}^{k+1}(t-1)\cdot X_t}{\sum_{t=1}^{k+1} X_t} > \frac{A+h\sum_{t=1}^{k}(t-1)\cdot X_t}{\sum_{t=1}^{k} X_t}\ \textbf{\textit{then}}\ Q_{1,1} = \sum_{t=1}^{k} X_t$$

As an alternative description for Least Unit Cost, find the minimum cost of:

$$C_n = \frac{A+h\sum_{i=0}^{n}(t_i-t_0)\hat{X}_i}{\sum_{i=0}^{n}\hat{X}_i}.$$

Cost of only ordering the necessary quantity in ***period 1***:

$$C_0 = \left(A+h\sum_{t=1}^{1}(t-1)X_t\right)/\sum_{t=1}^{1} X_t = A/X_1 = 200/100 = 2\ \text{MU/unit}.$$

Cost of also ordering the ***next*** demanded quantity in ***period 2***:

$$C_1 = \left(A + h\sum_{t=1}^{2}(t-1)X_t\right) / \sum_{t=1}^{2} X_t = (200 + 0.2(0 + 1 \cdot 100))/(100 + 100) = 1.10 \text{ MU/unit}$$

Cost of also ordering the ***zero*** demanded quantity in ***period 3***:

$$\left(A + h\sum_{t=1}^{3}(t-1)X_t\right) / \sum_{t=1}^{3} X_t = (200 + 0.2(1 \cdot 100 + 2 \cdot 0))/(100 + 100 + 0) = 1.10 \text{ MU/unit.}$$

A zero demand does not change the previous Least Unit Cost, so the search continues.
Cost of also ordering the ***second next***-demanded quantity in ***period 4***:

$$C_2 = \left(A + h\sum_{t=1}^{4}(t-1\right)X_t) / \sum_{t=1}^{4} X_t = \left(A + h\sum_{i=0}^{2}(t_i - t_0)\hat{X}_i\right) / \sum_{i=0}^{2} \hat{X}_i$$
$$= (200 + 0.2(1 \cdot 100 + (4-1)200))/(100 + 100 + 200) = 0.85 \text{ MU/unit.}$$

Cost of also ordering the ***third next***-demanded quantity in ***period 6***:

$$\left(A + h\sum_{t=1}^{6}(t-1\right)X_t) / \sum_{t=1}^{6} X_t = \left(A + h\sum_{i=0}^{3}(t_i - t_0)\hat{X}_i\right) / \sum_{i=0}^{3} \hat{X}_i$$
$$= (200 + 0.2(700 + (6-1)100))/(400 + 100) = 0.88 \text{ MU/unit}$$

0.88 > 0.85; therefore $Q_{1,1} = \sum_{t=1}^{4} X_t = \sum_{i=0}^{2} \hat{X}_i$ = 100 + 100 + 200 = 400 units.
Then, to calculate the next forthcoming order quantity, we start in period 6. Starting in period 5 makes no sense, with the necessary order quantity $X_6 = \hat{X}_3$ = 100 units.
Cost of only ordering the quantity in ***period 6***:

$$A/X_6 = 200/100 = 2 \text{ MU/unit.}$$

Cost of also adding the quantity in ***period 8*** when ordering the quantity necessary in period 6: $\left(A + h\sum_{i=3}^{4}(t_i - t_4)\hat{X}_i\right) / \sum_{i=3}^{4} \hat{X}_i = (200 + 0.2 \cdot 2 \cdot 350)/(100 + 350) = 0.76$ MU/unit

Therefore, $Q_{2,6} = \sum_{t=6}^{9} X_t = \sum_{i=3}^{4} \hat{X}_i$ = 100 + 350 = 450 units. This solution presents total set-up and inventory holding cost: $200 + 0.2(1 \cdot 100 + (4-1)200) + 200 + 0.2(8-6)350 =$ 680 MU.

2.3. Solution with Reformulated Silver-Meal (Lägsta Periodkostnad) (Segerstedt [12,21])

Periods with zero demand decrease the cost per period; periods with non-zero demand thereafter mostly increase the cost per period. This may lead to higher order quantities and set-up costs that are not necessary. Therefore, this solution only considers periods with non-zero demand. Segerstedt [12,21] calls this solution Silver-Meal and "Lägsta Periodkostnad".

$$\textbf{\textit{If}}\ \frac{A + h\sum_{i=0}^{n+1}(t_i - t_0)\hat{X}_i}{t_{n+1} - t_0 + 1} > \frac{A + h\sum_{i=0}^{n}(t_i - t_0)\hat{X}_i}{t_n - t_0 + 1}\ \textbf{\textit{then}}\ Q_{1,1} = \sum_{i=0}^{n} \hat{X}_i$$

which means the minimum of $C_n = \frac{A + h\sum_{i=0}^{n}(t_i - t_0)\hat{X}_i}{t_n - t_0 + 1}$ is sought.
Cost of only ordering the necessary quantity in ***period 1***:

$$C_0 = \left(A + h\sum_{i=0}^{0}(t_i - t_0)\hat{X}_i\right)/(t_0 - t_0 + 1) == 200 \text{ MU/period}$$

Cost of also ordering the ***first next***-demanded quantity in ***period 2***:

$$C_1 = \left(A + h\sum_{i=0}^{1}(t_i - t_0)\hat{X}_i\right)/(t_1 - t_0 + 1) = (200 + 0.2(2-1)100)/2 = 110 \text{ MU/period}$$

Cost of also ordering the ***second next***-demanded quantity in ***period 4***:

$$C_2 = \left(A + h\sum_{i=0}^{2}(t_i - t_0)\hat{X}_i\right)/(t_2 - t_0 + 1) = (200 + 0.2(0 + 1 \cdot 100 + (4-1)200)/4 = 85\ \text{MU}/\text{period}.$$

Cost of also ordering the ***third next***-demanded quantity in ***period 6***:

$$C_3 = \left(A + h\sum_{i=0}^{3}(t_i - t_0)\hat{X}_i\right)/(t_3 - t_0 + 1) = (200 + 0.2(1 \cdot 100 + (4-1)200) + (6-1)100))/6 = 73.33\ \text{MU}/\text{period}.$$

Cost of also ordering the ***forth next***-demanded quantity in ***period 8***:

$$C_4 = \left(A + h\sum_{i=0}^{4}(t_i - t_0)\hat{X}_i\right)/(t_4 - t_0 + 1)$$

116.25 > 77.33; therefore, $Q_{1,1} = \sum_{i=0}^{3} \hat{X}_i$ = 100 + 100 + 200 + 100 = 500 units.

Then, we start with $\hat{X}_4 = 350$ and see if we have any demand ($\hat{X}_5$) that it is favored to include. This is not the case, so $Q_{2,8} = 350$ units. The total set-up and inventory-holding cost for this solution becomes $200 + 0.2(1 \cdot 100 + (4-1)200) + (6-1)100) + 200$ = 640 MU.

2.4. Solution with Part-Period Balancing (DeMatteis [22])

The basic criterion is to select the number of demanded quantities covered by the replenishment so that the total inventory holding cost is as close as possible to the set-up cost.

When $h\sum_{i=0}^{n+1}(t_i - t_0)\hat{X}_i \geq A > h\sum_{i=0}^{n}(t_i - t_0)\hat{X}_i$ the search stops.

Then ***If*** $\frac{A}{h\sum_{i=0}^{n}(t_i - t_0)\cdot\hat{X}_i} - 1 \leq \frac{h\sum_{i=0}^{n+1}(t_i - t_0)\cdot\hat{X}_i}{A} - 1$ ***then*** $Q_{1,1} = \sum_{i=0}^{n} \hat{X}_i$ **else** $Q_{1,1} = \sum_{i=0}^{n+1} \hat{X}_i$.

This rule differs, e.g., from Axsäter [15], p. 69, which specifies that the inventory holding cost should be equal to or larger than the setup cost.

That means that the ratio between the set-up cost and the holding cost should be as close to one as possible. "As close to one as possible" is an idea involving several items with complementing restrictions that has also performed well: Economic Order Quantity (EOQ): Harris [23]; Economic Lot Scheduling Problem (ELSP): Segerstedt [12] and Holmbom and Segerstedt [24]; Joint Replenishment Problem (JRP): Nilsson et al. [25]; One-Warehouse N-Retailer (OWNR): Abdul-Jalbar et al. [26]. Table 3 presents the calculations for Part-Period Balancing.

Table 3. Solution: Part-Period Balancing, week-example.

i	$\hat{X}_i$	t_i	Inventory Holding Cost	Ratio	1/Ratio
0	100	1	0	-	-
1	100	2	0.2·1·100 = 20	200/20 = 10	0.1
2	200	4	20 + 0.2·3·200 = 140	200/140 = 1.43	0.699
3	100	6	140 +0.2·5·100 = 240	200/240 = 0.833	1.2
4	350	8	240 + 0.2·7·350 = 730	200/730 = 0.274	3.65

To order all quantities until period 6 presents the ration closest to one. Therefore, $Q_{1,1}$ is 500 units; left to deliver is $Q_{2,8}$ = 350 units. This solution presents the following total set-up and inventory holding cost: $200 + 0.2(1 \cdot 100 + (4-1)200) + (6-1)100) + 200$ = 640 MU.

2.5. Solution with Dynamic Programming (Wagner and Whitin [27])

An optimal solution to this problem with dynamic programming is what reformulated Silver-Meal (*rSM*) and Part-Period Balancing (*PPB*) have already shown: $Q_{1,1} = 500$ units, $Q_{2,8} = 350$ units. A better solution than 640 MU cannot be found.

3. A numerical Day-Example

We continue with a day-example. We know that we must order 300 units for a special project to be delivered in period 1. Five days later, in period 6, we have a new demand for 300 units of the same item; we have complementary future demands according to Table 4. The question is: should we order only these 300 units, or should we also order more, and how much of the future demand should we meet at the same time to avoid costs.

Table 4. Demand day-example, dates normalized to periods.

$\hat{X}_i$	i	t	$t_i - t_0$
300	0	1	0
300	1	6	5
400	2	16	15
500	3	31	32
200	4	37	36
300	5	45	44
600	6	52	51
600	7	62	61

Table 4 and Figure 2 show the current demand: $\hat{X}_0 = 300$, $t_0 = 1$; $\hat{X}_1 = 300$, $t_1 = 6$; $\hat{X}_2 = 400$, $t_2 = 16$; $\hat{X}_3 = 500$, $t_3 = 31$; $\hat{X}_4 = 200$, $t_4 = 37$; $\hat{X}_5 = 300$, $t_5 = 45$; $\hat{X}_6 = 600$, $t_6 = 52$; $\hat{X}_7 = 600$, $t_7 = 62$. The estimated order cost is A = 2000 MU, r = 15 %/year, and p = 150 MU/unit. h then becomes with 360 periods/days per year: $150 \cdot 0.15/360 = 0.0625$ MU per unit and day. $M = A/h = 32{,}000$.

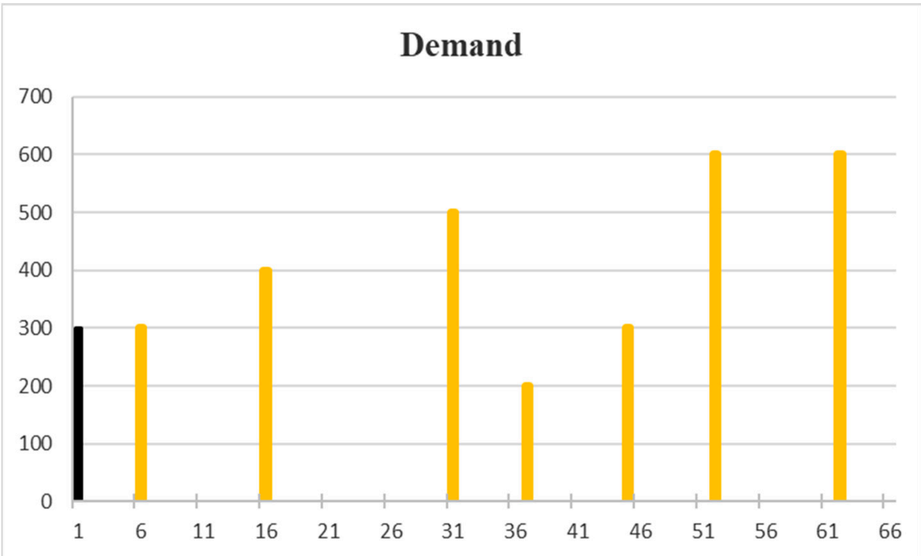

Figure 2. Future requirements, day-example.

We do not repeat the formulas; we present the solutions for the different methods in Table 5; Silver-Meal (*S-M*), Least Unit Cost (*LUC*), reformulated Silver-Meal (*rSM*), and Part-Period Balancing (*PPB*).

Table 5. Solutions day-example. Superscript i means i-replenishment.

i	$\hat{X}_i$	t	$t_i - t_0$	$h\sum_{j=0}^{i}(t_j - t_0)\hat{X}_j$	*S-M*	*LUC*	*rSM*	*PPB*
0	300	1	0	0	2000^{1}	6.667^{1}	2000.0^{1}	-
1	300	6	5	93.75	348.96^{1}	3.490^{1}	348.96^{1}	-
2	400	16	15	468.75	2000^{2}	2.469^{1}	154.30^{1}	-
3	500	31	30	1406.25	2000^{3}	2.271^{1}	109.87^{1}	-
4	200	37	36	1856.25	296.43^{3}	2.268^{1}	104.27^{1}	0.928; 1.08
5	300	45	44	2681.25	2000^{4}	6.667^{2}	104.03^{1}	0.746; 1.34
6	600	52	51	4593.75	282.81^{4}	2.514^{2}	2000.0^{2}	-
7	600	62	61	6881.25	2000^{5}	1.933^{2}	138.66^{2}	-

Silver-Meal (*S-M*) (as it is presented in Silver et al. [14], Nahmias [16], Axsäter [15], etc.) results in the fact that to meet the first demand, the next should also be ordered. For $t = 5$, the total cost per period is $2000/5 = 400$; for $t = 6 = t_{1,}$ the total cost per period is $(2000 + 93.75)/6 = 348.96$, still a decrease. For $t = 15$, the total cost per period is $(2000 + 93.75)/15 = 139.58$. For $t = 16 = t_2$, the total cost per period is $(2000 + 93.75 + 400 \cdot 15 \cdot 0.0625)/16 = (2000 + 468.75)/16 = 154.30$. $154.30 > 139.58$. Therefore, the "optimal" first lot size, according to *S-M*, is $300 + 300 = 600$. For $t = 30$, the total cost per period for the new lot size is $2000/(30 - 16 + 1) = 133.33$; for $t = 31 = t_3$, the total cost per period for the new lot size is $(2000 + 500 \cdot (30 - 15) \cdot 0.0625/(31 - 16 + 1)$ $= 154.30$, $154.30 > 113.33$. Therefore, the "optimal" second lot size according to *S-M* is only 400. A continuation of *S-M* results in the following order quantities presented in Table 4: $Q_{1,1} = 600$, $Q_{2,16} = 400$, $Q_{3,31} = 700$, $Q_{4,45} = 900$, and $Q_{5,62} = 600$.

Least Unit Cost (*LUC*) for $t = 1, 2 \ldots, 5$ the total cost per unit is $2000/300 = 6.667$. For $t = 6 = t_1$, the total cost per unit becomes $(2000 + 93.75)/(300 + 300) = 3.490$; similarly, for $t = 7, 8, \ldots, 15$. For $t = 16 = t_{2,}$ the total cost per unit becomes $(2000 + 468.75)/(600 + 400)$ = 2.469. *LUC* continues to decrease until $t = 45 = t_5$, then total cost per unit becomes $(2000 + 2681.25)/(1700 + 300) = 2.341$, $2.341 > 2.268$ (total cost per unit for $t = 44$); that means that the next four demands should be ordered, i.e., $300 + 300 + 400 + 500 + 200 = 1700$ units. Therefore, a new lot size will start for $t = 45 = t_5$ with a unit cost $2000/300 = 6.667$. For $t = 52 = t_6$, the unit cost for the new lot becomes $200 + 600 \cdot 0.0625(52 - 45)/(300 + 600) = 2.541$. Instead of five quantities for *S-M*, *LUC* leads to two: $Q_{1,1} = 1700$ and $Q_{2,45} = 1500$.

Reformulated Silver-Meal (*rSM*) starts with, like *S-M*, for $t = 1 = t_{0,}$ a period cost of $2000/1 = 2000$ MU/day. For $t = 6 = t_1$, the period cost is $(2000 + 0.0625 \cdot 5 \cdot 300)/6 = 348.96$. For $t = 16 = t_2$, the period cost is $(2000 + 468.75)/(15 + 1) = 154.30$. *rSM* continues to decrease to $t = 45 = t_5$, $(2000 + 2681.25)/45 = 104.03$. However, for $t = 52$ $= t_6$, *rSM* becomes $(2000 + 4593.75)/52 = 126.80$. Therefore, *rSM* results in the following order quantities: $Q_{1,1} = 2000$ and $Q_{2,52} = 1200$.

Net least period cost (*nLPC*) results in three order quantities: 1000, 1000, and 1200, Two less than *S-M* and one more than *rSM*.

Part-Period Balancing (*PPB*) recommends that $Q_{1,1} = 1700$; because first for $i = 5$ the inventory holding cost is larger than the setup cost; then a comparison is made with $i = 4$. Which ratio is closest to one? $i = 4$ is closest. For the rest of the planning horizon, the inventory holding costs will never exceed the new ordering costs; therefore, $Q_{2,45} = 1500$.

With these order quantities, the total cost for the complete planning horizon becomes:

S-M: $2000 + 93.75 + 2000 + 2000 + 75 + 2000 + 262.5 + 2000 = 10431.25$ MU.

LUC and *PPB*: $2000 + 1826.25 + 2000 + 900 = 6726.25$ MU.

rSM: $2000 + 2681.25 + 2000 + 375 = 7056.25$ MU.

nLPC: $2000 + 468.75 + 2000 + 337.50 + 2000 + 375 = 7181.25$ MU.

Dynamic Programming: 6726.25 MU.

4. Silver and Miltenburg's Examples

Silver and Miltenburg [8] present two interesting and informative examples that we will recapitulate here: Tables 5 and 6. The example in Table 6 shows a sharp decline in demand. The example illustrates *S-M*'s difficulty in treating decreasing demand, and it also triggers the modifications suggested in Silver and Miltenburg [8]. They state that $M = 100$ and *S-M* results in a single replenishment of 350 units at the start of period 1 and a normalized cost of 520. That means that $A = 100$ and $h = 1$. We show here the solutions with *LUC*, *rSM*, and *PPB*.

Table 6. Solutions: Silver and Miltenburg's declining demand. Superscript *i* means *i*-replenishment.

i	$\hat{X}_i$	t	$t_i - t_0$	$h(t_i - t_0)\hat{X}_i$	$h\sum_{j=1}^{i}(t_j - t_0)\hat{X}_j$	*S-M*	*LUC*	*rSM*	*PPB*
0	150	1	0	0	0	100 ^{1}	0.667 ^{1}	100 ^{1}	- ^{1}
1	95	2	1	95	95	97.5 ^{1}	0.796–1.053 ^{2}	97.5 ^{1}	0.95; 1.05 ^{1}
2	40	3	2	80	175	91.7 ^{1}	1.037 ^{2}	91.7 ^{1}	1.75; 0.57 ^{2}
3	30	4	3	90	265	91.3 ^{1}	1.212–3.333 ^{3}	91.3 ^{1}	- ^{2}
4	20	5	4	80	345	89 ^{1}	2.4 ^{3}	89 ^{1}	0.7; 1.43 ^{2}
5	15	6	5	75	420	86.7 ^{1}	2.308 ^{3}	86.7 ^{1}	1.15; 0.87 ^{2}

LUC finds the same solutions as Wagner-Whitin, $Q_{1,1} = 150$, $Q_{2,2} = 135$, and $Q_{3,4} = 65$; for a total cost of 390. *PPB* cannot stop at period 1 because it has no inventory holding costs; *PPB* therefore finds $Q_{1,1} = 245$ and $Q_{2,3} = 105$, with a total cost: 100 + 95 + 100 + 30 + 40 + 45 = 410. There are no periods of zero demand; therefore, *rSM* presents the same solution as *S-M*, $Q_{1,1} = 350$ and a total cost 520.

Silver and Miltenburg [8] present another example, Table 7, with sharply varying demand and periods with zero demand. $A = 70$ and $h = 0.25$; i.e., $M = 280$.

Table 7. Solutions: Silver and Miltenburg's varying demand. Superscript *i* means *i*-replenishment.

i	$\hat{X}_i$	t	$t_i - t_0$	$h(t_i - t_0)\hat{X}_i$	$h\sum_{j=1}^{i}(t_j - t_0)\hat{X}_j$	*S-M*	*LUC*	*rSM*	*PPB*
0	179	1	0	0	0	70 ^{1}	0.391 ^{1}	70 ^{1}	- ^{1}
1	44	2	1	11	11	40.5 ^{1}	0.363 ^{1}	40.5 ^{1}	- ^{1}
2	10	7	6	15	26	13.5→13.7–70 ^{2}	0.412–7 ^{2}	13.7 ^{1}	2.69; 0.37 ^{1}
3	123	11	10	307.5	333.5	17.5→38.6–70 ^{3}	1.45 ^{2}	36.7–70 ^{2}	0.210;4.76 ^{2}
4	55	15	14	192.5	526	23.3→25–70 ^{4}	1.61–1.27 ^{3}	25 ^{2}	0.78; 1.27 ^{2}
5	19	22	21	99.75	625.75	10→12.9–70 ^{5}	1.40–3.68 ^{4}	15.7 ^{2}	1.53; 0.65 ^{3}
6	174	25	24	1044	1669.75	23.3→50.1–70 ^{6}	1.038 ^{4}	52.7–70 ^{3}	
7	16	26	25	100	1769.75	37 ^{6}	1.035 ^{4}	37 ^{3}	

Silver-Meal (*S-M*) finds a solution:

$Q_{1,1} = 223$, $Q_{2,7} = 10$, $Q_{3,11} = 123$, $Q_{4,15} = 55$, $Q_{5,22} = 19$, and $Q_{6,25} = 190$; with a total cost: 70 + 11 + 70 + 70 + 70 + 70 + 70 + 4 = 435.

Least Unit Cost (*LUC*) finds a solution:

$Q_{1,1} = 233$, $Q_{2,7} = 133$, $Q_{3,15} = 55$, and $Q_{4,22} = 209$; with a total cost: 70 + 11 + 70 + 123 + 70 + 70 + 130.5 + 16 = 560.5.

Reformulated Silver-Meal (*rSM*) finds a solution:

$Q_{1,1} = 233$, $Q_{2,11} = 197$, and $Q_{3,25} = 190$; with a total cost: 70 + 26 + 70 + 55 + 52.25 + 70 + 4 = 347.25.

Part-Period Balancing (*PPB*) finds a solution:

$Q_{1,1} = 233$, $Q_{2,11} = 178$, $Q_{3,22} = 193$, and $Q_{4,26} = 16$; with a total cost: 70 + 26 + 70 + 55 + 70 + 130.5 + 70 = 491.5. However, the odd end can easily be adjusted to $Q_{1,1} = 233$, $Q_{2,11} = 178$, and $Q_{3,22} = 209$; with a total cost 437.5.

The Wagner-Whitin algorithm finds a solution:

$Q_{1,1} = 233$, $Q_{2,11} = 123$, $Q_{3,15} = 74$, and $Q_{4,25} = 190$; with a total cost: 70 + 26 + 70 + 70 + 33.25 + 70 + 4 = 343.25. Normalized to 343.25/0.25 = 1373.

Compared to the examples in Tables 1 and 4, examples in Tables 5 and 6 have small *M*-values, i.e., low order costs and/or high inventory holding costs. Solving the problem in Table 6, with $A = 70$ and $h = 0.025$; i.e., $M = 2800$, in our opinion with more realistic values, we find the following solutions:

S-M: $Q_{1,1} = 233$, $Q_{2,11} = 197$, and $Q_{3,25} = 190$; with a total cost: 223.725.

LUC: $Q_{1,1} = 356$ and $Q_{2,15} = 264$; with a total cost: 224.575.

rSM, *PPB*, and Wagner-Whitin: $Q_{1,1} = 430$ and $Q_{2,25} = 190$; with a total cost: 202.975.

5. Conclusions/Reflections

Literature and textbooks have traditionally not been considered in *periods of zero demand*. Today, a construction company and other companies with project-oriented production of products and services must satisfy demand for different quantities on different days. Between the delivery dates, there will be several days without demand. Lot-sizing methods must adapt to "days of inventory". Our examples verify the conclusions already formed by Silver and Miltenburg [8], Bookbinder and Tan [7], and Ho et al. [11]: the traditional way to present Silver-Meal makes it unsuitable to treat situations with zero demand periods. However, with a rather simple reformulation, Silver-Meal avoids this problem and calculates the true least-period cost. What Ho et al. [11] suggest does not calculate the true least-period cost. If it is a cost per period, the total cost must be divided by all involved and covered periods. (It is surprising that Ho et al. [11], or their reviewers, did not suggest *rSM*).

Our examples show that even a reformulated Silver-Meal still has difficulties with a *sharp decline in demand* (both *S-M* and *rSM*). In such a situation, it would be advantageous to include an additional constraint to eliminate set-ups covering a very large number of periods (Axsäter [28]). Otherwise, *LUC* or *PPB* may be preferable.

LUC has difficulties with *sharply varying demand*, especially with low order costs and high inventory holding costs. Therefore, it may not be a method suitable for a construction company or something similar with varying demand.

A reformulated Silver-Meal (*rSM*) avoids the problem *S-M* has with periods with zero demand. However, in some circumstances, *rSM*, like *S-M*, can stay at a local minimum and not the true minimum (noticed in Silver et al. [14], p. 212, and the example in Table 4). *LUC* is not harmfully affected by periods of zero demand. The same applies for *PPB*, which, in our investigation, has favorable characteristics.

The difference between *rSM* and *S-M* may look more complicated than it is. The problem we point to is that textbooks so far tell *S-M*: calculate a new least period cost for every period; we say: calculate a new least period cost only for periods with demand. For example, Segerstedt [12,21], call it *Silver-Meal*, but update the formulas.

In a practical application, there is generally a "rolling horizon"; when we have decided the first order quantity, we soon have new future demand to consider. An optimal solution only considering what we know presently (for example, in Tables 1, 3, 5 and 6) will seldom be optimal in the long run (Blackburn and Millen [5]). In a practical situation, the first necessary quantity is ordered, then after a while, another lot size is necessary to order, and then there are new future demands to consider. Furthermore, from Silver and

Miltenburg [8], "In many cases, in actual practice, the demand pattern continues beyond the planning horizon, and lot sizing is done on a rolling schedule basis. The basic heuristic can actually outperform the Wagner-Whitin approach in a rolling schedule environment". Therefore, heuristics, treated here and in textbooks, will help managers and performers in industry improve operations. A proper textbook should contain and treat *Silver-Meal*, *Least Unit Cost,* and *Part-Period Balancing*.

6. Suggested Extensions

We have not found a thorough literature review about what is published about lot sizing techniques the last 25–30 years, to the best of our knowledge. We have noticed other literature reviews that do not cover the techniques we treat here. Our target was to focus on how textbooks present these techniques. One hypothesis is that previous claims about the importance of lot-sizing techniques for Material Requirements Planning (MRP) were exaggerated. A forthcoming literature review must cover what is published and should cover what and how the techniques are used.

Previous studies of which technique is best, now appear obsolete. A future simulation study is suggested with different M values (=A/h), stochastic demand quantities, and stochastic periods with frequent zero demand, where these three lot sizing techniques are competing to show the least total cost. Examined with ANOVA analyses or similar. Compared to a reformulated Silver-Meal, a not reformulated Silver-Meal would be a loser in such a study.

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