

## Article

# Non-Parametric Hypothesis Testing for Unknown Aged Class of Life Distribution Using Real Medical Data

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**Abstract:** Over the last few decades, the statisticians and reliability analysts have looked at putting exponentiality to the test using the Laplace transform technique. The non-parametric statistical test used in this study, which is based on this technique, evaluates various treatment modalities by looking at failure behavior in the survival data that were gathered. Following use of the suggested strategy, patient survival times are recorded. In this investigation, it was presupposed that the Laplace transform order of (UBAC2) attribute or the constant failure rate would determine how the observed data behave (exponential scenario). If the survival data are exponential, the recommended treatment approach is ineffective. If the survival data are UBAC2L, the technique in use produces a better or a higher expected total present value than an older one governed by an exponential survival function (discussed in the Applications section). The efficiency and critical values of the test are calculated and compared to those of other tests in order to ensure that the suggested statistical test is applied correctly.

**Keywords:** reliability theory; medical statistics; UBAC2L; Laplace transform; Non-Parametric Hypothesis Testing; U-statistic and Pittman asymptotic efficiency

**MSC:** 62G10; 62G20



**Citation:** Bakr, M.E.; A. Al-Babtain, A. Non-Parametric Hypothesis Testing for Unknown Aged Class of Life Distribution Using Real Medical Data. *Axioms* **2023**, *12*, 369. <https://doi.org/10.3390/axioms12040369>

Academic Editor: Stelios Zimeras

Received: 15 February 2023

Revised: 24 March 2023

Accepted: 1 April 2023

Published: 11 April 2023



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## 1. Introduction

Reliability engineering uses a variety of ageing criteria to describe how cohesive engineering systems or their constituent elements decay. Reliability engineering introduced classes of life distributions and their various forms; these classes of life distributions have uses in biological research, biometrics, engineering, maintenance, and social sciences. Reliability engineering has also produced a number of ageing criteria that describe how coherent engineering systems or their component pieces deteriorate with time. Maintenance engineers and designers can use these components to create the finest maintenance methods. A fundamental idea in probability, statistics, and other related fields including reliability theory, survival analysis, and economics is the stochastic comparison of probability distributions. As a result, statisticians and reliability analysts are growing more and more Interested in modeling life distribution classes based on specific aging-related factors utilizing survival data. Over the past few decades, a variety of life distributions have been put forth in an effort to represent various aspects of aging: IFA, IFRA, UBA, UBAC, UBAC2, and UBAC2L are the most well-known of these classes. For more details, one can refer to Barlow and Proschan [1], Deshpand et al. [2], Klefsjo [3], Ahmed [4], Ali [5], and Abu-Youssef and El-Toony [6].

A key characteristic of the surviving distribution is the capacity to age. The studies provide many categories that may be used to classify distributions.

Let  $X$  be a non-negative random variable with cumulative distribution  $F(x)$ , survival functions  $\bar{F}(x)$ , and finite mean  $\mu = E(X)$ .

If  $\mu(\infty) > 0$  and  $\forall x, t \geq 0$ , Ahmad [4] introduce the used better than aged in concave (UBA) life distribution as

$$\bar{F}(t)e^{-\frac{x}{\mu(\infty)}} \leq \bar{F}(x+t), \quad x, t \geq 0.$$

Ali [5] introduce the used better than aged (UBAC2) life distribution as

$$_F(t)(1 - e^{-x}) \leq v(t) - v(x+t), \quad x, t \geq 0,$$

where  $_V(x) = \int_x^\infty _F(u)du$ .

**Definition 1.**  $F$  is said to be UBAC2L iff

$$\int_x^\infty e^{-su} \bar{F}(u)du \geq \frac{1}{s+1} e^{-st} \bar{F}(t) \text{ for all } s \geq 0. \quad (1)$$

It is obvious that  $UBA \implies UBAC2 \implies UBAC2L$ .

See Abu-Youssef and El-Toony [6].

From several aspects, statisticians and reliability analysts investigated testing exponentiality issues against various age classes of life distributions; for more information, see Ahmad [7], Rolski [8], Mahmoud and Diab [9], Abu-Youssef et al. [10], Ghosh and Mitra [11], Mahmoud et al. [12], Navarro and Pellerey [13] and Navarro [14].

Several authors, including Gadallah [15] and EL-Sagheer et al. [16], examined testing exponentiality using the Laplace transform technique after it was first investigated by Atallah et al. [17].

In this article, we will look upon the case where we have genuine data and want to compare ( $H_0$ : data is exponential against the alternative hypothesis  $H_1$ : data is UBAC2L). To test the null hypothesis against the alternative hypotheses, a test statistic must be defined. A random variable called the test statistic is used to gauge how closely a sample result adheres to one of the tested hypotheses.

The remaining portion of this article is divided into two key parts. For assessing whether  $H_0$  is exponential or whether  $H_1$  :  $F$  belongs to the UBAC2L class of all data and not exponential, we first provide a test statistic based on the Laplace transform technique. Pitman asymptotic efficiency for the LFR, Makeham, and Weibull distributions are produced. For sample sizes of  $n = 5(5)100$ , power estimates and Monte Carlo null distribution critical points are calculated. The majority of the topics we already looked at in the first section are covered in the second, which looks at right-censored data. A discussion of sets of both complete and censored real-world data then highlights the applicability of our test.

The following is how the paper is structured: In Section 2, we provide a test statistic for complete data based on the Laplace transform technique; Power estimates are calculated, and critical values are simulated for various sample sizes.. The test statistic for censored data is obtained in Section 3. Finally, in Section 4, we discuss some applications to demonstrate the utility of the proposed statistical test.

## 2. Exponentiality Departure Measure

In order to measure the departure from exponentiality in the approach of the UBAC2L class, we first develop a statistic. Pitman asymptotic efficiency, Monte Carlo null distribution critical points, and powers for common substitutes are used to do this.

### 2.1. Testing Exponentiality versus UBAC2L Class of Complete Data

Consider  $F(x) = 1 - e^{-\beta x}$  for  $\beta, x > 0$  as the exponential class. The official purpose of our experiment is to compare  $H_0$  :  $F$  is exponential with  $H_1$  :  $F$  is UBAC2L.

A measure of deviation from  $H_0$  in comparison to  $H_1$  is provided by the following lemma. As a result, a testing strategy may be created using it.

**Lemma 1.**

$$\delta(s) = \frac{1}{s^2} E \left[ (1 - e^{-sx}) - \frac{2s+1}{2(s+1)} (1 - e^{-2sx}) \right] \quad (2)$$

**Proof.** We suggest using the functional  $\delta(s)$  described below to calculate the UBAC2L distribution's departure from exponentiality:

$$\begin{aligned} \delta(s) &= \int_0^\infty \left[ \int_x^\infty e^{-su} \bar{F}(u) du - \frac{1}{s+1} e^{-st} \bar{F}(t) \right] e^{-sx} dx \\ &= I - II, \end{aligned}$$

where

$$\begin{aligned} I &= \int_0^\infty \int_x^\infty e^{-su} \bar{F}(u) e^{-st} du dt, \\ &= \frac{1}{s^2} E \left[ (1 - e^{-sx}) - \frac{1}{2} (1 - e^{-2sx}) \right], \end{aligned}$$

and

$$\begin{aligned} II &= \int_0^\infty e^{-st} \bar{F}(t) e^{-st} dt, \\ &= \frac{1}{2s} E [ (1 - e^{-2sx}) ]. \end{aligned}$$

Hence, the result follows.

Based on a random sample  $X_1, X_2, \dots, X_n$  from the distribution function  $F$ , then the empirical estimator form  $\delta(s)$  in (2) may thus be written as follows:

$$\hat{\delta}_n(s) = \frac{1}{s^2 n} \sum_i \left\{ (1 - e^{-sx_i}) - \frac{2s+1}{2(s+1)} (1 - e^{-2sx_i}) \right\}.$$

So,  $\hat{\delta}_n(s)$  can be obtained as

$$\hat{\delta}_n(s) = \frac{1}{s^2 n} \sum_i \varnothing(X_i),$$

where

$$\varnothing(X_i) = \left\{ (1 - e^{-sx_i}) - \frac{2s+1}{2(s+1)} (1 - e^{-2sx_i}) \right\}.$$

One may use the U-statistic theory to obtain the limiting distribution of  $\hat{\delta}_n(s)$ .

Set

$$\varnothing(X) = (1 - e^{-sx}) - \frac{2s+1}{2(s+1)} (1 - e^{-2sx}).$$

The next theorem establishes the statistic reported in (2) as having asymptotic normality.  $\square$

**Theorem 1.** As  $n \rightarrow \infty$ ,  $(\hat{\delta}_n(s) - \delta(s))$  is asymptotically normal with mean 0 and variance  $\sigma^2$  is given in (3), under  $H_0$ , the variance reduces to (4).

**Proof.** U-statistic theory (Lee [18]) yields the following:

$$E[\varnothing(X)] = E\left((1 - e^{-sx}) - \frac{2s+1}{2(s+1)}(1 - e^{-2sx})\right),$$

$$\sigma^2 = E\left(\left((1 - e^{-sx}) - \frac{2s+1}{2(s+1)}(1 - e^{-2sx})\right)^2\right) \quad (3)$$

One may demonstrate that  $\mu_0 = 0$  under  $H_0$ , and the variance is

$$\sigma_0^2(s) = \frac{2}{(1+s)^2(1+9s+26s^2+24s^3)}. \quad (4)$$

□

## 2.2. Monte Carlo Null Distribution Critical Points

The highest percentile points of our test  $\hat{\delta}_n(s)$ ,  $n = 5(5)100$  of UBAC2L class of life distributions are obtained using a 10,000-sample simulated sample from the exponential distribution using wolfram mathematica 10 in Table 1.

**Table 1.** The Higher Percentile Values of  $\hat{\delta}_n(0.09)$ .

n	90%	95%	99%
5	0.868731	1.2161	1.57217
10	0.488618	0.836443	1.23035
15	0.410873	0.611245	0.796065
20	0.361404	0.491752	0.696427
25	0.36791	0.485149	0.554637
30	0.298835	0.408053	0.433742
35	0.286803	0.404571	0.479514
40	0.235469	0.345774	0.396459
45	0.231345	0.322356	0.372959
50	0.252539	0.321946	0.377091
55	0.231475	0.330027	0.394961
60	0.250355	0.314627	0.360007
65	0.201125	0.283754	0.314362
70	0.186574	0.232399	0.27165
75	0.204348	0.266098	0.319055
80	0.187613	0.27441	0.332885
85	0.169602	0.229449	0.298501
90	0.18389	0.240486	0.298516
95	0.166333	0.213038	0.280515
100	0.161628	0.2132	0.24905

Table 1 shows that as the confidence level rises, the critical values rise, and as the sample size rises, the critical values fall.

### 2.3. Pittman Asymptotic Relative Efficiency

The PAE of our test  $\delta(s)$ , is investigated in this subsection, where

$$\text{PAE}(\delta) = \frac{\left| \frac{\partial}{\partial \theta} \delta \right|_{\theta \rightarrow \theta_0}}{\sigma_0} = \frac{1}{\sigma_0} \left| \frac{2s+1}{2s^2(s+1)} \int_0^\infty e^{-2sx} d\bar{F}'_{\theta_0}(x) - \frac{1}{s^2} \int_0^\infty e^{-sx} d\bar{F}'_{\theta_0}(x) \right|,$$

where  $\bar{F}'_{\theta_0}(x) = \frac{d}{d\theta} F_\theta(u) \Big|_{\theta \rightarrow \theta_0}$ .

Commonly used alternatives to the exponential model include the LFR, Makeham, and Weibull distributions. For  $\theta > 0$ , it is obvious that the LFR and Makeham distributions belong to the UBAC2L class, while the Weibull distribution belongs to the UBAC2L class for  $\theta > 1$ .

This leads to the following:

$$\text{PAE}(\delta, \text{LFR}) = \frac{1}{\sigma_0} \left| \frac{3+3s+s^2}{(1+s)^3} \right|.$$

$$\text{PAE}(\delta, \text{Makeham}) = \frac{1}{\sigma_0} \left| \frac{5+4s+s^2}{2(1+s)^2(2+s)} \right|.$$

$$\text{PAE}(\delta, \text{weibull}) = \frac{1}{\sigma_0} \left| \frac{1}{s} \left( \frac{\ln(1+s) + \gamma - 1}{(1+s)^2} - (\gamma - 1) \right) \right|, \quad \gamma(\text{Euler constant}).$$

A comparison of our test  $\hat{\delta}_n(s)$  to those of Ahmed et al. [19] ( $\delta_n^{(5)}$ ) and Abu-Youssef and El-Toony [6] ( $\delta_{U_{2L}}$ ) is proposed in Table 2 for selected values of  $s$ .

**Table 2.** PAE of  $\delta_n^{(1)}$ ,  $\delta_{U_{2L}}$  and  $\hat{\delta}(s)$ .

Distribution	$\delta_n^{(5)}$	$\delta_{U_{2L}}$	$\hat{\delta}_n(s)$			
			$s=0.09$	$s=0.1$	$s=0.3$	$s=0.5$
LFR	1.1456	1.3	1.385	1.379	1.262	1.225
Makeham	0.5455	0.58	0.564	0.565	0.562	0.573
Weibull	—	—	1.018	1.017	0.759	0.932

Table 2 shows that statistic  $\hat{\delta}_n(s)$  performs well for the three alternative families and is more efficient than statistics  $\delta_n^{(5)}$  and  $\delta_{U_{2L}}$ .

### 2.4. Power Estimates for Different Alternatives

The test statistic's power estimate is essential for determining how sensitive the test is to divergence from exponentiality towards the UBAC2L class. The greater the power estimates, the better the test statistic's ability to identify this deviation. At the significant level  $\alpha = 0.05$ , Table 3 evaluates the power of the test statistics  $\hat{\delta}_n(s)$  for the LFR, Gamma, and Weibull alternatives.

According to Table 3, our test yields good powers for the LFR, Gamma, and Weibull families. The estimated powers rise as the sample size expands.

**Table 3.** Estimates of powers at  $\alpha = 0.05$ .

Distribution	n	$\theta=1$	$\theta=2$	$\theta=3$
LFR	10	1	1	1
	20	1	1	1
	30	1	1	1
Gamma	10	0.0662	0.5736	0.9716
	20	0.0673	0.7831	0.9997
	30	0.068	0.8973	1
Weibull	10	0.7024	1	1
	20	0.9398	1	1
	30	0.9453	1	1

### 2.5. Applications for Complete Data

We use specific real-world data sets to illustrate how useful the study's results are. Table 4's results show that our test was in line with those of its rivals.

Data # 1: We consider the data set of 27 observations that shows the gaps in time between repeated failures of Airplane No. 7913's air cooling system (Proschan [20]). Table 4 makes it clear that the exponentiality null hypothesis is not rejected.

Data # 2: The data set representing "Fatigue life of 6061-T6 aluminium coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second" is introduced by Engelhardt et al. [21]. The same data are performed here as well. Our test rejects the exponentiality null hypothesis.

**Table 4.** Detailed statistics, for data sets.

Data # 1	$\hat{\delta}_n(s) = -149.074$
Data # 2	$\hat{\delta}_n(s) = 0.06575$

## 3. Testing Exponentiality for Censored Data

When subjects are unreachable after a research period, this is referred to as censored data or censored observations. Since the date of survival or the end of the sickness is unclear, some patients in some fields, such as biological sciences, may still be alive or disease-free at the conclusion of the study.

### 3.1. Test for UBAC2L in Case of Right-Censored Data

Here, a test statistic is suggested to compare:  $H_0$  and  $H_1$  using right censored data. Suppose  $n$  components are put on test, and  $X_1, X_2, \dots, X_n$  denote their complete-life time. Let that  $X_1, X_2, \dots, X_n$  be independent and identically distributed (i.i.d.) according to a continuous life distribution  $F$ .

The data in this subsection have been randomly right censored, and a test statistic is provided to test  $H_0$  against  $H_1$ .

Let's write the test statistic as follows:

$$\delta_c(s) = \frac{1}{ns^2} \left[ (1 - \theta(s)) - \frac{2s+1}{2(s+1)} (1 - \tau(2S)) \right], \quad (5)$$

where

$$\theta(s) = \int_0^\infty e^{-sx} dF(x), \quad \tau(2S) = \int_0^\infty e^{-2sx} dF(x)$$

$$\hat{\theta}(s) = \sum_{m=1}^l e^{-sZ(m)} \left( \prod_{p=1}^{m-2} C_p^{\delta_p} - \prod_{p=1}^{m-1} C_p^{\delta_p} \right)$$

$$\hat{\tau}(2s) = \sum_{m=1}^l e^{-2sZ_{(m)}} \left( \prod_{p=1}^{m-2} C_p^{\delta_p} - \prod_{p=1}^{m-1} C_p^{\delta_p} \right)$$

$$\eta = \sum_{j=1}^l \prod_{k=1}^{j-1} C_m^{\delta_m} (Z_{(j)} - Z_{(j-1)})$$

$$dF_n(Z_i) = \prod_{q=1}^{j-2} C_i^{\delta_i} - \prod_{q=1}^{j-1} C_i^{\delta_i},$$

$$\bar{F}_n(t) = \prod_{m < t} C_m^{\delta_m},$$

$$C_m = \frac{n-m}{n-m+1}, t \in [0, z_{(m)}].$$

For the censored data, let's simulate the upper percentiles once more in Table 5 using wolfram mathematica 10.

**Table 5.** The Higher Percentile Values of  $\delta_c(0.09)$ .

n	95%	98%	99%
5	9.06044	11.3263	11.3263
10	3.76773	4.24107	4.45821
15	2.16296	2.53999	2.78284
20	1.39771	1.64111	1.73429
25	0.946225	1.09012	1.23866
30	0.713409	0.852922	0.914456
35	0.599153	0.684245	0.774385
40	0.480519	0.543262	0.576404
45	0.424666	0.487289	0.534101
50	0.340545	0.394071	0.427245
55	0.300065	0.347867	0.374279
60	0.263235	0.29784	0.354673
65	0.241464	0.279319	0.306573
70	0.219645	0.253037	0.269572
75	0.194965	0.228086	0.255792
80	0.174686	0.209678	0.230627
85	0.158488	0.185419	0.205321
90	0.147107	0.16895	0.184741
95	0.136914	0.156613	0.166537
100	0.124793	0.142524	0.158562

The critical values roughly drop with increasing sample size and increase with increasing confidence levels as shown in Table 5.

### 3.2. Applications

We apply the findings to various real data sets to demonstrate the use of the conclusions presented in this article for censored data.

Data # 3: We take into account the data set in Kamran Abbas et al. [22] regarding the survival periods, in weeks, of 61 patients with incurable lung cancer treated with cyclophosphamide.

We calculate the statistic in (5) for  $s = 0.09$  and  $\delta_c = 0.6178$ , so we reject the null hypothesis as  $n = 61$ .

#### 4. Conclusions

A statistical test method has been developed in this study to assist in the quality assessment of possible diseases' treatment data. Our tests' outcomes showed whether the proposed methods had a favorable or unfavorable effect on the patients' survival times. The suggested statistical test's efficiency was calculated and compared to other tests to make sure it generates reliable results. Independent of the type of treatment method being utilized, the proposed test can be used to assess the success of any treatment approach in any area of study. However, it is not advised to compare many unique treatment programs using this non-parametric test. Moreover, it is suggested that novel, extremely efficient non-parametric statistical tests be developed and applied to evaluate the numerous suggested treatments. A statistical method should also be created to compare two or more different therapies for the same condition. Additionally, the recommended statistics' percentage points are simulated. The effectiveness of our suggested tests was compared to the efficacy of many other tests to demonstrate how well they work using Pitman's asymptotic relative efficiency and well-known life distributions including the LFR, Makeham, and Weibull families. The results of the research were then applied to real data sets.

**Author Contributions:** Conceptualization, M.E.B.; Methodology, M.E.B. and A.A.A.-B.; Software, M.E.B. and A.A.A.-B.; Validation, M.E.B.; Formal analysis, M.E.B. and A.A.A.-B.; Investigation, M.E.B.; Resources, M.E.B.; Data curation, M.E.B.; Writing original draft, M.E.B.; Writing—review & editing, M.E.B. and A.A.A.-B.; Supervision, A.A.A.-B. All authors have read and agreed to the published version of the manuscript.

**Funding:** This study was funded by Researchers Supporting Project number (RSP2023R156), King Saud University, Riyadh, Saudi Arabia.

**Data Availability Statement:** Data is contained within the article.

**Conflicts of Interest:** The authors declare there is no conflict of interest.

#### Abbreviations

IFR	Increasing failure rate.
IFRA	Increasing failure rate average.
UBA	Used better than age.
UBAC	Used better than age in convex order.
UBAC2	Used better than age in concave order.
UBAC2L	Laplace transform for used better than age in concave order.

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