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Existence and Stability of a Nonlinear Distributed Delayed Periodic AG-Ecosystem with Competition on Time Scales

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Abstract: The Ayala-Gilpin (AG) kinetics system is one of the famous mathematical models of ecosystem. This model has been widely concerned and studied since it was proposed. This paper stresses on a nonlinear distributed delayed periodic AG-ecosystem with competition on time scales. In the sense of time scale, our model unifies and generalizes the discrete and continuous cases. Firstly, with the aid of the auxiliary function having only two zeros in the real number field, we apply inequality technique and coincidence degree theory to obtain some sufficient criteria which ensure that this model has periodic solutions on time scales. Meanwhile, the global asymptotic stability of the periodic solution is founded by employing stability theory in the sense of Lyapunov. Eventually, we provide an illustrative example and conduct numerical simulation by means of MATLAB tools.

Keywords: competitive AG-ecosystem; distributed lags; existence and stability; time scales

MSC: 34K13; 34D23; 34N05

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1. Introduction

This paper mainly deals with the following nonlinear competitive periodic Ayala-Gilpin ecosystem with distributed lags on time scale

$$\begin{cases} \mathcal{U}^{\Delta}(\varsigma) = r_1(\varsigma) - c_{11}(\varsigma)[e^{\mathcal{U}(\varsigma)}]^{\theta_1} - c_{12}(\varsigma) \\ \quad \times \int_{-\xi_1(\varsigma)}^{0} k_1(s) e^{\mathcal{V}(\varsigma+s)} \Delta s - \phi_1(\varsigma) e^{-\mathcal{U}(\varsigma)}, \ \varsigma \in \mathbb{T}, \\ \mathcal{V}^{\Delta}(\varsigma) = r_2(\varsigma) - c_{22}(\varsigma)[e^{\mathcal{V}(\varsigma)}]^{\theta_2} - c_{21}(\varsigma) \\ \quad \times \int_{-\xi_2(\varsigma)}^{0} k_2(s) e^{\mathcal{U}(\varsigma+s)} \Delta s - \phi_2(\varsigma) e^{-\mathcal{V}(\varsigma)}, \ \varsigma \in \mathbb{T}, \\ \mathcal{U}(\varsigma) = \varphi_1(\varsigma), \ \mathcal{V}(\varsigma) = \varphi_2(\varsigma), \ \varsigma \in (-\xi, 0] \cap \mathbb{T}, \end{cases} \tag{1}$$

where $\mathbb{T}$ is a time scale, Δ is the delta (or Hilger) derivative on $\mathbb{T}$, $\mathcal{U}(\varsigma)$ and $\mathcal{V}(\varsigma)$ are the quantity densities of two vying species at moment ς, $r_i(\varsigma)(i = 1, 2)$ stands for the intrinsic birth rate, $c_{11}(\varsigma)$ and $c_{22}(\varsigma)$ are the intraspecific competition rates, $c_{12}(\varsigma)$ and $c_{21}(\varsigma)$ are the interspecific competition rates, the kernel function of distributed-lags is given by $k_i(\varsigma)(i = 1, 2)$, $\xi_i(\varsigma) > 0(i = 1, 2)$ is the distributed-lag function, $\phi_i(\varsigma)(i = 1, 2)$ is the manual control term, the constants $\theta_i > 0(i = 1, 2)$ measures the nonlinear interferences within species, $\varphi_i(\varsigma)(i = 1, 2)$ is the initial function, $\xi = \max\{\sup_{\varsigma\in\mathbb{T}} \xi_1(\varsigma), \sup_{\varsigma\in\mathbb{T}} \xi_2(\varsigma)\}$.

The proposal of the model is related to the famous experimental study of Drosophila competition. Combined with the experimental study of Drosophila competition, Ayala, Gilpin and Eherenfeld [1] put forward the following nonlinear dynamic model in 1973.

$$\begin{cases} \frac{d\mathcal{U}(\varsigma)}{d\varsigma} = r_1 \mathcal{U}(\varsigma) \left[1 - \left(\frac{\mathcal{U}(\varsigma)}{K_1} \right)^{\theta_1} - c_{12} \frac{\mathcal{V}(\varsigma)}{K_2} \right], \\ \frac{d\mathcal{V}(\varsigma)}{d\varsigma} = r_2 \mathcal{V}(\varsigma) \left[1 - \left(\frac{\mathcal{V}(\varsigma)}{K_2} \right)^{\theta_2} - c_{21} \frac{\mathcal{U}(\varsigma)}{K_1} \right], \end{cases} \tag{2}$$

where $r_i > 0(i = 1,2)$ represents inherent growth ratios. $K_i > 0(i = 1,2)$ stands for the maximum capacity of environment to species. The constant $\theta_i(i = 1,2)$ measures the nonlinear interferences within species. $c_{12} > 0$ and $c_{21} > 0$ mean the vying rate among species.

The nonlinear interferences within species are measured by constants $\theta_1 > 0$ and $\theta_2 > 0$. $c_{12} > 0$ and $c_{21} > 0$ are the measures of competition between species.

When $\theta_1 = \theta_2 = 1$, (2) becomes the below Lotka-Volterra competitive model

$$\begin{cases} \frac{d\mathcal{U}(\varsigma)}{d\varsigma} = r_1\mathcal{U}(\varsigma)\Big[1 - \frac{\mathcal{U}(\varsigma)}{K_1} - c_{12}\frac{\mathcal{V}(\varsigma)}{K_2}\Big], \\ \frac{d\mathcal{V}(\varsigma)}{d\varsigma} = r_2\mathcal{V}(\varsigma)\Big[1 - \frac{\mathcal{V}(\varsigma)}{K_2} - c_{21}\frac{\mathcal{U}(\varsigma)}{K_1}\Big]. \end{cases} \tag{3}$$

Therefore, the Ayala-Gilpin model is a generalization of Lotka-Volterra model. The parameters θ_1 and θ_2 can take any positive real number. Therefore, Ayala-Gilpin model has been widely favored since it was proposed. The kinetics properties of Ayala-Gilpin model have been extensively explored. In [2–7], the authors dealt with the persistence, extinction and attraction of AG-system. Amdouni et al. [8] investigated the existence and global exponential stability of pseudo almost periodic solutions for a generalized competitive AG-model. Korobenko et al. [9] handled the evolutionary stability of a diffusion AG-model. Zhao [10,11] studied the multiplicity of almost periodic solution and local exponential stability for two AG-systems with harvest term. If the time-lag, impulse and random effects are considered in the AG-system, many excellent results have been achieved (see [12–20]). with the exception of the classical AG-system, some extended AG-systems have also been studied (see [4,6,8,9,14,16,20–22]).

Moreover, the model (1) is more suitable for the actual situation of ecosystem. For example, the predation process is not instantaneous, but takes a period of time to complete. At the same time, the number of predators can not be increased immediately after predators prey. Therefore, time lag is common in the whole process of predation and transformation. As we all know, the environment of the ecosystem often presents certain periodic changes over time. For the harmonious coexistence of people and ecosystem, sometimes it is necessary to protect and intervene the ecosystem manually.

From the perspective of mathematical theory, it is also of great value to study model (1). According to the definition of time scale, the model (1) contains difference case and differential case. Indeed, when $\mathbb{T} = \mathbb{N}^+$, the model (1) becomes the following difference equation

$$\begin{cases} \mathcal{U}(\varsigma+1) - \mathcal{U}(\varsigma) = r_1(\varsigma) - c_{11}(\varsigma)[e^{\mathcal{U}(\varsigma)}]^{\theta_1} - c_{12}(\varsigma) \\ \quad \times \sum\limits_{s=-\xi_1(\varsigma)}^{0} k_1(s)e^{\mathcal{V}(\varsigma+s)} - \phi_1(\varsigma)e^{\mathcal{U}(\varsigma)}, \ \varsigma \in \mathbb{N}^+, \\ \mathcal{V}(\varsigma+1) - \mathcal{V}(\varsigma) = r_2(\varsigma) - c_{22}(\varsigma)[e^{\mathcal{V}(\varsigma)}]^{\theta_2} - c_{21}(\varsigma) \\ \quad \times \sum\limits_{s=-\xi_2(\varsigma)}^{0} k_2(s)e^{\mathcal{U}(\varsigma+s)} - \phi_2(\varsigma)e^{\mathcal{V}(\varsigma)}\Big], \ \varsigma \in \mathbb{N}^+, \\ \mathcal{U}(\varsigma) = \varphi_1(\varsigma), \ \mathcal{V} = \varphi_2(\varsigma), \ \varsigma \in \mathbb{N}^+. \end{cases} \tag{4}$$

When $\mathbb{T} = \mathbb{R}$, let $x(\varsigma) = e^{\mathcal{U}(\varsigma)}, y(\varsigma) = e^{\mathcal{V}(\varsigma)}$, then the model (1) becomes the following differential equation

$$\begin{cases} \frac{dx(\varsigma)}{d\varsigma} = x(\varsigma)\big[r_1(\varsigma) - c_{11}(\varsigma)[x(\varsigma)]^{\theta_1} - c_{12}(\varsigma) \\ \quad \times \int_{-\xi_1(\varsigma)}^{0} k_1(s)y(\varsigma+s)ds\big] - \phi_1(\varsigma), \ \varsigma \in \mathbb{R}, \\ \frac{dy(\varsigma)}{d\varsigma} = y(\varsigma)\big[r_2(\varsigma) - c_{22}(\varsigma)[y(\varsigma)]^{\theta_2} - c_{21}(\varsigma) \\ \quad \times \int_{-\xi_2(\varsigma)}^{0} k_2(s)x(\varsigma+s)ds\big] - \phi_2(\varsigma), \ \varsigma \in \mathbb{R}, \\ x(\varsigma) = e^{\varphi_1(\varsigma)}, \ y(\varsigma) = e^{\varphi_2(\varsigma)}, \ \varsigma \in \mathbb{R}. \end{cases} \tag{5}$$

In addition, a complex number set like $\mathbb{T} = \bigcup_{n=-\infty}^{+\infty}[n + \frac{1}{4}, n + \frac{1}{2}] \cup \{n\}$ is also a time scale. In 1988, Hilger first raised the time scale theory in his Ph.D. thesis [23], aiming at unifying difference and differential. For further study of time scale theory, please refer to the monographs [24,25] To the best my knowledge, no one has studied the periodic solution and stability of AG-system on time scales. Consequently, it is worthwhile to study the periodic dynamic behavior of (1).

The highlights of our work and the differences from previous published works are mainly manifested in two aspects. (i) We study the dynamic properties of AG-system in the sense of time scales, which can unify the differential and difference forms of AG-systems within the same framework. However, most previous studies on AG-systems have focused on the case of continuous differential equations (see some of them [6–16,22]). Previous papers dealing with pure difference AG-systems are rare. some of their papers involved discrete AG-systems containing impulsive terms (see [2,17,21]). (ii) Our model (1) is the closest to classical model (2). The model (1) only adds control harvest terms and distribution lags based on classical model (2). Compared to model (1), some previous papers on generalized AG-systems have made many modifications to classical model (2). For example, Wang et al. [4] added random terms and multiple species to their model. In [6,14,16], the authors not only added multiple species to their models, but also added nonlinear measures $\theta_{ij}(i \neq j)$ to the inter species competition term. In [20,21], the authors added Lévy and Markovian jumps to their models.

The surplus framework of the manuscript is arranged as follows. We mainly introduce the basic concepts and results of time scales, important assumptions and necessary propositions in Section 2. Section 3 focuses on finding the sufficient conditions for the existence of periodic solutions to system (1). In Section 4, the Lyapunov functional is applied to discuss the global asymptotic stability of system (1). Section 5 provides an example and numerical simulation to verify our main results. In the last Section 6, we made a brief conclusion and outlook.

2. Preliminaries

This section first concisely reviews the elementary knowledge of calculus on time scale. The following statements are taken from Refs. [24,25].

$\phi \neq \mathbb{T} \subset \mathbb{R}$ is called a time scale if $\mathbb{T}$ is closed. Define some operators as follows:

$$\sigma(\varsigma) = \inf\{\iota \in \mathbb{T} : \iota > \varsigma\}, \quad \rho(\varsigma) = \sup\{\iota \in \mathbb{T} : \iota < \varsigma\}, \quad \mu(\varsigma) = \sigma(\varsigma) - \varsigma, \ \forall\, \varsigma \in \mathbb{T},$$

then, we call that σ is jumping forward, ρ is jumping backward, and μ is the graininess.

For $\varsigma \in \mathbb{T}$, if $\varsigma > \inf \mathbb{T}$ and $\rho(\varsigma) = \varsigma$ ($\varsigma < \sup \mathbb{T}$ and $\sigma(\varsigma) = \varsigma$), then we call that ς is left-dense (right-dense). If $\rho(\varsigma) < \varsigma$ ($\sigma(\varsigma) > \varsigma$), then we call that ς is left-scattered (right-scattered). In addition, we have

$$\mathbb{T}^k = \begin{cases} \mathbb{T} \setminus \{M\}, & \mathbb{T} \text{ achieves a left-scattered maximum } M, \\ \mathbb{T}, & \text{otherwise,} \end{cases}$$

and

$$\mathbb{T}^k = \begin{cases} \mathbb{T} \setminus \{m\}, & \mathbb{T} \text{ reaches a right-scattered minimum } m, \\ \mathbb{T}, & \text{otherwise.} \end{cases}$$

For $\varpi > 0$, we call that a time scale $\mathbb{T}$ is ϖ-periodic, provided that $\forall\, \varsigma \in \mathbb{T} \Rightarrow \varsigma + \varpi \in \mathbb{T}$. Obviously, if $\mathbb{T}$ is ϖ-periodic, then $\mathbb{T}$ is unbounded above.

Definition 1. *We call that $u : \mathbb{T} \to \mathbb{R}$ is regulated iff $\lim_{t \to \varsigma^+} u(t)$ and $\lim_{t \to \varsigma^-} u(t)$ all exist (finite), $\forall\, \varsigma \in \mathbb{T}$.*

Definition 2. *We call that $u : \mathbb{T} \to \mathbb{R}$ is rd-continuous iff, for all right-dense point. $\varsigma \in \mathbb{T}$, we have $\lim_{t \to \varsigma^+} u(t) = u(\varsigma)$, and for all left-dense point $\iota \in \mathbb{T}$, we have that $\lim_{t \to \iota^-} u(t) = u(\iota^-)$ exists (finite). We denote the collection of all rd-continuous functions $u : \mathbb{T} \to \mathbb{R}$ as $C_{rd}(\mathbb{T}, \mathbb{R})$.*

Definition 3. *Let $u : \mathbb{T} \to \mathbb{R}$ and $\varsigma \in \mathbb{T}^k$. We call that a number $u^\Delta(\varsigma)$ (if exists) is the Δ-derivative of u at ς iff, for a given $\forall \epsilon > 0$, there has $\delta > 0$ such that*

$$|[u(\sigma(\varsigma)) - u(\iota)] - u^\Delta(\varsigma)[\sigma(\varsigma) - \iota]| < \epsilon|\sigma(\varsigma) - \iota|, \ \forall \iota \in (\varsigma - \delta, \varsigma + \delta) \cap \mathbb{T}.$$

The collection of first-order Δ-differentiable functions is denoted by

$$C^1_{rd}(\mathbb{T}, \mathbb{R}) = \{u : \mathbb{T} \to \mathbb{R} : \ u^\Delta(\varsigma) \text{ exists, and } u^\Delta(\varsigma) \in C_{rd}(\mathbb{T}, \mathbb{R}), \forall \varsigma \in \mathbb{R}\}.$$

According to the above definitions, one easily knows that u is Δ-differentiable $\Rightarrow$ u is continuous $\Rightarrow$ u is rd-continuous $\Rightarrow$ u is regulated.

Lemma 1. *If u is regulated, then there is a Δ-differentiable function U with differentiable region D that contents*

$$U^\Delta(\varsigma) = u(\varsigma), \ \ \forall \varsigma \in D.$$

Definition 4. *Assume that $u : \mathbb{T} \to \mathbb{R}$ is regulated, then we have the following concepts.*

(1) *We call that any function U as in Lemma 1 is a Δ-antiderivative of u.*
(2) *We define the Δ-indefinite integral of u as*

$$\int u(\varsigma)\Delta\varsigma = U(\varsigma) + c,$$

where an arbitrary constant c is a Δ-integral constant.
(3) *We define the Δ-definite integral as*

$$\int_\alpha^\beta u(\varsigma)\Delta\varsigma = U(\beta) - U(\alpha), \ \ \forall \alpha, \beta \in \mathbb{T}.$$

Lemma 2. *Let $\alpha, \beta \in \mathbb{T}$, $\lambda_1, \lambda_2 \in \mathbb{R}$, $u, v \in C_{rd}(\mathbb{T}, \mathbb{R})$, then we have the followings*

(1) $\int_\alpha^\beta [\lambda_1 u(\varsigma) + \lambda_2 v(\varsigma)]\Delta\varsigma = \lambda_1 \int_\alpha^\beta u(\varsigma)\Delta\varsigma + \lambda_2 \int_\alpha^\beta v(\varsigma)\Delta\varsigma.$
(2) $\forall \alpha \le \varsigma < \beta$, $u(\varsigma) \ge 0 \Rightarrow \int_\alpha^\beta u(\varsigma)\Delta\varsigma \ge 0.$
(3) $\forall \varsigma \in \{\varsigma \in \mathbb{T} : \alpha \le \varsigma < \beta\}$, $|u(\varsigma)| \le v(\varsigma) \Rightarrow |\int_\alpha^\beta u(\varsigma)\Delta\varsigma| \le \int_\alpha^\beta v(\varsigma)\Delta\varsigma.$

Lemma 3 ([26])**.** *For two Banach spaces $\mathfrak{X}$, $\mathfrak{Z}$, and nonempty bounded open subset $\Omega \subset \mathfrak{X}$, define some operators $\mathcal{L} : \mathfrak{X} \to \mathfrak{Z}$, $\mathcal{N} : \mathfrak{X} \times [0, 1] \to \mathfrak{Z}$, $\mathcal{Q} : \mathfrak{Z} \to \mathfrak{Z}$ and $\mathcal{J} : \mathfrak{Z} \to \mathfrak{Z}$. Assume that $\mathcal{L}$ is zero index Fredholm type, $\mathcal{N}$ is $\mathcal{L}$-compact, $\mathcal{Q}$ is projected, and $\mathcal{J}$ is homotopy. Further suppose that*

(a) *For all $0 < \eta < 1$, if x is a solution of $\mathcal{L}x = \eta\mathcal{N}(x, \eta)$, then $x \notin \partial\Omega \cap \mathrm{Dom}(\mathcal{L})$;*
(b) *If $x \in \partial\Omega \cap \mathrm{Ker}(\mathcal{L})$, then $\mathcal{QN}(x, 0)x \ne 0$;*
(c) $\deg(\mathcal{JQN}(x, 0), \Omega \cap \mathrm{Ker}(\mathcal{L}), 0) \ne 0.$

Then there has at least a $x^* \in \overline{\Omega} \cap \mathrm{Dom}(\mathcal{L})$ meeting with $\mathcal{L}x^* = \mathcal{N}(x^*, 1)$.

Lemma 4 ([11])**.** *Let $\alpha, \beta, \gamma, \vartheta > 0$, $F(x) = \alpha e^{(1+\vartheta)x} - \beta e^x + \gamma$. Assume that $\vartheta\alpha^{-\frac{1}{\vartheta}}\left(\frac{\beta}{1+\vartheta}\right)^{\frac{1+\vartheta}{\vartheta}} > \gamma$. Then we have the followings*

(1) *There exists a unique $x_0 = \frac{1}{\vartheta}\ln\left[\frac{\beta}{\alpha(1+\vartheta)}\right] \in \mathbb{R}$ such that*

$$F_{\min} = F(x_0) = -\vartheta\alpha^{-\frac{1}{\vartheta}}\left(\frac{\beta}{1+\vartheta}\right)^{\frac{1+\vartheta}{\vartheta}} + \gamma < 0,$$

and for all $x_1 < x_2$, $F(x_1) > F(x_2)$ when $x_1, x_2 \in (-\infty, x_0]$, $F(x_1) < F(x_2)$ when $x_1, x_2 \in [x_0, +\infty)$.
(2) *There are only two $x_1^*, x_2^* \in \mathbb{R}$ with $x_1^* < x_2^*$ such that $F(x_1^*) = F(x_2^*) = 0$.*

Now let's introduce some symbols.

$$\min\{[0,+\infty)\cap\mathbb{T}\}\triangleq\kappa,\ \ I_\varpi=[\kappa,\kappa+\varpi]\cap\mathbb{T},\ \ \sup_{\varsigma\in I_\varpi}w(\varsigma)\triangleq\overline{w},$$

$$\inf_{\varsigma\in I_\varpi}w(\varsigma)\triangleq\underline{w},\ \ \frac{1}{\varpi}\int_{I_\varpi}w(s)\Delta s=\frac{1}{\varpi}\int_\kappa^{\kappa+\varpi}w(s)\Delta s\triangleq\widehat{w},$$

where $w(\varsigma+\varpi)=w(\varsigma)$, $\forall\, w\in C_{rd}(\mathbb{T},\mathbb{R})$. The whole paper needs the following basic assumptions.

(H_1) Assume that $0<r_1(\varsigma)$, $r_2(\varsigma)$, $\xi_1(\varsigma)$, $\xi_2(\varsigma)$, $\phi_1(\varsigma)$, $\phi_2(\varsigma)$, $c_{11}(\varsigma)$, $c_{22}(\varsigma)$, $c_{12}(\varsigma)$, $c_{21}(\varsigma)$, $\varphi_1(\varsigma)$, $\varphi_2(\varsigma)$, $k_1(\varsigma)$, $k_2(\varsigma)\in C_{rd}(\mathbb{T},\mathbb{R})$ are all ϖ-periodic, and satisfy $\int_{-\xi}^0 k_1(s)\Delta s<\infty$, $\int_{-\xi}^0 k_1(s)\Delta s<\infty$, where $\xi=\max\{\overline{\xi_1},\overline{\xi_2}\}$.

3. Existence of Periodic Solution on Time Scales

In the portion, we shall put to use Lemma 3 to argue that system (1) has a periodic solution. Set $\mathfrak{X}=\mathfrak{Z}=W_1\oplus W_2$, where $W_1=\{w(\varsigma)\equiv(C_1,C_2)^T\in\mathbb{R}^2\}$,

$$W_2=\{w(\varsigma)=(w_1(\varsigma),w_2(\varsigma))^T:w_j(\varsigma)\in C_{rd}(\mathbb{T},\mathbb{R}),w_j(\varsigma+\varpi)=w_j(\varsigma),j=1,2\},$$

equipped with the norm

$$\|w\|=\max_{1\le j\le 2}\sup_{\varsigma\in I_\varpi}|w_j(\varsigma)|,\ \ \forall w=(w_1,w_2)^T\in\mathfrak{X}=\mathfrak{Z}. \tag{6}$$

In the manner of Ref. [27], it is easy to prove Lemmas 5–8. So, we omit their proofs.

Lemma 5. *$\mathfrak{X}=\mathfrak{Z}$ is the Banach space with the norm $||\cdot||$ defined as (6).*

Lemma 6. *$\mathcal{L}:\mathfrak{X}\to\mathfrak{Z}$ defined by*

$$\mathcal{L}w(\varsigma)=w^\Delta(\varsigma)=(w_1^\Delta(\varsigma),\,w_2^\Delta(\varsigma))^T,\ \forall\,w(\varsigma)=(w_1(\varsigma),\,w_2(\varsigma))^T\in\mathfrak{X},$$

then $\mathcal{L}$ is zero index Fredholm type.

Lemma 7. *For all $w=(\mathcal{U},\mathcal{V})^T\in\mathfrak{X}=\mathfrak{Z}$, $\mathcal{N}(w,\eta):\ \mathfrak{X}\times[0,1]\to\mathfrak{Z}$, $\mathcal{P}:\ \mathfrak{X}\to\mathfrak{Z}$ and $\mathcal{Q}:\mathfrak{Z}\to\mathfrak{Z}$ are defined by*

$$\mathcal{N}(w,\eta)=\begin{pmatrix} r_1(\varsigma)-c_{11}(\varsigma)e^{\theta_1\mathcal{U}(\varsigma)}-\eta c_{12}(\varsigma)\int_{-\xi_1(\varsigma)}^0 k_1(s)e^{\mathcal{V}(\varsigma+s)}ds-\phi_1(\varsigma)e^{-\mathcal{U}(\varsigma)}\\ r_2(\varsigma)-c_{22}(\varsigma)e^{\theta_2\mathcal{V}(\varsigma)}-\eta c_{21}(\varsigma)\int_{-\xi_2(\varsigma)}^0 k_2(s)e^{\mathcal{U}(\varsigma+s)}ds-\phi_2(\varsigma)e^{-\mathcal{V}(\varsigma)}\end{pmatrix},$$

$$\mathcal{P}w=\mathcal{Q}w=(\widehat{\mathcal{U}},\widehat{\mathcal{V}})^T=\left(\frac{1}{\varpi}\int_\kappa^{\kappa+\varpi}\mathcal{U}(\varsigma)\Delta\varsigma,\ \frac{1}{\varpi}\int_\kappa^{\kappa+\varpi}\mathcal{V}(\varsigma)\Delta\varsigma\right)^T,$$

$$\mathrm{Ker}(\mathcal{L})=\{w=(\mathcal{U},\mathcal{V})\in\mathfrak{X}:\ (\mathcal{U},\mathcal{V})=(C_1,C_2),\varsigma\in\mathbb{T}\},$$

$$\mathrm{Im}(\mathcal{L})=\{w=(\mathcal{U},\mathcal{V})\in\mathfrak{Z}:\ (\widehat{\mathcal{U}},\widehat{\mathcal{V}})=(0,0)\},$$

$$(\mathcal{L}^{-1}|_{\mathcal{P}})(w(\varsigma))=\begin{pmatrix}\int_\kappa^\varsigma\mathcal{U}(s)\Delta s-\frac{1}{\varpi}\int_\kappa^{\kappa+\varpi}\int_\kappa^\varsigma\mathcal{U}(s)\Delta s\Delta\varsigma\\ \int_\kappa^\varsigma\mathcal{V}(s)\Delta s-\frac{1}{\varpi}\int_\kappa^{\kappa+\varpi}\int_\kappa^\varsigma\mathcal{V}(s)\Delta s\Delta\varsigma\end{pmatrix}.$$

Then $\mathcal{N}(w,\eta)$ is $\mathcal{L}$-compact on $\overline{\Omega}\times[0,1]$.

Lemma 8. *Let $\mathbb{T}$ be an ϖ-periodic time scale. Suppose $\psi : \mathbb{T} \to \mathbb{R}$ be an ϖ-periodic function which is rd-continuous, then*

$$0 \le \sup_{s\in I_\varpi} \psi(s) - \inf_{s\in I_\varpi} \psi(s) \le \frac{1}{2}\int_\kappa^{\kappa+\varpi} |\psi^\Delta(s)|\Delta s.$$

(H_2) Suppose that all inequalities hold as follows:

$$\theta_1(\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})^{-\frac{1}{\theta_1}}\left(\frac{\overline{r_1}}{1+\theta_1}\right)^{\frac{1+\theta_1}{\theta_1}} > \underline{\phi_1},\ \theta_2(\underline{c_{22}}e^{-\varpi\theta_2\overline{r_2}})^{-\frac{1}{\theta_2}}\left(\frac{\overline{r_2}}{1+\theta_2}\right)^{\frac{1+\theta_2}{\theta_2}} > \underline{\phi_2}.$$

Theorem 1. *Assume that (H_1) and (H_2) are true, then there has at least an ϖ-periodic solution $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ satisfies model (1) on periodic time scale $\mathbb{T}$ such that $L^- - \varpi\overline{r_1} < \widetilde{\mathcal{U}}(\varsigma) < L^+$ and $M^- - \varpi\overline{r_2} < \widetilde{\mathcal{V}}(\varsigma) < M^+$, where $L^\pm$ and $M^\pm$ can be solved by the below equations*

$$\begin{aligned} f(L^\pm) =& (\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})e^{(1+\theta_1)L^\pm} - \overline{r_1}e^{L^\pm} + \underline{\phi_1} = 0,\\ g(M^\pm) =& (\underline{c_{22}}e^{-\varpi\theta_2\overline{r_2}})e^{(1+\theta_2)M^\pm} - \overline{r_2}e^{M^\pm} + \underline{\phi_2} = 0. \end{aligned}$$

Proof. Let $\mathfrak{X} = \mathfrak{Z}$ be same as Lemma 5, and $\mathcal{L}, \mathcal{N}, \mathcal{P}, \mathcal{Q}$ be same as Lemmas 6 and 7. In what follows, we shall prove that model (1) has an ϖ-periodic solution based on Lemma 3.

First of all, we find the existence region $\Omega \subset \mathfrak{X}$ of solution. Assume that an ϖ-periodic solution $w = (\mathcal{U}, \mathcal{V})^T \in \mathfrak{X}$ solves the operator equation $\mathcal{L}w = \eta\mathcal{N}(w, \eta)$, then we obtain

$$\begin{cases} \mathcal{U}^\Delta(\varsigma) = \eta\big[r_1(\varsigma) - c_{11}(\varsigma)e^{\theta_1\mathcal{U}(\varsigma)} - \eta c_{12}(\varsigma)\int_{-\xi_1(\varsigma)}^0 k_1(s)e^{\mathcal{V}(\varsigma+s)}\Delta s - \phi_1(\varsigma)e^{-\mathcal{U}(\varsigma)}\big],\\ \mathcal{V}^\Delta(\varsigma) = \eta\big[r_2(\varsigma) - c_{22}(\varsigma)e^{\theta_2\mathcal{V}(\varsigma)} - \eta c_{21}(\varsigma)\int_{-\xi_2(\varsigma)}^0 k_2(s)e^{\mathcal{U}(\varsigma+s)}\Delta s - \phi_2(\varsigma)e^{-\mathcal{V}(\varsigma)}\big]. \end{cases} \tag{7}$$

Integrating at both sides of (7) yields

$$\begin{cases} \int_\kappa^{\kappa+\varpi}\big[r_1(\varsigma) - c_{11}(\varsigma)e^{\theta_1\mathcal{U}(\varsigma)} - \eta c_{12}(\varsigma)\int_{-\xi_1(\varsigma)}^0 k_1(s)e^{\mathcal{V}(\varsigma+s)}\Delta s - \phi_1(\varsigma)e^{-\mathcal{U}(\varsigma)}\big]\Delta\varsigma = 0,\\ \int_\kappa^{\kappa+\varpi}\big[r_2(\varsigma) - c_{22}(\varsigma)e^{\theta_2\mathcal{V}(\varsigma)} - \eta c_{21}(\varsigma)\int_{-\xi_2(\varsigma)}^0 k_2(s)e^{\mathcal{U}(\varsigma+s)}\Delta s - \phi_2(\varsigma)e^{-\mathcal{V}(\varsigma)}\big]\Delta\varsigma = 0. \end{cases} \tag{8}$$

In view of periodicity of $\mathcal{U}(\varsigma)$ and $\mathcal{V}(\varsigma)$, there exist μ_1, μ_2, ν_1 and $\nu_2 \in I_\varpi$ satisfying $\mathcal{U}(\mu_1) = \overline{\mathcal{U}}, \mathcal{U}(\mu_2) = \underline{\mathcal{U}}$, $\mathcal{V}(\nu_1) = \overline{\mathcal{V}}$, $\mathcal{V}(\nu_2) = \underline{\mathcal{V}}$. The first equation in (7) and (8) leads

$$\int_\kappa^{\kappa+\varpi} |\mathcal{U}^\Delta(\varsigma)|\Delta\varsigma < 2\varpi\overline{r_1}. \tag{9}$$

By the first equation of (8) and (9) together with Lemma 8, we have

$$\begin{aligned} \varpi\overline{r_1} \ge& \int_\kappa^{\kappa+\varpi} r_1(s)\Delta s = \int_\kappa^{\kappa+\varpi} c_{11}(\varsigma)e^{\theta_1\mathcal{U}(\varsigma)}\Delta\varsigma + \eta\int_\kappa^{\kappa+\varpi} c_{12}(\varsigma)\\ &\times\left[\int_{-\xi_1(\varsigma)}^0 k_1(s)e^{\mathcal{V}(\varsigma+s)}\Delta s\right]\Delta\varsigma + \int_\kappa^{\kappa+\varpi}\phi_1(\varsigma)e^{-\mathcal{U}(\varsigma)}\Delta\varsigma\\ >& \varpi\underline{c_{11}}e^{\theta_1\mathcal{U}(\mu_2)} + \varpi\underline{\phi_1}e^{-\mathcal{U}(\mu_1)} \ge \varpi\underline{c_{11}}e^{\theta_1[\mathcal{U}(\mu_1)-\varpi\overline{r_1}]} + \varpi\underline{\phi_1}e^{-\mathcal{U}(\mu_1)}, \end{aligned}$$

which implies that

$$(\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})e^{(1+\theta_1)\mathcal{U}(\mu_1)} - \overline{r_1}e^{\mathcal{U}(\mu_1)} + \underline{\phi_1} < 0. \tag{10}$$

We derive from Lemma 4 that there has unique $L_0 \in \mathbb{R}$ such that

$$f_{\min} = f(L_0) = -\theta_1(\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})^{-\frac{1}{\theta_1}}\left(\frac{\overline{r_1}}{1+\theta_1}\right)^{\frac{1+\theta_1}{\theta_1}} + \underline{\phi_1},\ f'(L_0) = 0,$$

where $L_0 = \frac{1}{\theta_1}\ln\left[\frac{\overline{r_1}}{(\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})(1+\theta_1)}\right]$. From (H_2) and Lemma 4, one knows that $f(L_0) < 0$, and $f(x) = 0$ has only two roots L^- and L^+ satisfying

$$L^- < L_0 < L^+,\ f(L^\pm) = 0. \tag{11}$$

By (11) and Lemma 4, the Inequality (10) is solved by

$$L^- < \mathcal{U}(\mu_1) < L^+. \tag{12}$$

It follows from Lemma 8 and (12) that

$$L^- - \varpi\overline{r_1} < \mathcal{U}(\mu_2) \le \mathcal{U}(\mu_1) < L^+. \tag{13}$$

Similarly, the second equation of (7) gives

$$\begin{aligned}
\varpi\overline{r_2} > \int_\kappa^{\kappa+\varpi} r_2(\varsigma)\Delta\varsigma &= \eta\int_\kappa^{\kappa+\varpi} c_{21}(\varsigma)\left[\int_{-\xi_2(\varsigma)}^{0} k_2(s)e^{\mathcal{U}(\varsigma+s)}\Delta s\right]\Delta\varsigma \\
&+ \int_\kappa^{\kappa+\varpi} c_{22}(\varsigma)e^{\theta_2\mathcal{V}(\varsigma)}\Delta\varsigma + \int_\kappa^{\kappa+\varpi} \phi_2(\varsigma)e^{-\mathcal{V}(\varsigma)}\Delta\varsigma \\
&> \varpi\underline{c_{22}}e^{\theta_2[\mathcal{V}(\nu_1)-\varpi\overline{r_2}]} + \varpi\underline{\phi_2}e^{-\mathcal{V}(\nu_1)},
\end{aligned}$$

which indicates that

$$(\underline{c_{22}}e^{-\varpi\theta_2\overline{r_2}})e^{(1+\theta_2)\mathcal{V}(\nu_1)} - \overline{r_2}e^{\mathcal{V}(\nu_1)} + \underline{\phi_2} < 0. \tag{14}$$

From Lemma 4, one knows that there has unique $M_0 \in \mathbb{R}$ such that

$$g_{\min} = g(M_0) = -\theta_2(\underline{c_{22}}e^{-\varpi\theta_2\overline{r_2}})^{-\frac{1}{\theta_2}}\left(\frac{\overline{r_2}}{1+\theta_2}\right)^{\frac{1+\theta_2}{\theta_2}} + \underline{\phi_2},\ g'(M_0) = 0,$$

where $M_0 = \frac{1}{\theta_2}\ln\left[\frac{\overline{r_2}}{(\underline{c_{22}}e^{-\varpi\theta_2\overline{r_2}})(1+\theta_2)}\right]$. It follows from (H_2) and Lemma 4 that $g(M_0) < 0$, and $g(x) = 0$ has two roots M^- and M^+ satisfying

$$M^- < M_0 < M^+,\ g(M^\pm) = 0. \tag{15}$$

By (15) and Lemma 4, the solution of Inequality (14) is read as

$$M^- < \mathcal{V}(\nu_1) < M^+. \tag{16}$$

From Lemma 8 and (16), we have

$$M^- - \varpi\overline{r_2} < \mathcal{V}(\nu_2) \le \mathcal{V}(\nu_1) < M^+. \tag{17}$$

Based on (13) and (17), we take

$$\Omega = \left\{(\mathcal{U}(\varsigma), \mathcal{V}(\varsigma))^T \in \mathfrak{X} : L^- - \varpi\overline{r_1} < \mathcal{U}(\varsigma) < L^+, M^- - \varpi\overline{r_2} < \mathcal{V}(\varsigma) < M^+\right\}.$$

Obviously, $\Omega \subset \mathfrak{X}$ meets with Lemma 3 (a).

We next adopt the reduction to absurdity to prove that Lemma 3 (b) hold, i.e., $w \in \partial\Omega \cap \mathrm{Ker}(\mathcal{L}) = \partial\Omega \cap \mathbb{R}^2$ implies $\mathcal{QN}(w,0) \ne (0,0)$. Indeed, assume that the conclusion is opposite, then there has a constant vector $W^* = (U^*, V^*)$ with $W^* \in \partial\Omega \cap \mathbb{R}^2$ that fulfills

$$\begin{cases}
\int_\kappa^{\kappa+\varpi}\left[r_1(\varsigma) - c_{11}(\varsigma)e^{\theta_1 U^*} - c_{12}(\varsigma)e^{V^*}\int_{-\xi_2(\varsigma)}^{0} k_1(s)\Delta s - \phi_1(\varsigma)e^{-U^*}\right]\Delta\varsigma = 0, \\
\int_\kappa^{\kappa+\varpi}\left[r_2(\varsigma) - c_{22}(\varsigma)e^{\theta_2 U^*} - c_{21}(\varsigma)e^{U^*}\int_{-\xi_2(\varsigma)}^{0} k_2(s)\Delta s - \phi_2(\varsigma)e^{-V^*}\right]\Delta\varsigma = 0.
\end{cases} \tag{18}$$

A discussion for (18) analogue to (8)–(17) yields $W^* = (U^*, V^*) \in \Omega \cap \mathbb{R}^2$, this is contrary to $W^* = (U^*, V^*) \in \partial\Omega \cap \mathbb{R}^2$. So the Lemma 3 (b) holds.

We choose the identity operator $\mathcal{J} = \mathcal{I}$ and calculate it directly

$$\deg\left\{\mathcal{J}\mathcal{Q}\mathcal{N}(w,0), \Omega \cap \mathrm{Ker}(\mathcal{J}), (0,0)^T\right\} \neq 0,$$

which means that Lemma 3 (c) holds. Thus, one concludes from Lemma 3 that there has at least an ϖ-periodic function $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ satisfying model (1). The proof is completed. □

4. Global Asymptotic Stability

The portion concentrates on the global asymptotic stability of model (1). To this end, we need the following definitions.

Definition 5. *According to Lyapunov stability theory, we call that the ϖ-periodic positive solution $\widetilde{w}(\varsigma) = (\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))$ of (1) is globally asymptotically stable on a periodic time scale $\mathbb{T}$ iff, for any positive solution $w(\varsigma) = (\mathcal{U}(\varsigma), \mathcal{V}(\varsigma))$ of (1), and for any large real number $R > 0$, when $\|\widetilde{w}(0) - w(0)\| < R$, we have $\lim_{\varsigma \to +\infty} \|\widetilde{w}(\varsigma) - w(\varsigma)\| = 0$.*

Definition 6 ([28])**.** *Let U_ς be a neighborhood of ς, $\forall \varsigma \in \mathbb{T}$, and $V \in C_{rd}(\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+)$, $D^+V^\Delta(\varsigma, x(\varsigma))$ is called the Dini derivative of V iff, $\forall \epsilon > 0$, there has a right neighborhood $U_\epsilon \cap U_\varsigma$ of ς satisfying*

$$\frac{V(\sigma(\varsigma), x(\sigma(\varsigma))) - V(s, x(s))}{\sigma(\varsigma) - s} < D^+V^\Delta(\varsigma, x(\varsigma)) + \epsilon, \ \forall s \in U_\epsilon, s > \varsigma.$$

If $V(\varsigma, x(\varsigma))$ is continuous at right-scattered ς, then

$$D^+V^\Delta(\varsigma, x(\varsigma)) = \frac{V(\sigma(\varsigma), x(\sigma(\varsigma))) - V(\varsigma, x(\varsigma))}{\sigma(\varsigma) - \varsigma}.$$

By Theorem 1, we know that there has an ϖ-periodic function $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T \in \Omega$ satisfying model (1). Let $u(\varsigma) = e^{\mathcal{U}(\varsigma)}$, $v(\varsigma) = e^{\mathcal{V}(\varsigma)}$, then one has $\mathcal{U}^\Delta(\varsigma) = (\ln u(\varsigma))^\Delta$ and $\mathcal{V}^\Delta(\varsigma) = (\ln v(\varsigma))^\Delta$. Thus the system (1) becomes

$$\begin{cases} (\ln u(\varsigma))^\Delta = r_1(\varsigma) - c_{11}(\varsigma)[u(\varsigma)]^{\theta_1} - c_{12}(\varsigma)\int_{-\xi_1(\varsigma)}^0 k_1(s)v(\varsigma+s)\Delta s - \frac{\phi_1(\varsigma)}{u(\varsigma)}, \ \varsigma \in \mathbb{T}, \\ (\ln v(\varsigma))^\Delta = r_2(\varsigma) - c_{22}(\varsigma)[v(\varsigma)]^{\theta_2} - c_{21}(\varsigma)\int_{-\xi_2(\varsigma)}^0 k_2(s)u(\varsigma+s)\Delta s - \frac{\phi_2(\varsigma)}{v(\varsigma)}, \ \varsigma \in \mathbb{T}. \end{cases} \tag{19}$$

Similarly, there exists an ϖ-periodic positive function $(\widetilde{u}(\varsigma), \widetilde{v}(\varsigma))^T \in \widetilde{\Omega}$ that solves system (19), here

$$\widetilde{\Omega} = \left\{(u(\varsigma), v(\varsigma))^T : e^{L^- - \varpi\overline{r_1}} < u(\varsigma) < e^{L^+}, e^{M^- - \varpi\overline{r_2}} < v(\varsigma) < e^{M^+}\right\}.$$

Choose the constants $\rho > 0$ and $\theta > 0$ such that

$$0 < \rho < \min\left\{e^{L^- - \varpi\overline{r_1}}, \ e^{M^- - \varpi\overline{r_2}}\right\}, \ \ \theta \geq \max\{1, \theta_1, \theta_2\}.$$

We further assume that

(H_3) The followings are true:

$$-\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}} + \rho^{-\frac{1}{\theta}}\overline{\phi_1} + \rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^0 k_2(s)\Delta s < 0,$$

$$-\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}} + \rho^{-\frac{1}{\theta}}\overline{\phi_2} + \rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^0 k_1(s)\Delta s < 0.$$

Taking variable substitution $u(\varsigma) = (\rho X(\varsigma))^{\frac{1}{\theta}}$ and $v(\varsigma) = (\rho Y(\varsigma))^{\frac{1}{\theta}}$, then we have

$$(\ln u(\varsigma))^{\Delta} = \left[\frac{1}{\theta}\ln(\rho X(\varsigma))\right]^{\Delta} = \frac{1}{\theta}\Big[\ln\rho + \ln X(\varsigma)\Big]^{\Delta} = \frac{1}{\theta}(\ln X(\varsigma))^{\Delta},$$

and

$$(\ln v(\varsigma))^{\Delta} = \left[\frac{1}{\theta}\ln(\rho Y(\varsigma))\right]^{\Delta} = \frac{1}{\theta}\Big[\ln\rho + \ln Y(\varsigma)\Big]^{\Delta} = \frac{1}{\theta}(\ln Y(\varsigma))^{\Delta}.$$

Consequently, system (19) changes into

$$\begin{cases} (\ln X(\varsigma))^{\Delta} = \theta\big[r_1(\varsigma) - \rho^{\frac{\theta_1}{\theta}} c_{11}(\varsigma) X^{\frac{\theta_1}{\theta}}(\varsigma) - \rho^{\frac{1}{\theta}} c_{12}(\varsigma) \\ \quad \times \int_{-\xi_1(\varsigma)}^{0} k_1(s) Y^{\frac{1}{\theta}}(\varsigma+s)\Delta s - \rho^{-\frac{1}{\theta}}\phi_1(\varsigma) X^{-\frac{1}{\theta}}(\varsigma)\big], \\ (\ln Y(\varsigma))^{\Delta} = \theta\big[r_2(\varsigma) - \rho^{\frac{\theta_2}{\theta}} c_{22}(\varsigma) Y^{\frac{\theta_2}{\theta}}(\varsigma) - \rho^{\frac{1}{\theta}} c_{21}(\varsigma) \\ \quad \times \int_{-\xi_2(\varsigma)}^{0} k_2(s) X^{\frac{1}{\theta}}(\varsigma+s)\Delta s - \rho^{-\frac{1}{\theta}}\phi_2(\varsigma) Y^{-\frac{1}{\theta}}(\varsigma)\big]. \end{cases} \tag{20}$$

Clearly, system (20) has an ϖ-periodic positive solution $(\widetilde{X}(\varsigma), \widetilde{Y}(\varsigma))^T = (\frac{1}{\rho}\widetilde{u}^{\theta}(\varsigma), \frac{1}{\rho}\widetilde{v}^{\theta}(\varsigma))^T$ within $\widetilde{\Omega}'$, where

$$\widetilde{\Omega}' = \left\{(X(\varsigma), Y(\varsigma))^T : \frac{1}{\rho}e^{\theta(L^- - \varpi\overline{r_1})} < X(\varsigma) < \frac{1}{\rho}e^{\theta L^+}, \frac{1}{\rho}e^{\theta(M^- - \varpi\overline{r_2})} < Y(\varsigma) < \frac{1}{\rho}e^{\theta M^+}\right\}.$$

From Theorem 1 and $\widetilde{\Omega}'$, we have

$$1 < \frac{1}{\rho}e^{\theta(L^- - \varpi\overline{r_1})} < \widetilde{X}(\varsigma) < \frac{1}{\rho}e^{\theta L^+},\ 1 < \frac{1}{\rho}e^{\theta(M^- - \varpi\overline{r_2})} < \widetilde{Y}(\varsigma) < \frac{1}{\rho}e^{\theta M^+}. \tag{21}$$

Theorem 2. *Assume that* (H_1)–(H_3) *hold, then a unique ϖ-periodic solution* $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ *of* (1) *is globally asymptotically stable.*

Proof. Assume that the ϖ-periodic function $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ satisfying (1) is globally asymptotically stable, then $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ is attractive, that is, for each function $(\mathcal{U}(\varsigma), \mathcal{V}(\varsigma))^T$ satisfying (1), we have $\lim_{\varsigma\to+\infty}[\mathcal{U}(\varsigma) - \widetilde{\mathcal{U}}(\varsigma)] = 0$, $\lim_{\varsigma\to+\infty}[\mathcal{V}(\varsigma) - \widetilde{\mathcal{V}}(\varsigma)] = 0$. If the system (1) has another ϖ-periodic solution $(\mathcal{U}^*(\varsigma), \mathcal{V}^*(\varsigma))^T \in \Omega$ with $(\mathcal{U}^*(\varsigma), \mathcal{V}^*(\varsigma))^T \neq (\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$, without loss of generality, assume that $\mathcal{U}^*(\varsigma) \neq \widetilde{\mathcal{U}}(\varsigma)$, then we obtain $0 < |\widetilde{\mathcal{U}}(\varsigma) - \mathcal{U}^*(\varsigma)| \leq |\widetilde{\mathcal{U}}(\varsigma) - \mathcal{U}(\varsigma)| + |\mathcal{U}(\varsigma) - \mathcal{U}^*(\varsigma)| \to 0$, as $\varsigma \to +\infty$, which is an obvious fallacy. Thus, we prove that the ϖ-periodic function $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ satisfying model (1) is unique provided that $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ is globally asymptotically stable. In addition, since the global asymptotical stability of ϖ-periodic function $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T \in \Omega$ satisfying (1) and $(\widetilde{X}(\varsigma), \widetilde{Y}(\varsigma))^T$ satisfying (20) is equivalent, we just need to show that the ϖ-periodic function $(\widetilde{X}(\varsigma), \widetilde{Y}(\varsigma))^T$ satisfying (20) is globally asymptotically stable. Indeed, it follows from (H_1), (H_2) and Theorem 1 that system (20) has an positive ϖ-periodic solution $(\widetilde{X}(\varsigma), \widetilde{Y}(\varsigma))^T$. For each positive solution $(X(\varsigma), Y(\varsigma))^T$ of (20), build a Lyapunov functional $V(\varsigma) = V_1(\varsigma) + V_2(\varsigma)$, here

$$V_1(\varsigma) = |\ln X(\varsigma) - \ln \widetilde{X}(\varsigma)| + |\ln Y(\varsigma) - \ln \widetilde{Y}(\varsigma)|, \tag{22}$$

$$\begin{aligned} V_2(\varsigma) =& \theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0} k_1(s)\left[\int_{\varsigma+s}^{\varsigma}\left|Y^{\frac{1}{\theta}}(\zeta) - \widetilde{Y}^{\frac{1}{\theta}}(\zeta)\right|\Delta\zeta\right]\Delta s \\ &+ \theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0} k_2(s)\left[\int_{\varsigma+s}^{\varsigma}\left|X^{\frac{1}{\theta}}(\zeta) - \widetilde{X}^{\frac{1}{\theta}}(\zeta)\right|\Delta\zeta\right]\Delta s. \end{aligned} \tag{23}$$

Obviously, $V(0) < +\infty$ and $V(\varsigma) \geq V_1(\varsigma)$. By (21), a direct Δ-derivation along (20) gives

$$\begin{aligned}
&D^+(|\ln X(\varsigma) - \ln \widetilde{X}(\varsigma)|)^\Delta \leq -\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}|X(\varsigma) - \widetilde{X}(\varsigma)| \\
&+ \theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0} k_1(s)\left|Y^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s + \theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\left|X^{\frac{-1}{\theta}}(\varsigma) - \widetilde{X}^{\frac{-1}{\theta}}(\varsigma)\right| \\
\leq & -\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}|X(\varsigma) - \widetilde{X}(\varsigma)| + \theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0} k_1(s)\left|Y^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s \\
&+ \theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\left|X^{\frac{1}{\theta}}(\varsigma) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma)\right|X^{\frac{-1}{\theta}}(\varsigma)\widetilde{X}^{\frac{-1}{\theta}}(\varsigma) \\
\leq & -\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}|X(\varsigma) - \widetilde{X}(\varsigma)| + \theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0} k_1(s)\left|Y^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s \\
&+ \theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\left|X^{\frac{1}{\theta}}(\varsigma) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma)\right|, \qquad (24)
\end{aligned}$$

$$\begin{aligned}
&D^+(|\ln Y(\varsigma) - \ln \widetilde{Y}(\varsigma)|)^\Delta \leq -\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}|Y(\varsigma) - \widetilde{Y}(\varsigma)| \\
&+ \theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0} k_2(s)\left|X^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s + \theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\left|Y^{\frac{-1}{\theta}}(\varsigma) - \widetilde{Y}^{\frac{-1}{\theta}}(\varsigma)\right| \\
\leq & -\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}|Y(\varsigma) - \widetilde{Y}(\varsigma)| + \theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0} k_2(s)\left|X^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s \\
&+ \theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\left|Y^{\frac{1}{\theta}}(\varsigma) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\right|Y^{\frac{-1}{\theta}}(\varsigma)\widetilde{Y}^{\frac{-1}{\theta}}(\varsigma) \\
\leq & -\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}|Y(\varsigma) - \widetilde{Y}(\varsigma)| + \theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0} k_2(s)\left|X^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s \\
&+ \theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\left|Y^{\frac{1}{\theta}}(\varsigma) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\right|, \qquad (25)
\end{aligned}$$

$$\begin{aligned}
&D^+\left(\int_{-\xi}^{0} k_1(s)\left[\int_{\varsigma+s}^{\varsigma}\left|Y^{\frac{1}{\theta}}(\zeta) - \widetilde{Y}^{\frac{1}{\theta}}(\zeta)\right|\Delta\zeta\right]\Delta s\right)^\Delta = \int_{-\xi}^{0} k_1(s)\Delta s \cdot \left|Y^{\frac{1}{\theta}}(\varsigma) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\right| \\
&- \int_{-\xi}^{0} k_1(s)\left|Y^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s, \qquad (26)
\end{aligned}$$

and

$$\begin{aligned}
&D^+\left(\int_{-\xi}^{0} k_2(s)\left[\int_{\varsigma+s}^{\varsigma}\left|X^{\frac{1}{\theta}}(\zeta) - \widetilde{X}^{\frac{1}{\theta}}(\zeta)\right|\Delta\zeta\right]\Delta s\right)^\Delta = \int_{-\xi}^{0} k_2(s)\Delta s \cdot \left|X^{\frac{1}{\theta}}(\varsigma) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma)\right| \\
&- \int_{-\xi}^{0} k_2(s)\left|X^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s. \qquad (27)
\end{aligned}$$

Since when $\alpha, \beta > 0$ and $x \geq 1$ $q(x) = |\alpha^x - \beta^x|$ is monotonically increasing, and $0 < \frac{\theta_1}{\theta}, \frac{\theta_2}{\theta}, \frac{1}{\theta} \leq 1$, it follows from (21), (24)–(27) and (H_3) that

$$D^+V^\Delta(\varsigma) \leq -\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}|X(\varsigma) - \widetilde{X}(\varsigma)| + \theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0} k_1(s)\left|Y^{\frac{1}{\theta}}(\varsigma+s) - \widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\right|\Delta s$$

$$
\begin{aligned}
&+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\Big|X^{\frac{1}{\theta}}(\varsigma)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma)\Big|-\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}\big|Y(\varsigma)-\widetilde{Y}(\varsigma)\big|\\
&+\theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Big|X^{\frac{1}{\theta}}(\varsigma+s)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\Big|\Delta s+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\Big|Y^{\frac{1}{\theta}}(\varsigma)-\widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\Big|\\
&+\theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0}k_1(s)\Delta s\cdot\Big|Y^{\frac{1}{\theta}}(\varsigma)-\widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\Big|-\theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0}k_1(s)\Big|Y^{\frac{1}{\theta}}(\varsigma+s)-\widetilde{Y}^{\frac{1}{\theta}}(\varsigma+s)\Big|\Delta s\\
&+\theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Delta s\cdot\Big|X^{\frac{1}{\theta}}(\varsigma)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma)\Big|-\theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Big|X^{\frac{1}{\theta}}(\varsigma+s)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma+s)\Big|\Delta s\\
=&-\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}\big|X(\varsigma)-\widetilde{X}(\varsigma)\big|+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\Big|X^{\frac{1}{\theta}}(\varsigma)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma)\Big|-\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}\big|Y(\varsigma)-\widetilde{Y}(\varsigma)\big|\\
&+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\Big|Y^{\frac{1}{\theta}}(\varsigma)-\widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\Big|+\theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0}k_1(s)\Delta s\cdot\Big|Y^{\frac{1}{\theta}}(\varsigma)-\widetilde{Y}^{\frac{1}{\theta}}(\varsigma)\Big|\\
&+\theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Delta s\cdot\Big|X^{\frac{1}{\theta}}(\varsigma)-\widetilde{X}^{\frac{1}{\theta}}(\varsigma)\Big|\\
\le&-\theta\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}\big|X(\varsigma)-\widetilde{X}(\varsigma)\big|+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_1}\big|X(\varsigma)-\widetilde{X}(\varsigma)\big|-\theta\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}\big|Y(\varsigma)-\widetilde{Y}(\varsigma)\big|\\
&+\theta\rho^{\frac{-1}{\theta}}\overline{\phi_2}\big|Y(\varsigma)-\widetilde{Y}(\varsigma)\big|+\theta\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0}k_1(s)\Delta s\cdot\Big|Y(\varsigma)-\widetilde{Y}(\varsigma)\Big|\\
&+\theta\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Delta s\cdot\Big|X(\varsigma)-\widetilde{X}(\varsigma)\Big|\\
=&\theta\left[-\rho^{\frac{\theta_1}{\theta}}\underline{c_{11}}+\rho^{-\frac{1}{\theta}}\overline{\phi_1}+\rho^{\frac{1}{\theta}}\overline{c_{21}}\int_{-\xi}^{0}k_2(s)\Delta s\right]\Big|X(\varsigma)-\widetilde{X}(\varsigma)\Big|\\
&+\theta\left[-\rho^{\frac{\theta_2}{\theta}}\underline{c_{22}}+\rho^{-\frac{1}{\theta}}\overline{\phi_2}+\rho^{\frac{1}{\theta}}\overline{c_{12}}\int_{-\xi}^{0}k_1(s)\Delta s\right]\Big|Y(\varsigma)-\widetilde{Y}(\varsigma)\Big|<0. \qquad (28)
\end{aligned}
$$

Thus, from (22), (23) and (28), one concludes that $V(\varsigma)$ is positive definite and $D^+V^{\Delta}(\varsigma)<0, \forall\varsigma\ge 0$. Therefore, one draws a conclusion that the ϖ-periodic solution $(\widetilde{X}(\varsigma),\widetilde{Y}(\varsigma))^T$ of system (20) has global asymptotic stability based on Lyapunov stability theory. The proof is completed. □

5. Numerical Simulation

The portion considers the following nonlinear Ayala-Gilpin competitive ecosystem having distributed lags on time scale $\mathbb{T}=\mathbb{R}$

$$
\begin{cases}
\frac{d\mathcal{U}(\varsigma)}{d\varsigma}=\mathcal{U}(\varsigma)\big[r_1(\varsigma)-c_{11}(\varsigma)\mathcal{U}^{\theta_1}(\varsigma)-c_{12}(\varsigma)\int_{-\xi_1(\varsigma)}^{0}k_1(s)\mathcal{V}(\varsigma+s)ds\big]-\phi_1(\varsigma),\\
\frac{d\mathcal{V}(\varsigma)}{d\varsigma}=\mathcal{V}(\varsigma)\big[r_2(\varsigma)-c_{22}(\varsigma)\mathcal{V}^{\theta_2}(\varsigma)-c_{21}(\varsigma)\int_{-\xi_2(\varsigma)}^{0}k_2(s)\mathcal{U}(\varsigma+s)ds\big]-\phi_2(\varsigma),\\
\mathcal{U}(\varsigma)=\varphi_1(\varsigma),\ \mathcal{V}(\varsigma)=\varphi_2(\varsigma),\ \varsigma\in(-\xi,0],
\end{cases} \qquad (29)
$$

where $r_1(\varsigma)=8+2\cos(3\varsigma), r_2(\varsigma)=6+\sin(2\varsigma), c_{11}(\varsigma)=5+2\sin(\varsigma), c_{22}(\varsigma)=3+\cos(2\varsigma)$, $c_{12}(\varsigma)=\frac{6+\sin(2\varsigma)}{10}$, $c_{21}(\varsigma)=\frac{3+\cos(3\varsigma)}{10}$, $\xi_1(\varsigma)=k_1(\varsigma)=\frac{2+\sin(\varsigma)}{5}$, $\xi_2(t)=k_2(\varsigma)=\frac{2+\cos(\varsigma)}{4}$, $\phi_1(\varsigma)=\frac{3+\cos(2\varsigma)}{7}$, $\phi_2(\varsigma)=\frac{4+\sin(\varsigma)}{7}$, $\theta_1=\frac{1}{2}$, $\theta_2=\frac{1}{\sqrt{2}}$. Take the initial functions $\varphi_1(\varsigma)=7+\sin(\varsigma)$, $\varphi_2(\varsigma)=\frac{2+\cos(\varsigma)}{7}$, $\varsigma\in(-\xi,0]=(-\frac{3}{4},0]$, here $\xi=\max\{\overline{\xi_1},\overline{\xi_2}\}=\max\{\frac{3}{5},\frac{3}{4}\}=\frac{3}{4}$.

Obviously, $r_1(\varsigma)$, $r_2(\varsigma)$, $c_{11}(\varsigma)$, $c_{22}(\varsigma)$, $c_{12}(\varsigma)$, $c_{21}(\varsigma)$, $\xi_1(\varsigma)$, $\xi_2(\varsigma)$, $\varphi_1(\varsigma)$, $\varphi_2(\varsigma)$, $k_1(\varsigma)$, $k_2(\varsigma)$, $\phi_1(\varsigma)$ and $\phi_2(\varsigma)$ are all positive periodic functions with period $\varpi=2\pi$. So the conditions (H_1) holds. A direct computation gives $\overline{r_1}=10$, $\underline{r_1}=6$, $\overline{r_2}=7$, $\underline{r_2}=5$, $\overline{c_{11}}=7$, $\underline{c_{11}}=3$, $\overline{c_{22}}=4$, $\underline{c_{22}}=2$, $\overline{c_{12}}=\frac{7}{10}$, $\underline{c_{12}}=\frac{1}{2}$, $\overline{c_{21}}=\frac{2}{5}$, $\underline{c_{21}}=\frac{1}{5}$, $\overline{\phi_1}=\frac{4}{7}$, $\underline{\phi_1}=\frac{2}{7}$, $\overline{\phi_2}=\frac{5}{7}$, $\underline{\phi_2}=\frac{3}{7}$, $\xi=\frac{3}{4}$, $\int_{-\xi}^{0}k_1(s)ds\approx 0.2463$, $\int_{-\xi}^{0}k_2(s)ds\approx 0.5454$. To solve the following algebraic equation

$$
f(L^{\pm})=(\underline{c_{11}}e^{-\varpi\theta_1\overline{r_1}})e^{(1+\theta_1)L^{\pm}}-\overline{r_1}e^{L^{\pm}}+\underline{\phi_1}=0,
$$

we find that $L^- \approx -3.5553$, $L^+ \approx 65.2398$. From the algebraic equation

$$g(M^{\pm}) = (\underline{c_{22}} e^{-\varpi\theta_2\overline{r_2}}) e^{(1+\theta_2)M^{\pm}} - \overline{r_2} e^{M^{\pm}} + \underline{\phi_2} = 0,$$

we have $M^- \approx -2.7932$, $M^+ \approx 45.7540$. Thus we get

$$\Omega = \big\{ (\mathcal{U}(\varsigma), \mathcal{V}(\varsigma))^T : 1.4738 \times 10^{-29} < \mathcal{U}(\varsigma) < 2.1542 \times 10^{28}, \\ 4.8491 \times 10^{-21} < \mathcal{V}(\varsigma) < 7.4252 \times 10^{19} \big\}.$$

The condition (H_2) is verified as

$$\theta_1 (\underline{c_{11}} e^{-\varpi\theta_1\overline{r_1}})^{-\frac{1}{\theta_1}} \left(\frac{\overline{r_1}}{1+\theta_1} \right)^{\frac{1+\theta_1}{\theta_1}} \approx 3.1914 \times 10^{28} > \underline{\phi_1} = \frac{2}{7},$$

$$\theta_2 (\underline{c_{22}} e^{-\varpi\theta_2\overline{r_2}})^{-\frac{1}{\theta_2}} \left(\frac{\overline{r_2}}{1+\theta_2} \right)^{\frac{1+\theta_2}{\theta_2}} \approx 1.0105 \times 10^{20} > \underline{\phi_2} = \frac{3}{7}.$$

By now, (H_1) and (H_2) have been verified. From Theorem 1, one knows that (29) exists at least an 2π-periodic positive solutions $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T \in \Omega$.

Next, we prove periodic positive solution $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ to be globally asymptotically stable. Indeed, take $\rho = 1.5 \times 10^{-29}$, $\theta = 120$, we get

$$-\rho^{\frac{\theta_1}{\theta}} \underline{c_{11}} + \rho^{-\frac{1}{\theta}} \overline{\phi_1} + \rho^{\frac{1}{\theta}} \overline{c_{21}} \int_{-\xi}^{0} k_2(s) ds \approx -1.1551 < 0,$$

and

$$-\rho^{\frac{\theta_2}{\theta}} \underline{c_{22}} + \rho^{-\frac{1}{\theta}} \overline{\phi_2} + \rho^{\frac{1}{\theta}} \overline{c_{12}} \int_{-\xi}^{0} k_1(s) ds \approx -0.0104 < 0.$$

Thus the condition (H_3) holds. From Theorem 2, one knows that the periodic solution $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ is globally asymptotically stable. We have carried out numerical simulation for example (29) as shown in Figure 1.

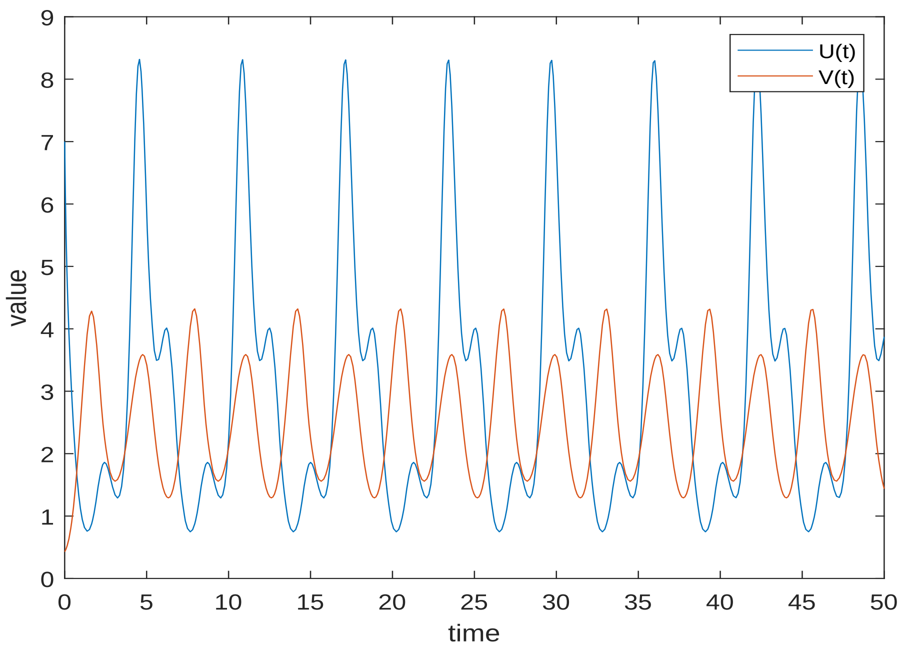

Figure 1. Existence and global asymptotic stability of solution $(\widetilde{\mathcal{U}}(\varsigma), \widetilde{\mathcal{V}}(\varsigma))^T$ to (29).

6. Summaries and Outlooks

The Ayala-Gilpin dfferential equation model is one of the successful patterns of applying mathematical theories and methods to study ecosystems. Its dynamic behavior has received significant attention and study from mathematicians and ecologists. In this work, we mainly investigate the existence and global asymptotic stability of periodic solutions for a class of nonlinear distributed-lag Ayala-Gilpin vying system (1) in the sense of time scales. By making use of Mawhin's coincidence degree theorem, we first gain some sufficient criteria for the existence of periodic solutions of model (1). Next, we construct an appropriate Lyapunov functional to demonstrate that model (1) is globally asymptotically stable. Subsequently, we examined the validity and applicability of our essential findings through theoretical analysis and numerical simulation of an example. Our conclusion reveals the existence of periodic oscillations in AG-ecosystem under certain conditions from a mathematical perspective. However, the long-term behavior of AG-ecosystem is globally asymptotically stable. As stated in studies [10,11], an ecosystem may have multiple stable states for different initial values, which means that the ecosystem has multiple positive solutions and is stable in their respective regions. Therefore, we can study the existence and local stability of multiple positive periodic solutions for model (1) in future works. In addition, some scholars have found that using fractional or partial differential models to study some practical problems is more accurate than using integer order differential models. Awaken by some recent papers [29–41], we plan to apply fractional calculus and PDE theory to further study the AG-ecosystem in the future, in order to explore more dynamic characteristics.

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