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A New Subclass of Bi-Univalent Functions Defined by a Certain Integral Operator

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Abstract: We introduce a comprehensive subfamily of analytic and bi-univalent functions in this study using Horadam polynomials and the q-analog of the Noor integral operator. We establish upper bounds for the absolute values of the second and the third coefficients and the Fekete–Szegö functional for the functions belonging to this family. Various observations of the results presented here are also discussed.

Keywords: analytic functions; bi-univalent functions; coefficient inequalities; Fekete–Szegö problem; Horadam polynomials; q-analog of the Noor integral operator

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1. Introduction

Let $\mathcal{A}$ denote the class of functions $l(\xi)$ normalized by the following Taylor–Maclaurin series:

$$l(\xi) = \xi + \sum_{k=2}^{\infty} m_k \xi^k \quad (\xi \in \mathbb{U}), \tag{1}$$

which are analytic in the open unit disk

$$\mathbb{U}=\{\xi : \xi \in \mathbb{C} \text{ and } |\xi| < 1\},$$

$\mathbb{C}$ being, as usual, the set of complex numbers. $\mathcal{S}$ denotes the subclass of $\mathcal{A}$ that are univalent in $\mathbb{U}$. It is well known that for $l \in \mathcal{S}$, $\left|m_3 - m_2^2\right| \leqslant 1$. A classical theorem of Fekete–Szegö (see [1]) states that for $l \in \mathcal{S}$ given by (1),

$$\left|m_3 - \zeta m_2^2\right| \leqslant \begin{cases} 3-4\zeta & \text{if} \quad \zeta \leqslant 0, \\ 1+2\exp\left(\frac{-2\zeta}{1-\zeta}\right) & \text{if} \quad 0<\zeta<1, \\ 4\zeta-3 & \text{if} \quad \zeta \geqslant 1. \end{cases}$$

This inequality is sharp in the sense that for each ζ, there exists a function in $\mathcal{S}$ such that equality holds. Later, Pfluger [2] considered the complex values of ζ and provided

$$\left|m_3 - \zeta m_2^2\right| \leqslant 1 + 2\left|\exp\left(\frac{-2\zeta}{1-\zeta}\right)\right|.$$

In connection with functions in the family $\mathcal{S}$, on the account of the Koebe one-quarter theorem (see, for example, [3]), it is clear that under every function $l \in \mathcal{S}$ the image of $\mathbb{U}$ contains a disk of radius $1/4$. Thus, clearly, every univalent function l in $\mathbb{U}$ has an inverse l^{-1} satisfying the following conditions:

$$l^{-1}(l(\xi)) = \xi(\xi \in \mathbb{U})$$

and

$$l(l^{-1}(w)) = w(|w| < r_0(l);\ r_0(l) \geqq 1/4),$$

where

$$l^{-1}(w) = w - m_2 w^2 + (2m_2^2 - m_3)w^3 - (5m_2^3 - 5m_2 m_3 + m_4)w^4 + \cdots. \tag{2}$$

It is said that a function $l \in \mathcal{A}$ is bi-univalent in $\mathbb{U}$ if both l and l^{-1} are univalent in $\mathbb{U}$. Let Σ represent the class of bi-univalent functions in $\mathbb{U}$ for which (1) holds.

Lewin [4] demonstrated in 1967 that the second coefficient of every function l of the form (1) fulfills the estimate $|m_2| < 1.51$. Brannan and Clunie [5] hypothesized in 1967 that $|m_2| \leqq \sqrt{2}$ for $l \in \Sigma$. Netanyahu [6] later showed that $\max_{l \in \Sigma} |m_2| = \frac{4}{3}$. Kedzierawski [7] established the Brannan–Clunie hypothesis for bi-starlike functions in 1985.

In 1985, Tan [8] found the constraint for m_2, meaning that $|m_2| < 1.485$, which is the best-known estimate for functions in the class Σ. Brannan and Taha [9] estimated the coefficients $|m_2|$ and $|m_3|$ for bi-starlike and bi-convex functions of order β, respectively.

The research of bi-univalent functions was revitalized in recent years by Srivastava et al. [10], and a significant number of follow-ups to their work have been published in the literature since then. Particularly, a number of coefficient estimates for the initial coefficients $|m_2|$, $|m_3|$, and $|m_4|$ were demonstrated for various subclasses of Σ (see, for example, [11–22]).

Recently, Deniz [23] and Kumar et al. [24] both extended and improved the results of Brannan and Taha [9] by generalizing their classes by means of the principle of subordination between analytic functions. The problem of estimating the coefficients $|m_k|$ $(k \geqq 2)$ is still open (see also [22] in this connection).

For analytic functions l and σ in $\mathbb{U}$, l is said to be subordinate to σ if there exists an analytic function w such that

$$w(0) = 0, \ \ |w(\xi)| < 1 \text{ and } l(\xi) = \sigma(w(\xi)) \quad (\xi \in \mathbb{U}).$$

This subordination will be denoted here by

$$l \prec \sigma \quad (\xi \in \mathbb{U}),$$

or, conventionally, by

$$l(\xi) \prec \sigma(\xi) \quad (\xi \in \mathbb{U}).$$

In particular, if the function σ is univalent in $\mathbb{U}$, then we have the following equivalence:

$$l(\xi) \prec \sigma(\xi) \Leftrightarrow l(0) = \sigma(0) \text{ and } l(\mathbb{U}) \subset \sigma(\mathbb{U}) \quad (\xi \in \mathbb{U}). \tag{3}$$

If l of form (1) and $\sigma(\xi) = \xi + \sum_{k=2}^{\infty} n_k \xi^k$ are two functions in $\mathcal{A}$, then the Hadamard product (or convolution) of l and σ is denoted by $l * \sigma$ and is given by

$$(l * \sigma)(\xi) = \xi + \sum_{k=2}^{\infty} m_k n_k \xi^k = (\sigma * l)(\xi).$$

In recent years, q-analysis (q-calculus) has greatly motivated researchers due to its numerous applications in mathematics and physics. Jackson [25,26] was the first to give some application of q-calculus and also introduced the q-analog of the derivative and integral operator. Later on, Aral and Gupta [27,28], defined the q-Baskakov–Durrmeyer operator by using the q-beta function, while in the papers of Anatassiou and Gal [29] and Aral [30], the authors discussed the q-generalization of complex operators known as q-Picard and q-Gauss–Weierstrass singular integral operators. Using the convolution of normalized analytic functions, Kanas and Raducanu [31] defined the q-analog of the Ruscheweyh differential operator and studied some of its properties. The application of this differential operator was further studied by Aldweby and Darus [32] and Mahmood and Sokol [33].

The q-derivative of function $l \in \mathcal{A}$ was defined in [34] for $q \in (0,1)$ by

$$\partial_q l(\xi) = \frac{l(q\xi) - l(\xi)}{(q-1)\xi} \quad (\xi \neq 0) \tag{4}$$

and

$$\partial_q l(0) = l'(0).$$

Thus, we have

$$\partial_q l(\xi) = 1 + \sum_{k=2}^{\infty} [k,q] m_k \xi^{k-1}, \tag{5}$$

where

$$[k,q] = \frac{1-q^k}{1-q}, \quad [0,q] = 0 \tag{6}$$

and

$$[k,q]! = \begin{cases} \prod_{j=1}^{k} [j,q], & (k \in \mathbb{N} = \{1,2,\ldots\}) \\ 1, & (k = 0). \end{cases} \tag{7}$$

In addition, the q-generalized Pochhammer symbol for $\mathfrak{p} \geq 0$ is represented by

$$[\mathfrak{p},q]_k = \begin{cases} \prod_{j=1}^{k} [\mathfrak{p}+j-1,q], & (k \in \mathbb{N}) \\ 1, & (k = 0). \end{cases} \tag{8}$$

If $q \to 1$, then we obtain $[k,q] \to k$. Thus, if we choose the function $\sigma(\xi) = \xi^k$, while $q \to 1^-$, then we have

$$\partial_q \sigma(\xi) = \partial_q \xi^k = [k,q]\xi^{k-1} = \sigma'(\xi),$$

where σ' is the ordinary derivative.

Arif et al. [35] recently defined the function $I_{q,\mu+1}^{-1}(\xi)$ as the following relation:

$$I_{q,\mu+1}^{-1}(\xi) * I_{q,\mu+1}(\xi) = \xi \partial_q l(\xi) \quad (\mu \geq -1), \tag{9}$$

where

$$I_{q,\mu+1}(\xi) = \xi + \sum_{k=2}^{\infty} \frac{[\mu+1,q]_{k-1}}{[k-1,q]!} \xi^k \quad (\xi \in \mathbb{U}). \tag{10}$$

Because the series defined in (10) is convergent absolutely in A, Arif et al. [35] defined the q-analogue of the Noor integral operator $\mho_q^{\mu}$:$\mathbb{U} \to \mathbb{U}$ by $\xi \in \mathbb{U}$ using the definition of the q-derivative through convolution

$$\mho_q^{\mu} l(\xi) = I_{q,\mu+1}^{-1}(\xi) * l(\xi) = \xi + \sum_{k=2}^{\infty} \phi_{k-1} m_k \xi^k \quad (\xi \in \mathbb{U}), \tag{11}$$

where

$$\phi_{k-1} = \frac{[k,q]!}{[\mu+1,q]_{k-1}}. \tag{12}$$

We note that

$$\mho_q^{0} l(\xi) = \xi \partial_q l(\xi), \; \mho_q^{1} l(\xi) = l(\xi) \tag{13}$$

and also

$$\lim_{q \to 1^-} \mho_q^{\mu} l(\xi) = \xi + \sum_{k=2}^{\infty} \frac{k!}{(\mu+1)_{k-1}} m_k \xi^k. \tag{14}$$

This demonstrates that taking $q \to 1^-$ reduces the operator specified in (12) to the well-known Noor integral operator discussed in [36,37]. See the work of Aldweby and Darus [32] for additional information on the q-analog of differential and integral operators.

The Horadam polynomials, $h_j(x, a, b; p, \tau)$, or briefly $h_j(x)$, are given by the following recurrence relation (see [38,39]):

$$h_j(x) = pxh_{j-1}(x) + \tau h_{j-2}(x), \quad (j \in \mathbb{N} \text{ and } j \geqslant 3) \tag{15}$$

with

$$h_1(x) = a, \ h_2(x) = bx \ \text{ and } \ h_3(x) = pbx^2 + a\tau, \tag{16}$$

where $a, b, p, \tau \in \mathbb{Z}$.

Some special cases of Horadam polynomial $h_j(x)$ are as follows:

1. For $p = \tau = a = b = 1$, the Horadam polynomials $h_j(x)$ reduce to the Fibonacci polynomials $F_j(x)$;
2. For $p = \tau = b = 1$ and $a = 2$, the Horadam polynomials $h_j(x)$ become the Lucas polynomials $L_j(x)$;
3. For $a = b = 1$, $p = 2$ and $\tau = -1$, the Horadam polynomials $h_j(x)$ reduce to the Chebyshev polynomials $T_j(x)$ of the first kind;
4. For $p = b = 2$, $a = 1$ and $\tau = -1$, the Horadam polynomials $h_j(x)$ become the Chebyshev polynomials $U_j(x)$ of the second kind;
5. For $\tau = a = 1$ and $p = b = 2$, the Horadam polynomials $h_j(x)$ reduce to the Pell polynomials $P_j(x)$;
6. For $p = a = b = 2$ and $\tau = 1$,the Horadam polynomials $h_j(x)$ become the Pell-Lucas polynomials $Q_j(x)$ of the first kind.

For bi-univalent functions associated with particular polynomials, such as the Fibonacci polynomials, Lucas polynomials, Chebyshev polynomials, and the Horadam polynomials, the coefficient estimates and Fekete–Szegö inequality are determined. Additionally, we remark that the aforementioned polynomials and other special polynomials may have applications in the mathematical, physical, statistical, and engineering sciences. Several articles have investigated these polynomials (see [40–46]).

Theorem 1. *([38]) Let $\mathcal{H}(x, \xi)$ be the generating function of the Horadam polynomials $h_j(x)$. Then,*

$$\mathcal{H}(x, \xi) = \sum_{j=1}^{\infty} h_j(x)\xi^{j-1} = \frac{a + (b - ap)x\xi}{1 - px\xi - \tau\xi^2}. \tag{17}$$

We define the following subclass of Σ in this paper using the q-analog of the Noor integral operator (11) and Horadam polynomials given by the recurrence relation (15) and the generating function (17).

Definition 1. *For $\alpha, \lambda \geq 1, \rho \in \mathbb{C} \backslash \{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, a function $l \in \Sigma$ given by (1) is said to be in the class $\mathcal{N}_{\Sigma}^{\mu,q}(\lambda, \alpha, \beta, \rho; x)$ if the following subordinations are satisfied:*

$$1 + \frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^{\mu} l(\xi)\right)' + \lambda\left\{\left(\mho_q^{\mu} l(\xi)\right)'\right\}^{\alpha}\left[\frac{\mho_q^{\mu} l(\xi)}{\xi}\right]^{\beta-1} - 1\right\} \prec \mathcal{H}(x, \xi) - a + 1$$

and

$$1 + \frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^{\mu} \sigma(w)\right)' + \lambda\left\{\left(\mho_q^{\mu} \sigma(w)\right)'\right\}^{\alpha}\left[\frac{\mho_q^{\mu} \sigma(w)}{w}\right]^{\beta-1} - 1\right\} \prec \mathcal{H}(x, w) - a + 1,$$

which is given by (3).

2. Coefficient Estimates for the Subclass $\mathcal{N}_{\Sigma}^{\mu,q}(\lambda,\alpha,\beta,\rho;x)$

In this section, we find the estimates on the coefficients $|m_2|$ and $|m_3|$ for functions in the above defined subfamily $\mathcal{N}_{\Sigma}^{\mu,q}(\lambda,\alpha,\beta,\rho;x)$. In addition, the Fekete–Szegö problem for this subfamily is solved.

Our first main result is asserted by Theorem 2 below.

Theorem 2. *For $\alpha,\lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(\lambda,\alpha,\beta,\rho;x)$. Then,*

$$|m_2| \leq \frac{|\rho||bx|\sqrt{2|bx|}}{\sqrt{\left|\begin{array}{c}\left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\right]b^2x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\rho\left(pbx^2+a\tau\right)\end{array}\right|}},$$

$$|m_3| \leq \frac{|\rho|^2 b^2x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2}+\frac{|\rho||bx|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3-\eta m_2^2\right| \leq \begin{cases} \frac{|\rho||bx|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta-\phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{2|\rho||bx||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta-\phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\begin{aligned}\mathcal{S}(x) &= \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\right] \\ &\quad -2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\left(\frac{pbx^2+a\tau}{b^2x^2\rho}\right).\end{aligned}$$

Proof. Let $l \in \mathcal{N}_{\Sigma}^{\mu,q}(\lambda,\alpha,\beta,\rho;x)$. From Definition 1, for some analytic functions ψ, φ such that $\psi(0)=\varphi(0)=0$ and $|\psi(\xi)|<1$, $|\varphi(w)|<1$ for all $\xi, w \in \mathbb{U}$, we can write

$$\begin{aligned}&1+\frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^\mu l(\xi)\right)'+\lambda\left\{\left(\mho_q^\mu l(\xi)\right)'\right\}^\alpha\left[\frac{\mho_q^\mu l(\xi)}{\xi}\right]^{\beta-1}-1\right\} \\ &= 1+h_1(x)-a+h_2(x)\psi(\xi)+h_3(x)\psi^2(\xi)+\ldots\end{aligned} \tag{18}$$

and

$$\begin{aligned}&1+\frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^\mu \sigma(w)\right)'+\lambda\left\{\left(\mho_q^\mu \sigma(w)\right)'\right\}^\alpha\left[\frac{\mho_q^\mu \sigma(w)}{w}\right]^{\beta-1}-1\right\} \\ &= 1+h_1(x)-a+h_2(x)\varphi(w)+h_3(x)\varphi^2(w)+\ldots.\end{aligned} \tag{19}$$

From the Equalities (18) and (19), we obtain

$$\begin{aligned}&1+\frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^\mu l(\xi)\right)'+\lambda\left\{\left(\mho_q^\mu l(\xi)\right)'\right\}^\alpha\left[\frac{\mho_q^\mu l(\xi)}{\xi}\right]^{\beta-1}-1\right\} \\ &= 1+h_2(x)r_1\xi+\left[h_2(x)r_2+h_3(x)r_1^2\right]\xi^2+\ldots\end{aligned} \tag{20}$$

and

$$\begin{aligned}&1+\frac{1}{\rho}\left\{(1-\lambda)\left(\mho_q^\mu \sigma(w)\right)'+\lambda\left\{\left(\mho_q^\mu \sigma(w)\right)'\right\}^\alpha\left[\frac{\mho_q^\mu \sigma(w)}{w}\right]^{\beta-1}-1\right\} \\ &= 1+h_2(x)t_1w+\left[h_2(x)t_2+h_3(x)t_1^2\right]w^2+\ldots.\end{aligned} \tag{21}$$

It is well known that if

$$|\psi(\xi)| = \left|r_1\xi + r_2\xi^2 + r_3\xi^3 + \dots\right| < 1, \quad (\xi \in \mathbb{U})$$

and

$$|\varphi(w)| = \left|t_1 w + t_2 w^2 + t_3 w^3 + \dots\right| < 1, \quad (w \in \mathbb{U}),$$

then

$$\left|r_j\right| < 1 \text{ and } \left|t_j\right| < 1 \text{ for } j \in \mathbb{N}. \tag{22}$$

Thus, comparing the coefficients in (20) and (21), we have

$$(\lambda(2\alpha + \beta - 3) + 2)\phi_1 m_2 = \rho h_2(x) r_1, \tag{23}$$

$$\begin{aligned} &\left[\lambda\left(\frac{(\beta-1)(\beta-2)}{2} + 2\alpha(\beta-1) + 2\alpha(\alpha-1)\right)\right]\phi_1^2 m_2^2 \\ &+[(\lambda(3\alpha+\beta-4)+3)]\phi_2 m_3 \\ = \;& \rho h_2(x) r_2 + \rho h_3(x) r_1^2, \end{aligned} \tag{24}$$

$$-(\lambda(2\alpha + \beta - 3) + 2)\phi_1 m_2 = \rho h_2(x) t_1 \tag{25}$$

and

$$\begin{aligned} &\left[\lambda\left(\frac{(\beta-1)(\beta-2)}{2} + 2\alpha(\beta-1) + 2\alpha(\alpha-1)\right)\right]\phi_1^2 m_2^2 \\ &+[(\lambda(3\alpha+\beta-4)+3)]\phi_2\left(2m_2^2 - m_3\right) \\ = \;& \rho h_2(x) t_2 + \rho h_3(x) t_1^2. \end{aligned} \tag{26}$$

It follows from (23) and (25) that

$$r_1 = -t_1 \tag{27}$$

and

$$2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2 m_2^2 = \rho^2 h_2^2(x)\left(r_1^2 + t_1^2\right). \tag{28}$$

If we add (24) and (26), we obtain

$$\begin{aligned} &\left\{\begin{array}{c} [\lambda(\beta^2 - 3\beta + 2 + 4\alpha(\beta-1) + 4\alpha(\alpha-1))]\phi_1^2 \\ +2[(\lambda(3\alpha+\beta-4)+3)]\phi_2 \end{array}\right\} m_2^2 \\ = \;& \rho h_2(x)(r_2 + t_2) + \rho h_3(x)\left(r_1^2 + t_1^2\right). \end{aligned} \tag{29}$$

Substituting the value of $\left(r_1^2 + t_1^2\right)$ from (28) in (29), we arrive at

$$\begin{aligned} &\left\{\begin{array}{c} ([\lambda(\beta^2 - 3\beta + 2 + 4\alpha(\beta-1) + 4\alpha(\alpha-1))]\phi_1^2 \\ +2[(\lambda(3\alpha+\beta-4)+3)]\phi_2)\rho^2 h_2^2(x) \\ -\left[2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\right]\rho h_3(x) \end{array}\right\} m_2^2 \\ = \;& \rho^3 h_2^3(x)(r_2 + t_2). \end{aligned} \tag{30}$$

Moreover, using (15) and (22) in (30), we find that

$$|m_2| \le \frac{|\rho||bx|\sqrt{2|bx|}}{\sqrt{\left|\begin{array}{c} [\lambda(4\alpha(\alpha+\beta-2) + (\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2]b^2x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\rho(pbx^2 + a\tau) \end{array}\right|}}.$$

Next, if we subtract (26) from (24), we obtain

$$2[(\lambda(3\alpha+\beta-4)+3)]\left(m_3-m_2^2\right)\phi_2=\rho h_2(x)(r_2-t_2)+\rho h_3(x)\left(r_1^2-t_1^2\right). \tag{31}$$

Then, in view of (27) and (28), Equation (31) becomes

$$m_3=\frac{\rho^2h_2^2(x)}{2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2}\left(r_1^2+t_1^2\right)+\frac{\rho h_2(x)}{(\lambda(3\alpha+\beta-4)+3)\phi_2}(r_2-t_2).$$

Thus, applying (16), we conclude that

$$|m_3|\le\frac{|\rho|^2b^2x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2}+\frac{|\rho||bx|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}.$$

From (30) and (31),

$$\begin{aligned}m_3-\eta m_2^2 &= \frac{(\phi_2-\eta)\rho^3h_2^3(x)(r_2+t_2)}{\begin{array}{c}\{\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2[(\lambda(3\alpha+\beta-4)+3)]\phi_2\}\rho^2h_2^2(x)\\ -\left[2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\right]\rho h_3(x)\end{array}}\\ &\quad+\frac{\rho h_2(x)(r_2-t_2)}{2(\lambda(3\alpha+\beta-4)+3)\phi_2}\\ &= \rho h_2(x)\left\{\begin{array}{c}\left[\gamma(\eta,\rho,\phi_1,\phi_2;x)+\frac{1}{(\lambda(3\alpha+\beta-4)+3)\phi_2}\right]r_2\\ +\left[\gamma(\eta,\rho,\phi_1,\phi_2;x)-\frac{1}{(\lambda(3\alpha+\beta-4)+3)\phi_2}\right]t_2\end{array}\right\},\end{aligned}$$

where

$$\gamma(\eta,\rho,\phi_1,\phi_2;x)=\frac{\rho^2h_2^2(x)(\phi_2-\eta)}{\begin{array}{c}\{[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))]\phi_1^2+2[(\lambda(3\alpha+\beta-4)+3)]\phi_2\}\rho^2h_2^2(x)\\ -\left[2(\lambda(2\alpha+\beta-3)+2)^2\phi_1^2\right]\rho h_3(x)\end{array}}.$$

Then, in view of (16), we arrive at

$$\left|m_3-\eta m_2^2\right|\le\begin{cases}\frac{|\rho||h_2(x)|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & 0\le|\gamma(\eta,\rho,\phi_1,\phi_2;x)|\le\frac{1}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}\\ 2|\rho||h_2(x)||\gamma(\eta,\rho,\phi_1,\phi_2;x)|, & |\gamma(\eta,\rho,\phi_1,\phi_2;x)|\ge\frac{1}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}.\end{cases}$$

This evidently completes the proof of Theorem 2. □

For specific choices of parameters in Theorem 2, we give the following consequence.

Corollary 1. *Let $\rho\in\mathbb{C}\backslash\{0\}$ and $l\in\mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(1,1,0,\rho;x)$. Then*

$$|m_2|\le\frac{|\rho||bx|\sqrt{|bx|}}{\sqrt{\left|(2\phi_2-\phi_1^2)b^2x^2\rho^2-\rho(pbx^2+a\tau)\phi_1^2\right|}},$$

$$|m_3|\le\frac{|\rho|^2b^2x^2}{\phi_1^2}+\frac{|\rho||bx|}{2\phi_2}$$

and

$$\left|m_3-\eta m_2^2\right|\le\begin{cases}\frac{|\rho||bx|}{2\phi_2}, & |\eta-\phi_2|\le\frac{\left|(2\phi_2-\phi_1^2)b^2x^2\rho-\phi_1^2(pbx^2+a\tau)\right|}{2\rho b^2x^2\phi_2}\\ \frac{|bx|^3|\eta-\phi_2||\rho|^2}{\left|(2\phi_2-\phi_1^2)b^2x^2\rho-\phi_1^2(pbx^2+a\tau)\right|}, & |\eta-\phi_2|\ge\frac{\left|(2\phi_2-\phi_1^2)b^2x^2\rho-\phi_1^2(pbx^2+a\tau)\right|}{2\rho b^2x^2\phi_2}.\end{cases}$$

Theorem 2 can be used to generate the following interesting results.

Specializing the values of $\alpha = \lambda = a = b = p = \tau = x = \rho = 1$ and $\beta = 0$ in Theorem 2 above, we can give the following example.

Example 1. *Let $\mu = 0$, $q = 1/2 \in (0,1)$, $\eta = 1$ and $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{0,\frac{1}{2}}(1,1,0,1;1)$. Then*

$$|m_2| \leq \frac{1}{\sqrt{5}} = 0,4472...,$$

$$|m_3| \leq \frac{64}{63} = 1,0158...$$

and

$$\left|m_3 - m_2^2\right| \leq \frac{4}{7} = 0,5714....$$

These results are sharp.

In Theorem 2, we obtain the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Fibonacci polynomials $F_j(x)$.

Corollary 2. *For $\alpha, \lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(\alpha, \lambda, \beta, \rho; x)$. Then*

$$|m_2| \leq \frac{|\rho||x|\sqrt{2|x|}}{\sqrt{\left|\begin{array}{c} \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right]x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\rho(x^2+1)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{|\rho|^2 x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2} + \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3 - \eta m_2^2\right| \leq \begin{cases} \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta - \phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{2|x||\rho||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta - \phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\begin{aligned} \mathcal{S}(x) &= \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right] \\ &\quad -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{x^2+1}{x^2\rho}\right)\phi_1^2. \end{aligned}$$

In Theorem 2, we obtain the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Lucas polynomials $L_j(x)$.

Corollary 3. *For $\alpha, \lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(\alpha, \lambda, \beta, \rho; x)$. Then*

$$|m_2| \leq \frac{|\rho||x|\sqrt{2|x|}}{\sqrt{\left|\begin{array}{c} \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right]x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\rho(x^2+2)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{|\rho|^2 x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2} + \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3 - \eta m_2^2\right| \leq \begin{cases} \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta - \phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{2|\rho||x||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta - \phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\begin{aligned} \mathcal{S}(x) &= \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right] \\ &\quad -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{x^2+2}{x^2\rho}\right)\phi_1^2. \end{aligned}$$

In Theorem 2, we obtain the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Chebyshev polynomials $T_j(x)$ of the first kind.

Corollary 4. *For $\alpha, \lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_\Sigma^{\mu,q}(\alpha, \lambda, \beta, \rho; x)$. Then*

$$|m_2| \leq \frac{|\rho||x|\sqrt{2|x|}}{\sqrt{\left|\begin{array}{c} \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right]x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\rho(2x^2-1)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{|\rho|^2 x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2} + \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3 - \eta m_2^2\right| \leq \begin{cases} \frac{|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta - \phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{2|\rho||x||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta - \phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\begin{aligned} \mathcal{S}(x) &= \left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right] \\ &\quad -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{2x^2-1}{x^2\rho}\right)\phi_1^2. \end{aligned}$$

In Theorem 2, we obtain the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Chebyshev polynomials $U_j(x)$ of the second kind.

Corollary 5. *For $\alpha, \lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_\Sigma^{\mu,q}(\alpha, \lambda, \beta, \rho; x)$. Then*

$$|m_2| \leq \frac{4|\rho||x|\sqrt{|x|}}{\sqrt{\left|\begin{array}{c} 4\left[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2 + 2(\lambda(3\alpha+\beta-4)+3)\phi_2\right]x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\rho(4x^2-1)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{4|\rho|^2 x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2} + \frac{2|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3 - \eta m_2^2\right| \leq \begin{cases} \frac{|\rho||2x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta - \phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{4|\rho||x||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta - \phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\mathcal{S}(x) = \Big[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\Big] -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{4x^2-1}{4x^2\rho}\right)\phi_1^2.$$

In Theorem 2, we obtain the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Pell polynomials $P_j(x)$.

Corollary 6. *For $\alpha,\lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(\alpha,\lambda,\beta,\rho;x)$. Then*

$$|m_2| \leq \frac{4|\rho||x|\sqrt{|x|}}{\sqrt{\left|\begin{array}{c} 4\big[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\big]x^2\rho^2 \\ -2(\lambda(2\alpha+\beta-3)+2)^2\rho(4x^2+1)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{4|\rho|^2x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2}+\frac{2|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3-\eta m_2^2\right| \leq \begin{cases} \frac{|\rho||2x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta-\phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{4|\rho||x||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta-\phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\mathcal{S}(x) = \Big[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\Big] -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{4x^2+1}{4x^2\rho}\right)\phi_1^2.$$

In Theorem 2, we have the following corollary if the Horadam polynomials $h_j(x)$ are replaced by the Pell–Lucas polynomials $Q_j(x)$.

Corollary 7. *For $\alpha,\lambda \geq 1$, $\rho \in \mathbb{C}\backslash\{0\}$, $\beta \in \mathbb{C}$ and $Re(\beta) \geq 0$, let $l \in \mathcal{A}$ belong to the class $\mathcal{N}_{\Sigma}^{\mu,q}(\alpha,\lambda,\beta,\rho;x)$. Then*

$$|m_2| \leq \frac{2|\rho||x|\sqrt{|x|}}{\sqrt{\left|\begin{array}{c} \big[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\big]x^2\rho^2 \\ -(\lambda(2\alpha+\beta-3)+2)^2\rho(2x^2+1)\phi_1^2 \end{array}\right|}},$$

$$|m_3| \leq \frac{4|\rho|^2x^2}{|\lambda(2\alpha+\beta-3)+2|^2\phi_1^2}+\frac{2|\rho||x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}$$

and

$$\left|m_3-\eta m_2^2\right| \leq \begin{cases} \frac{|\rho||2x|}{|\lambda(3\alpha+\beta-4)+3|\phi_2}, & |\eta-\phi_2| \leq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2} \\ \frac{4|\rho||x||\eta-\phi_2|}{|\mathcal{S}(x)|}, & |\eta-\phi_2| \geq \frac{|\mathcal{S}(x)|}{2|\lambda(3\alpha+\beta-4)+3|\phi_2}, \end{cases}$$

where

$$\mathcal{S}(x) = \Big[\lambda(4\alpha(\alpha+\beta-2)+(\beta-2)(\beta-1))\phi_1^2+2(\lambda(3\alpha+\beta-4)+3)\phi_2\Big] -2(\lambda(2\alpha+\beta-3)+2)^2\left(\frac{4x^2+1}{4x^2\rho}\right)\phi_1^2.$$

3. Conclusions

The target of the present investigation is to introduce, by using the concept of subordination, a new subfamily of bi-univalent functions in the open unit disk $\mathbb{U}$ associated with Horadam polynomials as well as deriving the initial estimations of coefficients and Fekete–Szegö inequalities for functions belonging to this subfamily. The majority of our findings are found in Theorem 2. By further specializing the criteria, a number of additional repercussions of these new families are indicated.

Basic (or q-)series and basic (or q-)polynomials, especially the basic q-hypergeometric functions and basic (or q-)hypergeometric polynomials, are applicable, particularly in several diverse areas (see, for example, [34], pp. 350–351 and [47], p. 328). Furthermore, in Srivastava's recent survey-cum-expository review article ([47], p. 328), the so-called (p, q)-calculus was revealed to be a rather trivial and insignificant variation of the classical q-calculus, with the additional parameter p being redundant (see, for details, [47], p. 328 and [48], pp. 1511–1512). Indeed, this remark by Srivastava ([47], p. 328) would also apply to any attempt to create the very basic (p, q)-variants of the results provided in this study.

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