

## Article

# Multi-Objective Non-Linear Programming Problems in Linear Diophantine Fuzzy Environment

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**Abstract:** Due to various unpredictable factors, a decision maker frequently experiences uncertainty and hesitation when dealing with real-world practical optimization problems. At times, it's necessary to simultaneously optimize a number of non-linear and competing objectives. Linear Diophantine fuzzy numbers are used to address the uncertain parameters that arise in these circumstances. The objective of this manuscript is to present a method for solving a linear Diophantine fuzzy multi-objective nonlinear programming problem (LDFMONLPP). All the coefficients of the nonlinear multi-objective functions and the constraints are linear Diophantine fuzzy numbers (LDFNs). Here we find the solution of the nonlinear programming problem by using Karush-Kuhn-Tucker condition. A numerical example is presented.

**Keywords:** linear Diophantine fuzzy numbers; linear Diophantine fuzzy nonlinear programming problem; linear Diophantine fuzzy multi-objective nonlinear programming problem

**MSC:** 90C30; 03E72



**Citation:** Iqbal, S.; Yaqoob, N.; Gulistan, M. Multi-Objective Non-Linear Programming Problems in Linear Diophantine Fuzzy Environment. *Axioms* **2023**, *12*, 1048. <https://doi.org/10.3390/axioms12111048>

Academic Editors: Ramasamy Kavikumar, Kaviarasan Boomipalagan, S. A. Karthick and Hsien-Chung Wu

Received: 25 September 2023  
Revised: 8 November 2023  
Accepted: 11 November 2023  
Published: 13 November 2023



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## 1. Introduction

Nonlinear programming is generally presenting far greater challenges than linear programming. It can be challenging, even when the objective function is the only nonlinear aspect of the problem and all constraints are linear. Zimmermann [1] proposed the idea of fuzzy non-linear programming. Fuzzy non-linear programming are helpful in resolving problems that are challenging, impossible to resolve due to the ambiguous, subjective nature of the problem formulation, or problems which do not have an exact solution.

The idea of decision making in a fuzzy environment was first put forth by Bellman and Zadeh [2]. Maleki et al. [3] introduced a new approach for solving linear programming problems with ambiguous constraints by using a ranking function. Tanaka et al. [4] embraced this idea for resolving mathematical programming problems. A novel approach for solving fully fuzzy linear programming problems with fuzzy variables was put forth by Pandian and Jayalakshmi [5]. Tanaka and Asai [6] discussed fuzzy linear programming problems with fuzzy numbers. Linear programming problems with fuzzy constraints and fuzzy coefficients in both the left and right hand of the constraints set were taken into consideration by Campos and Vardegay [7]. Nehi and Hajmohadi [8] presented a ranking function for solving fuzzy multi-objective linear programming problem. A novel method for solving the fuzzy nonlinear programming problem was presented by Ghadle and Pawar [9]. A method for solving fuzzy nonlinear programming problems using the ranking function was suggested by Loganathan and Kiruthiga [10]. Tang and Wang [11] has developed a method to solve fuzzy non-linear programming problem in manufacturing systems.

The generalization of fuzzy set theory known as intuitionistic fuzzy set (IFS) theory, which was developed by Atanassov [12], has proven to be useful in the field of uncertainty theory because it provides information about the decision maker's acceptance, rejection,

and degree of hesitation. When analyzing a problem, decision-makers frequently deal with linguistic constraints, ambiguity in resource availability, issues with judgement, hesitation when making decisions, lack of evaluation experience, etc. A review of the literature found that a number of authors, including [13–17] have discussed linear programming problem in an intuitionistic fuzzy environment.

El Sayed and Abo-Sinna [18] presented a novel approach for fully intuitionistic fuzzy multi-objective fractional transportation problem. Garg et al. [19] presented intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environment. Rani et al. [20] discussed multi-objective non-linear programming problem in intuitionistic fuzzy environment. Singh and Yadav [21] discussed an optimality of multi objective non-linear programming problem in intuitionistic fuzzy environment.

To the best of our knowledge there is no any approach in literature to solve the multi-objective non-linear programming problem in linear Diophantine fuzzy environment. Therefore, in the present study we discuss the non linear programming problems in which all of the problem's decision parameters are triangular linear Diophantine fuzzy numbers. First, we use the ranking approach to turn all of the triangular linear Diophantine fuzzy numbers into crisp values, and then we get a crisp non linear programming problem, which we solve by the Kuhn Tucker conditions.

## 2. Preliminaries and Basic Definitions

In this section we present some basic definitions.

**Definition 1** ([22]). Let  $X$  represent the universe. The following defines an LDFS  $\mathcal{L}_{\mathfrak{N}}$  on  $X$

$$\mathcal{L}_{\mathfrak{N}} = \{(\vartheta, \langle \zeta_{\mathfrak{N}}^{\tau}(\vartheta), \varkappa_{\mathfrak{N}}^{\nu}(\vartheta) \rangle, \langle \gamma(\vartheta), \delta(\vartheta) \rangle) : \vartheta \in X\},$$

where,  $\zeta_{\mathfrak{N}}^{\tau}(\vartheta), \varkappa_{\mathfrak{N}}^{\nu}(\vartheta), \gamma(\vartheta), \delta(\vartheta) \in [0, 1]$  such that

$$0 \leq \gamma(\vartheta)\zeta_{\mathfrak{N}}^{\tau}(\vartheta) + \delta(\vartheta)\varkappa_{\mathfrak{N}}^{\nu}(\vartheta) \leq 1, \quad \forall \vartheta \in X,$$

$$0 \leq \gamma(\vartheta) + \delta(\vartheta) \leq 1.$$

The hesitation part can be expressed as

$$\varrho\pi_{\mathfrak{N}} = 1 - (\gamma(\vartheta)\zeta_{\mathfrak{N}}^{\tau}(\vartheta) + \delta(\vartheta)\varkappa_{\mathfrak{N}}^{\nu}(\vartheta)),$$

where  $\varrho$  is the reference parameter.

We write briefly  $\mathcal{L}_{\mathfrak{N}} = (\langle \zeta_{\mathfrak{N}}^{\tau}, \varkappa_{\mathfrak{N}}^{\nu} \rangle, \langle \gamma, \delta \rangle)$  for

$$\mathcal{L}_{\mathfrak{N}} = \{(\vartheta, \langle \zeta_{\mathfrak{N}}^{\tau}(\vartheta), \varkappa_{\mathfrak{N}}^{\nu}(\vartheta) \rangle, \langle \gamma(\vartheta), \delta(\vartheta) \rangle) : \vartheta \in X\}.$$

**Definition 2** ([22]). One can write an absolute LDFS on  $X$  as

$$^1\mathcal{L}_{\mathfrak{N}} = \{(\vartheta, \langle 1, 0 \rangle, \langle 1, 0 \rangle) : \vartheta \in X\},$$

an LDFS that is empty or null can be written as

$$^0\mathcal{L}_{\mathfrak{N}} = \{(\vartheta, \langle 0, 1 \rangle, \langle 0, 1 \rangle) : \vartheta \in X\}.$$

**Definition 3** ([22]). Let  $\mathcal{L}_{\mathfrak{N}} = \{(\vartheta, \langle \zeta_{\mathfrak{N}}^{\tau}(\vartheta), \varkappa_{\mathfrak{N}}^{\nu}(\vartheta) \rangle, \langle \gamma(\vartheta), \delta(\vartheta) \rangle) : \vartheta \in X\}$  be an LDFS. For any constants  $p, q, r, s \in [0, 1]$  such that  $0 \leq pr + qs \leq 1$  with  $0 \leq r + s \leq 1$ , define the  $(\langle p, q \rangle, \langle r, s \rangle)$ -cut of  $\mathcal{L}_{\mathfrak{N}}$  as follows:

$$(\mathcal{L}_{\mathfrak{N}})_{\langle p, q \rangle, \langle r, s \rangle}^{(p, q)} = \{\vartheta \in X : \zeta_{\mathfrak{N}}^{\tau}(\vartheta) \geq p, \varkappa_{\mathfrak{N}}^{\nu}(\vartheta) \leq q, \gamma(\vartheta) \geq r, \delta(\vartheta) \leq s\}.$$

**Definition 4** ([23]). Let  $\mathcal{L}_{\mathfrak{R}}$  be a LDFS on  $\mathbb{R}$  with the following membership functions ( $\xi_{\mathfrak{R}}^{\tau}$  and  $\gamma$ ) and non-membership functions ( $\varkappa_{\mathfrak{R}}^{\nu}$  and  $\delta$ )

$$\xi_{\mathfrak{R}}^{\tau}(x) = \begin{cases} \frac{x-\vartheta_1}{\vartheta_3-\vartheta_1} & \vartheta_1 \leq x \leq \vartheta_3 \\ \frac{\vartheta_5-x}{\vartheta_5-\vartheta_3} & \vartheta_3 \leq x \leq \vartheta_5 \\ 0 & \text{otherwise} \end{cases}, \quad \varkappa_{\mathfrak{R}}^{\nu}(x) = \begin{cases} \frac{\vartheta_3-x}{\vartheta_3-\vartheta_2} & \vartheta_2 \leq x \leq \vartheta_3 \\ \frac{x-\vartheta_3}{\vartheta_4-\vartheta_3} & \vartheta_3 \leq x \leq \vartheta_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\gamma(x) = \begin{cases} \frac{x-\vartheta'_2}{\vartheta'_3-\vartheta'_2} & \vartheta'_2 \leq x \leq \vartheta'_3 \\ \frac{\vartheta'_4-x}{\vartheta'_4-\vartheta'_3} & \vartheta'_3 \leq x \leq \vartheta'_4 \\ 0 & \text{otherwise} \end{cases}, \quad \delta(x) = \begin{cases} \frac{\vartheta'_3-x}{\vartheta'_3-\vartheta'_1} & \vartheta'_1 \leq x \leq \vartheta'_3 \\ \frac{x-\vartheta'_3}{\vartheta'_5-\vartheta'_3} & \vartheta'_3 \leq x \leq \vartheta'_5 \\ 0 & \text{otherwise} \end{cases}$$

where  $\vartheta'_1 \leq \vartheta'_2 \leq \vartheta'_3 \leq \vartheta'_4 \leq \vartheta'_5$  for all  $x \in \mathbb{R}$ . Then  $\mathcal{L}_{\mathfrak{R}}$  is called a triangular LDFN,

- (i) of type-1 if  $\vartheta_3 = \vartheta'_3$  and  $\vartheta_1 \leq \vartheta_2 \leq \vartheta_3 \leq \vartheta_4 \leq \vartheta_5$ ;
- (ii) of type-2 if  $\vartheta_3 \neq \vartheta'_3$  and  $\vartheta_1 \leq \vartheta_2 \leq \vartheta_3 \leq \vartheta_4 \leq \vartheta_5$ ;
- (iii) of type-3 if  $\vartheta_3 = \vartheta'_3$  and  $\vartheta_2 \leq \vartheta_1 \leq \vartheta_3 \leq \vartheta_5 \leq \vartheta_4$ ;
- (iv) of type-4 if  $\vartheta_3 \neq \vartheta'_3$  and  $\vartheta_2 \leq \vartheta_1 \leq \vartheta_3 \leq \vartheta_5 \leq \vartheta_4$ .

We only take type-1 triangle LDFN into consideration throughout the study, and we refer to this kind as triangular LDFN (TLDFN). According to this TLDFN,

$$\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5), (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \right\}.$$

**Definition 5** ([23]). Consider a TLDFN  $\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5), (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \right\}$ . Then

- (i) p-cut set of  $\mathcal{L}_{\mathfrak{R}_{TLDFN}}$  is a crisp subset of  $\mathbb{R}$ , and it is defined as

$$\begin{aligned} \mathcal{L}_{\mathfrak{R}_{TLDFN}}^p &= \{x \in X : \xi_{\mathfrak{R}}^{\tau}(x) \geq p\} \\ &= [\xi_{\mathfrak{R}}^{\tau}(x)(p), \xi_{\mathfrak{R}}^{\tau}(x)(p)] \\ &= [\vartheta_1 + p(\vartheta_3 - \vartheta_1), \vartheta_5 - p(\vartheta_5 - \vartheta_3)]. \end{aligned}$$

- (ii) q-cut set of  $\mathcal{L}_{\mathfrak{R}_{TLDFN}}$  is a crisp subset of  $\mathbb{R}$ , and it is defined as

$$\begin{aligned} \mathcal{L}_{\mathfrak{R}_{TLDFN}}^q &= \{x \in X : \varkappa_{\mathfrak{R}}^{\nu}(x) \leq q\} \\ &= [\varkappa_{\mathfrak{R}}^{\nu}(q), \varkappa_{\mathfrak{R}}^{\nu}(q)] \\ &= [\vartheta_3 - q(\vartheta_3 - \vartheta_2), \vartheta_3 + q(\vartheta_4 - \vartheta_3)]. \end{aligned}$$

- (iii) r-cut set of  $\mathcal{L}_{\mathfrak{R}_{TLDFN}}$  is a crisp subset of  $\mathbb{R}$ , and it is defined as

$$\begin{aligned} \mathcal{L}_{\mathfrak{R}_{TLDFN}}^r &= \{x \in X : \gamma(x) \geq r\} \\ &= [\gamma(r), \gamma(r)] \\ &= [\vartheta'_2 + r(\vartheta'_3 - \vartheta'_2), \vartheta'_4 - r(\vartheta'_4 - \vartheta'_3)]. \end{aligned}$$

(iv)  $s$ -cut set of  $\mathcal{L}_{\mathfrak{R}_{TLDFN}}$  is a crisp subset of  $\mathbb{R}$ , which is defined as follows

$$\begin{aligned}\mathcal{L}_{\mathfrak{R}_{TLDFN}}^s &= \{x \in X : \delta(x) \leq s\} \\ &= [\underline{\delta(s)}, \overline{\delta(s)}] \\ &= [\vartheta_3 - s(\vartheta_3 - \vartheta'_1), \vartheta_3 + s(\vartheta'_5 - \vartheta_3)].\end{aligned}$$

We can denote the  $(\langle p, q \rangle, \langle r, s \rangle)$ -cut of  $\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \\ (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \end{matrix} \right\}$  by

$$(\mathcal{L}_{\mathfrak{R}_{TLDFN}})_{\langle p, q \rangle, \langle r, s \rangle} = \left\{ \left( \left[ \xi_{\mathfrak{R}}^T(p), \bar{\xi}_{\mathfrak{R}}^T(p) \right], \left[ \varkappa_{\mathfrak{R}}^V(q), \bar{\varkappa}_{\mathfrak{R}}^V(q) \right] \right), \left( \left[ \gamma(r), \bar{\gamma}(r) \right], \left[ \delta(s), \bar{\delta}(s) \right] \right) \right\}.$$

The set of all TLDFN on  $\mathbb{R}$  is represented by the symbol  $\mathcal{L}_{\mathfrak{R}_{TLDFN}}(\mathbb{R})$ .

**Definition 6 ([23]).** A TLDFN  $\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \\ (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \end{matrix} \right\}$  is said to be positive if and only if  $\vartheta_1 \geq 0$  and  $\vartheta'_1 \geq 0$ .

**Definition 7 ([23]).** Two TLDFNs  $\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \\ (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \end{matrix} \right\}$  and  $\beta_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5) \\ (\delta'_1, \delta'_2, \delta'_3, \delta'_4, \delta'_5) \end{matrix} \right\}$  are said to be equal if and only if  $\vartheta_1 = \delta_1, \vartheta_2 = \delta_2, \vartheta_3 = \delta_3, \vartheta_4 = \delta_4, \vartheta_5 = \delta_5, \vartheta'_1 = \delta'_1, \vartheta'_2 = \delta'_2, \vartheta'_4 = \delta'_4$  and  $\vartheta'_5 = \delta'_5$ .

**Definition 8 ([23]).** Consider two positive TLDFNs  $\mathcal{L}_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \\ (\vartheta'_1, \vartheta'_2, \vartheta'_3, \vartheta'_4, \vartheta'_5) \end{matrix} \right\}$  and  $\beta_{\mathfrak{R}_{TLDFN}} = \left\{ \begin{matrix} (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5) \\ (\delta'_1, \delta'_2, \delta'_3, \delta'_4, \delta'_5) \end{matrix} \right\}$ , then

$$\begin{aligned}(i) \quad \mathcal{L}_{\mathfrak{R}_{TLDFN}} + \beta_{\mathfrak{R}_{TLDFN}} &= \left\{ \begin{matrix} (\vartheta_1 + \delta_1, \vartheta_2 + \delta_2, \vartheta_3 + \delta_3, \vartheta_4 + \delta_4, \vartheta_5 + \delta_5) \\ (\vartheta'_1 + \delta'_1, \vartheta'_2 + \delta'_2, \vartheta'_3 + \delta'_3, \vartheta'_4 + \delta'_4, \vartheta'_5 + \delta'_5) \end{matrix} \right\}; \\ (ii) \quad \mathcal{L}_{\mathfrak{R}_{TLDFN}} - \beta_{\mathfrak{R}_{TLDFN}} &= \left\{ \begin{matrix} (\vartheta_1 - \delta_5, \vartheta_2 - \delta_4, \vartheta_3 - \delta_3, \vartheta_4 - \delta_2, \vartheta_5 - \delta_1) \\ (\vartheta'_1 - \delta'_5, \vartheta'_2 - \delta'_4, \vartheta'_3 - \delta'_3, \vartheta'_4 - \delta'_2, \vartheta'_5 - \delta'_1) \end{matrix} \right\}; \\ (iii) \quad \mathcal{L}_{\mathfrak{R}_{TLDFN}} \times \beta_{\mathfrak{R}_{TLDFN}} &= \left\{ \begin{matrix} (\vartheta_1 \delta_1, \vartheta_2 \delta_2, \vartheta_3 \delta_3, \vartheta_4 \delta_4, \vartheta_5 \delta_5) \\ (\vartheta'_1 \delta'_1, \vartheta'_2 \delta'_2, \vartheta'_3 \delta'_3, \vartheta'_4 \delta'_4, \vartheta'_5 \delta'_5) \end{matrix} \right\}; \\ (iv) \quad \mathcal{L}_{\mathfrak{R}_{TLDFN}} \div \beta_{\mathfrak{R}_{TLDFN}} &= \left\{ \begin{matrix} \left( \frac{\vartheta_1}{\delta_5}, \frac{\vartheta_2}{\delta_4}, \frac{\vartheta_3}{\delta_3}, \frac{\vartheta_4}{\delta_2}, \frac{\vartheta_5}{\delta_1} \right) \\ \left( \frac{\vartheta'_1}{\delta'_5}, \frac{\vartheta'_2}{\delta'_4}, \frac{\vartheta'_3}{\delta'_3}, \frac{\vartheta'_4}{\delta'_2}, \frac{\vartheta'_5}{\delta'_1} \right) \end{matrix} \right\}; \\ (v) \quad k \times \mathcal{L}_{\mathfrak{R}_{TLDFN}} &= \begin{cases} \left\{ \begin{matrix} (k\vartheta_1, k\vartheta_2, k\vartheta_3, k\vartheta_4, k\vartheta_5) \\ (k\vartheta'_1, k\vartheta'_2, k\vartheta'_3, k\vartheta'_4, k\vartheta'_5) \end{matrix} \right\} & \text{if } k > 0 \\ \left\{ \begin{matrix} (k\vartheta_5, k\vartheta_4, k\vartheta_3, k\vartheta_2, k\vartheta_1) \\ (k\vartheta'_5, k\vartheta'_4, k\vartheta'_3, k\vartheta'_2, k\vartheta'_1) \end{matrix} \right\} & \text{if } k < 0. \end{cases}\end{aligned}$$

**Definition 9 ([24]).** The ranking function for TrapLDFN  $A$  is determined by the following factors:

$$\mathfrak{R}(A) = \sqrt{[\xi_{\mathfrak{R}}^T(x) - \xi_{\mathfrak{R}}^T(y)]^2 + [\gamma(x) - \gamma(y)]^2 + [\varkappa_{\mathfrak{R}}^V(x) - \varkappa_{\mathfrak{R}}^V(y)]^2 + [\delta(x) - \delta(y)]^2}.$$

This represents the Euclidean distance.

The centroids of the triangular LDFN's membership functions and non-membership functions are defined as

$$\begin{aligned}\xi_{\mathfrak{R}}^{\tau}(x) &= \frac{1}{3}[\vartheta_1 + \vartheta_3 + \vartheta_5], \\ \varkappa_{\mathfrak{R}}^{\nu}(x) &= \frac{1}{3}[2\vartheta_2 - \vartheta_3 + 2\vartheta_4], \\ \gamma(x) &= \frac{1}{3}[\vartheta'_2 + \vartheta_3 + \vartheta'_4], \\ \delta(x) &= \frac{1}{3}[2\vartheta'_1 - \vartheta_3 + 2\vartheta'_5],\end{aligned}$$

and

$$\begin{aligned}\xi_{\mathfrak{R}}^{\tau}(y) &= \frac{1}{3}, \quad \varkappa_{\mathfrak{R}}^{\nu}(y) = \frac{2}{3}, \\ \gamma(y) &= \frac{1}{3}, \quad \delta(y) = \frac{2}{3}.\end{aligned}$$

### Karush-Kuhn-Tucker (KKT) Conditions:

The KKT conditions are first-order conditions for constrained optimization problems, which are a generalization of the first-order conditions we are already familiar with. These more general requirements offer an integrated approach to constrained optimization, where

- we permit inequality constraints,
- the number of constraints is unlimited; the constraints may be binding or not binding at the solution,
- non-negativity constraints can be used,
- boundary solutions are allowed,
- non-negativity and structural constraints are also addressed in the same way,
- dual variables, also known as Lagrange multipliers.

Linear programming is a special case covered by the KKT conditions. Consider the following optimization problem,

$$\text{Min } f(x) \text{ subjected to } g_k(x) \leq 0 \text{ for } k = 1, 2, \dots, m \text{ where } X = [x_1, x_2, \dots, x_n].$$

Then KKT condition for  $X^* = [x_1^*, x_2^*, \dots, x_n^*]$  to be local minimum are

- (i)  $\frac{\partial f}{\partial x_i} + \sum_{k=1}^n \lambda_k \frac{\partial g}{\partial x_i} = 0, i = 1, 2, \dots, n;$
- (ii)  $\lambda_k g_k = 0, k = 1, 2, \dots, m;$
- (iii)  $g_k \leq 0, k = 1, 2, \dots, m;$
- (iv)  $\lambda_k \leq 0, k = 1, 2, \dots, m.$

On the other hand KKT condition for local maximum are

- (i)  $\frac{\partial f}{\partial x_i} + \sum_{k=1}^n \lambda_k \frac{\partial g}{\partial x_i} = 0, i = 1, 2, \dots, n;$
- (ii)  $\lambda_k g_k = 0, k = 1, 2, \dots, m;$
- (iii)  $g_k \leq 0, k = 1, 2, \dots, m;$
- (iv)  $\lambda_k \geq 0, k = 1, 2, \dots, m.$

### 3. Linear Diophantine Fuzzy Nonlinear Programming Problem (LDFNLPP)

The following is the mathematical model of LDFNLPP

$$\text{Maximize (or Minimize) } Z^L = \sum_{k=1}^n \tilde{c}_k^L x_k^L$$

$$\text{subject to } \sum_{k=1}^n \tilde{a}_{jk}^L x_k^L \leq \tilde{b}_j^L, \quad 1 \leq j \leq m; \quad x_k \geq 0.$$

where  $\tilde{c}^L, \tilde{a}_{jk}^L$  and  $\tilde{b}^L$  are triangular LDFNs.

The LDFNLPP suggested algorithm goes as follows:

**Step 1.** Calculate the ranking index according to Definition 9 for each parameter of the provided problem.

**Step 2.** By their respective ranking indices derived from Step 1, replace the Linear Diophantine fuzzy parameters.

**Step 3.** Apply the KKT condition to the reduced problem to get the optimal solution.

The following example shows the steps involved in using the suggested approach to arrive at an optimal solution to the LDFNLPP problem.

Consider the following LDFNLPP problem

$$\begin{aligned} \text{Min } Z &= -\tilde{2}x_1 - \tilde{5}x_2^2 \\ \text{subject to } &\begin{cases} \tilde{4}x_1 + \tilde{7}x_2 \leq \tilde{12} \\ \tilde{9}x_1 + \tilde{10}x_2 \leq \tilde{15} \\ x_1, x_2 \geq 0. \end{cases} \end{aligned}$$

$$\begin{aligned} \tilde{2} &= \left\{ \begin{array}{l} (7, 9, 13, 15, 17) \\ (6, 8, 13, 18, 20) \end{array} \right\}, \tilde{5} = \left\{ \begin{array}{l} (8, 10, 15, 18, 22) \\ (7, 9, 15, 20, 25) \end{array} \right\}, \tilde{4} = \left\{ \begin{array}{l} (4, 6, 8, 12, 14) \\ (3, 5, 8, 15, 19) \end{array} \right\}, \\ \tilde{7} &= \left\{ \begin{array}{l} (6, 8, 12, 16, 18) \\ (5, 7, 12, 19, 23) \end{array} \right\}, \tilde{12} = \left\{ \begin{array}{l} (29, 32, 37, 40, 42) \\ (25, 30, 37, 43, 47) \end{array} \right\}, \tilde{9} = \left\{ \begin{array}{l} (7, 9, 11, 13, 15) \\ (6, 8, 11, 16, 19) \end{array} \right\}, \\ \tilde{10} &= \left\{ \begin{array}{l} (9, 10, 12, 13, 15) \\ (8, 9, 12, 14, 17) \end{array} \right\}, \tilde{15} = \left\{ \begin{array}{l} (30, 35, 38, 42, 45) \\ (27, 33, 38, 46, 50) \end{array} \right\}. \end{aligned}$$

Following are the ranking indices for the parameters corresponding to the provided LDFNLPP applying steps 1 and 2.

$$\begin{aligned} \text{Min } Z &= -24.02x_1 - 28.88x_2^2 \\ \text{subject to } &\begin{cases} 18.80x_1 + 24.42x_2 \leq 70.99 \\ 22.37x_1 + 22.84x_2 \leq 75.99 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned}$$

We define the Lagrangian function;

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2) &= -24.02x_1 - 28.88x_2^2 + \lambda_1(70.99 - 18.80x_1 - 24.42x_2) \\ &\quad + \lambda_2(75.99 - 22.37x_1 - 22.84x_2). \end{aligned}$$

Using KKT conditions:

$$\begin{aligned} -24.02 - 18.80\lambda_1 - 22.37\lambda_2 &= 0, \\ -57.76x_2 - 24.42\lambda_1 - 22.84\lambda_2 &= 0, \\ 70.99 - 18.80x_1 - 24.42x_2 &= 0, \\ 75.99 - 22.37x_1 - 22.84x_2 &= 0. \end{aligned}$$

We obtain the following optimal solution:

$$\begin{aligned} x_1 &= 2.891, \quad x_2 = 0.495 \text{ and} \\ \text{Min } Z &= -\left\{ \begin{array}{l} (22.197, 28.469, 41.258, 47.775, 54.537) \\ (19.061, 25.333, 41.258, 56.938, 63.945) \end{array} \right\}. \end{aligned}$$

#### 4. Linear Diophantine Fuzzy Multi-Objective Nonlinear Programming Problem (LDFMONLPP)

$$\text{Maximize (or Minimize) } Z^L = \sum_{k=1}^n \tilde{c}_k^L x_k^L \quad (n \geq 2 \text{ as the objective function is non linear})$$

$$\text{subject to } \sum_{k=1}^n \widetilde{a}_{jk}^L x_k^L \leq (\geq) \widetilde{b}_j^L \quad 1 \leq j \leq m; x_k \geq 0.$$

where  $\widetilde{c}^L, \widetilde{a}_{jk}^L$  and  $\widetilde{b}^L$  are triangular LDFNs.

In this paper we discuss three cases.

#### Case I:

First, we take multi objective problem where the coefficients of the decision variables in the objective function, coefficients of the decision variables in the constraints, and right-side constraints are modelled as TLDFN, and then we create as an NLPP with linear Diophantine fuzzy inequalities and objective function.

$$\begin{aligned} \text{Max } \theta &= \sum_{j=1}^n \left( \begin{array}{c} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^{3'}, c_j^{4'}, c_j^{5'}) \end{array} \otimes X_i \right) \\ &\quad (n \geq 2 \text{ as the objective function is non linear}) \\ \text{Max } \theta &= \sum_{j=1}^n \left( \begin{array}{c} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^{3'}, c_j^{4'}, c_j^{5'}) \end{array} \otimes X_i \right) \\ &\quad (n \geq 2 \text{ as the objective function is non linear}) \\ \text{subject to } &\left\{ \begin{array}{c} (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5) \\ (a_{ij}^{1'}, a_{ij}^{2'}, a_{ij}^{3'}, a_{ij}^{4'}, a_{ij}^{5'}) \end{array} \otimes X_i \leq \left\{ \begin{array}{c} (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) \\ (b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'}) \end{array} \right. \right. \\ &\quad X \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n. \end{aligned}$$

We propose the following strategy to resolve problems involving linear Diophantine fuzzy nonlinear programming:

- Step 1.** Assume the coefficient  $c^L$  and coefficients of  $a_{jk}^L$  and  $b^L$  to be triangular linear Diophantine fuzzy numbers.
- Step 2.** Apply the ranking function from Definition 9.
- Step 3.** The stationary points can be found using the KKT conditions.
- Step 4.** Verify the optimality at these stationary points.
- Step 5.** Find the optimal solution.

Consider the following LDFMONLPP:

$$\begin{aligned} \text{Max } Z &= \widetilde{6}x_1^2 + \widetilde{4}x_2^2 \\ \text{Max } Z &= \widetilde{9}x_1^2 + \widetilde{2}x_2^2 \\ \text{subject to } &\begin{cases} \widetilde{5}x_1 + \widetilde{7}x_2 \leq \widetilde{10} \\ \widetilde{8}x_1 + \widetilde{3}x_2 \leq \widetilde{11} \end{cases} \\ &x_1, x_2 \geq 0. \end{aligned}$$

where

$$\begin{aligned} \widetilde{6} &= \left\{ \begin{array}{c} (1, 3, 5, 8, 9) \\ (0, 3, 5, 7, 11) \end{array} \right\}, \quad \widetilde{4} = \left\{ \begin{array}{c} (0, 4, 5, 6, 9) \\ (2, 3, 5, 7, 11) \end{array} \right\}, \quad \widetilde{9} = \left\{ \begin{array}{c} (2, 4, 4, 5, 6) \\ (1, 3, 4, 8, 10) \end{array} \right\}, \\ \widetilde{2} &= \left\{ \begin{array}{c} (1, 2, 5, 9, 11) \\ (0, 3, 5, 8, 10) \end{array} \right\}, \quad \widetilde{5} = \left\{ \begin{array}{c} (4, 6, 9, 10, 12) \\ (3, 5, 9, 15, 19) \end{array} \right\}, \quad \widetilde{7} = \left\{ \begin{array}{c} (4, 5, 12, 13, 22) \\ (3, 4, 12, 23, 29) \end{array} \right\}, \\ \widetilde{10} &= \left\{ \begin{array}{c} (19, 26, 32, 44, 48) \\ (18, 24, 32, 49, 52) \end{array} \right\}, \quad \widetilde{8} = \left\{ \begin{array}{c} (3, 7, 10, 13, 15) \\ (2, 6, 10, 19, 22) \end{array} \right\}, \quad \widetilde{3} = \left\{ \begin{array}{c} (10, 15, 19, 23, 29) \\ (9, 13, 19, 27, 35) \end{array} \right\}, \\ \widetilde{11} &= \left\{ \begin{array}{c} (23, 30, 39, 49, 62) \\ (21, 27, 39, 57, 70) \end{array} \right\}. \end{aligned}$$

Applying the ranking function on the above problem;

$$\begin{aligned} \text{Max } Z &= 9.634x_1^2 + 9.971x_2^2 \\ \text{subject to } &\begin{cases} 17.918x_1 + 25.378x_2 \leq 69.034 \\ 20.988x_1 + 39.618x_2 \leq 84.051 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned} \quad (1)$$

and

$$\begin{aligned} \text{Max } Z &= 8.925x_1^2 + 9.859x_2^2 \\ \text{subject to } &\begin{cases} 17.918x_1 + 25.378x_2 \leq 69.034 \\ 20.988x_1 + 39.618x_2 \leq 84.051 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned} \quad (2)$$

Define the Lagrangian function for problem 1.

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2) &= 9.634x_1^2 + 9.971x_2^2 + \lambda_1(69.034 - 17.918x_1 - 25.378x_2) \\ &\quad + \lambda_2(84.051 - 20.988x_1 - 39.618x_2), \end{aligned}$$

using KKT conditions to get the optimal solution

$$\begin{aligned} 19.268x_1 - 17.918\lambda_1 - 20.988\lambda_2 &= 0, \\ 19.942x_2 - 25.378\lambda_1 - 39.618\lambda_2 &= 0, \\ 69.034 - 17.918x_1 - 25.378x_2 &= 0, \\ 84.051 - 20.988x_1 - 39.618x_2 &= 0. \end{aligned}$$

We obtain the following optimal solution:

$$\begin{aligned} x_1 &= 0.9006, \quad x_2 = 1.6424 \quad \text{and} \\ \text{Max } Z &= \begin{cases} (0.9006, 13.222, 17.542, 22.672, 31.576) \\ (5.394, 10.525, 17.542, 24.559, 38.593). \end{cases} \end{aligned}$$

Now solving problem 2,

$$\begin{aligned} \text{Max } Z &= 8.925x_1^2 + 9.859x_2^2 \\ \text{subject to } &\begin{cases} 17.918x_1 + 25.378x_2 \leq 69.034 \\ 20.988x_1 + 39.618x_2 \leq 84.051 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned}$$

Define the Lagrangian function for the above case

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2) &= 8.925x_1^2 + 9.859x_2^2 + \lambda_1(69.034 - 17.918x_1 - 25.378x_2) \\ &\quad + \lambda_2(84.051 - 20.988x_1 - 39.618x_2). \end{aligned}$$

The necessary KKT conditions are

$$\begin{aligned} 17.85x_1 - 17.918\lambda_1 - 20.988\lambda_2 &= 0, \\ 19.718x_2 - 25.378\lambda_1 - 39.618\lambda_2 &= 0, \\ 69.034 - 17.918x_1 - 25.378x_2 &= 0, \\ 84.051 - 20.988x_1 - 39.618x_2 &= 0. \end{aligned}$$

We obtain the following optimal solution:

$$\begin{aligned} x_1 &= 0.947, \quad x_2 = 1.6192 \quad \text{and} \\ \text{Max } Z &= \begin{cases} (4.413, 8.826, 16.689, 28.069, 34.207) \\ (0.896, 10.551, 16.689, 28.136, 35.17). \end{cases} \end{aligned}$$



### Case II:

In this particular case, we talk about the nonlinearity of the multi objective variables, the coefficients of the decision variables in the constraints, and the right side of the constraints are TLDFNs.

$$\begin{aligned} \text{Max } \theta &= \sum_{j=1}^n (c_j \otimes X_i^j) \\ \text{Max } \theta &= \sum_{j=1}^n (c_j \otimes X_i^j) \\ \text{subject to } &\left\{ \begin{array}{l} (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5) \\ (a_{ij}^{1'}, a_{ij}^{2'}, a_{ij}^{3'}, a_{ij}^{4'}, a_{ij}^{5'}) \end{array} \right\} \otimes X_i \leq \left\{ \begin{array}{l} (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) \\ (b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'}) \end{array} \right\} \\ &X \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n. \end{aligned}$$

Applying the ranking function which is defined in Definition 9, then solve by KKT condition.

Consider the following LDFMONLPP problem

$$\begin{aligned} \text{Max } Z &= 2x_1^2 + 3x_2^2 \\ \text{Max } Z &= 3x_1^2 + 4x_2^2 \\ \text{subject to } &\left\{ \begin{array}{l} \tilde{6}x_1 + \tilde{9}x_2 \leq \tilde{12} \\ \tilde{7}x_1 + \tilde{10}x_2 \leq \tilde{15} \end{array} \right. \\ &x_1, x_2 \geq 0. \end{aligned}$$

where

$$\begin{aligned} \tilde{6} &= \left\{ \begin{array}{l} (1, 2, 3, 5, 7) \\ (0, 1, 3, 9, 11) \end{array} \right\}, \quad \tilde{9} = \left\{ \begin{array}{l} (2, 4, 6, 8, 10) \\ (1, 3, 6, 11, 12) \end{array} \right\}, \quad \tilde{12} = \left\{ \begin{array}{l} (7, 15, 19, 32, 39) \\ (6, 12, 19, 27, 43) \end{array} \right\}, \\ \tilde{7} &= \left\{ \begin{array}{l} (3, 5, 7, 11, 12) \\ (1, 4, 7, 10, 13) \end{array} \right\}, \quad \tilde{10} = \left\{ \begin{array}{l} (5, 7, 9, 10, 11) \\ (4, 6, 9, 12, 15) \end{array} \right\}, \quad \tilde{15} = \left\{ \begin{array}{l} (12, 22, 25, 36, 43) \\ (11, 21, 25, 42, 49) \end{array} \right\}. \end{aligned}$$

Using the ranking function on the above problem;

$$\begin{aligned} \text{Max } Z &= 2x_1^2 + 3x_2^2 \\ \text{subject to } &\left\{ \begin{array}{l} 8.258x_1 + 11.689x_2 \leq 45.463 \\ 13.868x_1 + 16.699x_2 \leq 57.652 \end{array} \right. \\ &x_1, x_2 \geq 0. \end{aligned} \quad (3)$$

and

$$\begin{aligned} \text{Max } Z &= 3x_1^2 + 4x_2^2 \\ \text{subject to } &\left\{ \begin{array}{l} 8.258x_1 + 11.689x_2 \leq 45.463 \\ 13.868x_1 + 16.699x_2 \leq 57.652 \end{array} \right. \\ &x_1, x_2 \geq 0. \end{aligned} \quad (4)$$

Define the Lagrangian function for problem 3;

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2) &= 2x_1^2 + 3x_2^2 + \lambda_1(45.463 - 8.258x_1 - 11.689x_2) \\ &\quad + \lambda_2(57.652 - 13.868x_1 - 16.699x_2), \end{aligned}$$

using KKT conditions to get the optimal solution

$$\begin{aligned} 4x_1 - 8.258\lambda_1 - 13.868\lambda_2 &= 0, \\ 6x_2 - 11.689\lambda_1 - 16.699\lambda_2 &= 0, \\ 45.463 - 8.258x_1 - 11.689x_2 &= 0, \\ 57.652 - 13.868x_1 - 16.699x_2 &= 0. \end{aligned}$$

We obtain the following optimal solution:

$$x_1 = 2.111, \quad x_2 = 1.694 \quad \text{and} \quad \text{Max } Z = 17.521.$$

Similarly, solving problem 4 by the same method we obtain the following optimal solution;

$$x_1 = 1.989, \quad x_2 = 1.796 \quad \text{and} \quad \text{Max } Z = 24.770.$$

### Case III:

In this case, we talk about a multi-objective problem with right hand side constants and decision variable coefficients in the objective function are modelled as TLDFN.

$$\text{Max } \theta = \sum_{j=1}^n \left( \begin{matrix} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^{3'}, c_j^{4'}, c_j^{5'}) \end{matrix} \otimes X_i \right)$$

$$\text{Min } \theta = \sum_{j=1}^n \left( \begin{matrix} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^{3'}, c_j^{4'}, c_j^{5'}) \end{matrix} \otimes X_i \right)$$

$$\text{subject to } A_i X_i^j \leq \begin{matrix} (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) \\ (b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'}) \end{matrix}$$

$$X \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n.$$

Consider the following LDFMONLPP problem

$$\begin{aligned} \text{Max } Z &= \tilde{2}x_1 + \tilde{5}x_2 \\ \text{Min } Z &= \tilde{8}x_1 - \tilde{9}x_2 \\ \text{subject to } &\begin{cases} 3x_1 + 4x_2^2 \leq \tilde{7} \\ 2x_1 + 5x_2^2 \leq \tilde{12} \\ x_1, x_2 \geq 0. \end{cases} \end{aligned}$$

where,

$$\begin{aligned} \tilde{2} &= \begin{Bmatrix} (1, 1, 2, 3, 5) \\ (0, 1, 2, 6, 7) \end{Bmatrix}, \quad \tilde{5} = \begin{Bmatrix} (2, 4, 6, 9, 10) \\ (1, 3, 6, 11, 13) \end{Bmatrix}, \quad \tilde{8} = \begin{Bmatrix} (4, 5, 6, 7, 8) \\ (3, 4, 6, 9, 12) \end{Bmatrix}, \\ \tilde{9} &= \begin{Bmatrix} (3, 5, 8, 10, 11) \\ (2, 4, 8, 13, 15) \end{Bmatrix}, \quad \tilde{7} = \begin{Bmatrix} (6, 8, 14, 19, 25) \\ (5, 7, 14, 24, 30) \end{Bmatrix}, \quad \tilde{12} = \begin{Bmatrix} (11, 12, 14, 21, 25) \\ (12, 13, 14, 27, 35) \end{Bmatrix}. \end{aligned}$$

Using the ranking function on the above problem;

$$\begin{aligned} \text{Max } Z &= 5.043x_1 + 12.027x_2 \\ \text{subject to } &\begin{cases} 3x_1 + 4x_2^2 \leq 30.242 \\ 2x_1 + 5x_2^2 \leq 39.148 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned} \quad (5)$$

and

$$\begin{aligned} \text{Min } Z &= -12.260x_1 - 14.880x_2 \\ \text{subject to } &\begin{cases} 3x_1 + 4x_2^2 \leq 30.242 \\ 2x_1 + 5x_2^2 \leq 39.148 \\ x_1, x_2 \geq 0. \end{cases} \end{aligned} \quad (6)$$

Define the Lagrangian function for problem 5,

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2) &= 5.043x_1 + 12.027x_2 + \lambda_1(30.242 - 3x_1 - 4x_2^2) \\ &\quad + \lambda_2(39.148 - 2x_1 - 5x_2^2), \end{aligned}$$

using KKT conditions to get the optimal solution

$$\begin{aligned}
5.043 - 3\lambda_1 - 2\lambda_2 &= 0, \\
12.027 - 8x_2\lambda_1 - 10x_2\lambda_2 &= 0, \\
30.242 - 3x_1 - 4x_2^2 &= 0, \\
39.148 - 2x_1 - 5x_2^2 &= 0.
\end{aligned}$$

which implies

$$\begin{aligned}
x_1 &= 9.015, \quad x_2 = 0.894 \text{ and} \\
\text{Max } Z &= \begin{cases} (10.803, 12.591, 23.394, 35.091, 54.015) \\ (0.894, 11.697, 23.394, 63.924, 74.727) \end{cases}
\end{aligned}$$

Similarly, solving problem 6 by the same method we obtain the following optimal solution,

$$\begin{aligned}
x_1 &= 9.804, \quad x_2 = 0.455 \text{ and} \\
\text{Min } Z &= -\begin{cases} (40.581, 51.295, 62.464, 73.178, 83.437) \\ (30.322, 41.036, 62.464, 94.151, 124.473) \end{cases}
\end{aligned}$$

## 5. Conclusions

In this paper, we have proposed multi-objective non-linear programming problem in linear Diophantine fuzzy environment having mixed type of conflicting objectives. We used a linear ranking function for triangular LDFNs. We developed three approaches to solve the LDFMONLPP with the help of Kuhn-Tucker conditions. Using this method, many decision making and optimization problems of uncertain nature can be solved. Also the proposed modelling will be useful for decision-making problems involving hesitation and uncertainty with multiple objectives in manufacturing, production, planning, and scheduling systems. There are several applications for nonlinear programming. Some of the most common are data networks routing, production planning, resource allocation, computer-aided design, solution of equilibrium models, data analysis and least squares formulations and modeling human or organizational behavior. These applications usually share some attributes regarding problem structure that make convex optimization algorithms very effective.

**Author Contributions:** S.I. wrote the main manuscript text. N.Y. and M.G. supplied the main results. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Data Availability Statement:** The data presented in this study are available on request from the second author.

**Acknowledgments:** The authors are highly grateful to the referees for their valuable comments and suggestions.

**Conflicts of Interest:** The authors declare no conflict of interest.

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