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A Joint Location–Allocation–Inventory Spare Part Optimization Model for Base-Level Support System with Uncertain Demands

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Abstract: This paper copes with a joint Location-Allocation-Inventory problem in a three-echelon base-level spare part support system with epistemic uncertainty in uncertain demands of bases. The aim of the paper is to propose an optimization model under the uncertainty theory to minimize the total cost, which integrates crucial characterizations of the inventory control decisions and the location-allocation scheme arrangement under a periodic review order-up-to-S (T, S) policy. Uncertainty theory is introduced in this paper to characterize epistemic uncertainty, where demands are treated as uncertain variables and stockout loss is represented by value-at-risk in uncertain measurement. To solve the original uncertain optimization model, an equivalent deterministic model is derived and addressed by an improved bilevel genetic algorithm. Moreover, the proposed models and algorithm are encoded into numerical examples for supply chain programming. The results highlight the applicability of the model and the algorithm's effectiveness in approaching the optimal solution compared with traditional genetic algorithm. Sensitivity analyses are further made for the impacts of review time and inventory capacity on different cost components.

Keywords: Location-Allocation-Inventory; base-level spare part; uncertainty theory; bilevel genetic algorithm

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1. Introduction

Base-level support systems provide spare parts storage, replacement, and other services, which play an essential role in the current maintenance strategy of sophisticated military and engineering systems [1,2]. Moreover, a collection of industries has witnessed that inventory control of spare parts and physical distribution of supply sites consume more than half of the total cost statistically [2]. In this sense, it is of great value for base-level support systems to explore how to appropriately manage the location of supply sites, inventory strategies, and allocation relationships.

The Location-Allocation-Inventory model (LAIM) is a sort of joint model aiming to simultaneously address the problem of location, allocation, and inventory control. Recent developments have revealed the significance of building such models. One of the earliest studies is proposed by Yao et al. [3]. Considering customer demands and safety storage stock, they established a model to shrink the supply chain costs by specifying the number of warehouses along with their corresponding locations, the allocation quantity for customers, and the control of inventory levels. Likewise, in 2012, Tsao et al. [4] presented an integrated facility-inventory allocation network to maximize overhead reduction without breaking the regulated area coverage restraints. Dai et al. [5] attempted to make a tradeoff between expenditure involved and stipulating stock capacity with carbon emissions for a perishable product location–inventory supply chain. In addition, this kind of joint model can also be utilized to improve the efficiency of municipal solid waste collection systems [6]. It can be seen that LAIMs have been applied to a variety of issues in industries and environmental fields with good enforcement performance. However, these models can't be directly embedded in support systems due to the lack of consideration on the characteristics of support systems, such as supply availability, service level, and balanced dispatching regulations [7–9].

Uncertainty quantification of supply chain variables is also a non-negligible issue, which has a significant impact on the results of supply chain optimization. In the past decades, probability theory has been firstly employed to cope with such problems. For example, Asian and Nie [10] used normal distribution to describe demand's uncertainty with consideration of market volatility. Islam et al. [11] delineated Poisson distributed demand and exponentially distributed supplier capacity in a three-layer inventory-allocation management system merging. Errico et al. [12] assumed service times to be mutually independent discrete triangular distributions in a vehicle routing problem. Nevertheless, probability theory is not always appropriate to disentangle indeterminate scenarios of spare part provision, especially for military resource supplements, because, for armament systems, the burstiness and variability of military missions often cause the historical knowledge unable to completely characterize the demand information. As for the above-mentioned scene with sparse useful observational data, frequency probabilistic methods are limited by the law of the large numbers, namely the frequency tends to the probability when the number of independent repeated trials tends to infinity [13,14]. Moreover, it is epistemic uncertainty rather than aleatory uncertainty that becomes the principal uncertainty under such sparse or no data conditions. Therefore, two mathematical measures are introduced as alternatives to specify epistemic uncertainty.

One train of thought originating from fuzzy theory has been explored a lot for a few decades [15–18]. Nonetheless, the puzzle that fuzzy theory is not compliant with the duality axiom, has confused supply chain decision-makers for a long time. The other is the uncertainty theory built by Liu [19], which is regarded as a breakthrough in handling epistemic uncertainty problems that predicate normality, duality, subadditivity, and product axioms. In the wake of the theory's development, it has been gradually discovered to be an excellent way to tackle the epistemic uncertainty in supply chains. For instance, Sheng et al. [20] referred to an infinite-horizon production–inventory optimal control problem with regulated production rates which is transformed into the value function illustrated by uncertainty theory owing to the scarcity of affordable experiment samples. Asim et al. [21] demonstrated an integrated multi-echelon multi-item production-transportation close-loop network, where cost, demand, and capacity are characteristic as uncertain variables to vanquish the inapplicability of probability or fuzzy set theory for the model with belief degree. Shen [22] pointed out that many environmental and social emergencies in supply chains incur the demand, cost, and capacity acting as uncertain variables. Hosseini and Pishvaee [23] put forward an α-maximum capacity path (UMCP) problem with inaccurate transportation reliability and operational capacity of links appearing as 'belief degrees' caused by deficient information and computing simplification. In their work, links' capacities are deemed as uncertain variables, and the stability analysis is launched in the framework of uncertainty theory. To this end, they utilized the uncertainty theory to establish an uncertain purveyor selection model concerning the elements of cost, environmental impact, and social benefits evaluated by uncertainty theory. Therefore, in this paper, uncertainty theory is expropriated to quantify the uncertainties of demand caused by the insufficiency of historical information in the programming stage and the shortage cost loss deriving from exigencies of long-distance delivery. Specifically, we suppose demands as normal-distributed uncertain variables mingled into the supportability chance constraints, while the penalty cost is weighted by the uncertain value-at-risk.

Since LAIMs need to concurrently determine the depot locations, the allocation relationships, and inventory parameters, the problem itself tends to be high non-linearity and NP-hardness with huge feasible space. Methodically, the kind of NP-hardness difficulty yields the rise of plenty of heuristic and metaheuristic algorithms which do well in scaling down the CPU time and model size [24]. Up to now, a handful of latest works have proponed some algorithms for addressing LAIMs. For example, Tirkolaee et al. [25] developed a Self-Learning Particle Swarm Optimization (SLPSO) algorithm for solving a multi-echelon capacitated LAIM, which tackled the PSO algorithm's deficiency of lacking intelligent particles. Mousavi et al. [26] made a comparison of a Modified Genetic Algorithm (MGA) and a Particle Swarm Optimization (PSO) for gaining results of a random two-echelon LAIM. The gained results exemplified that the MGA with Taguchi parameter adjustment approach outperforms the PSO accordant with the fitness functions and computing time. Sahraei and Samouei [27] analyzed the distinct performance of genetic and electromagnetic meta-heuristic algorithms for a bi-level scenario-based LAIM and indicated the genetic algorithm is more effective. It can be seen that genetic algorithm and particle swarm optimization algorithm are both feasible approaches for solving the Location-Allocation-Inventory problems, and the introduction of parameter adjustment methods along with bilevel optimization methodology play positive roles in algorithm convergence and optimal solution acquisition. Consequently, we deploy an improved bilevel genetic algorithm to accomplish the model-solving, attempting to improve operation accuracy in contrast with the traditional genetic algorithm.

The highlights of this work include:

(i) Different from the previous literature, this work develops a joint Location-Allocation-Inventory model (LAIM) for a base-level spare part supply problem, which mingles with supportability indexes, namely, service level and supply availability.

(ii) Unlike the common way of demand characterization, the proposed model considers the vein with a lack of historical data about demand information, and thus the demands in the network were characterized by the uncertainty theory with value-at-risk introduced for stockout loss evaluation.

(iii) As for solving the uncertain optimization model, an equivalent deterministic model was derived with an improved bilevel genetic algorithm proposed for accelerating the optimal value acquirement.

The remainder of this paper is organized as follows: Section 2 presents the preliminaries on uncertainty theory. Section 3 is dedicated to describing the problem with an in-depth exploration of the provided three-echelon support system, and according to the hypothesis and analysis, a spare parts supply optimization model is developed with shortage events existing. Then, Section 4 emphasizes employing an improved bilevel genetic algorithm to help designate appropriate transportation and storage tactics. Section 5 shows an application of the proposed model in a specific case, a comparison between the proposed algorithm and the traditional GA, and a sensitivity analysis of various cost components. Concluding remarks are drawn along with a discussion for future research in Section 6.

2. Preliminaries on Uncertainty Theory

For the sake of appropriately forecasting the demand quantity based on subjective experience, we adopt the uncertainty theory which is a novel axiomatic mathematical theory proposed by Liu for resolving epistemic uncertainty [19]. Below are some definitions and theorems related to uncertain assessment.

Definition 1 (Uncertain variable)**.** *An uncertain variable ξ is a function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\xi \in \mathfrak{B}$ is an event for any Borel set $\mathfrak{B}$ of real numbers.*

Definition 2 (Uncertain distribution). *The uncertainty distribution Φ of an uncertain variable ξ is defined by*

$$\Phi(\alpha) = \mathcal{M}\{\xi \leq x\} \tag{1}$$

for any real number x

Definition 3 (Normal uncertainty distribution). *An uncertain variable ξ is called a normal variable if it has a normal uncertainty distribution*

$$\Phi(x) = \left(1 + exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1} \tag{2}$$

denoted by $\mathcal{N}(e, \sigma)$,where e and σ are real numbers with $\sigma > 0$.

Definition 4 (Inverse uncertain distribution). *Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then, the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ. A function Φ^{-1} is an inverse uncertainty distribution of an uncertain variable ξ if and only if*

$$\mathcal{M}\{\xi \leq \Phi^{-1}(\alpha)\} = \alpha \tag{3}$$

for all $\alpha \in [0, 1]$.

Definition 5 (Inverse normal uncertain distribution). *The inverse uncertainty distribution of normal uncertain variable $\mathcal{N}(e, \sigma)$ is*

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}. \tag{4}$$

Definition 6 (Inverse normal uncertain distribution). *Let ξ be an uncertain variables. Then, the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq x\}\, dx - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq x\}\, dx \tag{5}$$

provided that at least one of the two integrals is finite.

Definition 7 (Inverse normal uncertain distribution). *Assume that a system contains uncertain factors $\xi_1, \xi_2, \ldots, \xi_n$ and has a loss function f. Then, the value-at-risk is defined as*

$$\mathrm{VaR}(\alpha) = sup\{x | \mathcal{M}\{f(\xi_1, \xi_2, \ldots, \xi_n) \geq x\} \geq \alpha\}. \tag{6}$$

Note that VaR*(α) represents the maximum possible loss when α percent of the right tail distribution is ignored.*

Theorem 1 (Inverse uncertain distribution operational law). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$is continuous, strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an inverse uncertainty distribution*

$$\Psi(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \Phi_{m+2}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)). \tag{7}$$

Theorem 2 (Normal uncertain distribution operational law). *Let ξ_1 and ξ_2 be independent normal uncertain variables $\mathcal{N}(e_1, \sigma_1)$ and $\mathcal{N}(e_2, \sigma_2)$, respectively. Then, the sum $\xi_1 + \xi_2$ is also a normal uncertain variable $\mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2)$, i.e.,*

$$\mathcal{N}(e_1, \sigma_1) + \mathcal{N}(e_2, \sigma_2) = \mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2) \tag{8}$$

3. Problem Description and Model Formulation

In this section, we first collate the problem of a three-level supply chain network for the base-level support system. With the introduction of the assumptions and notations, an uncertain optimization model is formulated for the problem.

3.1. Problem Statement

This paper specializes in providing a three-echelon Location-Allocation-Inventory network for supporting a kind of base-level spare part. As Figure 1 demonstrated, the network is made up of one supplier and several bases scattered within a relatively concentrated region. Supplier is responsible for producing and offering spare parts required by bases, which is usually unique and relatively far from the bases in the support system. The bases are responsible for replacing spare parts for the machine [28]. Therefore, to realize the timely support of spare parts, it is essential for the bases to have a certain amount of storage capacity. Each base in the network has a depot, which is of course the ideal situation. However, due to the constraints of resources, only a certain number of depots can be allowed to be built. In this paper, we identify the number of depots as a given number n with the consideration of depot-covered areas and affordable support resources. As a result, the bases can be divided into two categories: bases without depot, and bases with depot. The latter should undertake the responsibility for launching periodical orders from the supplier and responding to real-time demand from bases including itself. Note that each base accepts spare parts from a single depot, each depot can serve more than one base.

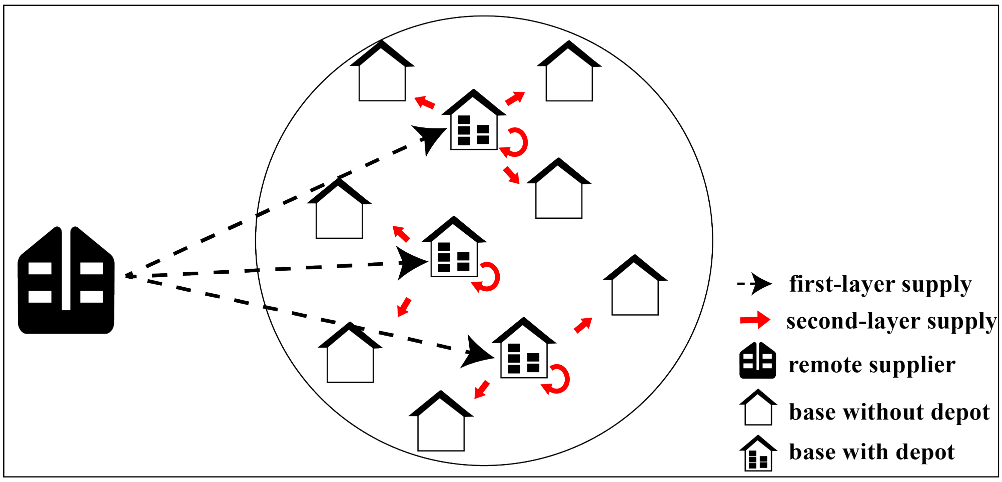


Figure 1. The framework of a three-echelon supply chain with product and information flow.

In respect of demand estimation, we confront the situation that historical demand data is scarce or untrustworthy with epistemic uncertainties domaining the parameter characterization. Because of this, we introduce uncertainty theory to mathematically describe the demand quantity and cost constituents.

In the articulated supply chain for base-level spare parts provision, the coordinates of bases are predefined, while depot locations and the depot-base supply relationships need to be determined. In addition, we suppose the selected centralized depots are amenable to the (T, S) policy defined as the periodic review order-up-to-S [29]. As Figure 2 demonstrated, the floating demand of bases (Figure 2a) transfers to upper-layer centralized depots, which brings about the descent of inventory level, whereas the periodical ordering and the followed inventory replenishment provoke the frequent rebounds of inventory level. It is worth noting that there is a certain length of time (the so-called lead time) between the ordering and inventory replenishment stages, which emanates from the manufacturing process and long-distance delivery [30]. For simplification, we set it to be a constant value. Figure 2 also underlines one scene where the net inventory level is inferior to zero, and the scene can be explained as the trigger of a risk shortage. From the abovementioned management of Location-Allocation-Inventory activities, we intend to minimize the total supply cost, which covers the amortized expenditure arising from depot setup/renovation along with safeguard, delivery expense, inventory holding cost, stockout risk loss, and order outlay.

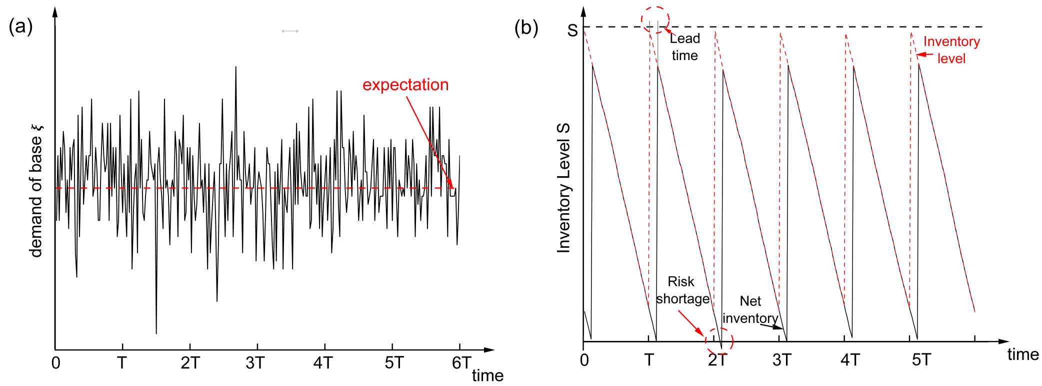


Figure 2. The time-variant characteristic sketch maps: (**a**) the distribution of demand from base ξ and (**b**) inventory level S.

Compared with the provided services of common merchandise, spare parts support systems are bound to meet some supportability standards. In this article, we pick up service level and supply availability as supportability evaluation indexes. Meanwhile, reasonable allotment of safeguard assets is amalgamated to the equilibrium supply relationship of depots.

3.2. Assumptions

In accordance with the problem background, the underlying assumptions are summarized as follows:

- Lead time is regarded as a constant/
- Bases have no storage capacity initially but have the probability to construct a depot and each of them should be supplied by only one depot including itself.
- The difference value between the number of service bases among centralized depots cannot exceed one for equilibrating provision;
- (T, S) inventory policy is adopted for each depot with a review period regulated within a certain range.
- Base demand is independent and normally distributed uncertain variable.
- Unmet demand in a period is reckoned as backlogging loss which needs to be replenishment in the next period.
- The total number of depots in the network is fixed.
- Depots are considered as distributing centers that place orders to the supplier and satisfy the real-time demand from depots.

3.3. Abbreviations and Notations

Abbreviations

LAIM: Location–Allocation–Inventory Model;
ULAIM: Uncertain Location–Allocation–Inventory Model;
EBO: Expected backorder (stockout order);
GA: Genetic Algorithm;

Parameters

I: set of bases which are also viewed as candidate depots;
b_i: fix the section of unit time safeguards expenditure of candidate depot i, $\forall i \in I$;
$c_i^{(1)}$: inventory-related section of unit time unit inventory capacity safeguards expenditure of candidate depot i, $\forall i \in I$;
$c_i^{(2)}$: unit distance unit spare part allocation expense of the candidate depot i, $\forall i \in I$;
$c_i^{(3)}$: unit inventory order outlay of the candidate depot i, $\forall i \in I$;
h_i: inventory cost per spare part of the candidate depot i, $\forall i \in I$;
g_i: risk shortage loss coefficient of the candidate depot i, $\forall i \in I$;
k_i: single inventory review outgoing of the candidate depot i, $\forall i \in I$;
(x_i, y_i): coordinate of base i, $\forall i \in I$;
n: number of depots, a constant;
L: lead time of spare part centralized depots, a constant;
A: lower limit of inventory supply availability for all bases;
α: belief degree for satisfying the criteria of depot service level for each depot;
β: belief degree for guaranteeing the supply available for each base;
γ: stockout risk level for the proposed system;
Z: unit time installation number of spare parts for each equipment;
N_i: equipment quantity of base i, $\forall i \in I$;
C: unit time total cost;
C_i: unit time total cost of depot i;
C_B: unit time overall maintenance expenditure of depots;
C_T: unit time overall allocation expense;
C_H: unit time overall holding cost of depots;
C_R: unit time overall stockout risk loss of depots;
C_O: unit time overall order outlay of depots;
ξ_i: unit time uncertain demands of the base i, $\forall i \in I$;
f_i: shortage value-at-risk quantity of the candidate depot i, $\forall i \in I$;
$\Phi_i(x)$: uncertainty distribution of ξ_i, $\forall i \in I$;
$\Phi_i^{-1}(x)$: inverse uncertainty distribution of ξ_i, $\forall i \in I$;
$E(x)$: expected value of the uncertain variable x;

Decision variable

S_i: designed inventory level of the selected base i with a depot, $\forall i \in I$;
T_i: designed review period of the selected base i with a depot, $\forall i \in I$;

$$X_i = \begin{cases} 1, & \text{if base } i \text{ has centralized depot} \\ 0, & \text{if base } i \text{ can't store spare parts} \end{cases}, \forall i \in I;$$

$$Y_{ij} = \begin{cases} 1, & \text{if base } i \text{ is supplied by selected base } j \\ 0, & \text{if base } i \text{ is not supplied by selected base } j \end{cases}, \forall i, j \in I;$$

3.4. Model

According to the problem background, we formulate an uncertain optimization model to realize the total cost minimality without violating the appointed constraints. The mission of the model is to determine the centralized depot locations, inventory parameters, and allocation relationships. The following subsection presents the objective and constraints of the model.

3.4.1. Objective Function

From the cost point of view, this paper summarizes five types of cost, namely, depot maintenance expenditure cost, allocation expense, inventory holding cost, risk shortage loss, and order outlay cost. As for the long-term service of the supply chain, we aspire to minimize the unit time total cost equal to the aggregation of the above five cost compositions, which can be expressed as:

$$C = C_B + C_T + C_H + C_R + C_O. \tag{9}$$

Explicitly, the detailed interpretations of different unit time cost components are listed below.

- **The unit time overall maintenance expenditure of depots**
  If base i is appointed as depot location equivalent to $X_i = 1$, it will undergo two types of cost-consuming maintenance activities. One contains the initial setup and multitudinous renovations which are believed to spend a certain expenditure that is in line with the depot capacity S_i. The linear correlation cost coefficient is recorded as $c_i^{(1)}$. Additionally, each support depot on guard should pay some safeguard cost for maintaining its normal operation, which is simplified to be a fixed value b_j. Therefore, the unit time overall maintenance expenditure of depots can be evaluated by the sum of the two cost sections of all depots, i.e.,

$$C_B = \sum_{i \in I} X_i (b_i + c_i^{(1)} \cdot S_i). \tag{10}$$

- **The unit time overall allocation expense**
  On the condition that base i obtains service from depot j $(Y_{ij} = 1)$, there will be an amount of allocation expense which indicates the cost spent on spare parts transportation between depot j and base i. The expense is associated with the average demands $E(\xi_i)$, the two sites transportation distance, and the unit distance unit spare part allocation expense of depot j, (denoted by $c_j^{(2)}$). Hence, the unit time overall allocation expense satisfies the expression:

$$C_T = \sum_{j \in I} \sum_{i \in I} c_j^{(2)} \cdot \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \cdot E(\xi_i) \cdot Y_{ij}. \tag{11}$$

- **The unit time overall holding cost of depots**
  The influences of the time-varying inventory level of depot j on cost consumption can be integrated into two aspects under the assumption that $X_j = 1$.
  On the one hand, when the depot j has a positive inventory level, it needs to pay for managing the inventory and the corresponding expense depending on the stock quantity. In particular, the unit time holding cost of depot j can be computed by multiplying its expected net inventory quantity with a given cost coefficient h_j. The expected net inventory is computed by the line integral. As Figure 2 displayed, the inventory level is replenished to S_j at the beginning of review periods while the expected inventory level at end of the periods is $S_j - \sum_{i \in I} E(\xi_i) \cdot Y_{ij} \cdot T_j$. Therefore, the expected inventory level is $S_j - \frac{1}{2} \sum_{i \in I} E(\xi_i) \cdot Y_{ij} \cdot T_j$. In addition, the spare parts at the lead time will not incur the inventory cost, the corresponding quantifies of which should be deducted from the expected inventory level. Thus, the unit time overall holding cost C_H can be assessed by:

$$C_H = \sum_{j \in I} X_j \cdot h_j \cdot \left(S_j - \frac{1}{2} \sum_{i \in I} E(\xi_i) \cdot Y_{ij} \cdot T_j - \sum_{i \in I} E(\xi_i) \cdot L \cdot Y_{ij} \right)^+ \tag{12}$$

  where the function $(x)^+$ represents the superior value of x and 0.

- **The unit time overall stockout risk loss of depots**
On the other hand, the negative inventory level attributes to the shortage loss. According to uncertainty theory, the shortage value-at-risk quantity f_j as a substitute for the risk index indicates the maximum possible loss number of spare parts in a review period. Through the multiplication of value f_j with an allotted coefficient g_j, we gain the whole review period shortage loss of depot j. Then, the unit time overall stockout risk loss of depots is demonstrated below:

$$C_R = \sum_{j \in I} X_j \cdot g_j \cdot \frac{1}{T_j} \cdot \left(\sup\left\{ f_j \middle| \mathcal{M}\{\sum_{i \in I} \xi_i \cdot Y_{ij} \cdot T_j - S_j \geq f_j\} \geq \gamma \right\} \right)^+ \quad (13)$$

where the function $(x)^+$ represents the superior value of x and 0.
- **The unit time overall order outlay of depots**
Order outlay hails from the rapid inventory counting and the followed order requesting procedures at the end of each period. For depot j, every inventory counting brings about a fixed payment k_j, plus an order batch related spend paid for launching periodical orders, where an order batch is the demand totality of the bases set that accords with $Y_{ij} = 1,\ i \in I$. Accordingly, the unit time order outlay can be written as:

$$C_O = \sum_{j \in I} (\sum_{i \in I} c_j^{(3)} \cdot E(\xi_i) \cdot Y_{ij} + k_j \cdot X_j / T_j). \quad (14)$$

In the formula, $c_j^{(3)}$ are already known.

3.4.2. Constraints

Subsequently, we will explicit constraints in the proposed model:

- **Supportability constraints**
As a matter of fact, the support system is bound to vouch for the effectiveness of supply. Here, we judge the relative system feature by two supportability criteria. The first is the service level, which can be interpreted as the chance of inventory shortage meeting a given belief degree α:

$$\mathcal{M}\{S_j - \sum_{i \in I} \xi_i \cdot Y_{ij} \cdot T_j \geq 0\} \geq \alpha \quad \forall j \in I. \quad (15)$$

Secondarily, the support demand-side often contracts with the service side that the machines requesting spare parts are in action to some degree, which implies the extent of task completion. In accordance with the definition equation proposed by Sherbrooke 2006, the 'expected availability' can be interpreted: if excluding cannibalization and failure coupling, the supply availability A can be indicated as [31]:

$$A = \left(1 - \frac{EBO}{NZ}\right)^Z, \quad (16)$$

where EBO indicates the expected stockout number of spare parts, N is the total number of machines, and Z denotes the number of spare parts per machine.
Through embedding 'supply availability' into our supportability measure architecture, we obtain the constraint as below:

$$\mathcal{M}\left\{ \sum_{i \in I} \xi_i \cdot Y_{ij} \cdot T_j - S_j \leq \inf\{(1 - A^{1/Z}) \cdot N_i \cdot Z \cdot T_j |\ Y_{ij} = 1\} \right\} \geq \beta \quad \forall j \in I, \quad (17)$$

where β is a predetermined confidence level.

- **Centralized depot number constraint**
In the preliminary scheme planning phase, we assume the number of depots n has been arranged, that is,
$$\sum_{i\in I} X_i = n. \tag{18}$$
- **Supply equilibrium constraint**
The balanced utilization can avoid overusing some of the depots while desolating others. Therefore, the difference in the number of bases served by each depot is to be less than 1. To this end, the supply equilibrium constraint can be written as:
$$X_j \cdot X_k \cdot \left(\sum_{i\in I} Y_{ij} - \sum_{i\in I} Y_{ik}\right) \leq 1 \quad \forall j,k \in I, j \neq k. \tag{19}$$
- **Demand serving constraint**
According to assumption NO. 2, one base should be provided by only one depot, which can be expressed as:
$$\sum_{j\in I} Y_{ij} = 1 \quad \forall i \in I. \tag{20}$$

3.4.3. Uncertain Location–Allocation–Inventory Model (ULAIM)

In summary, with the unit time multicomponent cost as the objective function, and the before-mentioned requirements as constraints, we establish the supply relationship selection and inventory parameters optimization model as follows:

$$\begin{cases}
\min\ C = \sum\limits_{i\in I} X_i \cdot \left(b + c_i^{(1)} \cdot S_i\right) + \sum\limits_{j\in I}(\sum\limits_{i\in I} c_j^{(3)} \cdot E(\xi_i) \cdot Y_{ij} + k_j \cdot X_j / T_j) \\
\qquad + \sum\limits_{j\in I}\sum\limits_{i\in I} c_j^{(2)} \cdot \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \cdot E(\xi_i) \cdot Y_{ij} \\
\qquad + \sum\limits_{j\in I} X_j \cdot h_j \cdot \left(S_j - \frac{1}{2}\sum\limits_{i\in I} E(\xi_i) \cdot Y_{ij} \cdot T_j - \sum_{i\in I} E(\xi_i) \cdot L \cdot Y_{ij}\right)^+ \\
\qquad + \sum\limits_{j\in I} X_j \cdot g_j \cdot \frac{1}{T_j} \cdot \left(\sup\left\{ f_j \middle| \mathcal{M}\left\{\sum\limits_{i\in I} \xi_i \cdot Y_{ij} \cdot T_j - S_j \geq f_j\right\} \geq \gamma \right\}\right)^+ \\
s.t. \\
\quad \mathcal{M}\{S_j - \sum\limits_{i\in I} \xi_i \cdot Y_{ij} \cdot T_j \geq 0\} \geq \alpha \quad \forall j \in I \\
\quad \mathcal{M}\left\{\sum\limits_{i\in I} \xi_i \cdot Y_{ij} \cdot T_j - S_j \leq \inf\{(1 - A^{1/Z}) \cdot N_i \cdot Z \cdot T_j |\, Y_{ij} = 1\}\right\} \geq \beta \quad \forall j \in I \\
\quad \sum\limits_{i\in I} X_i = n \\
\quad X_j \cdot X_k \cdot \left(\sum\limits_{i\in I} Y_{ij} - \sum\limits_{i\in I} Y_{ik}\right) \leq 1 \quad \forall j,k \in I, j \neq k \\
\quad \sum\limits_{j\in I} Y_{ij} = 1 \quad \forall i \in I \\
\quad Y_{ij} \in \{0,1\}, X_i \in \{0,1\} \quad \forall i,j \in I
\end{cases} \tag{21}$$

3.4.4. Deterministic Equivalence of the Model

In order to convenient model solving, the uncertain model is transformed into the equal deterministic form, which is expressed as:

$$\begin{cases} \min\ C = \sum\limits_{i\in I} X_i \cdot \left(b + c_i^{(1)} \cdot S_i\right) + \sum\limits_{j\in I} \left(\sum\limits_{i\in I} c_j^{(3)} \cdot E(\xi_i) \cdot Y_{ij} + k_j \cdot X_j / T_j\right) \\ \qquad + \sum\limits_{j\in I} \sum\limits_{i\in I} c_j^{(2)} \cdot \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \cdot E(\xi_i) \cdot Y_{ij} \\ \qquad + \sum\limits_{j\in I} X_j \cdot h_j \cdot \left(S_j - \frac{1}{2} \sum\limits_{i\in I} E(\xi_i) \cdot Y_{ij} \cdot T_j - \sum\limits_{i\in I} E(\xi_i) \cdot L \cdot Y_{ij}\right)^+ \\ \qquad + \sum\limits_{j\in I} X_j \cdot g_j \cdot \frac{1}{T_j} \cdot \left(\sum\limits_{i\in I} \Phi_i^{-1}(1-\gamma) \cdot Y_{ij} \cdot T_j - S_j\right)^+ \\ s.t. \\ \quad \sum\limits_{i\in I} \Phi_i^{-1}(\alpha) \cdot Y_{ij} \cdot T_j \le S_j \quad \forall j \in I \\ \quad \sum\limits_{i\in I} \Phi_i^{-1}(\beta) \cdot Y_{ij} \cdot T_j \le S_j + \inf\{(1 - A^{1/Z}) \cdot N_i \cdot Z \cdot T_j |\ Y_{ij} = 1\} \quad \forall j \in I \\ \quad \sum\limits_{i\in I} X_i = n \\ \quad X_j \cdot X_k \cdot \left(\sum_{i\in I} Y_{ij} - \sum_{i\in I} Y_{ik}\right) \le 1 \quad \forall j,k \in I, j \ne k \\ \quad \sum\limits_{j\in I} Y_{ij} = 1 \quad \forall i \in I \\ \quad Y_{ij} \in \{0,1\}, X_i \in \{0,1\} \quad \forall i,j \in I \end{cases} \tag{22}$$

4. Algorithm

So far, diverse modified genetic algorithms that import parameter optimization methods have been widely employed to puzzle out the NP-hard problems in supply chains [32]. In an effort to ease model resolving and optimal in the research, we propose an improved heuristic algorithm that adopted the population renewal idea in the GA and commingled the retrieval thought springing from neighborhood search. The algorithm intends to exhaust the possible search domain more directionally than the traditional GA. During simulation, we have dug out the reality that the supply–demand relationship makes a more remarkable difference to the inventory parameters than the opposite effect. It is the reason why the problem can be disassembled into nested bilevel population sequencing. In the outer layer, our optimization concentrates on determining depot location and supply–demand relationship. In contrast, the inter-layer optimization highlighted in the blue frame aims at programming the inventory parameters.

As shown in Figure 3, the detailed simulation steps are exhibited as follows:

1. Iterate over all X_j, Y_{ij} combinations to generate outer layer populations, and the number of the entire combining samples is treated as the population volume.
2. For all population samples, compute unit time depot supply quantity and the floor level of S_j/T_j that satisfies the supportability requirements.
3. According to the range and required precision, choose the appropriate interval $\triangle T$ of the review period.
4. Generate new populations owing distinctive T_j that displays as an arithmetic sequence with $\triangle T$ as the difference, like the left section in Figure 4.
5. For all depots in the generated new populations, initialize the inventory level S_j as the minimum supportable inventory level $S_{(lb,j)}$, compute the order outlay, stockout risk loss, and inventory cost of each depot, and then proceed to acquire the addictive total cost of each depot.
6. Record the current population, these cost components and the additive total costs.
7. Within the specified number of epochs, mutate S_j and solve the correspondent cost components. Compare them with the previous total cost of each depot. If any

depot's total cost is lower, replace the recorded inventory level and the corresponding cost values.

8. Pick up the population samples with the lowest total cost for each depot and reform the population set in *Step* 1 (Figure 4 right side shown).
9. If $\triangle T$ does not satisfy the T_j's precision, make $T_j - \triangle T$ and $T_j + \triangle T$ to be the lower and upper bound of the review period, and then repeat Step 3∼Step 9. If the precision regulation is met, exert the next step.
10. Sequence population samples and filtrate the best-centralized depot location and provision network architecture with optimized inventory parameters.

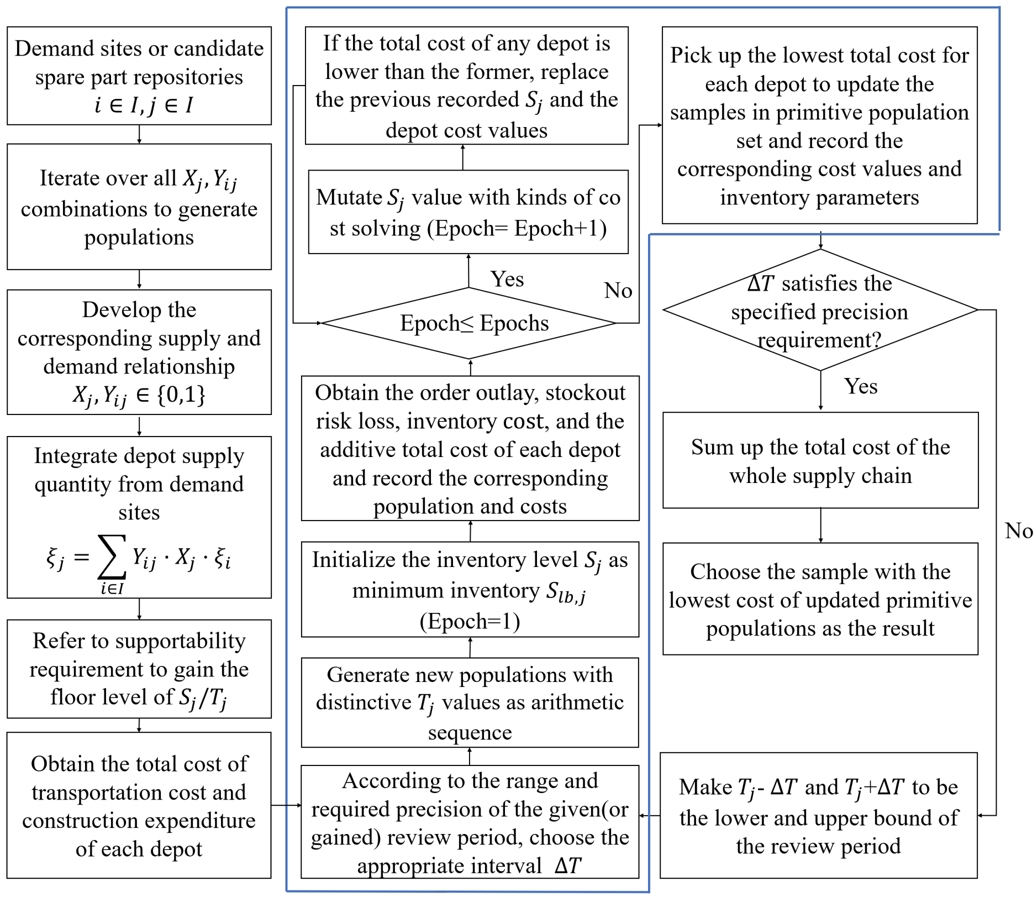


Figure 3. Key flows of the proposed heuristic algorithm.

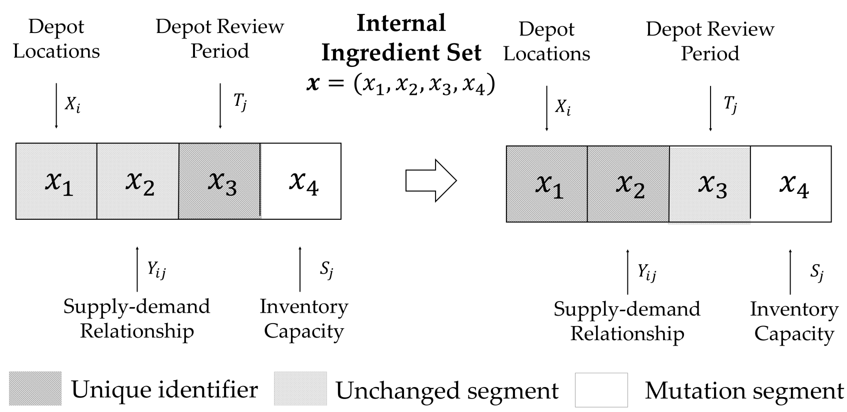


Figure 4. Internal structure of the bilevel population samples.

5. Numerical Analyses

In the present section, numerical examples of a base-level support system is discussed and analyzed to certify the validity of our proposed model and algorithm.

5.1. Settings

The sizes of the studied system are listed in Table 1 with known coefficients and parameters enumerated in Table 2. Noticeably, the bases' demands are assumed to be normal uncertain variables. For the optimization of the decision variable, the inventory capacity of each depot should be an integer variable, while, for simplicity, the review period is quoted in two decimal places. Moreover, the proposed algorithm does not utilize crossover process (crossover possibility = 0) and mutation of the inventory level is set to happen each epoch (mutation possibility = 1). This is because out-layer populations are gained by traversing all the possible location-allocation combinations, so that the crossover process is no longer needed. The other configuration parameters are listed in Table 3. The inner populations are generated through taking different values of parameter T at equal intervals period, the intervals $\triangle T$ are changed from 1 for roughly search to 0.1 for more accurate optimization and then to 0.01 which conforms to the regulated precision. For each inner population, we alter the inventory level value S for 30 epochs (epoch = 30). If the total cost is lower than the recorded value in any epoch, the corresponding S will replace the recorded value.

Table 1. The size of the problem.

Supplier	Depots	Base Zone
1	3	10

Table 2. Specified parameters for the supply chain.

i	(x_i,y_i)	ξ_i		h_i	g_i	k_i	N_i
		e_i	s_i				
1	(44, 98)	83	16	0.23	0.187	43	5
2	(16, 21)	78	13	0.24	0.171	40	6
3	(40, 53)	85	10	0.26	0.138	44	9
4	(96, 14)	84	15	0.22	0.199	44	8
5	(74, 67)	77	13	0.24	0.189	45	7
6	(24, 42)	84	12	0.25	0.185	42	3
7	(5, 2)	78	17	0.27	0.152	41	4
8	(54, 83)	83	12	0.25	0.172	46	8
9	(64, 48)	70	13	0.24	0.144	46	3
10	(98, 99)	85	16	0.26	0.184	47	7

ξ_i follows a normal uncertainty distribution, namely, $\xi_i \sim \mathcal{N}(e_i,s_i)$. Other parameters for the supply chain: $n = 3$, $L = 0.01$, $\alpha = 0.9$, $\beta = 0.9$, $\gamma = 0.01$, $A = 0.85$, $b = 5$. Cost set: $\forall\, i \in I$, $c_i^{(1)} = 0.01$, $c_i^{(2)} = 0.002$, $c_i^{(3)} = 0.5$. Limited by the ability about supplying and inventory checking, the review time T_i is in the range from 0.5 to 5.

Table 3. The configuration of the proposed algorithm.

Outer-Layer Populations	$\triangle T$	Inter-Layer Populations	Epoches of Mutating S_j
75,600	(1, 0.1, 0.01)	(5, 20, 20)	30

5.2. Results

According to the above settings, the procedure is coded in the MATLAB R2019a and tested on a workstation with 256 GB RAM and 2.9 GHz MAD EPYC Processor. It consumes 173.26928 s for solving the case and the results show that the minimum total cost is 416.08 with base 1, base 2, and base 4 chosen as the centralized depot sites. The according second supply–demand relationships are portrayed in Figure 5, while the relative parameters and detailed costs are presented in Tables 4 and 5.

Table 4 manifests the optimal parameters of the selected depot and their distinctive cost components, while Table 5 summarizes the outcomes of the cost distribution in the entire supply chain.

Table 4. The parameters and various cost values of centralized depots.

Parameter	Depot 1	Depot 2	Depot 3
Site selection	1(44, 98)	2(16, 21)	4(96, 14)
Supply relationship	1(1, 3, 8, 10)	2(2, 6, 7)	4(4, 5, 9)
Review period T_j	0.86	0.95	1.06
Stocked capacity S_j	346	277	298
Maintenance expenditure	8.46	7.77	7.98
Transportation expense	9.93	3.60	7.69
Inventory cost	45.58	38.54	38.12
Stockout risk loss	13.59	9.65	10.92
Order outlay	83.60	66.11	64.61
Total cost	161.15	129.51	129.31

Table 5. The different cost components.

C_B	C_T	C_H	C_R	C_O	Total Cost C
24.20	21.21	122.01	34.34	214.31	416.08

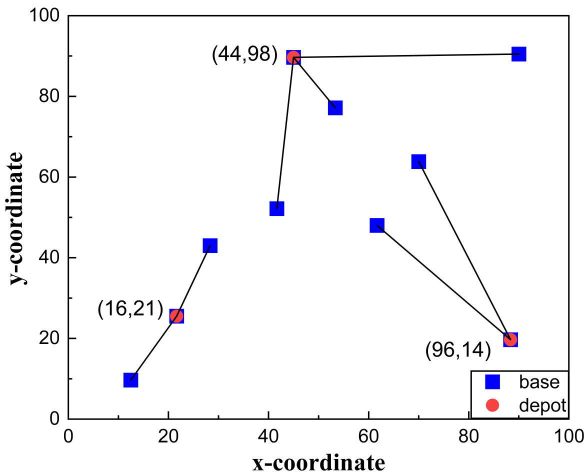


Figure 5. Allocation relationships gained from simulation.

5.3. Algorithm Comparison

In an effort to prominent the efficacy of the proposed algorithm, we itemize outcomes of the proposed heuristic algorithm and the typical genetic algorithm (populations: 75,600; Epochs: 30; crossover probability: 0.4; mutation probability: 0.95) in Table 6. The conclusion shows that the proposed algorithm is better at approaching the optimal value. Besides, it costs 1103.632415 s to complete computation which is roughly 7 times as long as the proposed algorithm. The results also verify the efficiency of the method.

Table 6. Configurations and outcomes of the two algorithms.

Outcomes	The Proposed Algorithm	The Typical Genetic Algorithm
Site selection	1, 2, 4	1, 2, 4
Supply relationship	1(1, 3, 8, 10), 2(2, 6, 7), 4(4, 5, 9)	1(1, 3, 8, 10), 2(2,6,7), 4(4,5,9)
Review time of depot	1(0.86), 2(0.95), 4(1.06)	1(0.92), 2(0.81), 4(0.98)
Stocked capacity	1(346), 2(582), 4(562)	1(371), 2(582), 4(562)
Maintenance expenditure	1(8.46), 2(7.77), 4(7.98)	1(8.71), 2(7.43), 4(8.13)
Transportation expense	1(9.93), 2(3.60), 4(7.69)	1(9.93), 2(3.60), 4(7.69)
Inventory cost	1(45.58), 2(38.54), 4(38.12)	1(49.01), 2(34.42), 4(43.45)
Stockout risk loss	1(13.59), 2(9.65), 4(10.92)	1(13.42), 2(8.21), 4(3.31)
Order outlay	1(83.60), 2(66.11), 4(64.61)	1(80.34), 2(73.38), 4(68.00)
Total cost	416.08	419.01

5.4. Extensive Case Studies

To better explore the general applicability of the model, we also try to change the supportability indexes and the results are exhibited in Table 7. Note that the bases' positions and demands remain unchanged and the varied parameters contain α, β, γ.

Table 7. Extensive case studies.

α	β	γ	Inventory Parameter	Total Cost	Running Time
0.9	0.9	0.01	1(0.86, 346); 2(0.95, 277); 4(1.06, 298)	416.083	173.269280 s
0.85	0.9	0.01	1(0.86, 345); 2(0.95, 276); 4(1.06, 297)	415.952	171.329017 s
0.9	0.95	0.01	1(0.85, 364); 2(0.95, 293); 4(1.05, 315)	418.338	172.475892 s
0.9	0.9	0.05	1(0.86, 346); 2(0.95, 277); 4(1.06, 298)	393.071	171.735646 s

The location-allocation relations keep unchanged in the programs.

From Table 7, we can summarize that the increase of belief degree α or β may lead to the adjustment of inventory parameters and the increase of total cost and vice versa.The higher belief degree α means the higher service level requirement, and then the corresponding cost will increase. Similarly, the rise of belief degree β equals to the stricter supply availability standard with the sacrifice of cost. In practice, the balanced level of cost and supportability depends on the selection of α and β. The decline of stockout risk level γ does not change the inventory parameter but it represents the descending penalty for shortage risk, which leads to the drop of stockout loss and causes a decrease of the total cost.

5.5. Sensitivity Analysis of Inventory Management

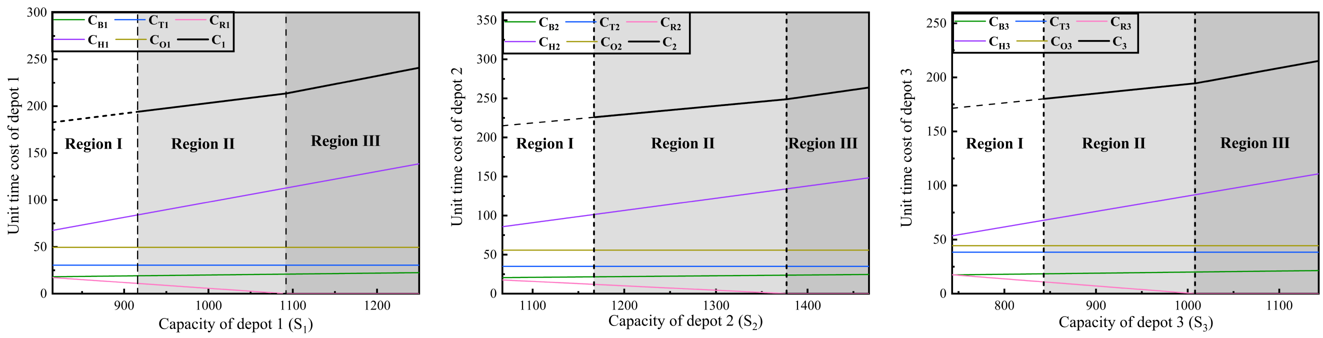
To clarify the impacts of inventory management on diverse cost components, we implement some sensitivity analysis of inventory parameters and then draw the results in Figures 6 and 7. Since the change rules among disparate costs and inventory parameters are similar for each depot, so the following analysis is applicable to every designated depot in the paper.

Firstly, we explore the impact of inventory capacity on disparate expense elements portrayed in Figure 6. According to the supportability constraints and the expression of stockout risk loss, the whole optimal process can be divided into three regions, namely, the unreliable region, the multifactor region, and the none stockout region. The detailed description is as follows:

- Region I: Unreliable region means that the centralized depot cannot meet the given supportability constraints, which makes the supply system unreliable. In this scene, owing to the substandard inventory level, comparatively frequent shortages and relatively long downtime cause the whole region undesirable.

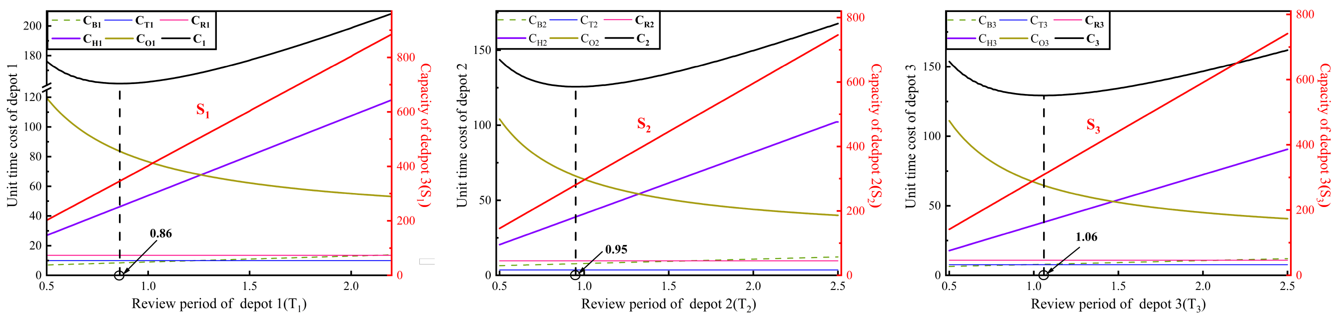
- Region II: Multifactor region indicates that the five cost components are in coexistence and exert their synergy effects on the total cost. Precisely, the increment of the total cost is identical to the escalation of inventory plus order expenditure minus the drop of the stockout risk loss. Because of the incompatible offset of the decline in contrast with the upward trend, the total cost exhibits continuing inflation.
- Region III: None stockout region signifies that shortage can be neglected. Although it is a fact that stockouts are likely to occur in any circumstance, we believe that if the inventory level reaches a certain high level, shortages are too rare to be ignored. Therefore, the transition of the total cost in the third region has a steeper amplification in contrast with region two.

To sum up, along with the accretion of inventory capacity, the cost spent on depot maintenance C_{Bj} (j = 1∼3) and parts allocation C_{Tj} (j = 1∼3) stays steady, while the expense for inventory holding C_{Hj}(j = 1∼3) and ordering C_{Oj} (j = 1∼3) keeps growing. The diversity of regions predominantly manifests in shortage cost C_{Oj} (j = 1∼3) and the total cost C_j (j = 1∼3).



(a) Sensitivity to depot 1capacity (b) Sensitivity to depot 2 capacity (c) Sensitivity to depot 3 capacity

Figure 6. Sensitivity analysis of inventory capacity.



(a) Sensitivity to depot 1 review period (b) Sensitivity to depot 2 review period (c) Sensitivity to depot 3 review period

Figure 7. Sensitivity analysis of inventory period.

From Figure 7, the red line manifested that the optimal inventory capacity rises linearly with the review period. Simultaneously, with the growth of the review period, the shortage loss (blue line) and the allocation expense (pink line) maintain immutable, whereas the expenditure for inventory (purple line) and maintenance (green line) remain boosting. Nonetheless, since the average per unit time inventory counting cost is proportional to counting frequency $1/T_j$, there exists an inversely proportional downward trend to order outlay (yellow line). The aforementioned trends add up to a dip preceding a persistent rise in total cost (the black curve shown). The minimum extreme value is regarded as the final choice of the parameter values marked on the horizontal axis.

Conclusively, the above discussions on the inventory parameter values help to evaluate the behavior of distinctive costs and find the optimal inventory policy that guarantees the system supportability requirements.

6. Conclusions

In this study, we came up with an optimization Location-Allocation-Inventory model for addressing a three-layer base-level spare part supply chain. The goal of the model is to minimize the total cost via appropriate management of centralized depot location, allocation relationships, and inventory policy. On the premise of practical conditions, depots were nominated to be controlled by (T, S) policy and balanced one-to-multiple allocation mode. Regarding supportability judgment, we supposed that the system was restricted by regulated service level and supply supportability. In view of the epistemic uncertainty in the model, uncertainty theory was introduced to quantify demand with stockout loss assessed by risk-at-value. To overcome the obstacle of solution acquirement, we put forward an improved bilevel genetic algorithm whose two stages target to accomplish supply–demand matching and inventory policy updating separately. Finally, numerical analyses are carried out to verify the efficiency of the proposed model and algorithm. The main conclusions can be summarized as follows:

(1) The joint location-inventory-allocation model was solvable with the gained decision variables containing the selected depot locations, allocation relations and inventory parameters.The results also demonstrated the distribution of different cost components.

(2) The suggested algorithm was validated to perform efficiently in optimal solution approaching compared to the traditional genetic algorithm.

(3) Extensive and sensitive analyses explored the general practicality of the model, and illustrated that the inventory parameters T, S had certain impacts on different cost components and the selection of suitable parameters could reduce the total expenditure.

With respect to future explorations, the following research can concentrate on the limitations of this paper. Firstly, other approaches for uncertainty measurement can be wielded for uncertain factor evaluation and compared with the current approach. Secondly, other supportability indicators, like logistics delay time and supply reliability [33], can also be mingled into the spare part provision models. Finally, production and maintenance affairs can also be conducted on the supply chain establishment for support systems.

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