

## Article

# Mathematical Modeling Research Output Impacting New Technological Development: An Axiomatization to Build Novelty

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**Abstract:** The mathematical modeling of research-based output impacting new technology development is crucial for a developing country. However, the complexity of modern mathematical modeling research output makes it unclear over how it can impact the development of new technology. Therefore, this study aims to explore, categorize and formulize the axioms of mathematical modeling research output that impacts the development of new technology. Seven participants were involved in this research. Interviews were conducted to explore their remarkable mathematical modeling output and how the output can impact the development of new technology. The categorization axioms are: i. mathematical modeling for theorizing, ii. mathematical modeling for simulations, iii. mathematical modeling for useable innovation and iv. patent and product commercialization. Finally, the categorization can be formulized as an axiom of mathematical modeling novelty, which is the desired research output. Moreover, patents and commercialization are the elements that mathematical modeling should possess for new technological development. The limited number of participants involved in this study makes this study formulation limited to only some types of mathematical modeling output. However, this substantive formulation could give some ideas in proposing the path and processes on how to enhance the effort for society to develop the culture of mathematical modeling in developing new technology.

**Keywords:** mathematical modeling research output; new technology development; qualitative axiom; patent and commercialization; building research novelty; mathematical modeling processes; applied sciences

**MSC:** 00A71; 06A75; 18C50; 18C35; 18D35; 34C60; 74G55



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## 1. Introduction

Mathematical models research output is diverse and have been used for studying many phenomenal conditions [1]. It is a powerful tool that can be used to investigate the effects of important aspects of unpredictable situation for a certain complex issue and for developing new technology [2]. The diversity of modern technologies nowadays has led to many types and processes of mathematical modeling due to the rise of practical problems which requires specific mathematical models. Some typical models can be used as the model for a specific physical phenomenon and some mathematical models can be used for specific biological phenomena, social and psychological phenomena, etc. [3,4]. Therefore, it is crucial to be clear on the types of mathematical model output categories, and how these categories could impact the development of new technology, as well as how it can be used as a guide to develop new technology. Furthermore, this formulization could help mathematical modeling researchers of developing countries to the path of

process for further technological development by increasing their effort on publishing their mathematical modeling research output. As an example, researchers from China have been publishing more research work than the United Kingdom and is projected to surpass the United States as the largest academic producer [5,6]. Other developing countries such as Brazil, India and South Korea are also expected to surpass France and Japan in terms of technological research output for the next six years [7–9]. In this regard, one of the most important tasks of a mathematical modeler is to understand the categories of mathematical modeling output and how these paths of modeling can contribute to the sustainable development of science, technology, engineering and mathematics or also known as STEM-based research output development [10].

The mathematical modeling of STEM-based research appears to be more important as it has been emphasized in many countries, especially in practical subjects [6]. However, concerns over the types and direction of mathematical modeling on output categorization and the impact of new technology development are still being studied [11,12]. In improving the quality of mathematical modeling of STEM-based research output, this study is crucial in giving a clear understanding for mathematical modeling researchers on the categorization and path of mathematical modeling for developing new technologies. Therefore, this study aims to qualitatively axiomatize and formulize the output categorization of mathematical modeling research output and explore how the output can impact the development of new technology.

In this article, we explore and axiomatize four types of mathematical modeling categorization that take into account the formulization of strategy on impacting new technological development under a specific path or situations. We explore a range of mathematical modeling scenarios with different modeling output, different modeling objectives and several levels of modeling. The aim of this study is to provide an empirical guideline for mathematical modeling scientists and researchers to further predict and develop their intended technology which impact mathematical modeling works. Furthermore, the aim here is not to precisely develop a model of technological impact using mathematical modeling, but rather to innovate the categorization, paths and strategies of new technological development from mathematical modeling research output. Thus, this article does not present an accurate mathematical formulization to forecast the modeling output for particular categories as there are many types of mathematical model outputs and factors that need to be considered. Therefore, including all types of models output in one study is complex and it might not provide clear evidence of its influence in the development of new technology.

The innovation presented in this study is the priority on the usability of all axiomatized categories where it can impact the development of new technology based on mathematical modeling research output. Here, we applied qualitative axiomatization of case scenarios where the path for new technology can be innovated by enhancing the technique and combining the use of mathematical modeling algorithms. This axiomatization shows that the development of new technology is most likely related to the availability of data, human capability, as well as resources attached to the modeling process and the willingness of researchers [13] to develop the new technology. It is also important to mention that many countries are increasing their efforts in developing their own new technology and mathematical modeling research output is crucial in producing new technology [14]. Hence, obtaining scientific support in designing the optimal path of mathematical modeling that impacts new technology development programs is of paramount importance under the certain limit of modeling categorization. An interesting previous work by [15] demonstrates a formulation of mathematical model categorization using a theory-based approach. Following this method, mathematical models are formalized in the mathematical language of categories, and relations between the models are formally defined. The author has presented new results and ideas by supporting the abstract modeling process.

One main advantage of the axiomatization and formulization in this study compared with previous studies is that we provide empirical concrete evidence of several different situations which can be replicated to other situations by presenting the investigation of

various mathematical models. In this article, we axiomatized four mathematical models of research output categories and formulized all of them on how the categories could impact new technology development based on respective evidence. Each evidence represents different categorization with regard to works of research in applied sciences. We qualitatively propose the differential axioms with different triangulated evidence to provide insight into the in-depth and trustable elements as well as strategy regarding priorities to enhance the production of new technology. The formulization includes elements of patent and commercialization of mathematical modeling research output as the motivation towards mathematical modeling output novelty use for the development of new technology. Thus, this study can provide further insights about mathematical modeling research output in impacting the development of new technology by introducing the elements, path and strategy using qualitative axiomatization.

### *Problem Statement*

Mathematical modeling has been extensively used in many different research fields [5,6]. Hence, scholarly mathematical modeling outputs which impact technology are also diverse and difficult to be categorized. Some mathematical modeling research outputs are abstract and implicit, while some are realistic [16,17]. Due to the abstractness of mathematical modeling output, the modeling objective can be unclear on how it impacts the development of new technologies [18] (i.e., the use of specific mathematical modeling to produce new technology). Research carried out by [15] proposed an abstract category using a theory-based approach to mathematical modeling research output; however, it does not provide qualitative elements of the categories nor the impacts on new technology development. Studies by [19] show that numerous problems from diverse disciplines can be converted using mathematical modeling by using equation(s) to define the suitable abstract spaces. The author postulated that the processes usually involve the  $n$ -dimensional Euclidean space, Hilbert space, Banach Space or other more general spaces. The solution of the proposed equations is in closed form and iterative algorithms is as an alternative way to solve the problem.

This situation still creates a question of “what are the processes and how can mathematical modeling research output be used to impact new technology development?”. This problem is typically prevalent amongst novice researchers in mathematical modeling [20] and due to the objectives of the model being misguided [21,22] (i.e., what are the output and how it can impact the development of new technology). Consequently, this problem will also affect the development of mathematical modeling research output as a product and further affect some countries on their agenda to produce new technology [23]. Therefore, the conceptualization of mathematical modeling output and how it can impact the development of new technology could give substantial axioms on mathematical modeling output categorization and how it impacts the development of new technology. Of course, it is impossible to cover diverse mathematical modeling output as a single categorization axiom, but this is an effort to bring a clear picture for the mathematical modeling community, especially for novice mathematical modelers so that they may focus on their modeling objective for further development. Therefore, this study aims to explore mathematical modeling output and conceptualization axioms on how it can impact specific technological development. Two research questions underpin this study: i. What are the axioms of mathematical modeling output categories? ii. How can the axioms impact technological development?

## **2. Methods and Source of Axiomatization**

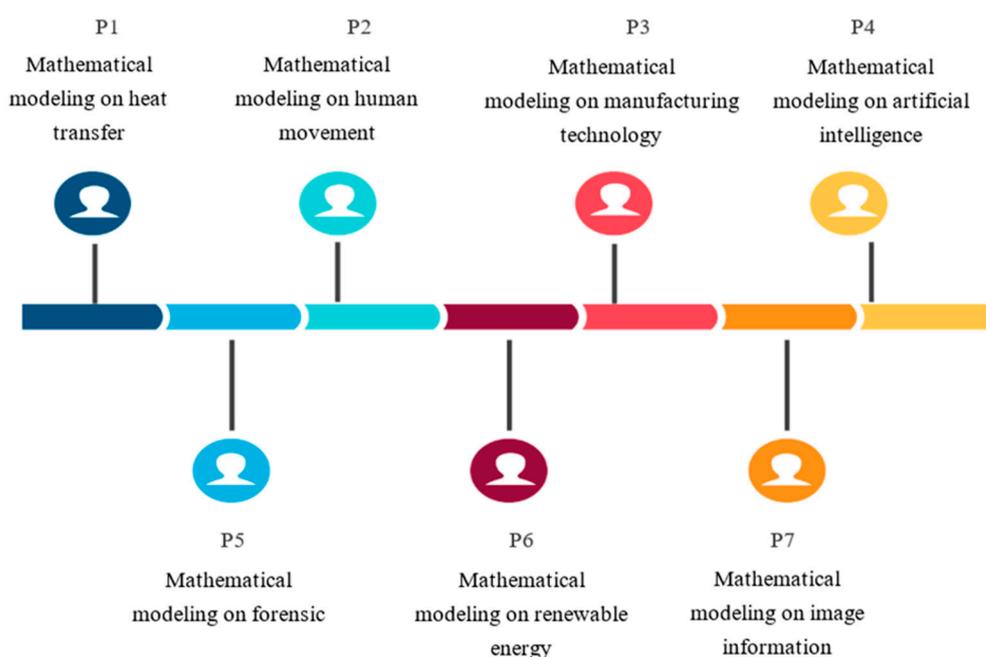
This study applied a qualitative case study [24–26] to determine the mathematical modeling research output axioms (A) [10] and the elements (E) on participants’ work in their fields. The first participant was selected based on his remarkable achievement as one of the top three World’s Most Influential Scientific Minds researchers on mathematical modeling. This was carried out by observing and searching his remarkable contribution

from newspapers and Google Scholar. The other six (6) participants were selected based on the snowballing technique [27], following suggestions from the previous participants. All suggested participants were firstly verified that they have published mathematical modeling work(s). As a result, several remarkable journal articles on mathematical modeling were collected. Participants were invited for an in-depth interview session (i.e., for verbal data source categorization axiom) and further document analysis (i.e., for concrete axiom evidence).

Categorization of qualitative data axiomatization was carried out using thematic analysis [28–30]. Participant’s verbal data were transcribed and the ideas about mathematical modeling research output with their elements were axiomatized from the interview transcript by making semantic interpretation into a statement [31]. According to Dimitrov, axiom is a statement that is so evident or well established and it is accepted without controversy or question [32]. Axiom is a starting point from which other statements are logically derived. Based on this rule, the interpretations that construct the axioms and elements were cross-checked amongst participants group to avoid individual misinterpretations or over interpretation. The emerging axioms and elements were triangulated with a secondary source of data which is the participant’s scholarly mathematical modeling output in the form of journal articles. This was carried out by cross-checking all the emerging axioms and elements with the respective participant’s articles. Formulation of the axioms and relationship elements of how they can impact technological development was carried out by applying an ontological relationship technique [33] to link all the axioms with the elements.

#### *Location and Participants’ Selection Technique*

This study was implemented at a university located in the center of the Peninsular of Malaysia. Participants from this university were selected based on a drive of one of the World’s Most Influential Scientific Minds researcher recorded as the global prominent scientist in mathematical modeling for the year 2014 of the university’s workplace. A snowball sampling technique was then performed to the next participant’s involvement at the same university. As a result, a total seven (7) participants were involved in this study. Figure 1 shows the participants’ mathematical modeling expertise on the snowballing processes and all participants are coded as P1 to P7.



**Figure 1.** Snowballing participant selection based on data collection processes.

In reference to Figure 1, participant P1 is a professor who is an expert in the modeling of heat transfer and fluid mechanics. Participant P2, who is an expert in the mathematical modeling of human movement was then nominated by P1 for their data to be collected. These processes were carried out for the remaining participants until the data were saturated in which there are no new emerging elements found or assumed to be found from this study perspective.

### 3. Results

#### 3.1. Qualitative Axiomatization of Mathematical Modeling Research Output

Collected participant interview data inductively axiomatized four mathematical modeling research output categories and are formulized as mathematical modeling novelty. The axiomatization and formulization statements are: i. For every modeling output, existing process of theorizing a specific system (natural or artificial phenomenon) in the form of a mathematical expression; ii. Existing mathematical model for simulation of a specific system; iii. Existing innovation in mathematical modeling in the tangible or intangible form; iv. Existing patent for commercialization from mathematical modeling output, and formulization statement: existing novelty in mathematical modeling research output. The emerging axiom categorization with these elements were supported by evidence in the form of documents published as article journals. Generally, the mathematical modeling research output depends on the modeling goal to solve real world problems, but the novelty factor is the axiom that they are trying to develop. Meanwhile, patents and commercialization are the paths for new technological development. The interpretations for axioms and their elements are shown below:

##### 3.1.1. Axiomatization 1 (A1): Mathematical Modeling for System Theorizing and the Impact on Technological Development

All participants mentioned that in every mathematical modeling process there is system theorizing which involving elements of observation and applies fundamental principles or theory (i.e., sciences, mathematics, engineering, social, economy, etc.) to develop an expanded system theory (i.e., either new or reconstructed). A system theory can be reproduced as a new mathematical modeling output by using observation. This developing theory which explains and predicts a phenomenon for the system under study uses a mathematical equation or a set of equations that govern together. Based on the interviews, as a mathematician, participants have been trained to be able to make observations and solve the phenomena being studied in the form of a mathematical formulation. A mathematical model representing an observed system explains and predicts changes in behavior(s). According to most participants, observing a specific system change can also be carried out based on a set(s) of previous data or factors of the system studied. By identifying the system's changing factors, modeling processes were then performed using a specific related basic mathematical formula to govern a new mathematical model. Furthermore, according to the participants, good mathematical modeling can also contribute to a useful theory for system improvement and helps develop new technology based on a set of past data as mentioned by P4:

*“ ... using past data, we can generate a mathematical model by observing changes ... using the simplest pattern that we observe ... we can generate an equation from the original equation to describe the system of a phenomenon ... for the complicated ones, we included other factors in the equation such as economic factors or price factor ... ”*

Source (P4, 4:28)

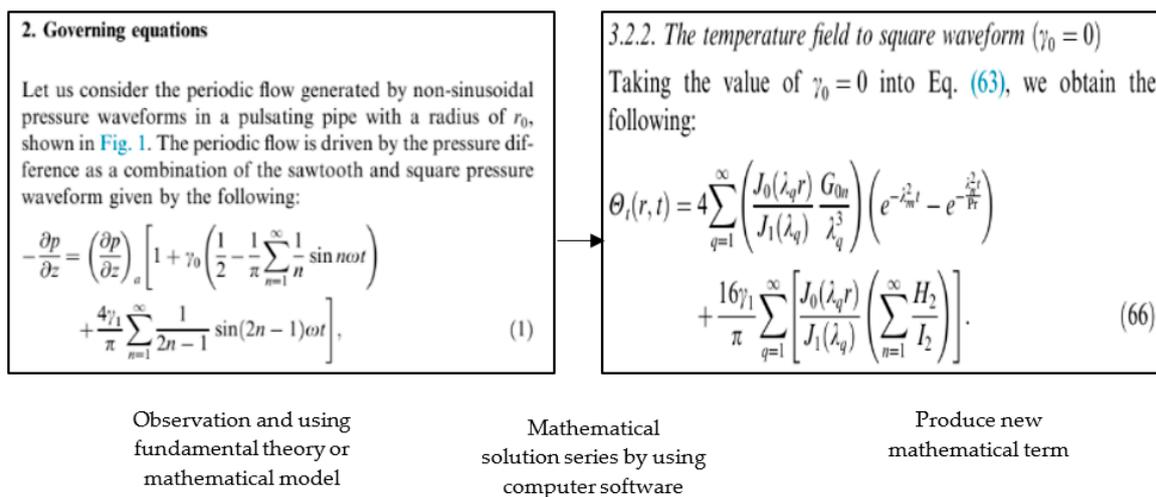
The second element of producing mathematical modeling for theorizing is the element of using digital technology to assist the problem-solving processes. A set of past system data will be solved using computer software and then specific pattern properties can be determined. This was mentioned by P2 as:

“We will solve the equation using computer software . . . but the danger is when we don’t really understand the software and we don’t know what we really want, and we just put in some random values . . . of course, the software will produce an output . . . but we don’t know what the value means—is it right? . . . it might just be due to a missing negative sign . . . if we don’t understand the system for example, and we just take the result at face value . . . the result might be reversed . . . for example, the result might cause a person to go backwards instead of going forward when we keep entering the wrong value. This can be caused by having a weak background in the topic or theory”.

Source (P2, 2:12)

The third element is the element of making assumption(s) which needs to be set for the system being studied. A set of assumptions is a very important element in order to help the modeling processes to be more reliable and simpler. Moreover, making a set of assumptions helps the mathematical modeler demonstrate a specific phenomenon correctly and a general equation or sets of the equation can then be produced.

Data on document analysis furthermore confirm that scholarly output in the form of a mathematical expression that contributes to the specific field of theory is one of the remarkable outcomes. Figure 2 shows the evidence on processes that were recorded in a published article where a new term of heat flow in a pulsating pipe with radius  $r$  [34] was produced, which was derived by P1 and his research group.



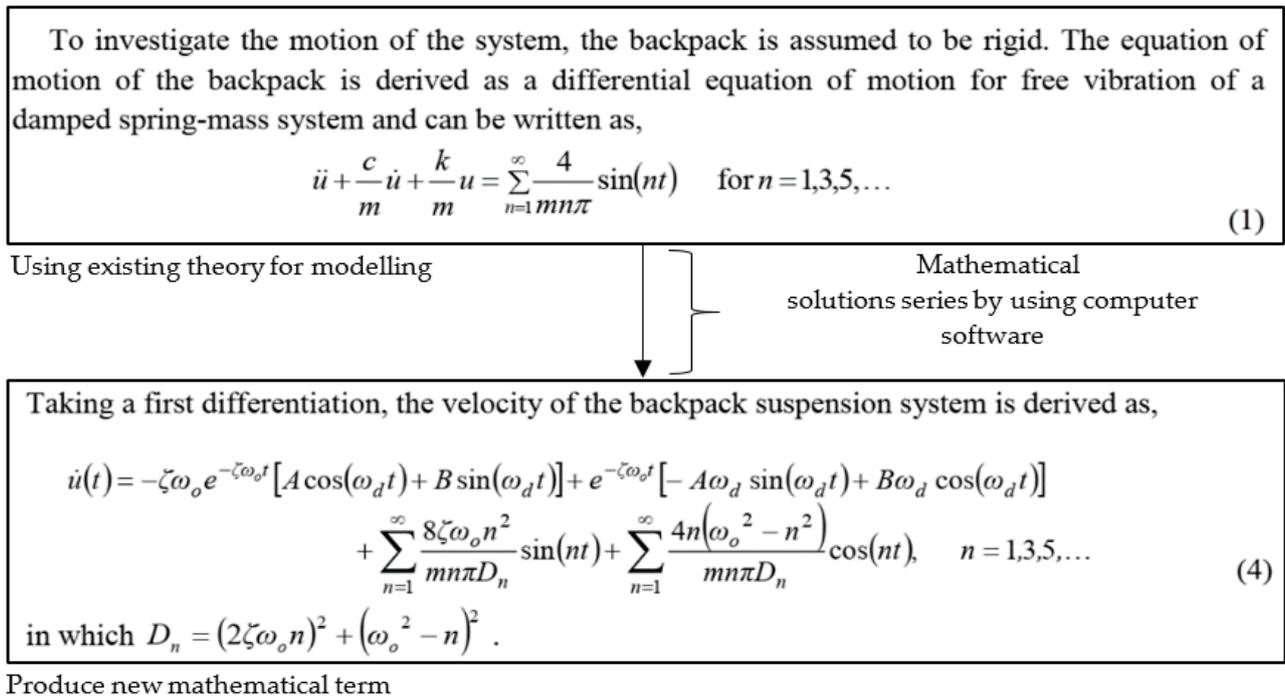
**Figure 2.** Mathematical modeling output from P1. Source: P1’s published article. Reprinted with permission from Ref. [34]. 2016, Ishak Hashim.

Based on the emerging elements that support axiomatization of the first category, it is clear that the fundamental method on the processes of producing new mathematical terms for a system studied is by governing it using basic fundamental theories or formulae. Therefore, mastering a specific knowledge that we need to theorize is another important key aspect in producing remarkable mathematical modeling output. These specific theories can then be expanded to new science and technology knowledge and application.

Participant P2, an expert in mathematical modeling for human movement [35], is one of the examples on how the development of the new technology based on the mathematical modeling output can be made. Figure 3 enforces the conceptualization of the new mathematical term as a mathematical modeling axiom. Similar to P1, the important element used by P2 is applying a set of assumptions before the modeling processes can continue further.

Furthermore, an important strategy in mathematical modeling is the use of a free body and schematic diagram. A phenomenon or artificial system can be modeled using a set of new mathematical expressions through this technique. The theorizing problem will consider existing theories, concepts and models under specific studies. The mathematical

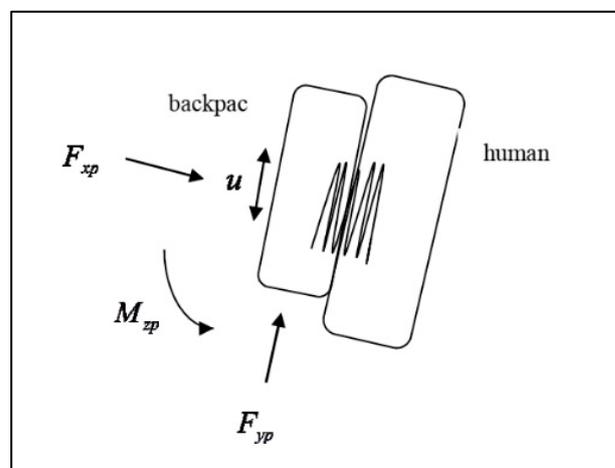
modeler needs to think about what aspect or new idea he or she needs to explore under the scope of the study. This method could generate a new mathematical model from the existing system and generate new ideas to produce new innovations. A creative and innovative idea of mathematical modeling will have the potential to produce new creative innovations. This mathematical modeling output categorization is derived from participant P1 and P2, and can be triangulated with document as in Figure 4 [35] and Figure 5 [36]:



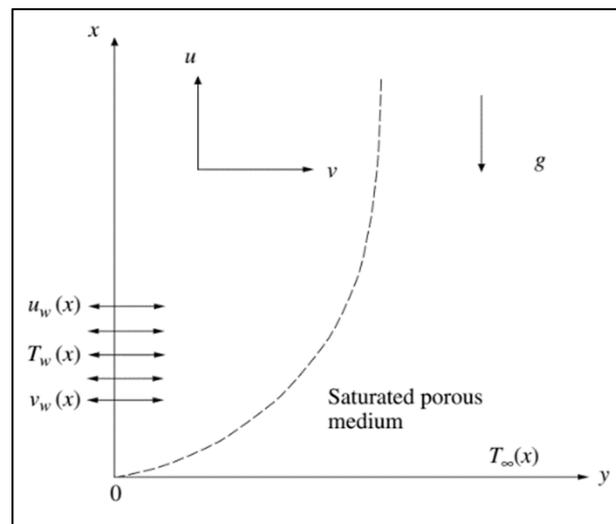
**Figure 3.** Mathematical modeling research output from P2. Source: P2’s published article. Reprinted with permission from Ref. [35]. 2012, Azmin Sham Rambely.

*“The important thing is to set up the geometry diagram . . . a free body diagram . . . then determine what other people have done on the studied phenomena, and what do you want to do on that particular phenomenon . . . based on that diagram, what’s a new term that can be generated . . . and what is the purpose of the equation for the new thing . . . then we can derive in mathematical terms, and we can get a new mathematical model . . . ”*

Source: (P1: 1:12)



**Figure 4.** Document analysis from participant P2 showing a free body diagram. Source: P2’s published article. Reprinted with permission from Ref. [35]. 2012, Azmin Sham Rambely.



**Figure 5.** Document analysis from participant P1 showing a schematic diagram. Source: P1's published article. Reprinted with permission from Ref. [36]. 2008, Ishak Hashim.

Finally, in order to generate a new theory using mathematical modeling, we would need a deep and strong background in mathematics in order to develop a mathematical model for a specific phenomenon's system. This element will help the modeler identify the current issues and problems to be explored and finally, solve them to determine the new mathematical term(s). This element was enforced by participant P2 as shown below:

*"We would need to have knowledge in mathematics and other disciplines . . . your math skills must be strong . . . if not, it may be difficult to even start working on the problem . . . but the first thing is the issue, how do we find the issue . . . "*

Source: (P2, 2:55)

### 3.1.2. Axiomatization 2 (A2): Mathematical Modeling for Simulation and the Impact on Technology Development

Normally, the research objective(s) of mathematical modeling is multi-disciplined, and one of the objectives is to simulate the system being studied. In the digital era, mathematical modeling for system simulation has an advantage whereby a computer can be used to solve and simulate a system. This axiom can be proven by participant P5, who researched the pattern image investigation on ink and a gun which fired bullets using statistical mathematical modeling for forensic purposes. Some investigations require this to be carried out using instruments found in laboratories such as vibrational spectroscopy to segregate data.

*"For example, in gun fire pattern recognition, yes, we have evidence for gun fire identification study, so how do we translate the image into mathematics, to the numbers, yes, in fact the image for example on the inner side of the bullet casing, we can translate it into a mathematical model and simulate the image . . . "*

Source: (P5: 10:16)

Furthermore, the image identification for forensic purposes used mathematical modeling to obtain the image pattern. Based on the participant's experience, the process of proofing of evidence is a difficult process. This is because the evidence is in the form of the image of a bullet shell that needs to be proved to match exactly the gun used. The research outcomes are typically 90% accurate, and the other 10% will depend on different analysis methods or approaches. The aspect of producing accuracy on visual images for forensics could be conceptualized as an element of the impact of mathematical modeling on developing new technology that is accurate and precise.

*“In the case of forensics, we can’t get a 100% accuracy, but we say that we only can get close to 90%, and we say it might be gang A, gang B, or gang C, let’s say in Malaysia that there are 7 gangs involved in gun-related crimes, so this might be done by gang A, gang B or gang C, so we go back to the lab and select a particular analysis using a microscope, and only then will we can get closer to suspect”*

Source: (P5: 10:28)

Furthermore, according to P5, the human being’s thinking limitation is the main factor for that to happen, not only in the image identification study but also in other research areas. This issue generally exists because of the lack of understanding of certain techniques used in mathematics.

*“But the court has not accepted the result of the analysis using computers so far, they also need human beings to verify the analysis, but we have made it easier, our real purpose is to screen people, or as we call it, the screening phase, it means that we have reduced 90% of the work and only 10% would require human intervention . . . ”*

Source: (P5: 10:29)

This developing new technology idea has been published by P5 in an article discussing the method and technique used for image identification [37]. Figure 6 shows how mathematics is incorporated into the bullet or ink identification process.

Furthermore, based on the information provided by P5, in some cases, the process of enhancing and improving the outcome takes a number of cycles and a long time to be completed, as well as involving several parties in the study of visual image technology using mathematical modeling.

*“The first cycle of the project was done manually, the second cycle was conducted automatically, and has managed to come out with important images from the inside of the bullet casing, meaning that before this we had to ask an expert, we had to hand draw a little bit, and in this second part, the part that I’m interested in, is done automatically . . . ”*

Source: (P5: 10:31)

Based on comments from previous mathematical modeling output studies published, P5 carried out an investigation and collaborated with the Royal Malaysia Police (PDRM) to widen the data and improve image simulation accuracy. In that sense, more samples were provided so that more data could be processed. As a result, the mathematical modeling research output had an impact on forensic imaging technology development by contributing accuracy improvements:

*“ . . . We can’t get a lot of data, and there were only a few new papers, and there were also some comments from outside on our paper that we did not have enough in terms of samples used, but these will incur a big cost, and it’s from PDRM . . . we get some contribution from them”*

Source: (P5: 10:26)

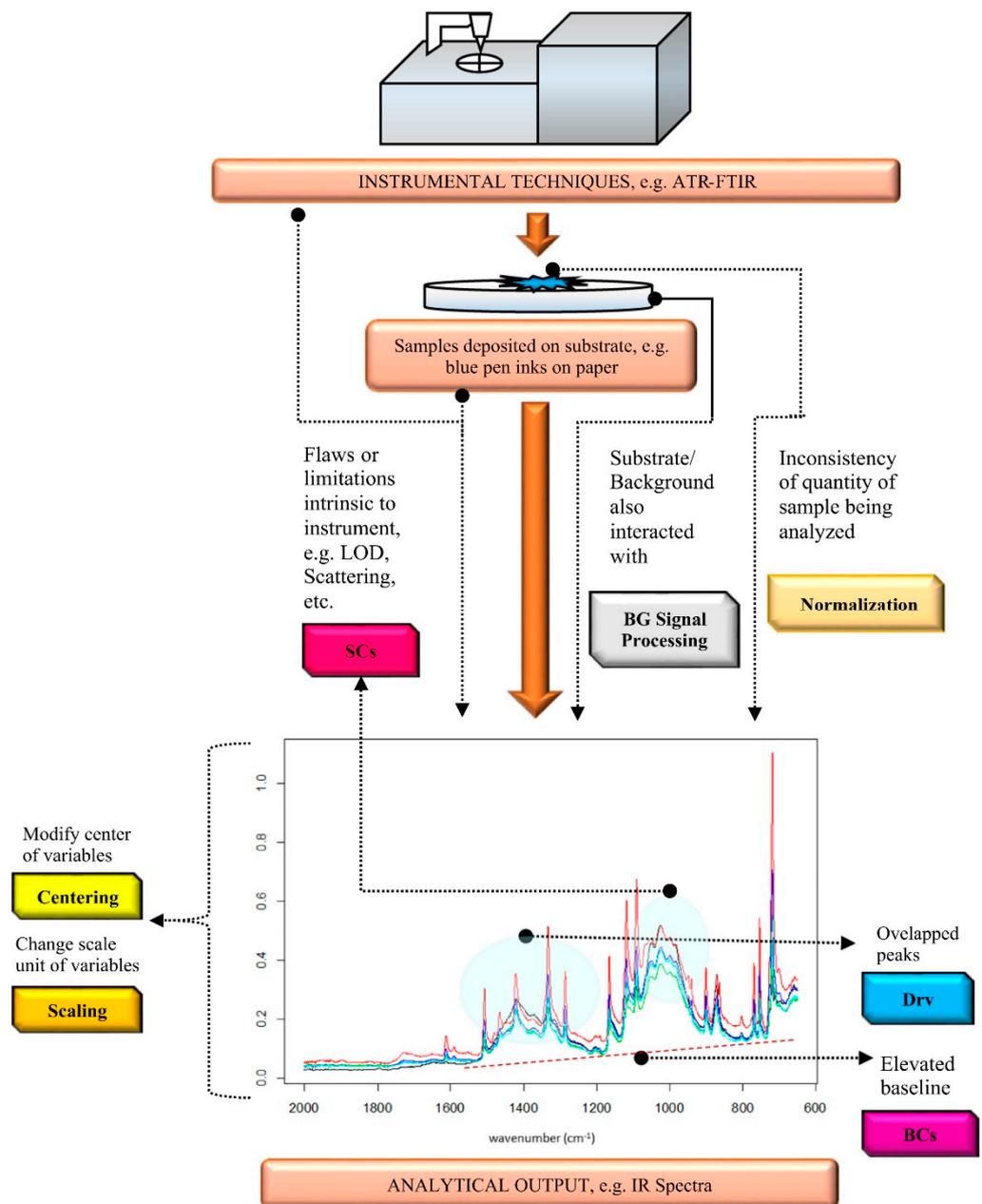
In the study of train simulation using the mathematical modeling approach that was carried out with Keretapi Tanah Melayu (KTM) to monitor the progress of the train system so that initial planning can be carried out for contingency case train scheduling:

*“ . . . Another interesting thing is the simulation, a simulation modelling that we have done was for the performance of the commuter system, our commuter system that is KTM, indeed the KTM has come to us to ask for consultant for modelling simulation, they wanted to see the train journey of KTM . . . ”*

Source: (P3: 10:12)

“Sometimes we don’t really understand the mathematics, it’s half of it . . . for example like scheduling, because we want to build a constraint for scheduling simulation . . . we have to understand . . . ”

Source: (P3: 10:56)



Typical characteristics of ATR-FTIR spectrum as exemplified with IR spectra collected from ink entries deposited on paper substrate. Each type of the interferences or noise could be resolved by different categories of DP methods.

**Figure 6.** Simulation processes using mathematical modeling output by participant P5. Source: P5’s published article. Reprinted with permission from Ref. [37]. 2017, Choong Yeun Liong.

### 3.1.3. Axiomatization 3 (A3): Mathematical Modeling for Useable Innovation Output the Impact on Technology Development

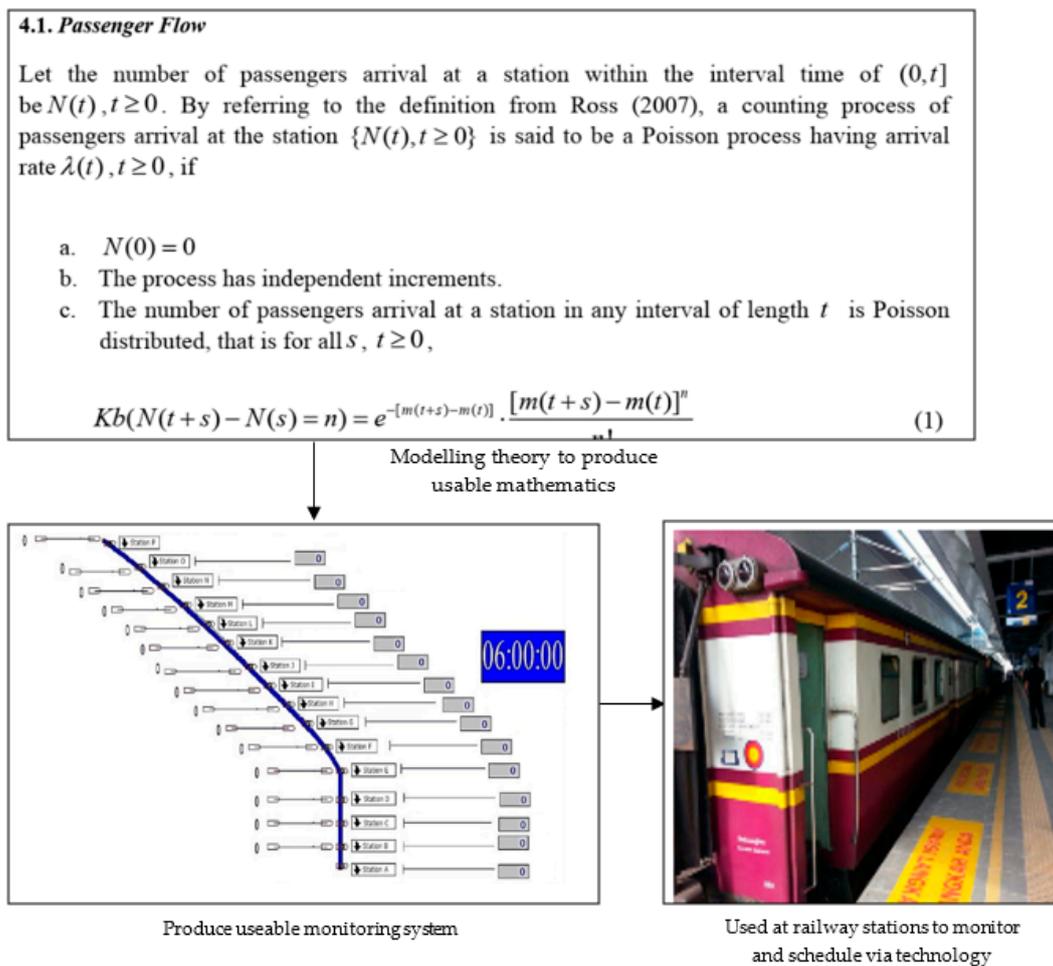
The mathematical modeling for useable innovation output in this study is the axioms of producing a new mathematical model and using the model for developing an innovation (i.e., tangible, or intangible innovation) and producing new technology. In the context of

mathematical modeling research, P3 explains that the mathematical modeling that he had been producing is a kind of model that is usable and will benefit society. This element is expressed below:

*“So, what I mean by mathematical model for me is a model that we try to benefit it to the public instead, we only use in self-books in the library . . . the example that I presented earlier is something that can actually be used for society . . . ”*

Source: (P3: 3:16)

This axiom can be triangulated using the document of P3. It is found that P3 has further used his output in the study of train simulation to produce a controlling system for KTM [38] to be used in the real application of the train system as shown in Figure 7:



**Figure 7.** Useable mathematical model output by P3 and P5. Source: P3’s published article. Reprinted with permission from Ref. [38]. 2015, Choong-Yeun Liong.

This axiom is enforced by the P2’s perspective by relating the process of solving real problems using mathematical modeling for a research output that is not just in the form of mathematical problem-solving. However, it is in the form of real situation problem solving. This situation justifies that mathematical modeling in the form that it can be applied to is more meaningful than the theoretical model.

*“Actually, by using mathematical modelling . . . in other words it is not a mathematical solution only . . . it’s more on the model that we can apply it”*

Source: (P2: 2:40)

In that sense, according to P7, to produce something that can be applied and usable, a collaboration would also need to be carried out with those involved in the field of the real situation. Collaboration can overcome many limitations encountered in a study and improve the capabilities of expanding a field to be studied. This element is confirmed by P7's collaboration with knee specialists to overcome the limitations of knowledge and expertise on the real application of imaging for medical usage. Furthermore, P7 indicated that it could improve the capabilities of the field of an expert on image visualization:

*"... There are not many knee experts... we were looking for experts to collaborate with... we could not find many knee experts in our University Hospital... I'd try searching on the website... people don't even know..."*

Source: (P7: 9:53)

According to him, the outcome of the mathematical modeling study on imaging visualization can further enhance the ability of the field of knowledge possessed to a product that is more useful in an effective way:

*"Because the model they have now is only used in the operating room, but we have it in the computer and software that can be taken anywhere, that's the advantage of the system itself..."*

Source: (P7: 10:14)

Using the collaboration method can increase a field's knowledge through larger-scale problem solving. For example, P5 was developing a software that can analyze large amounts of data on the loading bay optimization:

*"For the example, loading bay optimization, it is about methods of loading the goods. How practical is it? Past studies show to what extent? and why can't it be solved with the software or optimization package that is available now?"*

Source: (P5: 10:47)

#### 3.1.4. Axiomatization 4 (A4): Patent and Commercialization Output

The axiom of commercialization and patent for mathematical modeling output is that the output is very much dependent on establishing the elements of mathematical modeling for usability innovation for a researcher. For example, P7, a researcher on visual image information for medical purposes, has used interpretation to prove that the innovation from mathematical modeling-based research can produce some patents, making it possible to be commercialized.

*"Ortho-knee is ready, Ortho-Hip is being made... Ortho-Knee is being patented and also is being trademarked... it has been used at our University Hospital... now this product has been used for two years already... if two years our University Hospital is acceptable, then we can spread the use of this product to other hospitals..."*

Source: (P7: 9:68)

*"We developed, built and sold to people... we have used it for two years already... we made it and we are using it at the first stage..."*

Source: (P7: 9:69)

*"Now I already have five patents, but others have dozens already... I'm still a small fish... five patents..."*

Source: (P7: 9:70)

The element of generating ideas can also be seen when mathematical modeling outcomes can be commercialized from the study conducted. This element can be seen in the patent application for an idea or product from the research results. For that purpose, expertise from relevant agencies is required. According to P2, researchers need to submit simple and concise innovation ideas, and it is not necessarily needed for a complex engineering process to produce them:

*“When I commercialize my product, not really commercialize but what I mean is, when I patent my research outcome, they were surprised at how can mathematicians could come out with the product ... people talk like that ... then I said ... the reason is that the development of the product is actually very simple ... you don't need to be bombastic with development like you want to wait for the engineer to build that thing right ... it is not necessary ... that thing is very simple ... it is about what we learn ... because it involves the circuit, what we learned in school before ... and we have a program ... that's all ... ”*

Source: (P2: 2:65)

### 3.1.5. Formulization of Novelty in Mathematical Modeling

Experiencing new phenomena brings us to the element of new knowledge, learning new methods, skills and even innovating. From the aspect of mathematical modeling research output, there exists the elements that bring society to the betterment of their lives which can be axiomatized as a novel output. In that sense, a novelty is something that can give way for other research fields to be expanded and find uniqueness. In fact, there are many mathematical modeling outputs used to bring society to a new way of life. This axiom is interpreted from the interview data as shown below:

*“People never do this kind of simulation ... it's using FAP ... Frequency Assignment Problem ... mobile network ... I tried to solve it to optimize the frequency assignment ... to the channels, I managed to run a simulation of the cells. ... ”*

Source: (P2: 4:11)

Looking at the researcher's point of view, receiving critical comments on mathematical modeling output from other scholars brings specific research findings to the elements of novelty. For example, in the case of research in imaging for medical purposes carried out by P7, image accuracy is an important aspect that brings novelty to the research output.

*“According to the comments that we have received ... our image simulation is very accurate ... it has an error of just 0.02% ... they said that 0.02% is ok ... they said that if we could get it to 0.07% error, then it would be enough ... and getting 0.02%, thank God ... It's very good ... so they are confident with our findings ... We were able to give a solution where patients do not have to wait for a long period of time and reduce the time it takes for imaging to be available prior to surgery”*

Source: (P7: 9:72)

Developing a mathematical model for a system based on a phenomenon is one of the most challenging aspects of the novel output. As a university researcher, the involvement of PhD students in mathematical modeling research gives many advantages to novel research outputs, which was interpreted from P5, where his PhD students have come out with a novel output. Looking at this in-depth view, a novel finding can also be interpreted as a new finding (i.e., tangible, or intangible output), different outputs from existing ones, ease of use, and sometimes, a sophisticated product.

*“For PhD research, it needs to get something new, produce something new, and in science, we are really interested to discover something new, look for something ... something easy, in the case of gun loaded with bullets, the based-side bullet imaging is much easier than the inside, because the features of the inside shell are difficult to render, because there is lighting on the stretch that comes out, then we look at the mathematics, it is also sophisticated, it just so happens that in my PhD, I also used concept of moments, I know concept of moments can give unique features, the term now bitara, uniqueness for each image ... ”*

Source: (P5: 10:23)

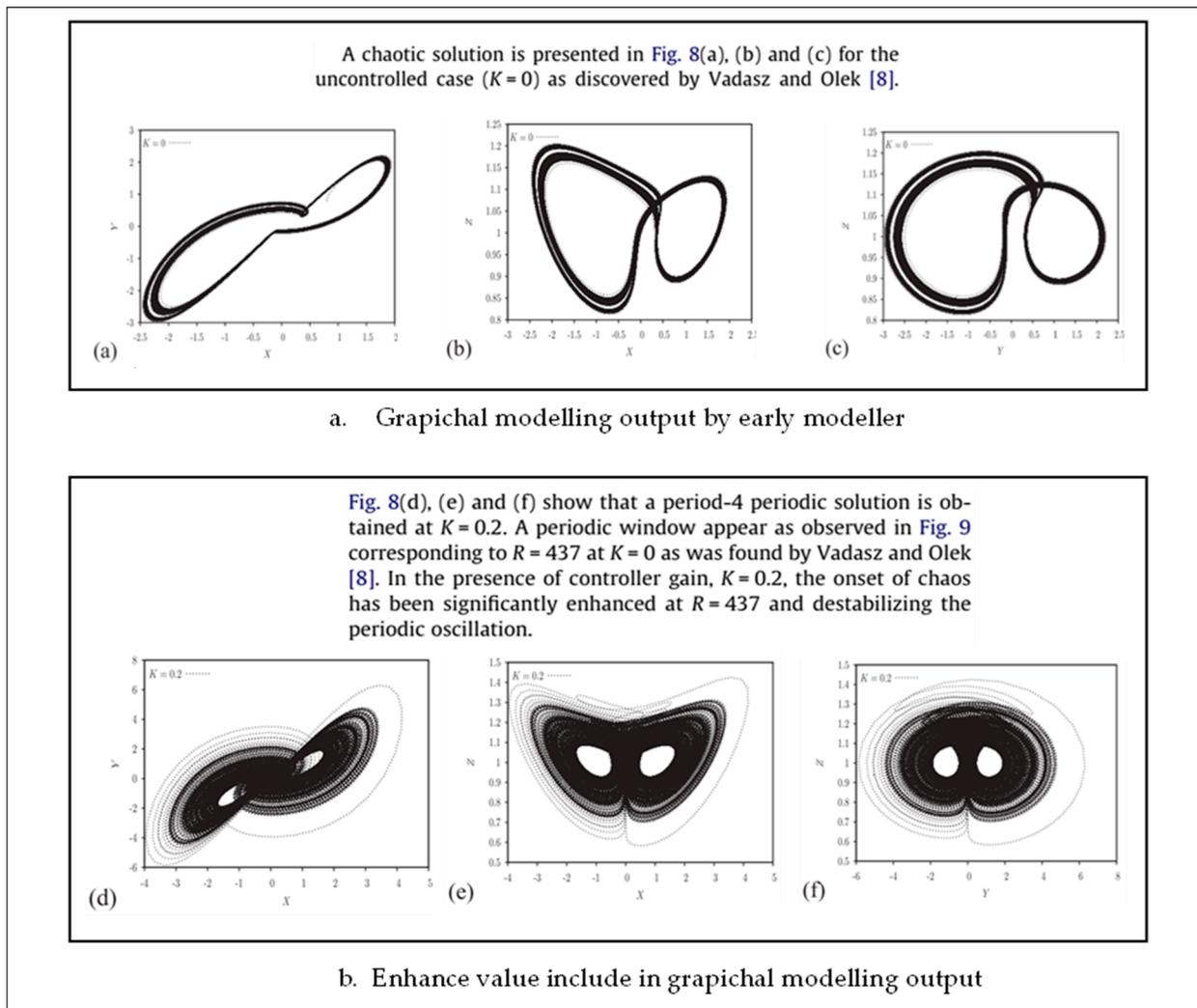
Comparing P5 and P7 with P1, research output in terms of theory in mathematical modeling can bring novelty to heat transfer and fluid mechanics research. It can be

concluded that the co-element for novelty for P5 and P7 to P1 is producing something new and useful to others on the theory of heat transfer.

“ ... There is a classic one ... there are areas that people don't study ... there is a boundary condition, the boundary that people always study on this type, but at the other areas, they don't study on it, and I tried to look into it, then I have the result that other researcher don't have ... I changed the boundary condition ... ”

Source: (P1: 8:53)

Evidence from document analysis proves that novelty on the research output by P1 and his group [39] has been carried out to expand the theory from Vadas and Olek's investigation on the chaotic behavior in the narrow, fluid-saturated porous layer under the influence of variations of the controller as shown in Figure 8.



**Figure 8.** The process to produce a unique mathematical model output by P1. Source: P1's published article. Reprinted with permission from Ref. [39]. 2011, Ishak Hashim.

Interpreted from a variety of data sources from the participants. Most of the data sources on the newness and uniqueness can be triangulated using journal articles. One article out of the data collected shows the uniqueness of the findings to show a novel mathematical model output produced by P1 and his team as shown in Figure 9.

In this paper, we have investigated periodic and chaotic behaviour in a narrow, fluid-saturated porous layer under the influence of various values of the controller gain  $K$ . We found that the effects of the controller are more pronounced, especially at the subcritical values of  $R = 24.9$  ( $R < R_{c2} = 27.78$ ) and  $R = 63$  ( $R < R_{c2} = 65.01$ ) which can be used to restrain chaotic behaviour in a thermal convection loop and sustain unidirectional flow. A particular case for the suppression and enhancement of chaotic convection in the presence of feedback control was demonstrated for low and moderate values of Vadasz number at  $R = 24.9, 60.5, 75$  and  $R = 63, 290, 437$ , respectively. As a conclusion, the presented computational results indicate that feedback control has stabilizing or destabilizing character according to the significantly slightly higher controller gain  $K$  than in an uncontrolled case and will confer a great advantage for controlling chaos in many industrial applications.

**Figure 9.** The novel mathematical research output by P1 and team members. Source: P1's published article. Reprinted with permission from Ref. [39]. 2011, Ishak Hashim.

In most of the participants' research, it can be seen that novelty is the element that was desired to be achieved. It is an important element for mathematical modeling scholar output and one of the approaches used to make sure that the output is novel is through publication, in which this aspect was highlighted by all participants. For P1, he was competing for novel mathematical modeling output on the study of heat transfer and fluid mechanics with other mathematical modeling scholars.

*"I once competed with a professor over the submission of a paper, I know that this professor is expert in mathematics, but he is not an expert in fluid mechanics . . . his expertise is in numerical methods . . . it so happened that when I submitted the manuscript, it was him who reviewed my manuscript . . . he held onto my manuscript for a long time . . . maybe he kept it . . . he is an expert in numerical methods but not in fluid mechanics . . . so when he saw my mathematical equation, he likely kept them from being published . . ."*

Source: (P1: 8:34)

One of the important elements in mathematical modeling scholarly output novelty is the potential of the output when it comes to the development of new technology. The interview session with P2 found that this element has to be carried out using the STEM integration approach due to some limitations in engineering and technical capabilities amongst scientists who use mathematical modeling. Additionally, according to P2, through collaboration, she had an opportunity to produce a unique idea to solve the problem of children carrying overweight bags to school in Malaysia:

*"In my case, at that time there are no other techniques available to detected things by weight . . . it used to be a problem . . . that was the problem that I had studied before . . . I did an experiment . . . of school children carrying heavy bags to school, so how much weight can a child carry . . . of course the weight of the each student is not the same . . . so we have to standardize it first . . . then we looked at how much weight schools children are able to carry, finding the critical value that would cause back aches . . . at that time carrying a heavy bag to school was an issue. Such a thing could result in children suffering from back problems."*

Source: (P2: 2:17)

Furthermore, according to P1, the element of the ability to use certain knowledge explains that, through the integration of disciplines, the capability of a knowledge field can be enhanced. According to P1, the application of a specific field knowledge allows the field knowledge in thermal physics modeling research to be used for other fields:

*“It’s like the research I’m doing . . . if there’s field integration . . . knowledge, it’ll take us further . . . if not, we’ll be stuck at that stage . . . for example, if you’re good at programming, but if you don’t relate it with real-world applications, you will remain at that stage . . . ”*

Source: (P1: 8:15)

Based on the elements that have been discussed, it can be concluded that mathematical modeling novelty research output is the axioms that every researcher tries to achieve. In mathematical modeling, novelty can be produced from many different research outputs such as in terms of theoretical modeling, simulation modeling and modeling for useable innovation output. Meanwhile, patents and commercialization are the final aspects that can be contributed to society.

#### 4. Formulization and Discussion

Comparing all the group axioms, the central axioms of mathematical modeling research output can be formulized to see on how they impact the development of new technology and the strategy to seek novelty [40] for society use. By constantly comparing [41] four categories of mathematical modeling research output axioms  $A(X)_n$ , the fundamental axiom modelling output is system theorizing and this can be strategized by inculcating elements of  $E(T)_n$  which finally impact new technology development as in Table 1.

System theorizing of mathematical modeling output [42,43] is the fundamental processes and creates most important general output formulae that can be used for new technological development. Meanwhile, a specific system theorizing modeling output can be derived from the general formulae to produce another new fundamental mathematical research output. This output is in the form of mathematical formula or a set of equations that can be used for further theory or technological development [44]. The general system’s theorizing mathematical modeling output is the basic idea for most technological development [45]. In many cases, to produce system theory, a modeler needs to have strong background and understanding of the field they are studying. For example, expertise in the specific field of heat transfer has developed a new model of theorizing output on a system about heat transfer. The system theory output normally is derived by rooting it on the phenomena being observed, and the observer should have ability to relate the elements being observed with specific fundamental principles or theory.

Besides work on real system observation, system theorizing can also be developed using a set of existing or past data analyses. An example of this element is system theorizing on the spread of COVID-19, which can be developed from recorded data or purely developed from the theory of past events [46]. The most important element for this mathematical modeling output is setting some system assumption(s) for the study, which functions as system constraints and limitations to make the model more reliable, explainable and able to give good prediction. By considering the existing theories, concepts or models under specific mathematical modeling studies, the theorizing of a system can then be developed for expansion or revision for a specific theory so that more reliable and remarkable output contributions could be made. For certain fields, this step will involve a free body or schematic diagram to illustrate the idea. These processes require a good knowledge and understanding of mathematics as it involves using a mathematical equation or a set of equations that govern the system [47]. One important characteristic that mathematical modelers should emphasize in regard to mathematical modeling is determining system pattern properties. The system pattern properties would give more information about the system’s behaviors if certain variables or assumptions were modified.

**Table 1.** List of categorical mathematical modeling outputs axiom  $A(X)_n$  and elements  $E(X)_n$ .

Central Axiom	Axiom $A(X)_n$ ; $n = 1, 2, 3, \dots$	Mathematical Modeling Axiom Element $E(X)_n$ ; $n = 1, 2, 3, \dots$
Mathematical modeling novelty towards building new technology $F(N)$	System theorizing output modeling $(X \sim T)$	<b><math>E(T)_n</math>:</b> <ol style="list-style-type: none"> <li>1. Mastering specific field of knowledge.</li> <li>2. Rooted from system observed and has fundamental principles.</li> <li>3. Observation can also be carried out based on sets of previous data.</li> <li>4. Involves assumptions that need to be set for system under review.</li> <li>5. To develop a theory for explanation and system phenomena prediction.</li> <li>6. Consideration of existing theories, concepts and models under specific study.</li> <li>7. Using free body and schematic diagrams.</li> <li>8. Strong mathematical background.</li> <li>9. Using mathematical equations or a set of equations that govern together.</li> <li>10. Involves system pattern properties determination.</li> <li>11. Using digital technology to assist the problem-solving processes.</li> <li>12. Contribution of theory to the specific field.</li> </ol>
	System simulation output modeling $(X \sim S)$	<b><math>E(S)_n</math>:</b> <ol style="list-style-type: none"> <li>1. Used to break human being limitation (i.e., thinking, visualizing and physical).</li> <li>2. Used in multi-discipline research objectives.</li> <li>3. Need to understand certain analyzing techniques.</li> <li>4. Some investigations need to be conducted in laboratories.</li> <li>5. Combination of methods that have the potential to develop new precise technology.</li> <li>6. Some different methods of analysis could enhance precession.</li> <li>7. Could involve cyclic investigations and several parties in the study.</li> <li>8. Has been advantageous due to the advancement of computer technology.</li> </ol>
	Useable innovation output (tangible/intangible) $(X \sim I)$	<b><math>E(I)_n</math>:</b> <ol style="list-style-type: none"> <li>1. Using the first and second axiom to develop and innovate new technology.</li> <li>2. Can be in tangible or intangible innovation.</li> <li>3. Useable and can benefit society.</li> <li>4. Solve real-world problems.</li> <li>5. Collaboration could also be carried out with those involved in different fields.</li> <li>6. Improve and expand the understanding of a field.</li> </ol>
	Patent and commercialization output (Product) $(X \sim C)$	<b><math>E(C)_n</math>:</b> <ol style="list-style-type: none"> <li>1. Establish mathematical modeling for usability innovation.</li> <li>2. Patents to protect uniqueness.</li> <li>3. Patent claims for idea or product of research output.</li> <li>4. Solve real-life problems and situation.</li> </ol>

Furthermore, new technology could be developed using the system properties with respect to the constraints. In the digital era, the world of mathematical modeling is advantageous, especially in the modeling of complex phenomena. Digital technology is used in mathematical modeling, to assist problem-solving processes since this technology provides fast, efficient and reliable results beyond human limitations. Therefore, it can be confirmed that the use of technology for modeling problems contributes to the specific field of theory.

The second mathematical modeling research output was categorized as system simulation modeling. A system simulation modeling is constructed by developing the first axiom—system theorizing output modeling. Using the system theorizing in the form

of formulization, simulation can be developed using computer technology to describe system behaviors for certain limits and approximations. This technique is powerful since it surpasses the limitations of human experience (i.e., thinking, visualizing and physical limitation). The impact of simulation output on technology development is vast for a wide range of fields and has been used for multi-discipline research objectives [48]. However, the purposes of simulation are more practical (i.e., application of the model to solve problems or answer questions) for complex problems. Modeling and simulation aim to simplify and avoid the useless production of complex copies of a complex reality.

A good simulation is the simplest simulation that can represent the actual system and serves to fulfil a purpose. It also has the characteristic to help us understand a complex system, and a solution for the specific problem could be determined. The simple model of a complex system will allow the complexity to no longer obstruct our view, and we will virtually be able to look through the complexity [49] of the system at the heart of things. We can further derive our strategy to solve complex real-world problems from the simulation. A competent hypothetical statement could be drowned out from simulation output about what is happening inside the system. As a user of the mathematical modeler, we should know that the data drawn out from a simulation are different from the actual system. From this study, we found that mathematical modeling simulation with the aid of computer technology in a laboratory impacts the development of new technology in pattern recognition for the forensic field. Although experimental data are not as accurate (superfluous) as mathematical models using computer simulations, using both techniques proves that the mathematical modeling output has a big impact on new technological developments if we are able to determine the best technique to be used.

It can be concluded that simulation modeling and experiments are dependent on one another. Many new technological developments could be made if the simulation and experimental techniques are integrated. For example, data from a tiny and complex system of forensic evidence can be extracted and simulated using mathematical modeling with the aid of laboratory techniques. The experimental data form can then be made to be more reliable and valid by the support of a mathematical simulation technique. This investigation cycle which uses a different analysis method could produce precise technology.

The third mathematical modeling research output was categorized as the usable innovation output, which can be tangible or intangible products. In this aspect, mathematical modeling operated behind the product as tangible or intangible. As we know, modeling involves a governing system of equations, and it very much relies on the purpose of problem solving [50]. Therefore, the goal of the output is to create new innovative technology. From this research perspective, it was proven that the mathematical modeling output has the potential to develop new technology, and the modeling technique is taken as a tool to solve a specific problem. Based on this principle, emphasizing the tangible or intangible new technology development objectives should be cultivated in the mathematical modeling curriculum. For most cases in this study, it can be proven that mathematical modeling is the best technique that mostly occupies for design and problem-solving curricula involving the sciences, engineering, programming and other related disciplines. It is rooted in applied mathematics and involves analyzing a system. Mathematical modeling plays a role as an analytical tool for designing a product or system conceptualization through a chain of a system by using mathematical derivation to the final output. From these research contexts, some mathematical modeling output comparisons show that the language of mathematics explaining the abstract system was used to represent a design that plays a vital role for a mathematical modeling output to be sufficiently structured for new technological developments. The representation can be carried out through causal hierarchies and network diagrams to show the sensible articulation of the system towards new technology with respect to the validation of the design at the corresponding level of detail as shown in Figure 10.

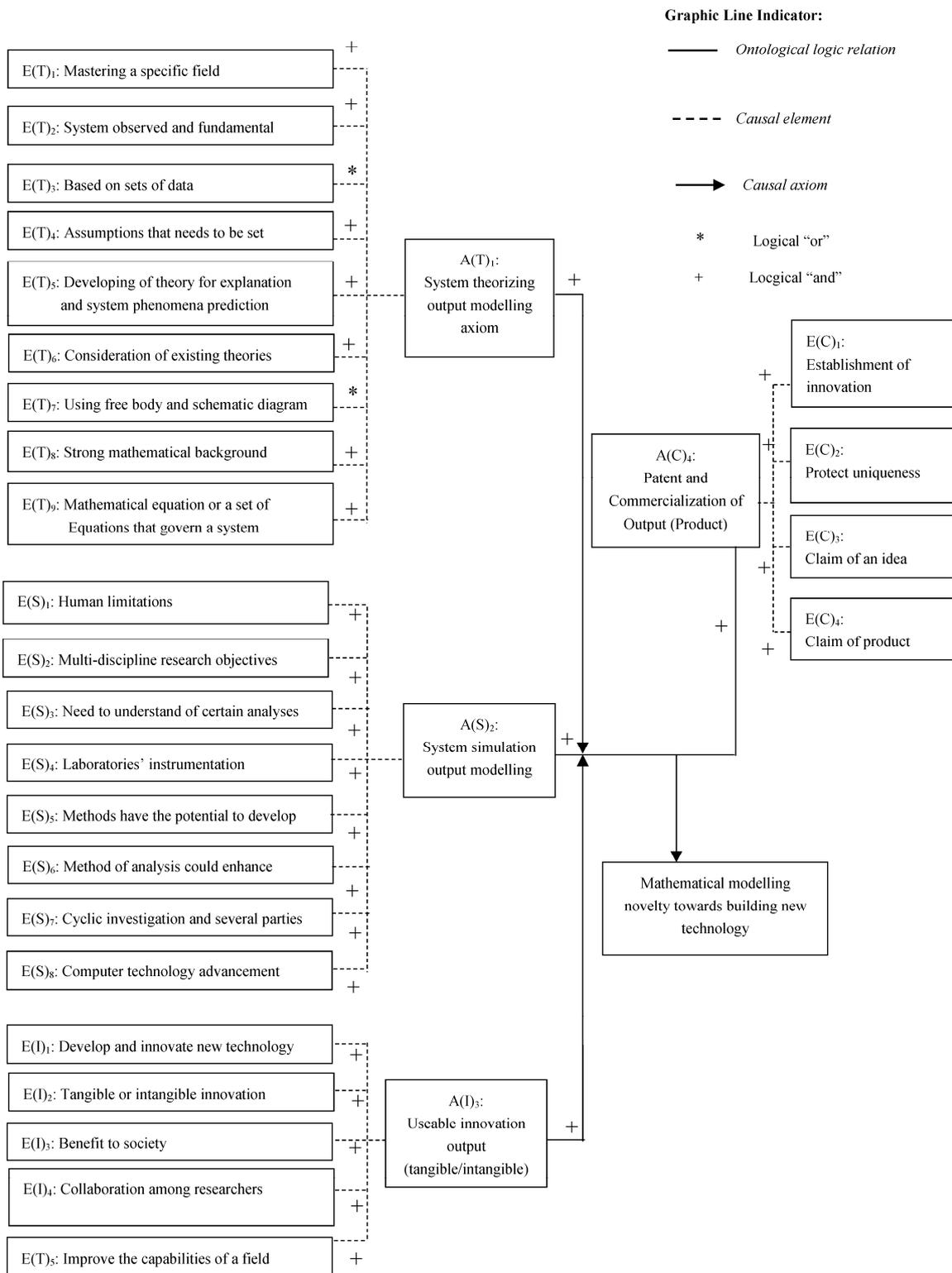


Figure 10. Mapping of mathematical modeling axioms and elements.

The drive behind mathematical modeling axioms from the context of applied science scientists is to obtain patents and commercialize products. Patent on mathematical modeling research output is applied based on claims to protect certain intellectual properties. The claim can be made to more than one of the first three sub-axioms discussed. The novelty of the mathematical modeling determines the chance of the output to be covered. For

mathematical modeling that operates behind a product’s system, patent is a path before the product can be commercialized. However, the finding shows that patent on mathematical modeling research output is not purely dependent on the mathematical algorithm. The patent for mathematical modeling research output is a device or software that implements the mathematical algorithm, and the patent covers in terms of application. It can be understood that even a patent does not cover the algorithm from mathematical modeling research output, but it covers every possible implementation of the mathematical algorithm. In that sense, a mathematical formula containing and using symbols and mathematical operations (i.e., addition, subtraction, division, etc.) is not valid for a patent claim without the implementation in a specific system or device. From the perspective of this case axiomatization, creating or translating the mathematical algorithms into a useable product for society is a challenge for developing new technology. This challenge can be overcome by enhancing the ability to research and developing mathematical modeling output.

*Data Triangulation and Consensus*

The mathematical modeling research output formulization using axioms  $A(X)_n$  for impacting new technological development with elements  $E(X)_n$  was triangulated using data from document analysis (mathematical modeling article journal) as presented in Table 2. Participant consensus is obtained by using a yes or no answer. The consistency of data triangulation from different sources confirmed all axioms and elements have consistency and reliability towards the elements that impact new technology development.

**Table 2.** List of axiom and element under the formulization of mathematical modeling research output.

Formulization	Axiom $A(X)_n$	Element $E(X)_n$	Source of Data (P1 to P7)		Participant Consensus
			Interview	Document	Yes or No
Mathematical modeling novelty towards building new technology	$A(T)_1$ : System theorizing output modeling	$E(T)_1$	all	all	Yes
		$E(T)_2$	P1, P2, P5	all	Yes
		$E(T)_3$	P2, P4, P5	P2, P4, P5	Yes
		$E(T)_4$	all	all	Yes
		$E(T)_5$	P1, P5	P1, P5	Yes
		$E(T)_6$	all	all	Yes
		$E(T)_7$	P1, P2	P1, P2, P5	Yes
		$E(T)_8$	P1, P2, P4	all	Yes
		$E(T)_9$	P1, P2, P5	all	Yes
		$E(T)_{10}$	P1, P4, P5	P1, P4, P5	Yes
		$E(T)_{11}$	all	all	Yes
		$E(T)_{12}$	P1, P5, P6	P1, P5	Yes
Mathematical modeling novelty towards building new technology	$A(S)_2$ : System simulation output modeling	$E(S)_1$	all	all	Yes
		$E(S)_2$	P2, P4, P7	P2, P4, P7	Yes
		$E(S)_3$	P1, P2, P4	P1, P2, P4	Yes
		$E(S)_4$	P1, P2, P7	P1, P2, P4	Yes
		$E(S)_5$	P1, P4, P7	P1, P4	Yes
		$E(S)_6$	P1, P4, P7	P4, P5, P7	Yes
		$E(S)_7$	P2, P4, P7	P1, P2, P5	Yes
		$E(S)_8$	all	all	Yes
Useable innovation output modeling (tangible/intangible)	$A(I)_3$ :	$E(I)_1$	P1, P2, P3, P5, P7	P2, P3, P5, P7	Yes
		$E(I)_2$	P2, P3, P4, P5, P7	P2, P3, P4, P5	Yes
		$E(I)_3$	P1, P2, P6	P1, P2, P5, P6	Yes
		$E(I)_4$	all	all	Yes
		$E(I)_5$	P1, P2, P4, P5, P7	all	Yes
Patent and commercialization of modeling output (Product)	$A(C)_4$ :	$E(C)_1$	P2, P3, P5, P7	P1–P6	Yes
		$E(C)_2$	P2, P3, P5, P7	P1–P6	Yes
		$E(C)_3$	P2, P5, P7	P1–P6	Yes
		$E(C)_4$	P2, P3, P4, P5, P7	P1–P6	Yes

Data triangulation shows that not every participant obtained data that contributed to the development of elements that exist. Some data were from interviews and some of the data can be found in their articles. This is because they are sharing their experiences spontaneously using the interview technique, while more rigorous and detailed data could be explored using document analysis to determine and make in-depth element exploration. Even though not all elements were found in all participants, but all of participants agree on every respective element as the construct for their respective axiom. This approach is trustworthy for every axiom and shows categories and path replication and transferability. Furthermore, the identification of elements may be performed through different angles of data sources and different interpretation. As such, conforming the elements through a secondary source of data can reduce the misinterpretation at which the information was emerging.

## 5. Conclusions

The current study presents categorical mathematical modeling output in the form of mathematical formulae [15]. However, it is unclear to determine and understand the modeling output, modeling purposes and its impact on new technological development. In this study, we axiomatize mathematical modeling output qualitatively. It is presented on the formulization based on stages to the novelty for new technology development. The formulation can be used for research and educational purposes by bridging mathematical modeling with path of new technological development. The formulization output framework path for developing technology is related to real-world problems, and finally mapping mathematical modeling research work output to the impact for new technological development. By emphasizing qualitative axiomatization and following the unique real problem-solving context focusing on new technological development, our framework puts great emphasis on the emergence of four main axioms, i.e., system theorizing output modeling, useable innovation output as tangible or intangible output, system simulation output modeling and patent for commercialization of research output. Meanwhile, mathematical modeling novelty is the final destination towards building new technological development by showing the ability to get patents and commercialization of output as products.

From this study, it can be formulized that the fundamental mathematical modeling axiom is system theorizing output models and these types of models have the ability to describe how a system functions. This mathematical modeling axiom translates a system into the language of mathematics, and it is fundamental to any further new technological development. The fundamental system theorizing has many advantages for science and technology development. This fundamental output has a very precise language and can help us formulate new ideas and identify underlying assumptions. The system theorizing is also built with a concise language, with well-defined rules that can be manipulated to other systems. This axiom is the result that mathematicians have proved over hundreds of years ago. In modern world, computers can be used to perform numerical calculations to validate and enhance the modeling output. Data have shown that there are many causal elements that drive participants on producing mathematical modeling output for system theorizing. The main causal elements are that they must have a strong fundamental knowledge about mathematics and its related fields. This element has to be complimented by other important elements such as the ability to use mathematical tools such as free body and schematic diagrams in order to visualize their mathematical model idea, has a strong and deep ability of observation and fundamental principles, knowing how to find existing sets of data, being able to determine assumptions that need to be set, has a strong ability of developing theory for explanation and consideration of existing theories and able to understand mathematical equations or a set of equation that govern a system.

Complementing the first axiom with logic “and” is the second axiom which is mathematical modeling output for system simulation. This axiom emerged from the elements of to meet the need to overcome human being limitations. One of the characteristics of these elements is the modeling of advanced work from the outcome of mathematical models

for system theorizing. Mathematical models for system simulation are also more practical in solving real-life problems, impacting a wide range of field studies and can be used in multi-disciplines. This can be indicated in the use of mathematical modeling for system simulation that has been produced by P6 in which it has been used in forensic science. The consequences from this are that the mathematical modeling outputs were used for system theorizing and simulation is the third axiom which is a usable innovation output, either intangible or tangible product. This is because the mathematical modeling outputs not only give the mathematical solutions but a model that can solve real-life problems. In most cases, the results also indicated that mathematical modeling outputs produced utilized the STEM integration approach to overcome some limitations in engineering and technical capability amongst mathematical modeling scientists. Using the STEM approach, collaborations with other scientists in other fields creates an opportunity to produce a unique idea to solve real-world problems such as Malaysian students carrying overweight bags to school. Therefore, it can be concluded that mathematical modeling outputs produced from the context of this study clearly could contribute to solving a specific real-life problem and the development of new technology.

The last emerging axiom from this research context is patent and commercialization, which is the desired output of mathematical modeling. Most of the cases clearly showed that the mathematical modeling outputs were seeking patents and commercialization of products. Patent is an advanced process where a mathematical modeling research output is in the form of product that solved a real-life problem where a certain claim can be made to protect it before commercialization. This element also showed an establishment of innovative research idea output which came from the impact of mathematical modeling research. It means that efforts should be taken for the further development of research findings on mathematical modeling outputs where it can enhance the output to the sufficient level to apply new technology that benefits society. It is also explained that the mathematical modeling creative processes and understanding on how innovations arise where novelty towards building new technology are the elements that are put into focus for further discoveries. The element of novelty could lead to effective interventions that nurture success for sustainable society growth. From the empirical findings obtained in this study, we conclude that novelties in mathematical modeling are discovered in a variety of different contexts that impact the development of new technology and we further introduce a framework that can be used to determine novelties that emerge in mathematical modelling. The framework clearly showed that the categories, elements and path used to produce mathematical models for the development of new technology and subsequently benefit society. Mathematical modeling also cannot stand alone in the mathematics field. It needs to be integrated with other research fields such as medical science, computer science and social science. However, the findings exhibit some limitations. One of the limitations faced in this study is the limited number of participants which led to the limited scope of mathematical modeling outputs. Therefore, it is suggested that more mathematical modeling outputs need to be explored in order to get a precise picture of the contribution of mathematical outputs to the development of new technology.

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