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Certain Unified Integrals Involving a Multivariate Mittag–Leffler Function

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Abstract: A remarkably large number of unified integrals involving the Mittag–Leffler function have been presented. Here, with the same technique as Choi and Agarwal, we propose the establishment of two generalized integral formulas involving a multivariate generalized Mittag–Leffler function, which are expressed in terms of the generalized Lauricella series due to Srivastava and Daoust. We also present some interesting special cases.

Keywords: generalized hypergeometric function ${}_pF_q$; generalized (Wright) hypergeometric functions $p\Psi_q$; generalized Lauricella series; Oberhettinger's integral formula; generalized Mittag–Leffler function



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1. Introduction and Preliminaries

The generalized Lauricella series (see, for example, Refs. [1] (p. 454) and [2] (p. 37)) is defined as follows:

$$\begin{aligned} F_{C:D^{(1)};\dots;D^{(n)}}^{A:B^{(1)};\dots;B^{(n)}} \left(\begin{array}{c} z_1 \\ \vdots \\ z_n \end{array} \right) &= F_{C:D^{(1)};\dots;D^{(n)}}^{A:B^{(1)};\dots;B^{(n)}} \left(\begin{array}{c} [(a):\theta^{(1)}, \dots, \theta^{(n)}] : \\ [(c):\psi^{(1)}, \dots, \psi^{(n)}] : \\ [(b)^{(1)}:\phi^{(1)}]; \dots; [(b)^{(n)}:\phi^{(n)}]; \\ [(d)^{(1)}:\delta^{(1)}]; \dots; [(d)^{(n)}:\delta^{(n)}]; \end{array} \right. \\ &\quad \left. z_1, \dots, z_n \right) \\ &= \sum_{k_1, \dots, k_n=0}^{\infty} \Omega(k_1, \dots, k_n) \frac{z_1^{k_1}}{k_1!} \cdots \frac{z_n^{k_n}}{k_n!}, \end{aligned} \quad (1)$$

where, for convenience,

$$\Omega(k_1, \dots, k_n) = \frac{\prod_{j=1}^A (a_j)_{k_1\theta_j^{(1)} + \dots + k_n\theta_j^{(n)}} \prod_{j=1}^{B^{(1)}} (b_j^{(1)})_{k_1\phi_j^{(1)}} \cdots \prod_{j=1}^{B^{(n)}} (b_j^{(n)})_{k_n\phi_j^{(n)}}}{\prod_{j=1}^C (c_j)_{k_1\psi_j^{(1)} + \dots + k_n\psi_j^{(n)}} \prod_{j=1}^{D^{(1)}} (d_j^{(1)})_{k_1\delta_j^{(1)}} \cdots \prod_{j=1}^{D^{(n)}} (d_j^{(n)})_{k_n\delta_j^{(n)}}}, \quad (2)$$

the coefficients

$$\begin{cases} \theta_j^{(m)} \ (j = 1, \dots, A); \ \phi_j^{(m)} \ (j = 1, \dots, B^{(m)}); \\ \psi_j^{(m)} \ (j = 1, \dots, C); \ \delta_j^{(m)} \ (j = 1, \dots, D^{(m)}); \ \forall m \in \{1, \dots, n\} \end{cases}, \quad (3)$$

are real and positive, (a) abbreviates the array of A parameters a_1, \dots, a_A , and (b^(m)) abbreviates the array of $B^{(m)}$ parameters

$$b_j^{(m)} \ (j = 1, \dots, B^{(m)}); \ \forall m \in \{1, \dots, n\},$$

with similar interpretations for (c) and (d^(m)) ($m = 1, \dots, n$).

The interested reader may refer to papers on the subject for more details [1,2].

The familiar generalized hypergeometric series pF_q is defined as (Ref. [3], Section 1.5)

$$\begin{aligned} {}_pF_q \left[\begin{matrix} \gamma_1, \dots, \gamma_p; z \\ \beta_1, \dots, \beta_q; \end{matrix} \right] &= \sum_{n=0}^{\infty} \frac{(\gamma_1)_n \cdots (\gamma_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \frac{z^n}{n!} \\ &= {}_pF_q(\gamma_1, \dots, \gamma_p; \beta_1, \dots, \beta_q; z), \end{aligned} \quad (4)$$

where $(\gamma)_n$ is defined as the Pochhammer symbol (for $\gamma \in \mathbb{C}$) and it is denoted by [3] (pp. 2, 4–6)

$$\begin{aligned} (\gamma)_n &:= \begin{cases} 1 & (n = 0) \\ \gamma(\gamma + 1) \dots (\gamma + n - 1) & (n \in \mathbb{N} := \{1, 2, 3, \dots\}) \end{cases} \\ &= \frac{\Gamma(\gamma + n)}{\Gamma(\gamma)} \quad (\gamma \in \mathbb{C} \setminus \mathbb{Z}_0^-), \end{aligned} \quad (5)$$

and \mathbb{Z}_0^- denotes the set of nonpositive integers.

Furthermore, Oberhettinger's integral formula [4]

$$\int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} dx = 2\lambda b^{-\eta} \left(\frac{b}{2} \right)^\mu \frac{\Gamma(2\mu) \Gamma(\eta - \mu)}{\Gamma(1 + \eta + \mu)}, \quad (6)$$

provided $0 < \Re(\mu) < \Re(\eta)$.

The well-known Mittag-Leffler function and its generalization were introduced and studied by Mittag-Leffler [5,6], Wiman [7,8], Agarwal [9], Humbert [10], Humbert and Agarwal [11] and other authors [12–15].

Motivated by above works here, with the same technique as Choi and Agarwal [12], we propose the establishment of two generalized integral formulas involving a multivariate generalized Mittag-Leffler function, which are expressed in terms of the generalized Lauricella series due to Srivastava and Daoust [1] given in Equation (1).

In a recent paper, Saxena and Kalla [16] introduced a more generalized Mittag-Leffler function as

$$E_{(\rho_j),\eta}^{\gamma_j}(z_1, \dots, z_m) \equiv E_{(\rho_1, \dots, \rho_m),\eta}^{(\gamma_1, \dots, \gamma_m)}(z_1, \dots, z_m) = \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(\gamma_1)_{k_1} \dots (\gamma_m)_{k_m} z_1^{k_1} \dots z_m^{k_m}}{\Gamma(\eta + \rho_1 k_1 + \dots + \rho_m k_m)(k_1)! \dots (k_m)!}, \quad (7)$$

where $\eta, \gamma_j, \rho_j, z_j \in \mathbb{C}, \Re(\rho_j) > 0, j = 1, \dots, m$.

Equation (7) is a generalization of well-known results.

On setting $m = 1$, Equation (7) reduces to the Mittag-Leffler function defined by Prabhakar [15]):

$$E_{\rho,\eta}^{\gamma}(z) = \sum_{k=0}^{\infty} \frac{(\gamma)_k z^k}{\Gamma(k\rho + \eta) k!}, \quad (8)$$

where $\rho, \eta, \gamma, z \in \mathbb{C}$, $\Re(\rho) > 0$, $\Re(\eta) > 0$ and $(\eta)_n$.

On setting $\gamma = 1$, Equation (8) reduces to the Mittag–Leffler function defined by Wiman [8]:

$$E_{\rho, \eta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\rho + \eta)k!}, \quad (9)$$

where $\rho, \eta, z \in \mathbb{C}$, $\Re(\rho) > 0$, $\Re(\eta) > 0$,

On setting $\eta = 1$, Equation (9) reduces to the Mittag–Leffler function defined by [5,6]

$$E_{\rho}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\rho + 1)k!}, \quad (10)$$

where $\rho \in \mathbb{C}$, $\Re(\rho) > 0$, $z \in \mathbb{C}$.

We also require the generalized hypergeometric function ${}_p\psi_q[z]$ (see [17,18]) defined by

$${}_p\psi_q[z] = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^p \Gamma(a_i + \alpha_i k)}{\prod_{j=1}^q \Gamma(b_j + \beta_j k)} \frac{z^k}{k!} \quad (11)$$

provided that $p, q \in N_0 = N \cup \{0\}$; $a_i, b_j \in \mathbb{C}$; $\alpha_i, \beta_j \in \mathbb{R}$; $\alpha_i, \beta_j \neq 0$; $i = 1, \dots, p$; $j = 1, \dots, q$.

2. Main Results

We establish two generalized integral formulas, which are expressed in terms of the generalized Lauricella functions (1), by inserting a generalized Mittag–Leffler function (7) with suitable arguments into the integrand of (6).

Theorem 1. *The following integral formula holds true: For $\eta, \mu, z_j \in \mathbb{C}$ and $x > 0$, where $j = 1, \dots, m$*

$$\begin{aligned} & \int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} E_{(\rho_j), \eta}^{(\gamma_j)} \left(\frac{z_j}{x + b + \sqrt{x^2 + 2bx}} \right) dx \\ &= 2^{1-\mu} b^{\mu-\eta} \frac{\Gamma(1+\eta)\Gamma(\eta-\mu)}{(\Gamma(\eta))^2 \Gamma(1+\eta+\mu)} \\ & \cdot F_{3:0;\dots,0}^{2:1;\dots,1} \left[\begin{matrix} [1+\eta:1,\dots,1], [\eta-\mu:1,\dots,1], \\ [\eta:1,\dots,1], [1+\eta+\mu:1,\dots,1], [\eta:\rho_1,\dots,\rho_m] \end{matrix} ; \begin{matrix} [\gamma_1:1]; \dots; [\gamma_m:1] \\ -; \dots; - \end{matrix}; \frac{z_1}{b}, \dots, \frac{z_m}{b} \right], \end{aligned} \quad (12)$$

where $0 < \Re(\mu) < \Re(\eta)$.

Theorem 2. The following integral formula holds true: For $\eta, \mu, z_j \in \mathbb{C}$ and $x > 0$, where $j = 1, \dots, m$

$$\begin{aligned} & \int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} E_{(\rho_j),\eta}^{(\gamma_j)} \left(\frac{xz_j}{x + b + \sqrt{x^2 + 2bx}} \right) dx \\ &= 2^{1-\mu} a^{\mu-\eta} \frac{\Gamma(\eta-\mu)\Gamma(1+\eta)}{(\Gamma(\eta))^2\Gamma(1+\eta+\mu)} \\ & \cdot F_{3:0;\dots;0}^{2:1;\dots;1} \left[\begin{matrix} [1+\eta:1,\dots,1], [2\mu:2,\dots,2], \\ [\eta:1,\dots,1], [1+\eta+\mu:2,\dots,2], [\eta:\rho_1,\dots,\rho_m] \end{matrix} ; \begin{matrix} [\gamma_1:1]; \dots; [\gamma_m:1] \\ -; \dots; - \end{matrix}; \begin{matrix} z_1 \\ \vdots \\ z_m \end{matrix}; \begin{matrix} z_1 \\ \vdots \\ z_m \end{matrix} \right], \end{aligned} \quad (13)$$

where $0 < \Re(\mu) < \Re(\eta)$.

Proof. For convenience, let the left-hand side of the assertion (12) be denoted by \mathcal{I} . By applying (7) to the integrand of (12), we obtain

$$\begin{aligned} \mathcal{I} &= \int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} \\ & \cdot \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(\gamma_1)_{k_1} \dots (\gamma_m)_{k_m}}{\Gamma(\eta + \rho_1 k_1 + \dots + \rho_m k_m)} \left(\frac{z_1}{x + b + \sqrt{x^2 + 2bx}} \right)^{k_1} \frac{1}{k_1!} \dots \left(\frac{z_m}{x + b + \sqrt{x^2 + 2bx}} \right)^{k_m} \frac{1}{k_m!} dx \end{aligned}$$

Then, interchanging the order of integration and summation,

$$\mathcal{I} = \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(\gamma_1)_{k_1} \dots (\gamma_m)_{k_m}}{\Gamma(\eta + \rho_1 k_1 + \dots + \rho_m k_m)} \frac{z_1^{k_1}}{k_1!} \dots \frac{z_m^{k_m}}{k_m!} \int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-(\eta+k_1+\dots+k_m)} dx, \quad (14)$$

we can apply the integral formula (6) to the integral in (14) and obtain the following expression:

$$\begin{aligned} \mathcal{I} &= \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(\gamma_1)_{k_1} \dots (\gamma_m)_{k_m}}{\Gamma(\eta + \rho_1 k_1 + \dots + \rho_m k_m)} \\ & 2(\eta + k_1 + \dots + k_m) b^{-(\eta+k_1+\dots+k_m)} \left(\frac{b}{2} \right)^{(\mu)} \frac{\Gamma(2\mu)\Gamma(\eta+k_1+\dots+k_m-\mu)}{\Gamma(1+\eta+\mu+k_1+\dots+k_m)} \frac{(z_1)^{k_1}}{k_1!} \dots \frac{(z_m)^{k_m}}{k_m!}. \end{aligned}$$

Now, arranging the constant term and using $\eta + k_1 + \dots + k_m = \frac{\Gamma(\eta+k_1+\dots+k_m+1)}{\Gamma(\eta+k_1+\dots+k_m)}$, we obtain

$$\begin{aligned} \mathcal{I} &= 2^{1-\mu} b^{\mu-\eta} \Gamma(2\mu) \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(\gamma_1)_{k_1} \dots (\gamma_m)_{k_m}}{\Gamma(\eta + \rho_1 k_1 + \dots + \rho_m k_m)} \frac{\Gamma(\eta+k_1+\dots+k_m+1)}{\Gamma(\eta+k_1+\dots+k_m)} \\ & \frac{\Gamma(\eta+k_1+\dots+k_m-\mu)}{\Gamma(1+\eta+\mu+k_1+\dots+k_m)} \left(\frac{z_1}{b} \right)^{k_1} \frac{1}{k_1!} \dots \left(\frac{z_m}{b} \right)^{k_m} \frac{1}{k_m!}. \end{aligned}$$

The above equation can be multiplied and devided with $\Gamma(\eta+1)$, $(\Gamma(\eta))^2$, $\Gamma(\eta-\mu)$, $\Gamma(1+\eta+\mu)$

Now, using the properties of the Gamma function as $(1 + \eta)_{k_1 + \dots + k_m} = \frac{\Gamma(1 + \eta + k_1 + \dots + k_m)}{\Gamma(1 + \eta)}$, we find that

$$\begin{aligned} \mathcal{I} = & 2^{1-\mu} b^{\mu-\eta} \Gamma(2\mu) \frac{\Gamma(1+\eta)}{\Gamma(\eta)} \frac{1}{\Gamma(\eta)} \frac{\Gamma(\eta-\mu)}{\Gamma(1+\eta+\mu)} \\ & \cdot \sum_{k_1, \dots, k_m=0}^{\infty} \frac{(1+\eta)_{k_1+\dots+k_m} (\eta-\mu)_{k_1+\dots+k_m}}{(\eta)_{k_1+\dots+k_m} (1+\eta+\mu)_{k_1+\dots+k_m}} \\ & \cdot \frac{(\gamma)_{k_1} \cdots (\gamma)_{k_m}}{(\eta)_{\rho_1 k_1 + \dots + \rho_m k_m}} \frac{(z_1/b)^{k_1}}{k_1!} \cdots \frac{(z_m/b)^{k_m}}{k_m!}. \end{aligned} \quad (15)$$

Finally, we interpret the multiple series in (15) as a special case of the general hypergeometric series in several variables defined by (1). Thus, we are led to the assertion (12). The assertion (13) of the Theorem 2.2 can be proved by a similar argument. \square

3. Special Cases

In this section, we derive certain new integral formulas involving Prabhakar-type Mittag-Leffler functions [15] in the integrands of (12) and (13), respectively.

By setting $m = 1$ in (12) and (13) and applying the expression in (1) to the identities, we obtain two integral formulas, as stated in Corollary 1 and 2, respectively.

Corollary 1.

$$\int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} \cdot E_{\rho,\eta}^\gamma \left(\frac{z}{x + b + \sqrt{x^2 + 2bx}} \right) dx = \frac{\Gamma(2\mu)}{\Gamma(\gamma)} 2^{(1-\mu)} a^{(\mu-\eta)} {}_3\psi_3 \left[\begin{matrix} (\gamma, 1), (\eta - \mu, 1), (\eta + 1, 1); \\ (\eta, \rho), (1 + \eta + \mu, 1), (\eta, 1); \end{matrix} z/b \right] \quad (16)$$

with the convergence conditions followed by Theorem 1.

Corollary 2.

$$\int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} \cdot E_{\rho,\eta}^\gamma \left(\frac{xz}{x + b + \sqrt{x^2 + 2bx}} \right) dx = \frac{\Gamma(\eta - \mu)}{\Gamma(\gamma)} 2^{(1-\mu)} b^{(\mu-\eta)} {}_3\psi_3 \left[\begin{matrix} (\gamma, 1), (\gamma + 1, 1), (2\mu, 2); \\ (\eta, \rho), (1 + \eta + \mu, 2), (\eta, 1); \end{matrix} z/2 \right], \quad (17)$$

with the convergence conditions followed by Theorem 2.

It is easily seen that, if we set $\gamma = 1$ in (16) and (17), we obtain new integral formulas, as stated in Corollary 3 and 4, respectively.

Corollary 3.

$$\int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} \cdot E_{\rho,\eta} \left(\frac{z}{x + b + \sqrt{x^2 + 2bx}} \right) dx = 2^{(\mu-1)} b^{(\mu-\eta)} \Gamma(2\mu) {}_2\psi_3 \left[\begin{matrix} (\eta - \mu, 1), (\eta + 1, 1); \\ (\eta, \rho), (1 + \eta + \mu, 1), (\eta, 1); \end{matrix} z/b \right], \quad (18)$$

with the convergence conditions followed by Theorem 1.

Corollary 4.

$$\int_0^\infty x^{\mu-1} \left(x + b + \sqrt{x^2 + 2bx} \right)^{-\eta} \cdot E_{\rho,\eta} \left(\frac{xz}{x + b + \sqrt{x^2 + 2bx}} \right) dx = 2^{(\mu-1)} b^{(\mu-\eta)} \Gamma(\eta - \mu) {}_2\psi_3 \left[\begin{matrix} (\eta+1, 1), (\eta+1, 1); \\ (\eta, \rho), (1+\eta+\mu, 2), (\eta, 1); \end{matrix} z/2 \right], \quad (19)$$

with the convergence conditions followed by Theorem 2.

4. Conclusions

We conclude our investigation by remarking that the results presented here can be easily converted in terms of the known and new integral formulas after small changes in parameters. We are investigating the main results to find potentially useful applications in a variety of areas.

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