



# Article Non-Axisymmetric Bouncing Dynamics on a Moving Superhydrophobic Surface

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**Abstract:** The phenomenon of droplet impact on moving surfaces is widely observed in fields such as transportation, rotating machinery, and inkjet printing. Droplets exhibit non-axisymmetric behavior due to the motion of solid surfaces which significantly determines core parameters such as contact time, maximum spreading radius, and bounding velocity, thereby affecting the efficiency of related applications. In this study, we focus on the kinetics and morphology of the non-axisymmetric bouncing behaviors for droplets impacting on a moving superhydrophobic surface (SHPS) within the normal (We<sub>n</sub>) and tangential (We<sub>t</sub>) Weber numbers. Considering the influences of the moving surface on the contact area and contact time, the previous scaling formula for the horizontal velocity of droplets has been improved. Based on the velocity superposition hypothesis, we establish a theoretical model for the ratio of the maximum spreading radius at both ends depending on We<sub>n</sub> and We<sub>t</sub>. This research provides both experimental and theoretical evidence for understanding and controlling the non-axisymmetric behavior of droplets impacting on moving surfaces.

Keywords: droplet; superhydrophobic substrate; contact time; spreading dynamics

## 1. Introduction

During the process of liquid droplet impact, various phenomena such as stacking, rebounding, and splashing can occur under the influence of the droplet properties and surface characteristics [1–3]. The rebound process refers to the lateral spreading of the droplet, with the conversion of kinetic energy to surface energy, gradually reaching the maximum spreading length  $D_{max}$ , after which the droplet recoils towards the center until it reaches a certain degree of contraction. Subsequently, the droplet rebounds from the surface [4–6]. The phenomenon of droplet impact has been extensively studied both numerically and experimentally [7,8]. The rebound behavior of liquid droplets is influenced by various factors such as droplet properties [9,10], droplet radius and velocity [11,12], surface properties [13,14], and incident angle [15,16].

In previous studies on droplet impact, the predominant focus was on stationary surfaces, and the maximum spreading length of the impacting droplet was dependent on the normal Weber number  $We_n$ , especially in cases involving low viscosity and limited wetting ability [17]. When a droplet impacts on a stationary flat surface, the contact time was found to be contingent upon the radius and independent of the impact velocity [18]. In this context, contact time is defined as the duration from the first contact of the droplet with



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). the surface to its complete detachment. Research into contact time has garnered practical applications in the field of self-cleaning surfaces [19]. Here, the design of macro-textures and microstructures can effectively reduce the contact time [20,21].

However, in practical applications, droplet impact phenomena mostly occur on moving surfaces, such as raindrops impacting aircraft wings and wind turbine blades [22], the size of the droplets influencing the process of extinguishing fires [23], and the directional transport of droplets in inkjet printing [24]. A few studies have found that on the SHPSs, as the speed of the moving surface increases, elevated surface velocities amplify the extension of impacting droplets, consequently leading to a reduction in contact time [25,26]. Similar findings have also been observed on moving surfaces with macroscopic structures and wetting patterns [27].

These conclusions mostly analyze droplets from a kinematic perspective, focusing on droplet spreading and contact time. There are still some conclusions related to momentum. Some studies suggest the main cause of momentum transfer is the aerodynamic Leidenfrost effect [28]. This phenomenon refers to the thin air film formed between the droplet and the moving surface, which is the primary factor generating viscous forces on the droplet. By analyzing the forces acting on the thin air film between droplet and surface, scaling relationships can be derived to summarize the contact time, spreading ratio, and horizontal distance. These dynamic conclusions have been well validated [29]. However, in these experiments, the presence of an air film between the liquid droplet and the surface was not distinctly observed. Droplets will inevitably come into contact with the surface, and the reason for this difference may be due to insufficient surface velocity to generate a noticeable air layer. We tend to the view that the transfer of horizontal momentum is caused by the viscous boundary layer developing in the liquid [30]. Throughout the impact process, energy dissipation due to viscous losses is observed. The fundamental physical principle of non-axisymmetric bouncing dynamics is the transfer of horizontal momentum during the impact process, gradually shifting from the bottom to the top of the droplet. Based on this, a formula related to the horizontal velocity of droplets has been proposed [31]; while the feasibility of this velocity formula has been well validated, there are still some sections for improvement. For example, in the derivation of this formula, the spreading region of the droplet is considered to be circular, which is accurate when impacting a stationary surface but imprecise on a moving surface, where the spreading region is elongated [29,32]. Furthermore, the contact time of the droplet is also assumed to be the same as the contact time on a stationary surface  $\tau_s$ , whereas on a moving surface the contact time  $\tau_m$  is shorter than  $\tau_s$  [25,26].

Moreover, there is still a lack of satisfactory solutions regarding how the spreading range of droplets is influenced by surface motion. In this study, the fundamental dynamic behaviors of droplet impact on moving surfaces are focused on being revealed, and a more accurate correction has been made to the scaling formula for the droplet horizontal velocity, as well as an analysis of the asymmetric evolution of the spreading radius at both ends of the droplet over time caused by surface motion.

## 2. Methodology

We conducted experimental studies on SHPSs with horizontal movement speeds of  $V_t = 0-2.2 \text{ m/s}$  (corresponding to We<sub>t</sub> = 0–116.08) using water droplets with impact velocities of  $V_n = 0.48-1.70 \text{ m/s}$  (corresponding to We<sub>n</sub> = 5.69–71.26), where We<sub>t</sub> =  $\rho V_t^2 D_0 / \gamma$  and We<sub>n</sub> =  $\rho V_n^2 D_0 / \gamma$  are defined as the normal and tangential Weber number, respectively. Water density was  $\rho \approx 1000 \text{ kg/m}^3$ , surface tension coefficient  $\gamma \approx 0.073 \text{ N/m}$ , and  $D_0 = 1.8 \pm 0.1 \text{ mm}$  was the initial diameter of the water droplets. Figure 1a shows the schematic diagram of the experimental apparatus, where the copper plate was polished, ultrasonically cleaned, and coated with an organic reagent three times. After drying at room temperature for 30 min, a solution mainly composed of nano-silica particles and silicone resin was sprayed, and the static contact angle of water droplets on the surface was about 160° (Figure 1a). The surface was fixed to a rotating motor to obtain the horizontal

speed, and the water droplets were pushed by the injection pump (LSP01-3A) and dropped onto the SHPS under the action of gravity after separating from the needle tip. The impact velocity of the droplets was adjusted by varying the distance between the needle tip and the surface. The position of the high-speed camera (GX-8E, NAC), the point of droplet impact, and the center point of the circular superhydrophobic surface are all located on the same straight line, as shown in Figure 1b (top view). For this experiment, the centrifugal force of the droplet on the disk can be neglected [33]; therefore, the motion of the surface during droplet impact can be considered as rectilinear. The environmental temperature was about 25  $^{\circ}$ C, and the humidity was around 30%.



**Figure 1.** (a) Schematic of the experimental setup. (b) View from above showing the position of the camera, droplet impact point, and center of the circular SHPS.

#### 3. Results and Discussion

Figure 2 shows the spreading and receding stages of water droplets on a SHPS at  $We_n = 29.80$ , with  $We_t = 0$  and  $We_t = 44.06$  (multimedia view). After the droplet hits the surface, a liquid film forms as it gradually spreads out towards the maximum spreading radius  $D_{max}$ . The droplet then retracts from both ends towards the center, with its height gradually increasing due to the conversion of surface energy into kinetic energy. The retraction process is dominated by inertia force and surface tension, and it is also influenced by the motion of the surface. Once sufficient kinetic energy has accumulated, the droplet will start to bounce off the surface.



**Figure 2.** (a) Snapshot of droplet impact on the SHPS for  $We_n = 29.80$  and  $We_t = 0$ . (b) Snapshot of droplet impact on the SHPS for  $We_n = 29.80$  and  $We_t = 44.06$ . (Multimedia view).

When the surface is stationary, the receding rates of the droplet from both ends are the same (Figure 2a, t = 3.83 ms), resulting in a symmetric bouncing phenomenon. When the surface is in motion, the receding rate on the upstream side is faster than on the downstream side. (Here, downstream refers to the side in the direction of surface movement and the upstream side refers to the opposite direction, as shown in Figure 2b. We will discuss the upstream and downstream in the subsequent sections.) This is due to the influence of surface velocity, causing more kinetic energy to accumulate on the upstream side resulting in a higher height than the downstream side, leading to an "L"-shaped droplet (Figure 2b,  $t = 5.01 \times 5.83$  ms). As the surface velocity increases, the spreading length of the droplet is stretched longer under the influence of the surface forces, making the "L"-shape more pronounced. Subsequently, the droplet begins to bounce off the surface. Compared to the stationary surface, the contact time is reduced from 8.17 ms to 7.17 ms when the surface is in motion. When the surface is stationary, both ends of the droplet simultaneously leave the surface, while in motion, the downstream side of the droplet will detach from the surface before the upstream side (Figure 2b, t = 4.17 ms). This reduces the portion of the droplet in contact with the surface, leading to a reduction in the contact time [34].

In addition, we conducted statistical analysis of experiments involving different We<sub>t</sub> and We<sub>n</sub> for the contact time. Figure 3a shows the normalized contact time  $\tau/\tau_0$  as a

function of We<sub>t</sub> under different We<sub>n</sub> conditions, where  $\tau_0 \approx \sqrt{\left(\frac{\rho D_0^3}{\sigma}\right)}$  is the inertialcapillary time [35]. As seen, the contact time gradually decreases as We<sub>t</sub> increases. The difference between the contact times at We<sub>t</sub> = 0 and We<sub>t</sub> = 116 is about 30%. These findings are consistent with previous conclusions on moving SHPSs [25,29]. The time from droplet impact surface to maximum spreading is defined as  $\tau_{spread}$ , and the time from maximum spreading to detaching from the surface is defined as  $\tau_{retract}$ . It can be observed that under different We<sub>t</sub>, the  $\tau_{spread}$  remains relatively constant, and the variations in  $\tau_{retract}$  and contact time are similar, as shown in Figure 3b.



**Figure 3.** (a) Normalized contact time  $\tau/\tau_0$  under different We<sub>n</sub> groups varies as the change of We<sub>t</sub>, where  $\tau_0$  is the inertial-capillary time. (b) Spreading time  $\tau_{spread}$  and retraction time  $\tau_{retract}$  vary as the change of We<sub>t</sub>.

#### 3.1. Momentum Transfer of Droplet on the Moving Surface

The bouncing and spreading of droplets are essentially the exchange of kinetic energy and surface energy. When a droplet hits a stationary surface, it bounces vertically along the impact direction, while during surface motion the rebound direction of the droplet tilts towards the direction of surface motion. Research has shown that there are two theories explaining the cause of this phenomenon. The first theory, based on the aerodynamic Leidenfrost effect, suggests it is due to the thin layer of air formed between the droplet and the surface [29], while the second theory proposes it is the result of liquid-surface contact forming a viscous boundary layer [31]. We are inclined towards the latter theory. Furthermore, this study is based on the no-slip boundary condition and derives the scaling relationship for the horizontal speed of droplet.

Accordingly, we calculated the center of mass from the shape of droplet in the side view, taking the moment of contact t = 0 ms as the starting point and the moment of rebound as the end point (Figure 4a). By measuring the displacement of the center of mass and the contact time, the averaged horizontal velocity  $V_a$  of droplet can be obtained.



**Figure 4.** (a) The blue dot represents the centroid of the droplet, from which the droplet velocity  $V_a$  is calculated based on the distance moved in the horizontal direction. (b) Side view of the droplet at different time instances. (c) Top view of the droplet at different time instances, where  $D_{\text{max-m}}$  denote the maximum spreading lengths of the droplet on stationary and moving surfaces, respectively.

Initially,  $V_a = 0$ , but due to the shear effect of the surface the momentum increases during the impact process. Considering the total viscous force and combining Newton's second law in the horizontal direction  $mdV_a/dt = F(t)$ , the average velocity in the horizontal direction can be obtained [31]:

$$V_a = \frac{3\mu V_t}{2\rho D_0^3} \frac{1}{\delta_0} \int_0^t D^2(t) dt$$
 (1)

where  $\mu$  is the viscosity of the liquid,  $\delta_0$  is the thickness of the boundary layer, *t* is the contact time of the droplet on the moving surface, and  $D^2(t)$  represents the contact area of the droplet. In previous studies,  $D^2(t)$  is considered to be the square of the maximum spreading length on a stationary surface  $D_{\text{max}-s}$ . However, the droplet is not circular on a moving surface. It will be stretched along the direction of surface movement, and the maximum spreading of the droplet in that direction is defined as  $D_{\text{max}-m}$ , as shown in Figure 4c. The maximum spreading length in the direction perpendicular to the horizontal is less influenced by surface motion, so it can be approximated as the spreading length  $D_{\text{max}-s}$  on a stationary surface [36]. Based on this, we can deduce:

$$V_a \sim \frac{3\mu V_t}{2\rho D_0^3} \frac{1}{\delta_0} D_{\max-m} D_{\max-s} t \tag{2}$$

With the maximal spreading ratio of the drop  $\beta_{max} = D_{max}/D_0$ , the dimensionless integral mean value of boundary layer thickness  $\delta = \delta_0/D_0$ , the dimensionless contact time  $\tau = V_n t/D_0$ , we can derive that:

$$V_a \sim \frac{3\mu V_t}{2\rho D_0 V_n} \frac{1}{\delta} \beta_{\max-m} \beta_{\max-s} \tau \tag{3}$$

On the moving surface, the maximum spreading ratio  $\beta_{\text{max-m}} \sim \text{We}_n^{1/4} \text{Ca}^{1/6}$  [29], where the capillary number Ca =  $\mu V_t / \gamma$ . (Despite this equation being based on the aerodynamic Leidenfrost effect, our experimental results have shown good agreement with it; see the comparative results in Supplementary Material Figure S1). On the stationary surface, the maximum spreading ratio  $\beta_{\text{max-s}} \sim \text{We}_n^{1/4} (1 - \cos\theta)^{-1/2}$  [37].

Another aspect that distinguishes our work from previous research is that they considered the contact time  $\tau$  in Equation (3) as the contact time on a stationary surface  $\tau_s$ , while we consider  $\tau$  as the contact time on a moving surface  $\tau_m$ . In Figure 3a, we can clearly observe a significant difference in contact time between  $\tau_s$  and  $\tau_m$ . Based on volume conservation, the relationship between these two is given by [25]:

$$\frac{\tau_m}{\tau_s} = \sqrt{\frac{D_{\max-s}}{D_{\max-m}}} \tag{4}$$

The validation of Equation (3) can be observed in Supplementary Material Figure S2. The contact time on the stationary surface  $\tau_s \sim We_n^{1/2}(1 - \cos\theta)^{-1/2}$ . By combining Equations (3) and (4) with the above scaling relationships, we can deduce:

$$V_a \sim \frac{3}{2\delta} (1 - \cos\theta)^{-\frac{5}{4}} \operatorname{Re}^{-1} \operatorname{We}_n \operatorname{Ca}^{\frac{1}{12}} V_t$$
(5)

where the dimensionless Reynolds number is defined as  $\text{Re} = \rho V_n D_0 / \mu$ . Equation (5) is derived using the contact time  $\tau_m$  and maximum spreading ratio  $\beta_{\text{max-m}}$  on a moving surface. If we were to derive it using the contact time  $\tau_s$  and maximum spreading ratio  $\beta_{\text{max-s}}$  on a stationary surface [31], we could derive:

$$V_a \sim \frac{3}{2\delta} (1 - \cos\theta)^{-\frac{3}{2}} \operatorname{Re}^{-1} \operatorname{We}_n V_t$$
(6)

When comparing Equations (5) and (6), the difference is the additional influence of the capillary number Ca in Equation (5). This inclusion of Ca enhances the influence of surface motion on the droplet velocity, making it a more accurate representation. The boundary layer thickness can be regarded as a constant value. Therefore, Equation (5) can be expressed as:

$$V_a \sim (1 - \cos\theta)^{-\frac{3}{4}} \text{Re}^{-1} \text{We}_n \text{Ca}^{\frac{1}{12}} V_t$$
 (7)

In Figure 5, we can observe a good consistency between the experimental data and Equation (7).



**Figure 5.** Variation of  $V_a$  as a function of Re, We<sub>n</sub>, Ca, and  $V_t$ , with the solid line representing the best fit of Equation (7).

As previously mentioned, the conversion of momentum during droplet impact involves the consumption of energy. This implies that the averaged horizontal velocity of the droplet  $V_a$  should be lower than the surface velocity  $V_t$ . We found that the velocity of the droplet is only about 1% of the surface movement velocity. As a result, we defined the averaged restitution coefficient  $\varepsilon_t = V_a/V_t$ . Previous research has demonstrated a negative correlation between We<sub>n</sub> and  $\varepsilon_t$  because higher We<sub>n</sub> impacts can cause more significant droplet deformation, leading to more energy dissipation during rebound and a decrease in  $\varepsilon_t$  [38–42]. However, our study on the moving surface revealed an opposite result; we found that  $\varepsilon_t$  increases with the increase in We<sub>n</sub>, as shown in Figure 6a, and there is no significant change with the increase in  $We_t$ , as shown in Figure 6b. The black line in the box represents the median line, and the point in the middle is the mean value. The upper and lower boundaries of the box represent the upper interquartile and lower interquartile, respectively, while the whiskers represent the extreme values. The reasons for this different trend are as follows. Firstly, previous studies focused on instantaneous velocity, while this study specifically examines the average velocity throughout the entire impact process. Secondly, a significant factor is that as the droplet impacts a moving surface, with an increase in We<sub>*n*</sub>, the spreading length increases by approximately 20%. This results in the viscous boundary layer length between the droplet and the surface also increasing, leading



to an increase in the force exerted on the droplet in the horizontal direction. Ultimately, this is manifested as an increase in the coefficient of restitution.

**Figure 6.** (a) The boxplot of tangential recovery coefficient  $\varepsilon_t$  with respect to We<sub>n</sub> whose range corresponding to each We<sub>t</sub> is 5.2–116.1. (b) The boxplot of tangential recovery coefficient  $\varepsilon_t$  with respect to We<sub>t</sub> whose range corresponding to each We<sub>n</sub> is 5.7–60.9.

### 3.2. Morphology of Droplet on Moving Surface

We conducted an analysis of changes in droplet behavior from a momentum perspective. In addition, the most significant difference between the stationary and moving surfaces is not only the contact time, but also the asymmetric spreading and receding of the droplet on the upstream and downstream sides.

Therefore, we divided the droplet into two portions  $D_{up}$  and  $D_{down}$  based on the impact center point. Initially, the positions of the center points  $D_s = 0$  is represented by the red dot (Figure 7a). It varies at different moments; it moves with the horizontal velocity  $V_a$  of the droplet, as indicated by the green point in Figure 7a. The distance from the point along the moving direction to the edge of the droplet is referred to as  $D_{uv}$ , and the distance from the point along the opposite direction of motion to the edge of the droplet is referred to as  $D_{\text{down}}$ . When We<sub>t</sub> = 0, the spreading length of the droplet on the upstream and downstream sides is symmetrically distributed along  $D_s = 0$ . However, as We<sub>t</sub> increases, the spreading length of the droplet on the upstream side gradually increases, and its spreading time becomes longer, as shown in Figure 8a,b The negative values in Figure 8a indicate that the droplet is located to the right of the boundary point. In addition, the time of initiating receding on the upstream side is later than that on the downstream side; the spreading time of the downstream side is around 2 ms. For different Wet values, there is almost no effect on the length of spreading. By contrast, the length during the receding phase gradually reduces with the increase in We<sub>t</sub>. In Figure 8c,d, with a fixed We<sub>t</sub> = 44.06 and different  $We_n$ , the spreading length on both the upstream and downstream sides increases with  $We_n$ , with the increment being more significant on the downstream side.  $D_{up}$  during both the retraction and spreading phases is increasing with  $We_{\mu}$  but the increment during the retraction phase is relatively small.



**Figure 7.** (a) Schematic diagram distinguishing the upstream and downstream spreading of the droplet. The red dot represents the initial center of impact of the droplet, while the green dot indicates the position after moving with velocity  $V_a$ . The green dot serves as the reference for defining the boundary between the upstream and downstream regions, where  $D_s = 0$ . (b) Side view of the droplet at different time instances. (c) Schematic representation of the velocity superposition of  $V_s$  and  $V_t$ .

Thus, we postulate that when the droplet reaches its maximum spreading length the length of the upstream and downstream portions should be a function of the impact velocity  $V_n$  and surface velocity  $V_t$ . Assuming that the averaged spreading velocity is  $V_s$  and the surface velocity is  $V_t$ , the averaged velocity of the upstream can be treated as  $V_{up} = V_s - V_t$ ; similarly, the averaged velocity of the downstream is  $V_{down} = V_s + V_t$ , as shown in Figure 7c. Taking into account that the droplet will also move a certain distance caused by the moving substrate, the maximal spread length ratio of the upstream to downstream can be expressed as:

$$\frac{D_{\rm up}}{D_{\rm down}} \sim \frac{(V_s - V_t)\tau_{\rm spread} - V_a\tau_{\rm spread}}{(V_s + V_t)\tau_{\rm spread} + V_a\tau_{\rm spread}}$$
(8)

If the droplet's upstream and downstream are distinguished based on the calculated moving distance of the green dot, then the term  $V_a \tau_s$  representing the sliding distance of the droplet in Equation (7) can be eliminated, and the formula can be simplified as:

$$\frac{D_{\rm up}}{D_{\rm down}} \sim \frac{V_s - V_t}{V_s + V_t} \tag{9}$$



**Figure 8.** (a) Variation of upstream and downstream spread lengths of the droplet with time for different values of We<sub>t</sub> at We<sub>n</sub> = 49.8, with the center of the droplet at  $D_s = 0$  at the beginning of impact used to distinguish between the upstream and downstream regions. (b) Actual deformation of the droplet corresponding to (a). (c) Variation of upstream and downstream spread lengths of the droplet with time for different values of We<sub>n</sub> at We<sub>t</sub> = 44.06. (d) Actual deformation of the droplet corresponding to (c).

Obviously,  $V_t \sim \alpha We_t^{1/2}$ , and for  $V_s$ , it is a function of the maximum spreading length and spreading time,  $V_s \sim D_{max}/\tau_{spread}$ . As can be observed in Figure 3b, the spreading time  $\tau_{spread}$  is independent of surface velocity and impact velocity, which is consistent with previous research findings [3,29]. Therefore, we can treat  $\tau_{spread}$  as a constant, while  $D_{max} \sim D_0 We_n^{1/4}$ ; based on these, we can derive the expression for  $V_s \sim \beta We_n^{1/4}$ . Substituting these values into Equation (8) we have:

$$\frac{D_{\rm up}}{D_{\rm down}} \sim \frac{\alpha \mathrm{We}_n^{1/4} - \beta \mathrm{We}_t^{1/2}}{\alpha \mathrm{We}_n^{1/4} + \beta \mathrm{We}_t^{1/2}}$$
(10)

where coefficients  $\alpha$  and  $\beta$  are related constants. Equation (10) can be written as Equation (11):

$$\frac{D_{\rm up}}{D_{\rm down}} \sim 1 - \frac{2W e_t^{1/2}}{kW e_u^{1/4} - W e_t^{1/2}}$$
(11)

where coefficient  $k = \alpha/\beta$ . In Figure 9, we can observe a good consistency between the experimental data and Equation (11). This scaling relationship has practical applications such as using water to cool rotating machinery. Understanding the spreading range of droplet holds significance for such applications [43].



**Figure 9.** We<sub>t</sub> and We<sub>n</sub> as functions of the normalized ratio of upstream and downstream spread lengths, the solid line representing the best fit of Equation (11).

## 4. Conclusions

In this study, experiments have shown that the contact time between water droplets and surfaces is inversely proportional to surface velocity. Our main conclusions are as follows. 1. We have refined the scaling relationship for the horizontal velocity  $V_a$  of the droplet, substituting the contact time and spreading length of the stationary surface in the scaling relationship with those on a moving surface. These modifications enhance the accuracy of the results. These corrections make the scaling relationship more accurate. 2. Furthermore, an investigation into the variation relationship between the droplet's tangential restitution coefficient  $\varepsilon_t$  and Weber numbers  $We_n$  and  $We_t$  was conducted. Additionally, we were surprised to discover that the velocity of the droplets was only about 1% of the SHPSs speed. We will focus on exploring ways to improve the exchange rate of droplet momentum in future work. 3. Based on the asymmetry of droplet spreading, a coupling scale relationship between the maximum spreading length ratio of water droplets upstream and downstream was revealed by superimposing the spreading velocity and the surface velocity. This has provided us with new insights into the energy transfer mechanisms of small droplets. **Supplementary Materials:** The following supporting information can be downloaded at: https: //www.mdpi.com/article/10.3390/sym16010029/s1, Figure S1: The validation of the maximum spreading ratio  $\beta_{max-m}$  on a moving surface; Figure S2: The validation of the contact time  $\tau_m$  on a moving surface.

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