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Editorial

New Trends on the Mathematical Models and Solitons Arising in Real-World Problems

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The essence of mathematical tools used to exemplify the practical problems that exist in daily life is as old as the world itself. Mathematical models in science and technology have recently attracted an increased amount of research attention with the aim to understand, describe, and predict the future behaviors of natural phenomena. Recent studies on fractional calculus have been particularly popular among researchers due to their favorable properties when analyzing real-world models associated with properties such as anomalous diffusion, non-Markovian processes, random walk, long range, and, most importantly, heterogeneous behaviors [1–17]. The concept of local differential operators along with power law settings and non-local differential operators were suggested in order to accurately replicate the above-cited natural processes. The complexities of nature have led mathematicians and physicists to derive the most sophisticated and scientific–mathematical operators to accurately replicate and capture pragmatic realities [18–25].

In response to the call for papers, 49 submissions were received. All submissions were reviewed by at least three experts in the field. Finally, 15 papers were accepted for publication in this Special Issue, all of which are of high quality and well representative of the areas covered by this Special Issue. This corresponds to an acceptance rate of 30%.

The published papers in this Special Issue are herein briefly studied as follows:

In [26], the authors developed a functional integration matrix via the Hermite wavelets and proposed a novel technique called the Hermite wavelet collocation method (HWM). Here, they studied two models. The coupled system of an ordinary differential equation (ODE) was modeled on the digestive system by considering different parameters such as sleep factors, tension, food rates, death rates, and medicine. Here, they discussed how these parameters influence the digestive system and showed the results through figures and tables. Another fractional model was used to study the COVID-19 pandemic. This model is defined by a system of fractional ODEs including five variables, labeled S (susceptible), E (exposed), I (infected), Q (quarantined), and R (recovered). The proposed wavelet technique investigates these two models. Here, they expressed the modeled equation in terms of the Hermite wavelets along with the collocation scheme. Then, using the properties of the wavelets, the modeled equation was converted into a system of algebraic equations. They used the Newton–Raphson method to solve these nonlinear algebraic equations. The obtained results were compared with numerical solutions and the Runge–Kutta method (R–K method), expressed through tables and graphs. The HWM's computational time (consumes less time) is better than that of the R–K method.

In [27], the Sumudu decomposition method (SDM) was used as a way to find approximate solutions to two-dimensional fractional partial differential equations. A numerical algorithm used for solving the fractional Riccati equation was investigated. The authors formed a combination of the Sumudu transform method and decomposition method. The fractional derivative was described in the Caputo sense.

In [28], it was found that the accuracy of control systems applied to motors is influenced by uncertainties and abrupt variations in the load and system parameters. Some

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robust control strategies for responding to disturbances and uncertainties, parametric variations, and nonlinearities were proposed in the literature, adding complex control rules and considerable computational efforts. Therefore, this paper presented the application of a sliding mode control method based on a washout filter (SMC-w) for speed control in a permanent magnet DC motor. In addition, the dynamic behavior of the SMC-w was evaluated under changes in the reference speed and load torque. The responses of the control system under variations in the speed reference signal and load torque were studied. The results were contrasted with conventional proportional integral derivative (PID) control to evaluate the efficiency and improvement of the SMC-w. The qualitative shape of the transient response of the speed and the current concerning changes in the reference speed were symmetric for the SMC-w controller, but the overshoot, settling time, and steady-state error values were different. This technique has great potential for industrial application as it can be controlled efficiently with a low computational cost and a simple design, which benefits its implementation in practical environments.

In [29], the authors state that the existence of humans is dependent on nature, and that this existence can be disturbed by either human-made devastations or by natural disasters. As a universal phenomenon in nature, symmetry has attracted scholars' attention. The study of symmetry provides insights into physics, chemistry, biology, and mathematics. One of the most important characteristics in the expressive assessment and development of computational design techniques is symmetry. Yet, the use of mathematical models is an important method of studying real-world systems. The symmetry reflected by a mathematical model reveals the inherent symmetry of real-world systems. This study focused on the contagious model of pine wilt disease and symmetry, employing the q-HATM (q-Homotopy Analysis Transform Method) to the leading fractional operator, Atangana–Baleanu (AB), to arrive at a better understanding. The outgrowths are exhibited in figures and tables. Finally, the paper helps to analyze the practical theory, assisting the prediction of its manner that corresponds to the guidelines when contemplating the replica.

In [30], the novel auxiliary equation methodology (NAEM) was employed to scrutinize various forms of solitary wave solutions for the modified equal-width wave (MEW) equation. M-truncated derivatives along with Atangana–Baleanu (AB)-fractional derivatives were employed to study the soliton solutions of the problem. The fractional MEW equations are important for describing hydro-magnetic waves in cold plasma. A comparative analysis was utilized to study the influence of the fractional parameter on the generated solutions. The secured solutions include bright, dark, singular, periodic, and many other types of soliton solutions. Compared to other methods, the solutions demonstrate that the proposed technique is particularly effective, straightforward, and trustworthy as it contains families of solutions. In addition, the symbolic soft computation was used to verify the obtained solutions. Finally, the system was subjected to a sensitive analysis. The integer-order results calculated using the symmetry method and presented in the literature can be addressed as limiting cases of the present study.

In [31], a mathematical model for Streptococcus suis infection was improved by using the fractional order derivative. The modified model also investigated the transmission between pigs and humans. The proposed model can classify the pig population density into four classes, including the pig susceptible class, pig infectious class, pig quarantine class, and pig recovery class. Moreover, the human population density was separated into three classes, including the human susceptible class, human infectious class, and human recovery class. The spread of infection was analyzed by considering the contact between humans and pigs. The calculation of the basic reproduction number (R0), the infectious indicator, was carried out using the next-generation matrix. The disease-free equilibrium is locally asymptotically stable if $R0 < 1$, and the endemic equilibrium is locally asymptotically stable if $R0 > 1$. The theoretical analyses of the fractional order derivative model, existence, and uniqueness, were proposed. The numerical examples were illustrated to support the proposed stability theorems. The results show that the fractional order derivative model provides the various possible solution trajectories with different fractional orders for the same parameters. In addition, transmission between pigs and humans resulted in the spread of streptococcus suis infection.

In [32], the author's aim was to achieve new soliton solutions of the Gilson–Pickering equation (GPE) with the assistance of Sardar's subequation method (SSM) and the Jacobi elliptic function method (JEFM). The applications of the GPE are wider because we studied some valuable and vital equations such as the Fornberg–Whitham equation (FWE), Rosenau–Hyman equation (RHE), and Fuchssteiner–Fokas–Camassa–Holm equation (FFCHE), which were obtained by making particular choices of the parameters involved in the GPE. Many techniques are available to convert PDEs into ODEs to extract wave solutions. Most of these techniques are a case of symmetry reduction, known as nonclassical symmetry. In our work, this approach is used to convert a PDE to an ODE and obtain the exact solutions of the NLPDE. The obtained solutions are unique, remarkable, and significant for readers. The Mathematica 11 software was used to derive the solutions of the presented model. Moreover, the diagrams of the acquired solutions used to obtain distinct values of the parameters were demonstrated in two and three dimensions along with contour plots.

In [33], the authors analyzed the dynamical behavior and chaos control of an atmospheric circulation model, known as the Hadley circulation model, in the frame of Caputo and Caputo–Fabrizio fractional derivatives. The fundamental novelty of this paper is the application of the Caputo derivative with equal dimensionality to models that include memory. A sliding mode controller (SMC) was developed to control chaos in this fractional-order atmospheric circulation system with uncertain dynamics. The proposed controller was applied to both commensurate and non-commensurate fractional-order systems. To demonstrate the intricacy of the models, the authors plotted some graphs of various fractional orders with appropriate parameter values. They observed the influence of the thermal force on the dynamics of the system. The outcome of the analytical exercises was validated using numerical simulations.

In [34], several types of solitary wave solutions of (3+1)-dimensional nonlinear extended and modified quantum Zakharov–Kuznetsov equations were established successfully via the implantation of three mathematical methods. The concerned models have many fruitful applications to describe the waves in quantum electron–positron–ion magnetoplasmas and the weakly nonlinear ion acoustic waves in plasma. The results derived via the MEAEM method, ESE method, and modified F expansion were retrieved and shown to be expedient to illuminate the collaboration between lower nonlinear ion acoustic waves in the future. To analyze the physical behaviors of the models, some solutions were plotted graphically in 2D and 3D by imparting particular values to the parameters under the given condition at each solution. Hence, the explored solutions have profitable rewards in the field of mathematical physics.

In [35], the authors considered time-fractional Emden–Fowler-type equations and solved them using the rational homotopy perturbation method (RHPM). The RHPM is based on two power series in rational form. The existence and uniqueness of the equation were proven using the Banach fixed-point theorem. Furthermore, the authors approximated the term h(z) with a polynomial of a suitable degree and then solved the system using the proposed method, obtaining an approximate symmetric solution. Two numerical examples were investigated using this proposed approach. The effectiveness of the proposed approach was checked by comparing the graphs of the exact and approximate solutions. The table of absolute error was also presented to understand the method's accuracy.

In [36], the fractional–space stochastic (2+1)-dimensional breaking soliton equation (SFSBSE) was taken into account in the sense of the M-Truncated derivative. To obtain the exact solutions to the SFSBSE, the authors used the modified F-expansion method. There are several varieties of the obtained exact solutions, including trigonometric and hyperbolic functions. The attained solutions of the SFSBSE established in this paper extend a number of previously attained results. Moreover, in order to clarify the influences of multiplicative noise and the M-Truncated derivative on the behavior and symmetry of the solutions for the SFSBSE, the authors employed Matlab to plot three-dimensional and two-dimensional diagrams of the exact fractional–stochastic solutions. In general, a noise term that destroys the symmetry of the solutions increases the solutions' stability.

In [37], the authors marked that the half-logistic modified Kies exponential (HLMKEx) distribution is a novel three-parameter model that is introduced in the current work to expand the modified Kies exponential distribution and improve its flexibility in modeling real-world data. Due to its versatility, the density function of the HLMKEx distribution offers symmetrical, asymmetrical, unimodal, and reverse J-shaped forms, as well as increasing reverse J-shaped and upside-down hazard rate forms. An infinite linear representation can be used to represent the HLMKEx density. The HLMKEx model's fundamental mathematical features were obtained, such as the quantile function, moments, incomplete moments, and moments of residuals. Additionally, some measures of uncertainty as well as stochastic ordering were derived. To estimate its parameters, eight estimation methods were used. With the use of detailed simulation data, they compared the performance of each estimating technique and obtained partial and total ranks for the accuracy measures of absolute bias, mean squared error, and mean absolute relative error. The simulation results demonstrate that, in contrast to other competing distributions, the proposed distribution can fit the data more accurately. Two actual data sets were investigated in the field of engineering to demonstrate the adaptability and application of the suggested distribution.

In [38], the authors obtained sufficient conditions to prove the existence and uniqueness of solutions (EUS) for FDEs in the sense of the ψ-Caputo fractional derivative (ψ-CFD) in the second-order $1 < \alpha < 2$. They know that ψ-CFD is a generalization of previously familiar fractional derivatives: Riemann–Liouville and Caputo. By applying the Banach fixed-point theorem (BFPT) and the Schauder fixed-point theorem (SFPT), they obtained the desired results, and to embody the theoretical results, they provided two examples that illustrate the theoretical proof.

In [39], the susceptible–infected–quarantined–recovered–vaccinated (SIQRV) epidemic model was used. The symmetrical aspects of the proposed dynamic model, disease-free equilibrium, and stability were analyzed. The symmetry of the population size over time allows the model to find stable equilibrium points for any parameter value and initial conditions. The assumption that the initial conditions and parameter values demonstrate strong symmetry plays a key role in the analysis of the fractional SIQRV model. In order to combat the pandemic nature of the disease, control the disease in the population, and increase the possibility of eradicating the disease, effective control measures including quarantine and immunization need to be implemented. Fractional derivatives were used in the Caputo sense. In the model, vaccination and quarantine are two important applications to manage the spread of the pandemic. Although some of the individuals who were vaccinated with the same type and dose of vaccine gained strong immunity, the vaccine could not give sufficient immunity to the other part of the population. This is thought to be related to the structural characteristics of individuals. Thus, although some of the individuals who were vaccinated with the same strategy will be protected against the virus for a long time, others may become infected soon after vaccination. Appropriate parameters were used in the model to reflect this situation. In order to validate the model, the model was run by using the official COVID-19 data from Türkiye from about a year ago which was the time that this study was conducted. In addition to the stability analysis of the model, numerical solutions were obtained using the fractional Euler method.

In [40], the authors investigated the existence and uniqueness of implicit solutions in a coupled symmetry system of hybrid fractional order differential equations, along with hybrid integral boundary conditions in Banach algebra. The methodology centers on a hybrid fixed-point theorem that involves mixed Lipschitz and Carathéodory conditions, serving to establish the existence of solutions. Moreover, it derives sufficient conditions for solution uniqueness and establishes the Hyers–Ulam types of solution stability. This study contributes valuable insights into complex hybrid fractional order systems and their practical implications.

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References

1. Tartaglione, V.; Farges, C.; Sabatier, J. Fractional Behaviours Modelling with Volterra Equations: Application to a Lithium-Ion Cell and Comparison with a Fractional Model. *Fractal Fract.* **2022**, *6*, 137. [CrossRef]
2. Ortigueira, M.D.; Magin, R.L. On the Equivalence between Integer- and Fractional Order-Models of Continuous-Time and Discrete-Time ARMA Systems. *Fractal Fract.* **2022**, *6*, 242. [CrossRef]
3. Ata, E.; Kıymaz, I.O. New generalized Mellin transform and applications to partial and fractional differential equations. *Int. J. Math. Comput. Eng.* **2023**, *1*, 45–66. [CrossRef]
4. Brandibur, O.; Kaslik, E. Stability Analysis for a Fractional-Order Coupled FitzHugh–Nagumo-Type Neuronal Model. *Fractal Fract.* **2022**, *6*, 257. [CrossRef]
5. Jafari, H.; Goswami, P.; Dubey, R.S.; Sharma, S.; Chaudhary, A. Fractional SIZR model of Zombie infection. *Int. J. Math. Comp. Eng.* **2023**, *1*, 91–104. [CrossRef]
6. Ciancio, A.; Ciancio, V.; D'onofrio, A.; Flora, B.F.F. A Fractional Model of Complex Permittivity of Conductor Media with Relaxation: Theory vs. Experiments. *Fractal Fract.* **2022**, *6*, 390. [CrossRef]
7. Karner, T.; Belšak, R.; Gotlih, J. Using a Fully Fractional Generalised Maxwell Model for Describing the Time Dependent Sinusoidal Creep of a Dielectric Elastomer Actuator. *Fractal Fract.* **2022**, *6*, 720. [CrossRef]
8. Dipesh; Kumar, P.; Cattani, C. Optimizing industrial growth through alternative forest biomass resources: A mathematical model using DDE. *Int. J. Math. Comput. Eng.* **2023**, *1*, 187–200. [CrossRef]
9. Rosa, S.; Torres, D.F.M. Fractional Modelling and Optimal Control of COVID-19 Transmission in Portugal. *Axioms* **2022**, *11*, 170. [CrossRef]
10. Messina, E.; Pezzella, M.; Vecchio, A. Positive Numerical Approximation of Integro-Differential Epidemic Model. *Axioms* **2022**, *11*, 69. [CrossRef]
11. Singh, R.; Mishra, J.; Gupta, V.K. Dynamical analysis of a Tumor Growth model under the effect of fractal fractional Caputo-Fabrizio derivative. *Int. J. Math. Comp. Eng.* **2023**, *1*, 115–126. [CrossRef]
12. Bhatter, S.; Kumawat, S.; Jangid, K.; Purohit, S.D.; Baskonus, H.M. Fractional Differential Equations Related to an Integral Operator Involving the Incomplete I-function as a Kernel. *Math. Methods Appl. Sci.* **2023**, *46*, 15033–15047. [CrossRef]
13. Wang, Y.; Gao, W.; Baskonus, H.M. An efficient computational approach for fractional order model describing the water transport in unsaturated porous media. *Mod. Phys. Lett. B* **2023**, *37*, 2350059. [CrossRef]
14. Chen, Q.; Sabir, Z.; Raja, M.A.Z.; Gao, W.; Baskonus, H.M. A fractional study based on the economic and environmental mathematical model. *Alex. Eng. J.* **2023**, *65*, 761–770. [CrossRef]
15. Hezenci, F.; Budak, H. Certain Simpson-type inequalities for twice-differentiable functions by conformable fractional integrals. *Korean J. Math.* **2023**, *31*, 217–228.
16. Bhatter, S.; Purohit, S.D.; Nisar, K.S.; Munjam, S.R. Some fractional calculus findings associated with the product of incomplete ℵ-function and Srivastava polynomials. *Int. J. Math. Comp. Eng.* **2024**, *2*, 97–116.
17. Morín-Castillo, M.M.; Arriaga-Hernández, J.; Cuevas-Otahola, B.; Oliveros-Oliveros, J.J. Analysis of Dipolar Sources in the Solution of the Electroencephalographic Inverse Problem. *Mathematics* **2022**, *10*, 1926. [CrossRef]
18. Srinivasa, K.; Mundewadi, R.A. Wavelets approach for the solution of nonlinear variable delay differential equations. *Int. J. Math. Comp. Eng.* **2023**, *1*, 139–148. [CrossRef]
19. Kavya, K.; Veeresha, P.; Baskonus, H.M.; Alsulami, M. Mathematical modeling to investigate the influence of vaccination and booster doses on the spread of Omicron. *Commun. Nonlinear Sci. Numer. Simul.* **2023**, *130*, 107755. [CrossRef]
20. Gasmi, B.; Moussa, A.; Mati, Y.; Alhakim, L.; Baskonus, H.M. Bifurcation and exact traveling wave solutions to a conformable nonlinear Schrödinger equation using a generalized double auxiliary equation method. *Opt. Quantum Electron.* **2024**, *56*, 18. [CrossRef]
21. Raihen, M.N.; Akter, S. Prediction modeling using deep learning for the classification of grape-type dried fruits. *Int. J. Math. Comput. Eng.* **2024**, *2*, 1–12. [CrossRef]
22. Rodrigo, M.; Zulkarnaen, D. Mathematical Models for Population Growth with Variable Carrying Capacity: Analytical Solutions. *AppliedMath* **2022**, *2*, 466–479. [CrossRef]
23. Sivasundaram, S.; Kumar, A.; Singh, R.K. On the complex properties to the first equation of the Kadomtsev-Petviashvili hierarchy. *Int. J. Math. Comp. Eng.* **2024**, *2*, 71–84. [CrossRef]
24. Gao, W.; Baskonus, H.M. The modulation instability analysis and analytical solutions of the nonlinear Gross-Pitaevskii model with conformable operator and Riemann wave equations via recently developed schemes. *Adv. Math. Phys.* **2023**, *2023*, 4132763. [CrossRef]

25. Guirao, J.L.G. On the stochastic observation for the nonlinear system of the emigration and migration effects via artificial neural networks. *Int. J. Math. Comp. Eng.* **2023**, *1*, 177–186. [CrossRef]
26. Srinivasa, K.; Baskonus, H.M.; Sánchez, Y.G. Numerical Solutions of the Mathematical Models on the Digestive System and COVID-19 Pandemic by Hermite Wavelet Technique. *Symmetry* **2021**, *13*, 2428. [CrossRef]
27. Trujillo, S.C.; Candelo-Becerra, J.E.; Hoyos, F.E. Numerical Validation of a Boost Converter Controlled by a Quasi-Sliding Mode Control Technique with Bifurcation Diagrams. *Symmetry* **2022**, *14*, 694. [CrossRef]
28. Velasco-Muñoz, H.; Candelo-Becerra, J.E.; Hoyos, F.E.; Rincón, A. Speed Regulation of a Permanent Magnet DC Motor with Sliding Mode Control Based on Washout Filter. *Symmetry* **2022**, *14*, 728. [CrossRef]
29. Padmavathi, V.; Magesh, N.; Alagesan, K.; Khan, M.I.; Elattar, S.; Alwetaishi, M.; Galal, A.M. Numerical Modeling and Symmetry Analysis of a Pine Wilt Disease Model Using the Mittag–Leffler Kernel. *Symmetry* **2022**, *14*, 1067. [CrossRef]
30. Riaz, M.B.; Wojciechowski, A.; Oros, G.I.; Rahman, R.U. Soliton Solutions and Sensitive Analysis of Modified Equal-Width Equation Using Fractional Operators. *Symmetry* **2022**, *14*, 1731. [CrossRef]
31. Prathumwan, D.; Chaiya, I.; Trachoo, K. Study of Transmission Dynamics of *Streptococcus suis* Infection Mathematical Model between Pig and Human under ABC Fractional Order Derivative. *Symmetry* **2022**, *14*, 2112. [CrossRef]
32. Allahyani, S.A.; Rehman, H.U.; Awan, A.U.; Tag-ElDin, E.M.; Hassan, M.U. Diverse Variety of Exact Solutions for Nonlinear Gilson–Pickering Equation. *Symmetry* **2022**, *14*, 2151. [CrossRef]
33. Premakumari, R.N.; Baishya, C.; Veeresha, P.; Akinyemi, L. A Fractional Atmospheric Circulation System under the Influence of a Sliding Mode Controller. *Symmetry* **2022**, *14*, 2618. [CrossRef]
34. Areshi, M.; Seadawy, A.R.; Ali, A.; AlJohani, A.F.; Alharbi, W.; Alharbi, A.F. Construction of Solitary Wave Solutions to the (3 + 1)-Dimensional Nonlinear Extended and Modified Quantum Zakharov–Kuznetsov Equations Arising in Quantum Plasma Physics. *Symmetry* **2023**, *15*, 248. [CrossRef]
35. Albalawi, K.S.; Alkahtani, B.S.; Kumar, A.; Goswami, P. Numerical Solution of Time-Fractional Emden–Fowler-Type Equations Using the Rational Homotopy Perturbation Method. *Symmetry* **2023**, *15*, 258. [CrossRef]
36. Mohammed, W.W.; El-Morshedy, M.; Moumen, A.; Ali, E.E.; Benaissa, M.; Abouelregal, A.E. Effects of M-Truncated Derivative and Multiplicative Noise on the Exact Solutions of the Breaking Soliton Equation. *Symmetry* **2023**, *15*, 288. [CrossRef]
37. Alghamdi, S.M.; Shrahili, M.; Hassan, A.S.; Gemeay, A.M.; Elbatal, I.; Elgarhy, M. Statistical Inference of the Half Logistic Modified Kies Exponential Model with Modeling to Engineering Data. *Symmetry* **2023**, *15*, 586. [CrossRef]
38. Tayeb, M.; Boulares, H.; Moumen, A.; Imsatfia, M. Processing Fractional Differential Equations Using ψ-Caputo Derivative. *Symmetry* **2023**, *15*, 955. [CrossRef]
39. Öztürk, Z.; Bilgil, H.; Sorgun, S. Application of Fractional SIQRV Model for SARS-CoV-2 and Stability Analysis. *Symmetry* **2023**, *15*, 1048. [CrossRef]
40. Awad, Y.; Alkhezi, Y. Analysis of Implicit Solutions for a Coupled System of Hybrid Fractional Order Differential Equations with Hybrid Integral Boundary Conditions in Banach Algebras. *Symmetry* **2023**, *15*, 1758. [CrossRef]

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