

The Origin of the Difference between Space and Time

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Abstract: All differences between the role of space and time in nature are explained by proposing principles in which none of the spacetime coordinates has an a priori special role. Spacetime is treated as a non-dynamical manifold, with a fixed global $\mathbb{R}^D$ topology. The dynamical theory of gravity determines only the metric tensor on a fixed manifold. All dynamics is treated as a Cauchy problem, so it follows that one coordinate takes a special role. It is proposed that *any* boundary condition that is finite everywhere leads to a solution which is also finite everywhere. This explains the $(1, D - 1)$ signature of the metric, the boundedness of energy from below, the absence of tachyons, and other related properties of nature. The time arrow is explained by proposing that the boundary condition should be ordered. The quantization is considered as a boundary condition for field operators. Only the physical degrees of freedom are quantized.

Keywords: space; time; spacetime

1. Introduction

One of the most fundamental principles of modern theoretical physics is the principle of Lorentz covariance. This principle essentially says that all fundamental physical theories should treat space and time coordinates in the same way, up to a negative relative sign in the metric of spacetime. However, it is known that space and time coordinates are not really treated in the same way, and that these different treatments cannot be explained only from the negative relative sign in the metric. One has to introduce some additional principles in order to explain and describe the observed different roles of space and time in nature. Let us make a list of some very well-known principles and observational facts that explicitly state that space and time should be treated in different ways:

- There are a few space coordinates, but there is precisely one time coordinate.
- There is a time arrow, but there is nothing like a space arrow.
- Psychologically, we experience time and space in completely different ways; we remember the past and not the future, which refers to time, not to space.
- We can travel in space in all directions, but we cannot do that in time.
- Entropy grows with time, but not with space.
- There is a causality principle, which refers to time, not to space; in classical electrodynamics, one uses only retarded solutions and disregards advanced solutions, which again refers to the sign of time, not that of space.
- The separation of causally connected events should be timelike or light-like, but cannot be spacelike; the 4-momentum of a physical particle should be timelike or light-like, but cannot be spacelike.
- Time has a special role in the canonical (i.e., Hamiltonian) formalism; in field theory (of real scalar fields, for simplicity), the set of all degrees of freedom is given by all space points $\mathbf{x}$, not by all space-time points $(\mathbf{x}, t)$; in order to quantize fields, we propose equal-time (anti)commutators, not equal-space (anti)commutators; field operators (anti)commute for spacelike separations, not for timelike separations.
- The time component of the 4-momentum (energy) must be positive (or zero), while the space components of the 4-momentum can have both signs; the quantum operator



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of the space inversion is unitary, while the quantum operator of the time inversion is anti-unitary.

- In the quantum theory of particles (i.e., first quantization) there is an $\hat{x}$ -operator, but there is no $\hat{t}$ -operator.

If one believes that the fundamental laws of nature should possess a certain simplicity and symmetry, then it is reasonable to believe that the fundamental laws should have such a form that none of the space-time coordinates has an a priori special role. If this is so, none of the itemized laws can be fundamental. From some more fundamental laws it should rather follow that one of the coordinates *must* take a special role, by a mechanism which can be viewed as some kind of spontaneous symmetry breaking.

The idea that the different roles of space and time are consequences of spontaneous symmetry breaking is not new. In [1–3] the possibility is considered that this is achieved via the Higgs mechanism. The aim of this article is to give a proposal for a different mechanism which gives different roles to space and time, a mechanism which does not require the introduction of the Higgs field.

It is a tradition among almost all physicists that only finding the correct equations of motion is regarded as a really fundamental task, while the question of the boundary conditions is regarded as a secondary problem. Here I leave such a viewpoint. I consider the question of the boundary conditions as an equally fundamental question as the question of the equations of motion themselves. Therefore, I postulate some principles which the boundary conditions of the Universe should obey. These principles I choose in such a way that none of space-time coordinates has an a priori special role, but that they can still explain the known differences of the role of space and time in the Universe.

The difference between space and time emerges from the viewpoint that nature *must* choose some $(D - 1)$ -dimensional sub-manifold on which the boundary condition will be imposed. This automatically gives a special status to one particular coordinate, the coordinate which is constant on this sub-manifold. This is the mechanism of spontaneous symmetry breaking in my approach. I propose essentially three additional principles. First, spacetime is a non-dynamical manifold, with a fixed global $\mathbb{R}^D$ topology. Dynamical theory of gravity determines only the metric tensor on it. Second, I propose that *any* boundary condition which is finite everywhere leads to a solution which is also finite everywhere. This explains the hyperbolicity, i.e., $(1, D - 1)$ signature of the metric. (It is interesting to note that there is an attempt to explain the hyperbolicity by certain anthropic arguments [4]. My approach is based on the same mathematical properties of hyperbolic and non-hyperbolic equations exploited in that work, but I choose different arguments to favor hyperbolic equations only.) This second principle also explains the boundedness of energy from below, the absence of tachyons, and other related properties of nature. The third principle states that the boundary condition is ordered, rather than random. It explains the time arrow. The quantization is considered as a boundary condition for the field operators. Only physical degrees of freedom are quantized. This, together with the treatment of spacetime as a non-dynamical background, resolves the problem of time in quantum gravity, at least at the conceptual level. Possible paradoxes connected with the possibility of time travel are excluded by my choice of topology.

In Section 2 I present the main physical and mathematical ideas which led me to find the principles which can describe the nature of space and time and explain the differences between them. In Section 3 I give a precise formulation of these principles, as a set of axioms which classical physics should obey. The purpose of Section 4 is to discuss in more detail the origin of various differences between the role of space and time in classical physics, emphasizing that they all emerge from the axioms of Section 3. In Section 5 I discuss the origin of the difference between the role of space and time in quantum physics. The connection with classical physics is most manifest in the Heisenberg picture, which I use to formulate the quantization as a boundary condition for field operators. In Section 6 I discuss whether the second principle that I propose is satisfied for the known physical theories and what new consequences can emerge from this principle. In Section 7 I discuss

whether some of my axioms can be rejected or weakened. In addition, I make some remarks on the question of dimensionality of space. Section 8 is devoted to concluding remarks.

2. The Main Ideas

In this section I give the main physical and mathematical ideas which led me to find the principles proposed in Section 3. Section 2 is intended to be very pedagogical, but not too exhaustive. It is also intended to be intuitive, rather than rigorous.

Let us start from the origin of the time arrow. Most physicists agree that all manifestations of the time arrow (except the arrow connected with the direction of the expansion of the Universe) are consequences of the thermodynamic time arrow, i.e., of the fact that disorder increases with time. The fact that disorder grows with time is equivalent to the statement that the Universe was quite ordered in the past. Thus, the only real problem with the time arrow is to explain *why* the Universe was so ordered at some instant of time of its evolution. Since I cannot find any convincing explanation of this (except the anthropic principle [5]), I shall take this as one of my fundamental postulates. It is enough to postulate that at some “initial” instant of time (not necessarily the earliest instant) all fields and matter must be in some partially ordered configuration in all space regions, but in such a way that “initial” velocities have random space-directions. I require random directions of velocities because then *both time directions are equivalent*, in the sense that disorder increases in both directions from this “initial” instant. The present velocities are obviously not random, since they lead to increasing order in the negative time direction. The “initial” instant is actually the instant of minimal entropy.

The next question considered is why is *time* the coordinate which takes a special role? Why is this is not the *z*-coordinate, for example? Or why is there no more than one coordinate which takes a role similar to that of time? The answer to this question can be easily found if one treats the dynamics of the Universe as a Cauchy problem. To solve a partial differential equation in D dimensions, one first needs to fix some $(D - 1)$ -dimensional sub-manifold (Cauchy surface) on which the Cauchy data will be imposed. This automatically gives a special status to one particular coordinate, the coordinate which is constant on this sub-manifold. If we, in addition, require that the differential equation should provide a stable evolution of the Cauchy data, then for a second-order differential equation two necessary conditions must be fulfilled [4]: First, the equation must be hyperbolic, which corresponds to the $(1, D - 1)$ signature of the metric. Second, the Cauchy surface must be spacelike, i.e., the boundary condition must be the initial condition.

Let us illustrate this on a free-field equation

$$\partial_\mu \partial^\mu \phi(x) + m^2 \phi(x) = 0. \quad (1)$$

All known free fields satisfy this equation, including the Dirac field too. We assume that the metric has the form $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. m^2 is some real parameter that can be positive, zero, or negative. If we are looking for the solution of the form $\phi(x) = \exp(ik \cdot x)$, we find the dispersion relation

$$k_0^2 - \mathbf{k}^2 = m^2. \quad (2)$$

In general, any component of k can be complex. However, if we require that the solution is finite for any value of x , including the cases when some of the components of x are $\pm\infty$, we conclude that all components of k must be real. Now let us suppose that $m^2 > 0$. In this case, the real vector $\mathbf{k}$ can be arbitrary, since then k_0 is also real. However, k_0 cannot take an arbitrary real value, but must rather satisfy $k_0^2 \geq m^2$. We can construct the general solution of (1) which is finite everywhere as a Fourier expansion over plane-wave solutions. For some fixed t , it can have an *arbitrary* (finite everywhere) dependence on $\mathbf{x}$. However, for fixed z , for example, it cannot have an arbitrary time-dependence, because the spectrum of k_0 is truncated. Therefore, if we require that an *arbitrary* boundary condition that is

finite everywhere leads to a solution which is also finite everywhere, then the boundary condition must be the initial condition.

Using a similar argument one can also see that m^2 cannot be negative, because otherwise, owing to the fact that there is more than one space coordinate, no three components of k could take arbitrary real values, without leading to the fourth component of k being imaginary.

One can also easily generalize the analysis to the flat metric with the (n, m) signature, and conclude that an arbitrary boundary condition that is everywhere finite can lead to a solution which is also finite everywhere only if n or m is equal to 1.

Now we can already see the main idea why one coordinate, so-called time, takes a special role. The dynamics is described by some partial differential equations in D dimensions that treat all coordinates in the same way, up to some signs in the metric, which can generally take the (n, m) signature ($n + m = D$). Thus the differential equations are covariant with respect to the $SO(n, m)$ group of coordinate transformations. However, the differential equations do not describe the dynamics uniquely; one must also fix some boundary condition. To accomplish that, one first must fix the boundary itself, which is some $(D - 1)$ -dimensional sub-manifold. This defines the remaining one coordinate which has the same value at the whole $(D - 1)$ -dimensional subspace. By imposing that the *arbitrary* finite everywhere boundary condition leads to a solution which is also finite everywhere, we obtain that all coordinates on this boundary must have the same sign of the metric and that the remaining one coordinate must have the opposite sign of the metric. Thus we *derive* the Lorentz invariance $SO(1, D - 1)$ (isomorphic to $SO(D - 1, 1)$). This also leads to some constraints on the form of the differential equations, including the sign of m^2 . In addition, we impose that the boundary condition must be *ordered* in a described sense, from which we *derive* the second law of thermodynamics and thus the causal role of the time coordinate.

I also want to clarify some conceptual details that are important for a deeper understanding of gravity. Physicists are used to thinking that there is a great difference between the gravitational field and all other fields, because other fields describe some dynamics for which spacetime serves as a background, while the gravity field describes the dynamics of spacetime itself. So, they often imagine that spacetime itself cannot exist without the existence of the gravity field $g_{\mu\nu}(x)$, whereas it can exist without other fields (which corresponds to $T_{\mu\nu} = 0$ in the Einstein equation), and without the dark energy (which can be absorbed into a term contributing to the total $T_{\mu\nu}$). However, a manifold with coordinates x^μ can be well defined even without the metric being defined. This leads to the possibility of interpreting the gravitational field in such a way that it differs much less from the other fields. Such an interpretation could be useful in order to formulate a consistent theory of quantum gravity.

When solving the Einstein equation, one can forget that $g_{\mu\nu}(x)$ represents the metric tensor; it can be viewed just as some second-rank tensor field. Moreover, solving the Einstein equation as a Cauchy problem requires that the topology of spacetime should be fixed before the actual solving. More precisely, the Cauchy problem is well posed only if the topology takes the form $\Sigma \times \mathbb{R}$ on the *global* level, where Σ represents the topology of the Cauchy surface. (Note that, in practice, the Einstein equation is usually not solved as a Cauchy problem; it is solved by imposing some symmetry conditions of the metric on the *whole* spacetime. The various solutions satisfying these conditions are then recognized as representing various topologies.) In this article I propose that the whole dynamics *should* be treated as a Cauchy problem, so I propose that the topology of spacetime is not a dynamical entity. Since I require that none of space-time coordinates should have an a priori special role, the topology should also be symmetrical in that sense. Therefore, I choose $\mathbb{R}^D$ as a *global* topology. Note finally that the condition $D = 4$, as well as the (1,3) signature of the metric, must also be imposed by the initial condition in the Cauchy-problem approach to the Einstein equation.

3. The Formulation of Principles

In this section the precise formulation of principles that I propose is given as a set of axioms. These axioms refer mainly to classical physics, while the transition to quantum physics is discussed in Section 5.

Axiom 1. *There exists a manifold $\mathcal{M}$ which can be globally bijectively mapped to the set $\mathbb{R}^D$, where D is a fixed positive integer.*

This axiom says that spacetime is continuous, D -dimensional, infinite, and predynamical. The mapping in Axiom 1 defines the coordinates $x \equiv \{x^1, \dots, x^D\} \in \mathbb{R}^D$. Next we introduce a metric tensor on $\mathcal{M}$ which is a symmetric second-rank tensor which must satisfy the following axiom.

Axiom 2. *For each point x there exists a neighborhood U , non-negative integers n, m satisfying $n + m = D$, and coordinates such that the metric tensor possesses n positive and m negative eigenvalues on U .*

This axiom says that for each point there exist numbers n, m and coordinates such that the metric is invariant with respect to $SO(n, m)$ coordinate transformations at this point. This is a generalization of the Lorentz $SO(1, 3)$ invariance. It is also important to note that from Axiom 2 it follows that the metric possesses the global decomposition into $D = n + m$, i.e., if, for example, the manifold $\mathcal{M}$ has the (1,3) signature of the metric at some point, then it has the same signature on the whole $\mathcal{M}$.

Now we introduce dynamics, described by some fields $\varphi_a(x)$. The metric tensor can also be one of the dynamical fields, but this is not necessary. For dynamical fields we require the following axiom:

Axiom 3. *Dynamical fields satisfy partial differential equations (with derivatives with respect to x^μ) and for each point x there exist coordinates such that the equations are covariant with respect to $SO(n, m)$ coordinate transformations at this point, where n, m are determined by Axiom 2.*

To construct such differential equations, we do not usually have to worry about the precise values of n and m , since these equations look formally the same for various n, m when written in a manifestly covariant form. Use of Lagrangian techniques further simplifies the construction of such equations.

The knowledge of the differential equations does not determine dynamical fields uniquely. We want to understand the principles which nature obeys in order to pick up a particular solution that corresponds to the actual Universe. Now the essence of my philosophy is as follows: It is redundant for nature to choose some differential equations and some particular solution. Nature actually chooses some differential equations and some boundary condition. The crucial point is that nature *must* first choose some $(D - 1)$ -dimensional sub-manifold $\mathcal{M}_B \subset \mathcal{M}$ on which the boundary condition will be imposed, so nature really *does* choose it. This choice is not considered as a mathematical convenience, but rather as a real event in nature. Such a viewpoint can look slightly metaphysical, but we shall see that such a viewpoint leads to a natural explanation of the known differences between the roles of space and time, as well as to some new predictions. Furthermore, we shall see that, for a given universe, this “canonical” sub-manifold $\mathcal{M}_B$ can be uniquely identified, at least in principle. In the following I propose some axioms that refer to the properties of this “canonical” $\mathcal{M}_B$ and the corresponding boundary condition, which nature should obey.

If the differential equations are of the k -th order in the field $\varphi_a(x)$, then it is convenient to choose some connected boundary $\mathcal{M}_B$ and to fix $\varphi_a(x)$ and all its normal derivatives on it, up to the $(k - 1)$ -th derivative. If this is carried out for all fields appearing in the differential equations, the Cauchy–Kowalevski theorem provides that the solution is then

unique. (Strictly speaking, this theorem also requires the analyticity of the boundary condition and provides the analyticity of the solution. However, I shall assume that the Cauchy problem is well posed also for smooth enough boundary conditions which are not necessarily analytic.) Therefore, I propose the following axiom:

Axiom 4. *The boundary $\mathcal{M}_B$ is a connected $(D - 1)$ -dimensional sub-manifold which can be globally bijectively mapped to the set $\mathbb{R}^{D-1}$.*

It is understood that the boundary condition fixes the fields $\varphi_a(x)$ and all its normal derivatives up to the $(k - 1)$ -th derivative. Thus, because of Axiom 1, the topology of $\mathcal{M}_B$ proposed in Axiom 4 is the only one that can lead to a well-posed boundary-condition problem.

Let us now introduce the following definition:

Definition 1. *A function $\varphi : X \rightarrow \mathbb{C}$ is **regular** on a domain $X \subseteq \mathcal{M}$ if $|\varphi(x)|$ is bounded from above for every $x \in X$.*

In other words, a regular function is a function which is finite everywhere. It is quite reasonable to require that physical fields should be regular. However, it is known that some fields, such as the metric tensor $g_{\mu\nu}$, the connection $\Gamma_{\mu\nu}^\rho$, and the vector potential A_μ do not have to be regular. I shall refer to such fields as gauge fields. Only physical fields, such as the scalar curvature R and the field strength $F_{\mu\nu}$ have to be regular, whereas the gauge fields can possess only such irregularities which do not lead to irregularities of the corresponding physical fields. Having this in mind, I introduce the following definition:

Definition 2. *The field φ_a is **essentially regular** on X if its corresponding physical field is regular on X . The metric field is essentially regular on X if it is essentially regular as a field and satisfies Axiom 2 on X .*

I have no intention to give a rigorous definition of a physical field. Let me just note that fields appearing in the Lagrangians which do not possess any kind of gauge symmetry are their own physical fields.

Now we are ready to propose the following axioms:

Axiom 5. *For a given signature of the metric there exists $\mathcal{M}_B$ such that **every** boundary condition essentially regular on $\mathcal{M}_B$ leads to a solution essentially regular on $\mathcal{M}$.*

Axiom 6. *The Cauchy surface $\mathcal{M}_B$ is chosen in such a way as to satisfy the requirement of Axiom 5. The boundary condition is essentially regular on $\mathcal{M}_B$.*

Axiom 5 is central and the most important axiom of this article. This is actually not the constraint on the boundary condition, but rather on the signature and on the possible forms of the dynamics, i.e., on the possible equations of motion. As we shall see, this Axiom explains the hyperbolicity of the equations of motion, i.e., the $(1, D - 1)$ signature of the metric. It also explains a lot of known differences between the role of space and time itemized in the Introduction. Finally, it leads to some new predictions. All that will be discussed in later sections. Here I want to explain that axioms of this section lead to a new philosophy of the logical (not temporal) order which nature must follow when it chooses the conditions which uniquely determine the Universe.

Nature first chooses the dimension D of the manifold $\mathcal{M}$, according to Axiom 1. (Of course, the word “chooses” should not be understood in the anthropomorphic sense.) Then it chooses the signature (n, m) according to Axiom 5. After that it chooses $\mathcal{M}_B$ according to Axiom 6. The next step is to choose a set of fields $\{\varphi_a\}$ which will describe the dynamics, making a difference between the physical and the gauge fields, but not yet specifying its specific dependence on x . The very next step, the central one in my philosophy, is to choose

the differential equations (or Lagrangian) which will provide that *any* essentially regular boundary condition will lead to an essentially regular solution, according to Axiom 5. Of course, these differential equations must also satisfy some additional principles, such as covariance (Axiom 3) and probably some other principles, which are not important here. At the end it only remains to choose some particular boundary condition on $\mathcal{M}_B$, according to Axiom 6, which then uniquely determines the Universe.

The six axioms proposed so far still cannot explain all the differences between the role of space and time itemized in the Introduction. We need one additional axiom which will provide that disorder increases with time and thus explains the time arrow. This axiom must essentially say that the boundary condition is not completely random, but rather ordered somehow, as discussed in Section 2. It is not easy to formulate this axiom in a mathematically rigorous way. Thus I formulate this in a way which is not very rigorous, but rather intuitive:

Axiom 7. *The boundary condition on $\mathcal{M}_B$ is partially ordered, rather than random. In particular, absolute values of various fields are not homogeneous, but rather lumped in localized lumps. However, the field derivatives in the normal direction to $\mathcal{M}_B$, needed for the uniqueness of the solution of the Cauchy problem, are random.*

The last sentence in Axiom 7 corresponds to the assumption that the initial velocities are random, which provides that disorder increases in both time directions from the so-called initial hypersurface $\mathcal{M}_B$, so both time directions are equivalent.

4. The Differences between Space and Time in Classical Physics

In this section I discuss how all the differences between the role of space and time in classical physics emerge from the axioms of Section 3. However, it is important to note that most of the discussion is valid even if novel principles of this article are not realized in nature. Only Axioms 1 and 5 are really novel principles, in the sense that they differ from the conventional point of view and can be tested, at least in principle. In order to provide a complete and clear picture, I find it necessary to review some already-known results.

If dynamical fields satisfy second-order differential equations, then it follows from Axioms 3 and 5 that the differential equations must be hyperbolic, i.e., the signature of the metric must be $(1, D - 1)$ (see, for example, [4] and references therein). We shall see in Section 6 that Axiom 5 also explains why dynamical fields are not described by differential equations of order higher than second.

In order to satisfy Axiom 5, the hyperbolicity is necessary, but not sufficient. For free fields, for example, we have seen in Section 2 that m^2 cannot be negative. The fact that m^2 cannot be negative explains why the D -momentum of a free physical particle cannot be spacelike. If we assume that the propagation velocity of a free wave packet is given by the so-called group velocity

$$\mathbf{v}_g = \frac{d\omega}{d\mathbf{k}}, \quad (3)$$

where $\omega = \sqrt{\mathbf{k}^2 + m^2}$, then we see that there are no velocities greater than $c \equiv 1$, which then explains why the separation of causally connected events cannot be spacelike, at least for the free case.

However, it is fair to mention that the propagation velocity of a free wave packet is not always given by (3). Thus it is not strange that there are solutions of all known free relativistic wave equations (such as free Klein–Gordon, Maxwell, and Dirac equations) which propagate with superluminal velocities, i.e., velocities that are greater than c [6–9]. However, there are no real paradoxes with these solutions because it turns out that the corresponding physical quantities (such as the Poynting vector for the electromagnetic field) do not propagate faster than c . Thus the principle that no energy or information can propagate faster than c is not violated, and this is what we understand when we claim that fields do not propagate faster than c .

The requirement that $m^2 \geq 0$, which was obtained for free fields, in the case of interacting fields generalizes to the requirement that the energy should be bounded from below. To see this, we consider the Lagrangian for the real scalar field $\phi(x)$:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) , \quad (4)$$

where

$$V(\phi) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 . \quad (5)$$

The parameters μ^2 and λ are real constants. The corresponding equation of motion can be written in the form

$$\partial_\mu \partial^\mu \phi(x) + m_{\text{eff}}^2(x) \phi(x) = 0 , \quad (6)$$

where

$$m_{\text{eff}}^2(x) = -\mu^2 + \lambda \phi^2(x) . \quad (7)$$

If we require stable time evolution, and if $\lambda \neq 0$, then for a large $\phi^2(x)$ the relation $m_{\text{eff}}^2(x) \geq 0$ must be fulfilled [10]. This means that the relation $\lambda > 0$ must be fulfilled, which is actually the consequence of Axiom 5, because the stability requirement is essentially the same requirement as Axiom 5.

Let us now see what it has to do with the sign of energy. We introduce the canonical energy–momentum tensor

$$\Theta_\nu^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - g_\nu^\mu \mathcal{L} . \quad (8)$$

The corresponding energy density for the Lagrangian (4) is

$$\mathcal{H} = \Theta_0^0 = \Theta^{00} = \left[\frac{\dot{\phi}^2}{2} + \frac{(\nabla \phi)^2}{2} \right] + V(\phi) . \quad (9)$$

The term in the square brackets represents the kinetic part of Θ_0^0 . We see that it has the definite (positive) sign. It is easy to see that, owing to the $(1, D-1)$ signature of the metric, no other component of Θ_ν^μ or $\Theta^{\mu\nu}$ has definite sign of its kinetic part for $D > 2$. Since $\lambda > 0$, we see that $V(\phi)$ is bounded from below. Thus we see that the boundedness of the energy from below is actually the consequence of Axiom 5 (i.e., the stability requirement). A similar connection between Axiom 5 and the boundedness of the energy from below can be seen in a similar way for most of the other Lagrangians. The positivity of energy is then obtained from the appropriate energy shift, which does not change the physical laws (except gravity, at least in the conventional approach).

We see that if E is some admissible energy, then $-E$ may not be admissible energy. The consequence of this is that energy does not transform as a time component of a D -vector with respect to time inversion.

Let us discuss now the consequences of Axiom 7. (In the following I use the term “disorder” rather than “entropy”, because the former is a more general concept, while the latter corresponds to some particular measure of disorder, which can be inappropriate for some purposes). According to this axiom, the so-called initial state of the Universe is quite ordered. We assume that the degree of orderliness is homogeneous on the initial spacelike manifold $\mathcal{M}_B$. This manifold defines the natural foliation of spacetime into the class of spacelike manifolds $\Sigma(t)$, with the property $\Sigma(0) = \mathcal{M}_B$. We choose t in such a way that the orderliness is homogeneous (at least at some large scale) on the whole Σ for any fixed t . Disorder increases in both time directions from $t = 0$, so there is no special time direction. The causal, psychological, and electrodynamic time arrows are consequences of the disorder increase. The positive time direction is defined as a direction from $\mathcal{M}_B$ to the present time. Thus t defines the natural cosmological time, but still not uniquely, because t can be replaced by some $h(t)$, where h is some strictly increasing function, satisfying $h(0) = 0$. In order to

define the time coordinate uniquely, we can require $g_{tt} = 1$. For a given universe, $\mathcal{M}_B$ can be uniquely identified as a $(D - 1)$ -dimensional spacelike manifold Σ with the smallest measure of disorder (entropy). The instant $t = 0$ can be considered as the instant of the “creation” of the Universe (whatever this means) by some yet-unknown mechanism.

According to Axiom 7, the fields are initially lumped. Since fields cannot propagate faster than c , no part of the boundary of a D -dimensional lump can be spacelike. Therefore, from the covariant conservation laws of the form

$$\partial_\mu J^\mu = \partial_t J^0 + \nabla \mathbf{J} = 0, \quad (10)$$

it follows that various quantities are conserved in time, but not in space:

$$\frac{d}{dt} \int_V d^{D-1}x J^0(\mathbf{x}, t) = 0. \quad (11)$$

Let us now consider the question why we cannot travel in time. This question can be answered from several points of view, corresponding to slightly different definitions of the notion of time travel. First, one can argue that a time traveler can observe that he arrived at the past only if he remembers the future, which is extremely improbable. The second approach is based on the consideration of the difference between space and time travel. The fact that material objects can travel in both space directions but only in one time direction can be stated rigorously as: the trajectory of a material object $\mathbf{x}(t)$ is a single-valued function, whereas its inverse $t(\mathbf{x})$ is not necessarily a single-valued function. To clarify this, let us consider a $1 + 1$ dimensional example of a trajectory which would correspond to the time travel in that sense: $t(x) = -x^2$. This can be viewed as an object traveling first in the positive time direction, but at $t = 0$ it starts to travel in the negative time direction. However, this is how it really would look like for an independent observer: two *identical* objects (which is rather improbable by itself if these are not two elementary particles) approach each other, they finally collide at $t = 0$ and then disappear for $t > 0$, thus violating the conservation laws. In other words, objects can travel in both space directions, but only in one time direction because they are localized in space and thus conserved in time.

The third approach to time travel, based on the possibility that the Universe can possess topology or a metric tensor which admits closed timelike curves, is the subject of many current theoretical investigations. One of the most important contributions against time travel is given in [11], where it is argued that various conditions (topological defects and metric tensors which do not possess the $(1, D - 1)$ signature everywhere) needed for various mechanisms of time travel cannot be realized in practice, essentially because their realizations require infinite energy. However, in my opinion, the strongest argument against the time travel, discussed also in [11] and particularly clearly in [12], is the consistency requirement: for any space-time point x , all physical fields $\varphi_a(x)$ must be *uniquely* determined. The consistency in the Cauchy-problem approach is automatically provided by Axioms 1 and 4. The time travel based on metric tensors which do not possess the $(1, D - 1)$ signature everywhere is also excluded by Axiom 2. In [11] the possibility of time travel if it is possible to travel in space faster than light is discussed, but we have already excluded the possibility of traveling faster than light.

Now a few notes on the different roles of time and space in the Hamiltonian formalism. Historically, the Hamiltonian formalism was first developed for pointlike particles, i.e., for the objects which are strictly localized in space and exist for all times. This is the difference between the role of space and time already at the kinematic level. Thus, it is not strange that particle mechanics has a formulation, such as the Hamiltonian formalism, which treats space and time in different ways.

However, such an argument cannot be directly applied to field theories. The Hamiltonian formulation of them was probably partly influenced by our intuitive notion of time, which is the consequence of the time arrow, leading to the intuitive picture that dynamics

is something that changes with *time*. This leads to the notion of “degree of freedom” as a real variable which can (at the kinematic level) possess arbitrary dependence on time. Thus, the set of all degrees of freedom of a real scalar field is given by all space points $\mathbf{x}$, not by all space-time points $(\mathbf{x}, t)$. Dynamics, i.e., an equation of motion, is something that determines the actual time dependence. In the Hamiltonian approach to field theory, dynamics is given by the Hamiltonian density $\mathcal{H} = \mathcal{H}(\phi(\mathbf{x}), \pi(\mathbf{x}))$. The Poisson brackets among functions of $\phi(\mathbf{x})$ and $\pi(\mathbf{x})$ are actually equal-time Poisson brackets [13]. They can be viewed as Poisson brackets among initial conditions. Thus, the phase space is space of all initial conditions. In the spirit of the axioms of Section 3, it is most natural to consider *the degrees of freedom as variables which can be arbitrarily chosen on $\mathcal{M}_B$* (except that they must be essentially regular and ordered). Such a viewpoint will be exploited for the formulation of the canonical quantum theory.

I want to emphasize that the canonical formalism in *classical* field theory is only a convenience of calculation. Nothing is really lost if one does not at all introduce Hamiltonians and Poisson brackets, but rather uses only Lagrangians and corresponding manifestly covariant equations of motion. The existence of the Hamiltonian formalism in classical field theory still does not mean that space and time take different roles. For example, one could also formulate a variant of the canonical formalism in which the x^1 coordinate takes a special role, by introducing the Legendre transformation

$$\mathcal{H}^{(1)} = \pi^{(1)}(x) \frac{\partial \phi(x)}{\partial x^1} - \mathcal{L}, \quad (12)$$

where

$$\pi^{(1)}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_1 \phi(x))}. \quad (13)$$

(Note that (12) is equal to Θ_1^1 in (8)). In particular, this would lead to a new kind of Poisson brackets which would be interpreted as equal- x^1 Poisson brackets.

At the end of this section let me give a few notes on theories with constraints. The constraints appear in the Lagrangians which are invariant with respect to some local gauge transformations [14]. For such systems, some of the equations of motion are interpreted as constraint equations, which can be understood as constraints to the initial condition. Thus the initial condition is not arbitrary, i.e., the number of fields which can be arbitrarily fixed on the initial Cauchy surface is smaller than it seems at first sight. Axiom 5 refers to these physical degrees of freedom, which are actually the fields for which the initial condition can be arbitrarily chosen (this refers to their initial time derivatives too), whereas the initial values of other fields are determined via the constraint equations. In order to provide a well-posed Cauchy problem, some additional gauge conditions must be chosen before determining the time evolution.

5. The Differences between Space and Time in Quantum Physics

In this section I discuss the origin of the differences between space and time in quantum physics. The connection with classical physics is the most manifest in the Heisenberg picture, which I use to formulate the quantization as a boundary condition for field operators. As in Section 4, for the sake of completeness and clarity, I also review some already-known results. The main new idea of this section is a suggestion that quantum physics cannot remove, in a satisfactory way, singularities of the corresponding classical theory, so we need to modify the *classical* theory in order to remove the singularities.

The Heisenberg-picture quantization is based on equal-time commutators among canonical coordinates and conjugated momenta, which gives different roles to time and space. One could wonder whether we can use the equal-space Poisson brackets resulting from the formalism based on (12) to propose the corresponding equal-space commutation relations, without changing the physical content of the resulting theory. The answer is *no*, owing to the fact that the Poisson brackets are *defined* to be what they are, while the corresponding commutation relations are *postulated*. In other words, introduction of the

Poisson brackets does not change the physics, while introduction of the commutation relations does change the physics. Thus the difference between space and time in quantum physics is even deeper than in classical physics.

Let me stress some other important facts about the Heisenberg-picture quantization. The “general” solution of the equation of motion for a free real scalar field, which is usually used, is

$$\phi(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}2\omega} [a(k)e^{-ik \cdot x} + a^\dagger(k)e^{ik \cdot x}] , \quad (14)$$

where $\omega = \sqrt{\mathbf{k}^2 + m^2}$, and integration is performed over all real vectors $\mathbf{k}$. Let us emphasize once again that this is not really the general solution, because there are also other solutions connected with imaginary ω and $\mathbf{k}$. However, this is the general solution if we restrict ourselves to the solutions which are consistent with Axioms 5 and 6. A more general solution would lead to different physical results. In particular, fields would not commute for spacelike separations.

There is one more important property of the operator $\hat{\phi}(\mathbf{x}, t)$ and its corresponding Hilbert space. For any fixed instant $t = t_0$ and for any regular function $\phi(\mathbf{x})$, there is a Hilbert state $|\psi\rangle$ such that

$$\hat{\phi}(\mathbf{x}, t)|\psi\rangle = \phi(\mathbf{x})|\psi\rangle , \quad \text{for } t = t_0 . \quad (15)$$

A similar statement is true for the operator $\hat{\pi}(\mathbf{x}, t)$. However, similar statements are not true if the roles of time coordinate and one of the space coordinates are exchanged.

This fact leads to an important additional physical motivation for Axiom 5. This axiom essentially says that singular field configurations can never form in a proper classical theory. We know very well that Einstein’s theory of gravity does not possess this property, because it leads to cosmological and black-hole-like singularities. Almost everyone agrees that singularities do not really exist in the real world. However, there is a wide belief that *quantum* theory of gravity, when found one day, could remove such pathologies, even if the corresponding classical theory does possess these pathologies. I want to argue that quantum physics cannot remove, in a satisfactory way, the singularities of the corresponding classical theory; we should rather modify the theory of gravity for strong fields already at the classical level.

For this reason, I consider a simple example: a particle moving in a spherically symmetric potential $V(r)$, such that $V(\infty) = 0$ and $V(0) = -\infty$. Classically, the particle can fall into the potential well, thus reaching infinite kinetic energy (but finite total energy, which is the constant of the motion and is the sum of the kinetic and the potential energy). It is often said that quantum physics prevents such pathological behavior because it prevents the particle falling into the center of the potential well. However, is this really true? The Schrödinger equation gives a set of eigenfunctions of the Hamiltonian $\{\Psi_n(\mathbf{x}; t) = \psi_n(\mathbf{x})e^{-iE_n t}\}$, which serves as a basis for the general solution of the Schrödinger equation. This means that the particle can be found everywhere, including the singular point $r = 0$. The set of functions $\{\psi_n(\mathbf{x})\}$ is complete, which means, in particular, that the wave function at some particular instant can be proportional to $\delta^{D-1}(\mathbf{x})$, or to $e^{i\mathbf{p} \cdot \mathbf{x}}$ for any particular $\mathbf{p}$, which are eigenstates of the operators $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$, respectively. In other words, the particle can attain any position or any momentum. The only restriction is that these two quantities are not mutually independent, because the corresponding operators do not commute. In the language of energy, the particle can possess any mean potential energy or any mean kinetic energy, including the infinite one. The only restriction is that it cannot possess a mean total energy smaller than the ground-state energy E_0 , and this is not a much better situation than in classical physics, because in a typical physical classical situation we do not expect a negative infinite total energy either.

However, why is an atom still stable? This is because the probability density for states with a fixed energy $P_n(\mathbf{x}) = |\psi_n(\mathbf{x})|^2$ does not depend on time, so lasts forever. On the other hand, if the wave function is proportional, for example, to $\delta^{D-1}(\mathbf{x})$ at some instant

t , then it is a state which possesses components of many admissible energies. Thus $P(\mathbf{x})$ changes with time, being strictly localized only at one particular instant t . Thus we have much better chances to find the particle in a state with a fixed energy.

Similarly, if the classical theory of gravity possesses a singular solution $g_{\mu\nu}(\mathbf{x})$ for some instant t , then we must expect that in the corresponding quantum theory there exists a state $|\psi\rangle$ which corresponds to this solution at some instant t . The best we can expect is that we shall never observe such a state because its duration is too short. However, I believe that singular states should not exist at all, so I require that singularities should not appear even in classical physics.

Now we are finally ready to propose an axiom for the quantization of fields. It must explain, rather than postulate, why time has a special role in quantization and why in (14) we take only real ω and $\mathbf{k}$. We do not know how to canonically quantize theories with higher than second derivatives in the equations of motion, but it seems that such theories cannot be consistent with Axiom 5, as I discuss in Section 6. Thus I assume that all fields that can be arbitrarily chosen (except that they must be essentially regular and ordered) on $\mathcal{M}_B$, can be divided into a set of fields $\{\varphi_a\}$ and conjugate momentum fields $\{\pi_a\}$, where

$$\pi_a = \frac{\partial \mathcal{L}}{\partial(\partial_t \varphi_a)} \quad (16)$$

and t is the coordinate defined as in Section 4. Having all this in mind, I propose:

Axiom 8. Let $\mathbf{x}, \mathbf{x}' \in \mathcal{M}_B$. All fields $\{\varphi_a\}$ and $\{\pi_a\}$ are quantized in such a way that

$$\begin{aligned} [\hat{\varphi}_a(\mathbf{x}, 0), \hat{\pi}_b(\mathbf{x}', 0)]_{\pm} &= i\delta_{ab}\delta^{D-1}(\mathbf{x} - \mathbf{x}') , \\ [\hat{\varphi}_a(\mathbf{x}, 0), \hat{\varphi}_b(\mathbf{x}', 0)]_{\pm} &= [\hat{\pi}_a(\mathbf{x}, 0), \hat{\pi}_b(\mathbf{x}', 0)]_{\pm} = 0 . \end{aligned} \quad (17)$$

Furthermore, the field operators $\{\hat{\varphi}_a(x)\}$ and $\{\hat{\pi}_a(x)\}$ satisfy classical equations of motion and they are quantized in such a way that for given functions $\varphi_a(\mathbf{x})$ and $\pi_a(\mathbf{x})$ there exist states $|\psi_{\varphi_a}\rangle$ and $|\psi_{\pi_a}\rangle$ such that

$$\begin{aligned} \hat{\varphi}_a(\mathbf{x}, 0)|\psi_{\varphi_a}\rangle &= \varphi_a(\mathbf{x})|\psi_{\varphi_a}\rangle , \\ \hat{\pi}_a(\mathbf{x}, 0)|\psi_{\pi_a}\rangle &= \pi_a(\mathbf{x})|\psi_{\pi_a}\rangle , \end{aligned} \quad (18)$$

if and only if $\varphi_a(\mathbf{x})$ and $\pi_a(\mathbf{x})$ are essentially regular functions.

It is, of course, understood that we use anti-commutators if both fields possess half-integer spin and commutators otherwise. For fermion degrees, $\varphi_a(\mathbf{x})$ and $\pi_a(\mathbf{x})$ are products of a complex essentially regular function and a Grassmann number. Since this quantization is canonical, it is not manifestly covariant. However, we expect that covariance is preserved because the field operators satisfy the covariant equations of motion. This can be explicitly proved for free fields and on the perturbative level for fields in interaction, but I shall not consider these rather technical problems. One of the most important consequences of covariance is that the statements of Axiom 8 are valid not only for $t = 0$, but also for all other times, obtained by time evolution or coordinate transformation.

Axiom 8 can be understood as an initial condition for the field operators. It proposes that we have to quantize those classical variables which can be arbitrarily chosen on $\mathcal{M}_B$. It can also be viewed as an explanation why in the quantum theory of particles (i.e., first quantization) there is an $\hat{\mathbf{x}}$ -operator, but there is no $\hat{t}$ -operator.

An important ingredient of Axiom 8 is that it proposes that *only physical degrees of freedom should be quantized*. This is extremely important for quantum gravity, because the quantum theory of gravity in which both physical and nonphysical degrees are quantized is not equivalent to the theory in which only physical degrees are quantized. The quantization of the physical degrees only is also one of the ways how to solve the problem of time in quantum gravity [15].

It is also important to note that the consistency of the canonical quantization requires the topology of spacetime to be $\Sigma \times \mathbb{R}$, which is provided by Axiom 1.

It is straightforward to convert operators and states from the Heisenberg to the Schrödinger picture. This leads to the functional Schrödinger equation, which determines the wave functional $\Psi[\phi(\mathbf{x}); t]$, being a functional with respect to $\phi(\mathbf{x})$ and a function with respect to t . Both the Heisenberg and the Schrödinger picture of quantum field theory manifestly express the fact that space and time are not treated in the same way. On the other hand, it is usually stated that the functional-integral formulation is manifestly covariant, which might seem to be in contradiction with the fact that space and time have different roles in quantization. However, space and time have different roles even in the functional-integral formulation, because it is given by

$$\langle \phi_f(\mathbf{x}), t_f | \phi_i(\mathbf{x}), t_i \rangle = \int [d\phi(\mathbf{x}, t)] [d\pi(\mathbf{x}, t)] \times \exp \left\{ i \int_{t_i}^{t_f} dt \int d^{D-1}x [\pi(\mathbf{x}, t) \dot{\phi}(\mathbf{x}, t) - \mathcal{H}(\phi(\mathbf{x}, t), \nabla \phi(\mathbf{x}, t), \pi(\mathbf{x}, t))] \right\}, \quad (19)$$

where $|\phi(\mathbf{x}), t\rangle \equiv \Psi[\phi(\mathbf{x}); t]$. The left-hand side obviously gives different roles to space and time. This is manifested on the right-hand side in the fact that the functional integral is not performed over all functions $\phi(\mathbf{x}, t)$, but only over functions which satisfy

$$\phi(\mathbf{x}, t_f) = \phi_f(\mathbf{x}), \quad \phi(\mathbf{x}, t_i) = \phi_i(\mathbf{x}). \quad (20)$$

Furthermore, t takes values from the finite interval $t \in [t_i, t_f]$, while $\mathbf{x}$ takes values from the infinite interval $\mathbf{x} \in \mathbb{R}^{D-1}$. At the end, the sub-integral function $\pi \dot{\phi} - \mathcal{H}(\phi, \nabla \phi, \pi)$ is not Lorentz invariant. The invariant form is obtained only when the π -dependence is integrated out, the vacuum-to-vacuum amplitude is considered, and $t_i \rightarrow -\infty$, $t_f \rightarrow \infty$;

$$\langle \phi_f(\mathbf{x}) = 0, t_f \rightarrow \infty | \phi_i(\mathbf{x}) = 0, t_i \rightarrow -\infty \rangle \equiv Z = \int [d\phi(x)] \exp \left\{ i \int d^D x \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \right\}. \quad (21)$$

However, the left-hand side still treats space and time in different ways and the functional integral on the right-hand side is still restricted to functions which satisfy (20). The general expression (19) is equivalent to the Schrödinger equation, while the Lorentz-invariant expression (21) is only a special case, from which the Schrödinger equation cannot be derived.

It is also important to note that in (19), for a given space-time point $(\mathbf{x}, t \neq t_i, t_f)$, the integration is performed over *all* possible finite real values of ϕ and π . This is the direct consequence of the fact that for any regular functions $\phi(\mathbf{x})$, $\pi(\mathbf{x})$ and for any t there exist states such that these functions are eigen-values of the corresponding field operators. This means that in theories with constraints the functional integral is performed only over the physical degrees of freedom, which is important for quantum gravity. Note also that, according to my axioms, in the case of quantum gravity there is no sum over topologies; only the global $\mathbb{R}^D$ topology is included.

Let us now discuss the meaning of the discussion presented in Section 4 from the point of view of quantum field theory. Although the whole Section 4 refers to the classical field theory, all arguments are correct at the macroscopic level, because we know that classical theory is a good approximation at the macroscopic level. In particular, the law of disorder increasing, as a statistical law, is valid only on the macroscopic level. On the other hand, there are arguments that quantum mechanics possesses the intrinsic, fundamental time arrow, connected with the “fact” that wave functions collapse. However, excellent arguments against such conclusions are given in [16]. There are also arguments, based on the considerations of the wave function of the Universe, that entropy would start to decrease when the Universe starts to contract. It is remarkable to note that Hawking was the first that came to such a conclusion [17], but later he corrected himself [5], claiming that

his conclusions were based on certain misinterpretations. A general discussion on various misunderstandings of the time arrow is given in [18]. It seems to me that all conclusions made by some authors about the different status of the time arrow in classical and quantum physics, if not incorrect, are at least interpretation dependent, because there are various interpretations of quantum mechanics and no one knows yet which is the correct one. The origin of the collapse of the wave function is still not understood. My personal belief is that quantum mechanics is just some effective, incomplete theory, while the underlying more fundamental theory is some deterministic nonlocal hidden variable theory, which obeys some laws not very different from the axioms of Section 3. Actually, it is very likely that only Axiom 3 should be modified. For example, in the de Broglie–Bohm interpretation of quantum field theory [19], the classical equations of motion are modified by adding an external force proportional to $\hbar^2$, in which fields are integrated over space, but not over time. This term breaks Lorentz covariance and locality, but the resulting theory still possesses a well-posed initial-value problem. In this interpretation, Lorentz covariance and locality are statistical effects, which are the only ones measured in present experiments.

Having in mind the remarks of the last paragraph, we may conclude that quantum mechanics probably does not change the origin of the time arrow.

6. Do Our Theories Satisfy Axiom 5?

We have argued that equations of motion must obey some properties, such as hyperbolicity and boundedness of energy from below, in order to satisfy Axiom 5. However, nothing provides that these properties are enough. We have to check whether our theories really satisfy this axiom, and if they do not, whether they can be modified in such a way as to still agree with present observations. I give only some qualitative discussion of this, without intention to be rigorous.

Let us start from electrodynamics. Electromagnetic fields and charges obey Lenz's law, which essentially states that any change tends to be canceled. This speaks in favor of satisfying Axiom 5. One could argue that classical electrodynamics has problems with infinities connected with pointlike charges. However, one should not forget that we are considering a *field* theory of charges, i.e., continuous distributions of charge. Because of Axiom 6, there are no initial infinite charge densities and thus there are no initial pointlike charges. Since the force among charges of the same sign is repulsive, pointlike charges will never form. Of course, both classical and quantum electrodynamics still cannot determine the size of the electron and its electromagnetic mass, but the important thing is that classical electrodynamics does not *predict* the singularities of this kind.

However, it seems that classical electrodynamics can still lead to some divergences under very specific initial conditions. For example, one can consider a free electromagnetic wave which is exactly spherically symmetric and moves toward the center of the sphere. This will result in an infinite energy density in the center when the wave arrives there. However, the Lagrangian of electrodynamics is certainly not correct for very strong fields, so it is very likely that formation of such infinities is prevented on high energy scales, by some yet-unknown interactions. Similar discussion can be conducted for all other non-gravitational interactions.

The inconsistency of Einstein's theory of gravity with Axiom 5 is more obvious than that of other theories, because it is shown by Hawking and Penrose [20,21] that singularities will develop under very general initial conditions in Einstein's classical theory of gravity. This is one of the motivations to find an alternative theory of gravitation. The status of singularities in various alternative theories of gravitation is reviewed in [22].

One class of alternative gravity theories are higher derivative theories, based on addition of higher powers of the curvature tensor to the Lagrangian of Einstein's theory. However, even if these terms can prevent cosmological and black-hole-like singularities, their inconsistency with Axiom 5 is even more obvious. It turns out [23] that in such theories the energy is not bounded from below and thus runaway solutions appear. Similar problems appear in various non-gravitational higher derivative theories as well. There

is no general theorem which provides that every higher derivative partial differential equation possesses such problems, but such problems are found in physically interesting cases. This is why we usually disregard higher derivative theories. It is also often claimed that such theories violate causality. This is because one needs to impose the boundary conditions at $t \rightarrow \pm\infty$ in order to remove these runaway solutions. The presence of runaway solutions is also connected to the violation of Einstein causality, i.e., to non-vanishing (anti)commutators outside the light-cone. This connection can be easily seen on the example of tachyon fields [24].

Another class of generalizations of Einstein's theory of gravitation are gauge theories of gravity [1,25]. The most important of them is the Einstein–Cartan theory, which leads to the existence of torsion. It turns out that singularities in such theories do not develop under such wide conditions as in Einstein's theory, but they can still appear, for example, for spin-less matter, which does not feel torsion.

The third class of alternative gravity theories, perhaps most in the spirit of the philosophy of this article, are bi-metric theories. The main idea is to separate the metric tensor into two parts

$$g_{\mu\nu} = \gamma_{\mu\nu} + \Phi_{\mu\nu}, \quad (22)$$

where $\gamma_{\mu\nu}$ is a non-dynamical, background metric, while $\Phi_{\mu\nu}$ is a dynamical field, determined by some differential equations. Such theories are often called “field theories of gravitation” because such theories are the most similar to other field theories, describing a field in a fixed background metric. In theories of this kind it is manifest that topology is not dynamical, but rather fixed by the background metric $\gamma_{\mu\nu}$.

One of the variants of bi-metric theories is the theory developed by Logunov and others [26–30]. The motivation for this theory has been criticized because this theory was motivated by some incorrect criticism of Einstein's theory of gravity. However, Logunov's theory itself is self-consistent and still possesses some advantages with respect to Einstein's theory. The background metric in this theory is flat Minkowski metric $\gamma_{\mu\nu} = \eta_{\mu\nu}$. The metric $g_{\mu\nu}$ of a spherically symmetric object with a mass M takes the same form as a Schwarzschild solution for $r \gg 2MG$. However, a small mass m is attributed to the gravitational field $\Phi_{\mu\nu}$, whose effect is that the gravitational force becomes repulsive for strong fields, thus preventing black-hole and cosmological singularities. There are arguments that even a small mass cannot be attributed to a graviton because it would significantly deviate from experiments even in a small mass limit [31,32]. However, these arguments are applicable only to Einstein's theory of gravity, not to any theory of gravity. The effects of a small enough graviton mass in Logunov's theory are in agreement with experiments [33]. A homogeneous and isotropic universe is infinite in space, exists for an infinitely long time and oscillates. Thus it seems that this theory satisfies Axiom 5 and is manifestly in agreement with Axiom 1. However, I am far from saying that this is the right theory. For example, the corresponding quantum variant is certainly not renormalizable, essentially for the same reasons as Einstein's theory, because the same dimensional coupling constant G appears in the Lagrangian. I am just arguing that this theory could be closer to the right theory which we do not know yet.

Let us discuss at the end why there are no fields with spin higher than 2. Their status is similar to the theories with derivatives higher than second; there is no general theorem, but the simplest theories constructed, for example, in [34–36], possess some pathologies. They violate Einstein causality, i.e., they propagate faster than light and (anti)commutators do not vanish outside the light-cone. They also violate Cauchy causality, i.e., the Cauchy problem is not well posed. The axioms of Section 3 assume, of course, that the Cauchy problem must be well posed. The general relation between Einstein and Cauchy causality is discussed in [37].

7. Discussion

As discussed already, only Axioms 1 and 5 are really novel principles, in the sense that they differ from the conventional point of view and can be tested, at least in principle. Here

I want to discuss whether these axioms can be rejected or weakened and what consequences of this would be. I shall also give a few comments on the dimensionality of space.

Axiom 5 essentially says that for any finite everywhere initial condition the solution is also finite everywhere. This axiom explains the hyperbolicity, i.e., the $(1, D - 1)$ signature of the metric. It also explains the absence of tachyons, the positivity of energy, and other related properties of nature. However, from these properties Axiom 5 certainly cannot be derived.

First, there is a possibility that infinities do exist, but almost no one believes that.

A much more probable possibility is that nature somehow chooses only those initial conditions that will not lead to infinities. However, such a principle is quite unaesthetic; Axiom 6 seems much simpler and more natural than this one.

The best alternative is probably the assumption that singularities can occur in classical physics as long as quantum physics prevents them. However, as we have already discussed, quantum physics cannot prevent the existence of states which correspond to the singular behavior at some particular instant of time. The best we can expect from quantum physics is that it is practically impossible to observe such states. One can be satisfied with this, but Axiom 5, together with Axiom 8, is more satisfying, because it provides that singular states do not exist at all.

It is difficult to test Axiom 5 experimentally, because we cannot measure infinities. However, finding tachyons, for example, would be a strong argument against this axiom, but this would also violate some widely accepted principles, such as Einstein causality.

A more serious question is whether the topology is really a non-dynamical entity, as proposed in Axiom 1. I want to emphasize once again that the topology is a more fundamental concept than the metric tensor, in the sense that the former can be defined without the latter. Moreover, the Einstein equation is manifestly a theory of the metric tensor, not of the topology. If the Einstein equation is treated as a Cauchy problem, for example, by numerical computation, the manifold of space-time points and its topology *must* be defined before any computation of the metric tensor is performed. If the Cauchy problem is well posed, then the space topology cannot change during the time evolution [38]. The fact that some solutions of the Einstein equation correspond to some topologies still does not mean that the Einstein equation describes the topology; it merely means that the solution must be consistent with a given topology. Moreover, the metric tensor does not even uniquely determine the topology. For example, the flat metric $\eta_{\mu\nu}$ does not necessarily imply that the corresponding manifold is infinite; it can also correspond to a torus or a cylinder. A similar statement is true for any other differential equation; if the solution $\phi(x)$ satisfies some periodicity conditions, we still do not know whether this solution corresponds to a closed or an infinite manifold (i.e., set of points $\{x\}$). If the Einstein equation can say anything at all about the topology, it can do that only indirectly. At least, this is so in classical gravity. Can quantum gravity change this? The set of space-time points and its topology is certainly a non-dynamical entity in all non-gravitational theories, both classical and quantum. We just argued that this is also so in classical gravity. So I really do not see why quantum gravity would change this, at least if quantum gravity is based on the quantization of some classical theory of the *metric tensor*, such as Einstein's theory. This can be seen most explicitly in the Heisenberg picture; one writes the general solution of classical equations consistent with a given topology and then just promotes all free parameters to the operators (assuming that one can solve technical problems connected with this). This can also be seen from the kinematics of the wave function $\Psi[g_{\mu\nu}(x); t]$; since it is a functional of $g_{\mu\nu}(x)$, it can be defined only if the set of points $\{x\}$ is previously well defined. However, even if one proposes that various topologies must be allowed in quantum theory, for example, by summing over topologies in a functional integral (although it is not clear what would then stay on the left-hand side of the analog of (19) and what the analog of the condition (20) would be), then one would expect that the sum over various signatures, or even dimensionalities of spacetime should be performed as well, because the Einstein equation itself does not fix them either. However, for some reason, such a possibility is

not usually considered. The sum over dimensionalities in quantum gravity would imply that even in nongravitational quantum theories the sum over dimensionalities should be performed.

If the topology must be fixed, as I just argued, the next question is what *is* the topology of the Universe? In order for the Cauchy problem to be well posed and canonical quantization possible, it is necessary that the topology is of the form $\Sigma \times \mathbb{R}$. There are no inconsistencies (as far as I know) for any choice of a connected, orientable $(D - 1)$ -dimensional manifold Σ without a boundary. Closed Σ s would still allow only oscillatory solutions, such as $e^{ik \cdot x}$, no longer by the finiteness requirement, but rather by the periodicity requirement. However, the choice $\Sigma = \mathbb{R}^{D-1}$ is the simplest and leads to the highest degree of symmetry between space and time. Thus Axiom 1 seems to be very natural. Of course, this axiom still allows effective closed topologies by an “accident”, if solutions of the equations of motion satisfy some periodicity conditions. If all fields (and wave functions) satisfy appropriate periodicity conditions, no observation can distinguish the “really” closed universe from the periodic one.

At the end, let me make a few comments on the dimensionality of space. The axioms of this article certainly cannot explain why space is three-dimensional. The answer to this question should be sought elsewhere. For example, superstring theory predicts that $D = 10$. It still cannot explain why six coordinates are compactified, but if they are, this is not in contradiction with Axiom 1, as I just discussed. Some types of effective compactifications, such as torus T^6 , are still possible.

There are interesting attempts to explain why space is three-dimensional based on certain anthropic considerations [4]. However, such arguments do not seem too convincing to me.

8. Conclusions

All differences between the role of space and time in nature can be explained by proposing a set of principles in which none of the space-time coordinates has an a priori special role. The essence of my approach is a proposal that all dynamical field equations *must* be treated as a Cauchy problem. This requires that the topology of spacetime must be fixed at the predynamical level. Various choices of topology of the form $\Sigma \times \mathbb{R}$ are admissible, but the choice $\mathbb{R}^D$ is the most natural and is the only one that does not give an a priori special role to any coordinate. The hyperbolicity, i.e., $(1, D - 1)$ signature of the metric, can be explained by proposing that *any* boundary condition that is finite everywhere must lead to a solution which is also finite everywhere. It also explains the boundedness of energy from below, the absence of tachyons, and other related properties of nature. It is quite likely that this principle must be realized in nature because it automatically prevents all kinds of physical singularities. The time arrow can be explained by proposing that the boundary condition is ordered, rather than random. The quantization can be considered as a boundary condition for the field operators. It appears natural to quantize the physical degrees of freedom only. This, together with the treatment of spacetime as a non-dynamical background, resolves a lot of conceptual problems in classical and quantum gravity, including the problem of time in quantum gravity.

It was no intention of this article to be mathematically rigorous. The main intention was to provide a complete conceptual understanding. A more rigorous treatment, as well as many technical details of some questions considered here, can be found in references cited. I hope that future investigations will also put all other ideas of this article into a more rigorous framework.

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