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Editorial

Review of Contributions to the Special Edition: Symmetry in Integrable Systems: Theory and Application

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Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of science. Finding the exact solutions for nonlinear differential equations has always been one of the central themes of perpetual interest in mathematics and physics. Symmetry reductions can be carried out using the classical Lie group approach, the nonclassical Lie group approach and the Clarkson–Kruskal direct method, which are effective methods for obtaining exact solutions of nonlinear differential equations. This Special Issue focuses on the symmetry reductions of the NPDEs and other miscellaneous applications of nonlinear integrable systems. For these topics, seven papers are published in this Special Issue "Symmetry in Integrable Systems: Theory and Application".

The papers can be roughly classified into several topics: symmetry reductions of the NPDEs [1–3], the construction of new systems using the Lie algebra [4,5], symmetry structures of the nonlinear systems [6] and studies on the extended Temme integral [7]. The following is a brief summary of each classified topic.

The third-order and fourth-order dispersions of the model, which are the cubic-quartic optical solitons with dispersive reflectivity in fiber Bragg gratings and the parabolic law of nonlinearity, have been studied by the Lie symmetry method. A variety of soliton solutions such as bright, dark, singular and combo solitons have been found by a Lie symmetry analysis coupled with Kudryashov and Arnous approaches [1]. The symmetry properties of the basic equations of atmospheric motion have been proposed. The results regarding symmetries show that the basic equations of atmospheric motion are invariant under the space–time translation transformation, Galilean translation transformations and scaling transformations. Eight one-parameter invariant subgroups and eight one-parameter group invariant solutions have been demonstrated. The large-scale atmospheric motion equations not only show the characteristics of physical quantities changing with time, but also describe the characteristics of large-scale atmospheric vertical motion [2]. The Sharma–Tasso–Olver–Burgers (STOB) system has been analyzed by the Lie point symmetry method. The hypergeometric wave solution of the STOB equation has been derived by symmetry reductions and the consistent tanh expansion (CTE) method has been applied to the STOB equation. A nonauto Bäklund (BT) theorem that includes the over-determined equations and the consistent condition has been obtained using the CTE method. By using the nonauto BT theorem, the interactions between one soliton and the cnoidal wave and between one soliton and the multiple resonant soliton solutions have been constructed. The dynamics of these novel interaction solutions were demonstrated both in analytical and graphical forms. The results are potentially useful for explaining ocean phenomena [3].

Based on a kind of $N \times N$ non-semisimple Lie algebra, the Lie algebraic structures of zero curvature representations for general multilayer integrable couplings have been explored. These systems possess τ-symmetries (a type of time-dependent symmetry) [4]. The Hamiltonian operators for the integrable couplings of the Ablowitz–Kaup–Newell–Segur hierarchy and the Kaup–Newell hierarchy have been presented, and the corresponding Hamiltonians were found to be nontrivial degenerations. Multi-Hamiltonian structures have also been investigated. The involutive property has been proven for the new and known Hamiltonians with respect to the two Poisson brackets defined by the new and known Hamiltonian operators [5].

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A correct set of solutions to the Maxwell equations for a rotational pair of left and right waves has been established. The key bilinear parameters of energy densities, Poynting vectors, field helicities and the spin angular momentum density were studied. By evaluating these key parameters on the parametric plane of the medium chirality and the oblique rotation angle, a collaborative and symmetric response has been obtained for a pair of counter-propagating plane waves. Several ways of exploiting such symmetric properties are suggested for applications in optics and physics [6].

A new triple integral containing a modified Bessel function along with some interesting definite integrals has been studied, and representations in terms of the Hurwitz–Lerch zeta function, polylogarithm function and Riemann zeta functions have been evaluated. The presented results have been numerically verified for real, imaginary and complex values of the parameters in the integrals. Almost all Lerch functions possess an asymmetrical zero distribution [7].

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