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A Trapezoidal Fuzzy Number-Based VIKOR Method with Completely Unknown Weight Information

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Abstract: Multi-attribute group decision-making plays an important role in modern politics, economy, culture, and life. The multi-attribute decision-making process is limited by policymakers' experience, and knowledge of uncertainty factors, such as weight information, is difficult to directly provide. Moreover, many types of complex rescue information are difficult to accurately describe in numerical terms, which reduces the accuracy and objectivity of the decision results, although the fuzzy theory to solve these problems provides a suitable tool. In order to solve the above problems, a new VIKOR decision method based on trapezoidal fuzzy numbers (TFNs) is proposed. Firstly, the expert weight is calculated by the distance measurement method, the criterion weight is calculated by the deviation maximization method, and then the VIKOR method is used to solve the problem. In order to verify the effectiveness and feasibility of this method, it is applied to the emergency alternative selection problem. Finally, the symmetry analysis of the method is carried out by contrast experiment and sensitivity test.

Keywords: trapezoidal fuzzy number; VIKOR; multi-attribute group decision making; weight

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Citation: Liu, G.; Wang, X. A Trapezoidal Fuzzy Number-Based VIKOR Method with Completely Unknown Weight Information. *Symmetry* **2023**, *15*, 559. https://doi.org/10.3390/sym15020559

Academic Editors: Kuo-Ping Lin, Chien-Chih Wang, Chieh-Liang Wu and Liang Dong

Received: 10 January 2023
Revised: 13 February 2023
Accepted: 16 February 2023
Published: 20 February 2023

1. Introduction

Multi-attribute decision-making is an essential part of modern decision-making science and operations research, including multiple decision attributes and multiple decision schemes. The purpose of multi-attribute decision-making (MADM) is to find the ideal solution by sorting or selecting the best alternative plan according to the multiple attributes in the case of conflicting and unshared multiple attributes. As society develops, decision-making problems become increasingly complex, making it unlikely that a single decision-maker will consider all aspects of a problem. Therefore, knowing how to make an accurate and optimal choice of alternative plans in a fuzzy environment has attracted great attention from researchers in recent years.

In decision-making in the real emergency plan selection, because of the uncertainty of the information and other factors, the decision maker is difficulty using the accurate numerical description of complex rescue information; determining the value used to describe the rescue plan of attribute weights is not completely accurate, so people often prefer to use natural language or uncertain or multi-granularity linguistic preference information to evaluate decision object. In the process of converting decision information to an exact number, there is often data distortion and information asymmetry between decision information and expert opinion. In order to make the expert opinion and decision data symmetrical, we use a fuzzy set to solve the above problems. Since Zadeh [1] proposed fuzzy set theory in 1965 to deal with fuzzy problems, linguistic variables have been used to describe complex evaluation information. A fuzzy set is a kind of tool to deal with fuzzy information and imprecise information, but due to various uncertainties, there are always various problems in practical applications. Considering that a decision-maker's personal preference information is often affected by different degrees of lack of experience or relevant professional knowledge, Atanassov [2,3] proposed a method including membership degree, non-membership degree, and hesitation degree. Compared with ordinary fuzzy sets, intuitive fuzzy sets are simpler and more effective in expressing preference information. The intuitive fuzzy set [4–7] cannot express the decision maker's preference. It can only roughly express the degree of membership, the membership of the scheme attribute, or the degree of "good" and "bad" of the fuzzy concept. Sriramdas V [8] proposed a trapezoidal fuzzy number representing the distribution factors in the initial stage of design optimization and proposed an approximation method of trapezoidal fuzzy number according to linear programming. The document [9] introduces ITrFN and puts forward the concept of a continuous set, which is a generalization of a discrete set and a fuzzy number. The intuitive trapezoidal fuzzy number can indicate the degree of "good" and "bad", as well as the different dimensions of decision information [10]. In recent years, multi-attribute decision-making based on linguistic variables has attracted great attention from researchers. Khalifa and Kumar [11] provide a new solution to the allocation problem. It introduces an interval trapezoidal neutral number to express parameters and uses the weighted Chebyshev program of the ideal target to solve the optimal allocation problem. Xu Zeshui [12–14] proposed the concept of triangular fuzzy linguistic variables. The fuzzy language average operator (FLA), fuzzy language weighted average operator (FLWA), fuzzy language ordered weighted average operator (FLOWA), and induced fuzzy ordered weighted average operator (IFLOWA) are proposed and used in group decision-making. In 2005, Xu Zeshui [15] proposed the concept of trapezoidal fuzzy language variables, established the similarity formula between two trapezoidal fuzzy language values, and gave the scheme ranking based on trapezoidal fuzzy language variables multi-attribute decision-making based on similarity. The trapezoidal fuzzy number is also applied to the problem of optimal strategy for fuzzy inventory. Kumar [16] provides an optimal decision method for a fuzzy inventory model considering holding. Liang Xuechun [17] puts forward a trapezoidal fuzzy language weighted average operator (TFLWA) for trapezoidal fuzzy language variables and proposes a ranking method of a trapezoidal fuzzy language variable multi-attribute decision scheme based on a probability formula. The trapezoidal bipolar fuzzy number is proposed in [18], which enables decision-makers to assign preference information of substitutes with different attributes in the form of trapezoidal bipolar fuzzy number.

The VIKOR method is a compromise ranking method proposed by Opricovic [19] in 1998, and it was first applied by Opricovic and Tzeng [20] to solve fuzzy multi-attribute decision-making problems. The VIKOR method focuses on sorting and selecting from a set of alternatives: either the TOPSIS method cannot reflect the scheme of the service and the shortcoming of the proximity of the positive and negative ideal solution; then, considering the maximization of the utility of the group and the personal regret of minimization, it can also give full consideration to the preferences and the properties of conflict problem determination compromise, to help decision makers make rational decisions . Therefore, the VIKOR method is widely used to solve fuzzy multi-attribute decision-making problems [21–23]. However, the traditional VIKOR method cannot deal with the MADM problem with language criterion weight information, and knowing how to combine the calculation of decision maker weight and attribute weight with the VIKOR method given a set of decision information is important . Aiming at the above problems, this paper proposes a VIKOR method in which the weights of decision experts and attributes are completely unknown and the attribute values are trapezoidal fuzzy numbers.

In the multi-attribute decision-making process, weight can be divided into expert weight and attribute weight. The accuracy of weight will directly affect the accuracy of the decision results. Therefore, this paper not only considers the subjective preferences of decision-makers but also makes full use of objective information on decision objects to achieve the unity of subjective and objective. The following is a brief introduction to subjective and objective weights. Expert weight can also be called subjective weight, which is the important weight information obtained by decision-makers based on their own experience and subjective understanding of the importance of each attribute. It reflects the accumulation of previous experience and the professional knowledge of decision-makers as well as a subjective grasp of the existing decision-making background based on objective facts. Attribute weight, also known as objective weight, is an important parameter of multi-attribute decision-making, reflecting the objective importance of the attributes of decision-making objects. Accurate objective weight will greatly improve the accuracy of decision-making results.

The rest of this article is organized as follows. Section 2 introduces the basic concept of trapezoidal fuzzy numbers, arithmetic operations, distance measures, and the VIKOR method. Section 3 presents an overall framework and a VIKOR method for calculating expert weight and attribute weight based on the trapezoidal fuzzy numbers. In Section 4, the proposed method is applied to a case and the comparative analysis and sensitivity test are carried out. Finally, Section 5 discusses the main implications of this research and summarizes the main advantages, limitations, and future research directions of this research work.

2. Preliminaries

In fuzzy set theory, there are various numerical forms to describe fuzzy information, but sometimes the scientificity and accuracy of the final decision result may be greatly reduced because of the particularity of the decision problem and the complexity of the decision information. To solve the above problems, the relevant concepts of triangular interval fuzzy number [24,25] are proposed. The triangular fuzzy number can deal with the fuzzy problems of insufficient data or low precision very well [26] and has been successfully applied to the reliability analysis of air missile systems [27] and the reliability distribution research of high-speed punch press [28]. The numerical form can be used to describe the decision information more accurately, which can effectively reduce the phenomenon of inaccurate decision results caused by the inaccurate representation of decision information. However, there are still some problems in applying a triangular fuzzy number to decision-making on practical issues [29,30], namely, because the membership function curve of a triangular fuzzy number [31–33] is triangular, and only one point can be used to represent the most likely value, as shown in Figure 1 below. However, due to the influence of fuzzy factors in practical issues, there is often no clear peak value, which shows a flat distribution. Therefore, using triangular fuzzy numbers to deal with these parameters will lead to large simulation errors, which will affect the results. The trapezoidal fuzzy number can easily solve the above problems. While having the advantages of triangular fuzzy numbers, its membership function curve shows the trapezoidal distribution, which has a good fitting effect for parameter distributions with a relatively wide peak value. This is shown in Figure 2 below. Therefore, the multi-attribute decision-making method based on the trapezoidal fuzzy number is used to study emergency plan selection.

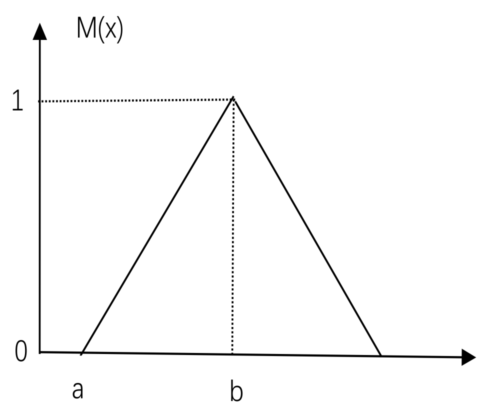

Figure 1. Triangular fuzzy number.

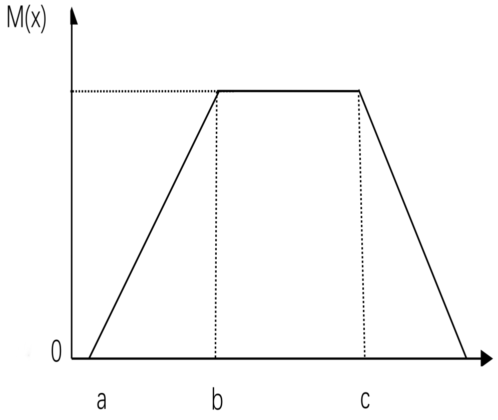

Figure 2. Trapezoidal fuzzy number.

2.1. Trapezoidal Fuzzy Numbers

Definition 1 (The TFNs). *Given the final domain U, let A be the set of real numbers in the domain, where r_1, r_2, r_3, and r_4 are the real numbers in the set A; $0 < r_1 \le r_2 \le r_3 \le r_4$; and then $M = (r_1, r_2, r_3, r_4)$ is A trapezoidal fuzzy number. Its membership function is denoted as $M_A(x)$.*

$$M_A(x) = \begin{cases} 0 & x < r_1 \\ \frac{x-r_1}{r_2-r_1} & r_1 \le x < r_2 \\ 1 & r_2 \le X \le r_3 \\ \frac{r_4-x}{r_4-r3} & r_3 \le x \le r_4 \\ 0 & x \ge r_4 \end{cases} \quad (1)$$

In particular, if $r_2 = r_3$, the trapezoidal fuzzy number M degenerates to a triangular fuzzy number, and if $r_1 = r_2 = r_3 = r_4$, the trapezoidal fuzzy number M degenerates to a real number. The membership function combines every U in the domain U with A number $M_A(U)$ in the interval [0, 1]. $M_A(U)$ is called the membership degree, which represents the qualification of U in A, that is, the membership degree of elements to fuzzy sets. When $M_A(U)$ is closer to 1, the degree of U belonging to A is higher. The fuzzy combination is reduced to the classical regular set.

Definition 2 (The algorithms). $M_1 = [a_1, b_1, c_1, d_1], M_2 = (a_2, b_2, c_2, d_2), M_3 = [a_3, b_3, c_3, d_3]$ *for the three trapezoidal fuzzy numbers, and $M_A(x) \in [0, 1]$; the trapezoid fuzzy number algorithm is as follows:*

- $M_1 \oplus M_2 = [a_1, b_1, c_1, d_1] \oplus [a_2, b_2, c_2, d_2] = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]$
- $M_1 \otimes M_2 = [a_1, b_1, c_1, d_1] \otimes [a_2, b_2, c_2, d_2] = [a_1a_2, b_1b_2, c_1c_2, d_1d_2]$
- *If* $0 < a \le b \le c \le d$, *then* $\frac{M_1}{M_2} = \left[\frac{a_1}{a_2}, \frac{b1}{b_2}, \frac{c_1}{c_2}, \frac{d_1}{d_2}\right]$
- $\lambda M_1 = \lambda[a_1, b_1, c_1, d_1] = [\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1]$
- $M_1 \oplus M_2 = M_2 \oplus M_1$
- $(\lambda + \lambda_1)M_1 = \lambda M_1 \oplus \lambda_1 M_1$
- $\lambda(M_1 + M_2) = \lambda M_1 + \lambda M_2$

Definition 3 (The distance of TFNs). *Let $M_1 = [a_1, b_1, c_1, d_1]$ and $M_2 = [a_2, b_2, c_2, d_2]$ be two trapezoidal fuzzy numbers*

$$d(M_1, M_2) = \frac{|a1 - a2| + |b1 - b2| + |c1 - c2| + |d1 - d2|}{4t} \quad (2)$$

is the distance between M_1 and M_2. Where t is the number of linguistic variables in the linguistic variable set M. The smaller the value of $d(M_1, M_2)$, the closer the distance between M_1 and M_2

Definition 4 (Distance between decision matrices)**.** *Let* $R_1 = \left(r_{ij}^1\right)_{m\times n}, R_2 = \left(r_{ij}^2\right)_{m\times n}$ *be two decision matrices, and the distance between the two decision matrices is as follows:*

$$D_K = D(R_1, R_2) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(r_{ij}^1, r_{ij}^2\right), k = 1, 2, \ldots, t \tag{3}$$

where k is the number of experts.

2.2. Vikor Method

Opricovic and Tzeng [34] first proposed the method. This method is mainly used to solve multi-attribute decision-making problems with coexistence, incompatible, and conflicting criteria. It focuses on sorting and selecting among a range of alternatives and determining the best solution, to solve the conflict of norms by compromise, to help decision-makers to make a final decision. Setting: $X = \{x_1, x_2, \ldots, x_m\}$ is the set of alternatives, $C = \{c_1\ , c_2, \ldots, c_n\}$ is the set of n attributes corresponding to each scheme, $w = (w_1, w_2, \ldots, w_n)$ is the attribute weight vector, and $w_j \geq 0$. Let $A = (a_{ij})_{\mathrm{m}\times\mathrm{n}}$ be the decision matrix with trapezoidal fuzzy language evaluation information,

$$A = \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix}$$

where a_{ij} represents the scheme $M_i \in M(i = 1, 2, \ldots, m)$, which corresponds to the attribute $C_j \in C(j = 1, 2, \ldots, N)$, an evaluation of the results, while $f_{ij} = [f_{ij}^{(T)}, f_{ij}^{(U)}, f_{ij}^{(V)}, f^{(Z)_{ij}}]$ gives policymakers a trapezoidal fuzzy language variable value . $f_{ij}^{(T)}, f_{ij}^{(U)}, f_{ij}^{(V)}, f^{(Z)_{ij}}$ is predefined language evaluation elements. According to the above evaluation information, the comprehensive evaluation steps of the trapezoidal fuzzy number multi-attribute decision scheme are as follows:

- Sept 1: Determine the positive ideal solution f_j^+ and the negative ideal solution f_j^-

$$f_j^+ = \max_{1\leq i\leq m} f_{ij}, j = 1, 2, \ldots, n$$
$$f_j^- = \min_{1\leq i\leq m} f_{ij}, j = 1, 2, \ldots, n$$

- Step 2: Calculate the maximum group utility value S_i and the minimum individual loss degree R_i

$$S_i = \sum_{j=1}^{n}\left[\frac{w_j d\left(f_j^+, f_{ij}\right)}{d\left(f_j^+, f_j^-\right)}\right], i = 1, 2, \cdots, m \tag{4}$$

$$R_i = \max_{1\leq i\leq m}\left[\frac{w_j d\left(f_j^+, f_{ij}\right)}{d\left(f_j^+, f_j^-\right)}\right], i = 1, 2, \cdots, m \tag{5}$$

Among them, $w_j(j = 1, 2, \ldots, n)$ represents the weight of the attribute, f_j^+ stands for positive ideal solution, f_j^-stands for negative ideal solution, and f_{ij} stands for expert ideal solution.

- Step 3: Calculate the compromise solution Q_i

$$Q_i = \lambda\frac{(S_i - S^-)}{(S^+ - S^-)} + (1-\lambda)\frac{(R_i - R^-)}{(R^+ - R^-)} \tag{6}$$

where $S^+ = \max S_i, S^- = \min S_i, R^+ = \max R_i$ $R^- = \min R_i$, and λ represent mechanism parameters, which are generally set as 0.5

- Step 4: Sort according to the compromise solution Q_i

All of the alternatives are ranked in descending order according to the value of the compromise solution Q_i. The first one is the optimal compromise solution in the decision problem, the second one is the suboptimal compromise solution, and the others are recursed in turn.

2.3. ELECTRE Method

Benavoun, Roy, and Sussman first proposed the ELECTRE [35–37] (the Elimination Et Choice Translation Reality) method in the 1960s. Later, Roy Skarka, Bertil, Hugonard, Yu, and others further developed this method to form different ELECTRE families Variation [38]. The basic idea of the Electre method is to eliminate inferior schemes in the following way, a series of weak predominance relations are constructed to gradually reduce the scheme set until decision-makers can choose the most satisfactory option. Since the construction method of weak dominance relationship is based on the test of "harmony" and "disharmony", the Electre method is also called the harmony analysis method.

2.4. Topsis Method

The Topsis [39–42] method is a multi-attribute decision-making method based on geometry. It evaluates m schemes under n attributes, which is similar to m points in n-dimensional space. In this n-dimensional space, an optimal ideal point (denoted as V^+) is found, each attribute value of which reaches the best value in each point. Find an imaginary worst negative ideal point (denoted as V^-) in the n-dimensional space, and each attribute value of it is the worst value of each point. In the original scheme set X, each scheme V^+ is compared with V^-, and the distance information between them is used as the standard for sorting m schemes.

3. Vikor Based on Completely Unknown Weight Information of Trapezoidal Fuzzy Number

In real life, limited by the experience, knowledge reserve, and thinking inertia of the decision maker, it is often difficult to give the exact value of the attribute, and the weight of the decision maker and the weight of the attribute are difficult to determine, so it is more suitable to express them with linguistic variables. To solve the above problems, this paper proposes the TFN-VIKOR method, which determines the expert weight based on the distance measure and determines the attributes using the deviation maximization method.

3.1. Framework of Decision-Making Method

This section describes the framework of decision-making method, this is shown in Figure 3 below,and we can see the whole method in this flowchart.

3.2. Identify Alternatives and Criteria

Let $X = (x_1, x_2, \ldots, x_m)$ be the set of N alternatives, $C = (C_1, C_2, \ldots, C_n)$ be the set of N attributes corresponding to each scheme, and $w = (w_1, w_2, \ldots, w_n)^T$ be the attribute weight. In fuzzy multiple attribute decision-making problems, the decision makers usually use value and attribute the value of a fuzzy number to represent [43–46] in dealing with complicated language variables or definitions that are very useful in which the situation is not clear; in such cases, it is difficult to use the traditional number for the appropriate expression and description . Policymakers can use it to describe language variables (LV) and then language variables in trapezoidal fuzzy numbers, as shown in Tables 1 and 2.

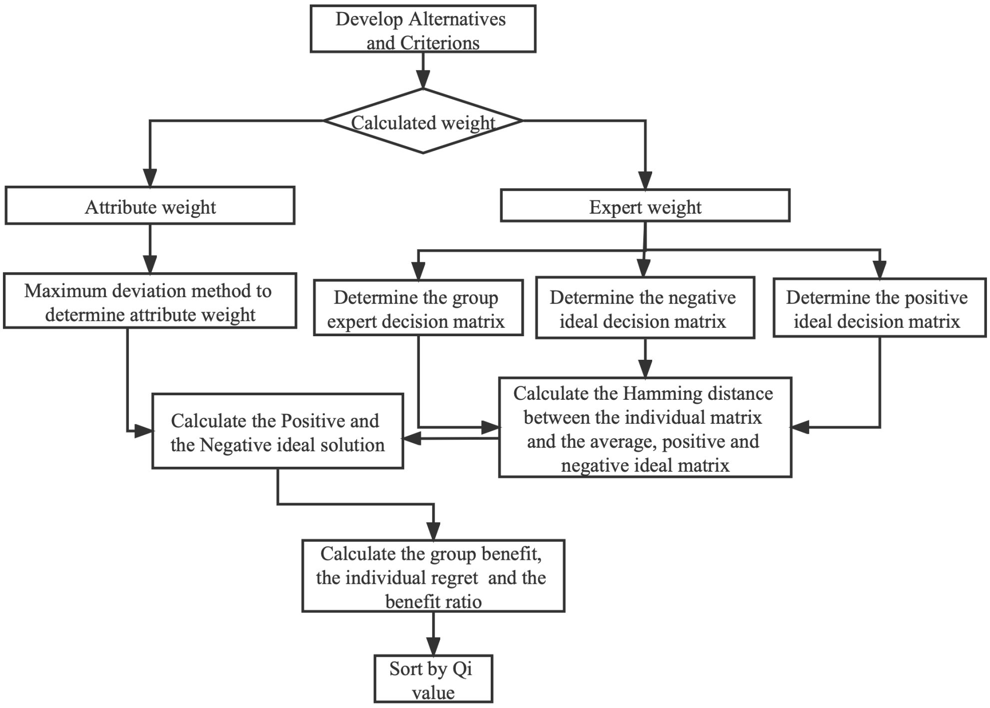

Figure 3. The framework of the decision-making method.

Table 1. Definitions of linguistic variables for the importance of each criterion.

Very Low (VL)	(0.0, 0.0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.2, 0.3)
Medium Low (ML)	(0.2, 0.3, 0.4, 0.5)
Medium (M)	(0.4, 0.5, 0.5, 0.6)
Medium High (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.8, 0.9)
Very High (VH)	(0.8, 0.9, 1.0, 1.0)

Table 2. Definitions of linguistic variables for the ratings.

Very Poor (VP)	(0, 0, 0, 0.091)
Poor (P)	(0, 0.091, 0.182, 0.273)
Moderately Poor (MP)	(0.182, 0.273, 0.364, 0.455)
Fair (F)	(0.364, 0.455, 0.545, 0.636)
Moderately Good (MG)	(0.545, 0.636, 0.727, 0.818)
Good (G)	(0.727, 0.818, 0.909, 1)
Very Good (VG)	(0.909, 1, 1, 1)

3.3. Calculate the Weight of Decision Experts

Due to information uncertainty, a lack of experience, and inaccuracy of human knowledge, it is impossible for decision experts to accurately calculate alternatives without considering various attributes. This paper innovatively uses the method based on the trapezoidal fuzzy matrix distance measure to solve the expert weight. The specific steps are as follows:

- Step 1: Determine the group expert decision matrix $R^* = \left(R^*_{ij}\right)_{m\times n}$ and the positive and negative ideal decision matrices $R^+ = \left(r^+_{ij}\right)_{m\times n}, R^- = \left(r^-_{ij}\right)_{m\times n}$

$$\boldsymbol{R}^*_{ij} = \frac{r^1_{ij} + r^2_{ij} + \cdots + r^t_{ij}}{t} \tag{7}$$

Among them, $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, where t is the number of experts, and r_{ij}^{t} is the individual expert decision matrix.

$$R_{ij}^{+} = \left(\left[\max_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{1i}^{(k)}, \max_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{2i}^{(k)}, \max_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{4i}^{(k)}, \max_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{1i}^{(k)} \right] \right) \tag{8}$$

$$R_{ij}^{-} = \left(\left[\min_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{1i}^{(k)}, \min_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{2i}^{(k)}, \min_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{4i}^{(k)}, \min_{\substack{1 \le k \le t \\ 1 \le i \le m}} r_{1i}^{(k)} \right] \right) \tag{9}$$

- Step 2: Determine the distance D_{K}^{+}, D_{k}^{-} between the individual expert decision matrix and the positive and negative ideal decision matrix

$$D_{k}^{+} = D\left(R_k, R^{+}\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} d\left(r_{ij}^{(k)}, r_{ij}^{+}\right), k = 1, 2, \cdots, t \tag{10}$$

$$D_{K}^{-} = D\left(R_k, R^{-}\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} d\left(r_{ij}^{(k)}, r_{ij}^{-}\right), k = 1, 2, \cdots, t \tag{11}$$

where the k represents the expert's serial number.

- Step 3: Give the expert weight λ_k according to the distance between decision matrices

$$\lambda_k = \frac{\lambda_k^*}{\sum_{k=1}^{t} \lambda_k^*}, \lambda_k^* = \frac{D_k^{+} + D_k^{-}}{D_k^*} \tag{12}$$

where the k represents the expert's serial number.

3.4. Calculate Attribute Weights

In this paper, the maximum deviation method is used to determine the weight of attributes. According to the core idea of the spread maximization method, from the point of view conducive to ranking schemes, the greater the deviation of the attribute value of the alternative scheme, the greater the weight of the attribute should be. On the contrary, the smaller the deviation of the scheme attribute value, the smaller the weight of the attribute should be. In particular, if all schemes have no difference in attribute value under a certain attribute, then this attribute has no effect on the sorting of schemes, and its weight can be set to zero.

For attributes $C_j, D_{ij}(w)$, the said plan of x_i and all other deviations can be defined as

$$D_{ij}(w) = \sum_{k=1}^{m} d\left(\boldsymbol{r}_{ij}, \boldsymbol{r}_{kj}\right) \boldsymbol{W}_j, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$$

and let

$$D_j(w) = \sum_{i=1}^{n} D_{ij}(w) = \sum_{i=1}^{n} \sum_{k=1}^{n} d\left(r_{ij}, r_{kj}\right) w_j, j = 1, 2 \cdots, n \tag{13}$$

Attribute C_j, $D_j(w)$ represents the total deviation of all schemes from other schemes, and the attribute weight vector should be selected to maximize the total deviation of all attributes from all schemes. To this end, the deviation function is constructed.

$$\max D(w) = \sum_{j=1}^{n} D_j(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right) w_j \tag{14}$$

Therefore, solving the weight vector W problem is equivalent to solving the following single objective optimization problem

$$\begin{cases} \max D(w) = \sum_{j=1}^{n} D_j(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right) w_j \\ \text{s.t. } \sum_{j=1}^{m} w_j = 1 \end{cases}$$

Construct the Lagrangian function as follows:

$$L(\boldsymbol{W}_j, \lambda) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} d\left(\boldsymbol{r}_{ij}, \boldsymbol{r}_{kj}\right) \mathcal{W}_j + \lambda \left(\sum_{j=1}^{n} w_j^2 - 1 \right)$$

let

$$\begin{cases} \frac{\alpha L(w_j,\lambda)}{\alpha w_j} = \sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right) + 2\lambda w_j = 0 \\ \frac{\alpha L(w_j,\lambda)}{\alpha \lambda} = \sum_{j=1}^{n} w_j^2 - 1 = 0 \end{cases}$$

Solving the established deviation maximization method model can be obtained as follows:

$$\begin{cases} 2\lambda = \sqrt{\sum_{j=1}^{n} \left(\sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right) \right)^2} \\ w_j = \frac{\sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right)}{\sqrt{\sum_{j=1}^{n} \left(\sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right) \right)^2}} \end{cases}$$

After normalizing the above solution, the weight value of each attribute can be obtained

$$w_j = \frac{\sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right)}{\sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{m} d\left(r_{ij}, r_{kj}\right)}, j = 1, 2, \cdots n \tag{15}$$

3.5. Calculate the Positive Ideal Solution f^+ and the Negative Ideal Solution f^-

The Positive Ideal Solution f^+ and the Negative Ideal Solution f^- as follows:

$$\boldsymbol{f}^+ = [\boldsymbol{f}_1^+, \boldsymbol{f}_2^+, \boldsymbol{f}_3^+, \boldsymbol{f}_4^+], \boldsymbol{f}^- = [\boldsymbol{f}_1^-, \boldsymbol{f}_2^-, \boldsymbol{f}_3^-, \boldsymbol{f}_4^-]$$

among them

$$\begin{aligned} \boldsymbol{f}_j^+ &= \left[\max r_{ij}^{(T)}, \max r_{ij}^{(U)}, \max r_{ij}^{(V)}, \max r_{ij}^{(Z)} \right], \quad j \in I_1 \\ \boldsymbol{f}_j^+ &= \left[\min r^{(T)}, \min r_{ij}^{(U)}, \min r_{ij}^{(V)}, \min r_{ij}^{(Z)} \right], \quad j \in I_2 \\ \boldsymbol{f}_j^+ &= \left[\max r_{ij}^{(T)}, \max r_{ij}^{(U)}, \max r_{ij}^{(V)}, \max r_{ij}^{(Z)} \right], \quad j \in I_1 \\ \boldsymbol{f}_j^+ &= \left[\min r_{ij}^{(T)}, \min r_{ij}^{(U)}, \min r_{ij}^{(V)}, \min r_{ij}^{(Z)} \right], \quad j \in I_2 \end{aligned}$$

I_1 indicates that the attribute is a benefit indicator, and I_2 indicates that the attribute is a cost indicator.

3.6. Group Benefit S_i, Individual Regret R_i, and Benefit Ratio Q_i Generated by Each Scheme Are Calculated

According to the definition of distance in Equation (2), the group benefit S_i, the individual regret R_i, and the benefit ratio $Q_i$generated by each scheme are calculated.

The decision is made according to the decision mechanism of maximizing the group effect. If $\lambda < 0.5$, the decision is made according to the decision mechanism that minimizes individual regret. In general, $\lambda = 0.5$ means that the decision is made from the perspective of equilibrium.

3.7. Choose the Best Solution

According to the order of Q_i value from small to large, the compromise ranking of finite decision schemes is carried out, and then the optimal scheme is selected.

4. Application

In this section, we use an example to illustrate the application of the TFN-VIKOR method in emergency plan selection.

4.1. An Illustrative Example

This paper uses the example in literature [29] to verify the effectiveness of the above decision-making methods. In order to implement the rescue operation of a city fire in China, a committee composed of three decision-makers (DM_1, DM_2 and DM_3) shall evaluate and select the most appropriate emergency plan. Select five possible emergency plans A_i (i = 1, 2, . . . , 5) and six standard C_j (j = 1, 2, . . . , 6) for further evaluation, namely, the information transmission capability (C_1), the command capability (C_2), the speed of the rescue department (C_3), the emergency plan and simulation exercise (C_4), the cooperation capability (C_5), and the prediction capability (C_6). The weights of the six criteria are described by using the linguistic term set LT_1 = very low (VL), low (L), medium Low (ML), medium (M), medium high (MH), high (H), and very high (VH), which are defined in Table 3. The performance ratings of the alternatives with respect to criteria are characterized by the linguistic term set: LT_2 = Very poor (VP), poor (P), medium poor (MP), fair (F), medium good (MG), good (G), and very good (VG), which is defined in Table 4. Among them, considering the difference in experience and knowledge reserved between experts, it is difficult to make a subjective evaluation of the weight of experts, and it is impossible to give the weight of the experts. Similarly, the weight of evaluation criteria cannot be given. It is necessary to determine the expert weight and criterion weight in a fuzzy multi-criteria decision-making process. In view of the fact that objective weights cannot be given above, this paper expresses attribute weight information and expert evaluation information in the form of TFN and combines it with the TFN-Vikor method proposed in this paper to calculate objective weights; then, the best solution is chosen.

Table 3. Attribute weights.

Criterion	DM_1	DM_2	DM_3
C_1	H	H	H
C_2	VH	VH	H
C_3	VH	VH	VH
C_4	H	H	MH
C_5	H	H	H
C_6	H	MH	M

Table 4. Decision makers' assessments based on each criterion.

Criterion	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
	A_1	G	G	VG
C_1	A_2	VG	G	G
	A_3	VG	VG	G
	A_4	G	G	G
	A_5	MG	MG	MG
	A_1	VG	VG	VG
C_2	A_2	MP	G	G
	A_3	G	G	VG
	A_4	G	G	MG
	A_5	F	F	F

Table 4. *Cont.*

Criterion	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
C_3	A_1	G	G	G
	A_2	G	F	MP
	A_3	F	F	F
	A_4	MG	MG	G
	A_5	MG	MG	MG
C_4	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	VG	VG	VG
	A_4	G	G	G
	A_5	MG	MG	G
C_5	A_1	G	G	G
	A_2	VG	MG	VG
	A_3	G	VG	G
	A_4	G	G	VG
	A_5	MG	MG	MG
C_6	A_1	VG	G	VG
	A_2	G	MG	G
	A_3	G	G	VG
	A_4	MG	F	MG
	A_5	F	F	MG

4.2. Decision Process

Next, we will use the TFN-VIKOR method proposed in this paper to analyze the emergency alternatives.

- Step 1 Decision makers can use linguistic term sets to transform linguistic variables into trapezoidal fuzzy numbers, LV1 = very low (VL), low (L), medium low (ML), medium (M), medium high (MH), high (H), and very high (VH), as shown in Table 1 to assess the importance of criteria. Additionally, the linguistic terms set LV2 = very poor (VP), poor (P), medium poor (MP), fair (F), medium good (MG), good (G), and very good (VG) were used to evaluate the rating of each criterion for deciding alternatives, and then the fuzzy decision matrix given by expert DM_k transformed into a normalized matrix R_{ij}^k=(r_{ij}^k)$_{mn}$, in which $r_{ij}^k = [r_{ij}^1, r_{ij}^2, r_{ij}^3, r_{ij}^4]$
for the benefit attribute index :

$$\begin{cases} x_{ij}^1 = r_{ij}^1 / \sqrt{\sum_{i=1}^m \left(r_{ij}^1\right)^2 + \left(r_{ij}^2\right)^2 + \left(r_{ij}^3\right)^2 + \left(r_{ij}^4\right)^2} \\ x_{ij}^2 = r_{ij}^2 / \sqrt{\sum_{i=1}^m \left(r_{ij}^1\right)^2 + \left(r_{ij}^2\right)^2 + \left(r_{ij}^3\right)^2 + \left(r_{ij}^4\right)^2} \\ x_{ij}^3 = r_{ij}^3 / \sqrt{\sum_{i=1}^m \left(r_{ij}^1\right)^2 + \left(r_{ij}^2\right)^2 + \left(r_{ij}^3\right)^2 + \left(r_{ij}^4\right)^2} \\ x_{ij}^4 = r_{ij}^4 / \sqrt{\sum_{i=1}^m \left(r_{ij}^1\right)^2 + \left(r_{ij}^2\right)^2 + \left(r_{ij}^3\right)^2 + \left(r_{ij}^4\right)^2} \end{cases}$$

and for the cost attribute index:

$$\begin{cases} x_{ij}^1 = \left(1/r_{ij}^4\right) / \sqrt{\sum_{i=1}^m \left(1/r_{ij}^1\right)^2 + \left(1/r_{ij}^2\right)^2 + \left(1/r_{ij}^3\right)^2 + \left(1/r_{ij}^4\right)^2} \\ x_{ij}^2 = \left(1/r_{ij}^3\right) / \sqrt{\sum_{i=1}^m \left(1/r_{ij}^1\right)^2 + \left(1/r_{ij}^2\right)^2 + \left(1/r_{ij}^3\right)^2 + \left(1/r_{ij}^4\right)^2} \\ x_{ij}^3 = \left(1/r_{ij}^2\right) / \sqrt{\sum_{i=1}^m \left(1/r_{ij}^1\right)^2 + \left(1/r_{ij}^2\right)^2 + \left(1/r_{ij}^3\right)^2 + \left(1/r_{ij}^4\right)^2} \\ x_{ij}^4 = \left(1/r_{ij}^1\right) / \sqrt{\sum_{i=1}^m \left(1/r_{ij}^1\right)^2 + \left(1/r_{ij}^2\right)^2 + \left(1/r_{ij}^3\right)^2 + \left(1/r_{ij}^4\right)^2} \end{cases}$$

- Step 2 It is very important to determine the expert weight in the decision-making process, and the objective expert weight can ensure the accuracy of the decision result. Equations (7)–(12) are used to calculate the weight of experts, and the results are shown in Table 5

Table 5. Weight of experts.

λ_j	λ_1	λ_2	λ_3
Value	0.235346	0.426254	0.338399

- Step 3 Due to objective reasons, it is difficult for experts to directly give attribute weight, but attribute weight is essential becuse it can describe the importance of the criterion and ensure the smooth progress of the decision. Calculate the weight of attributes according to Equation (15), and the results are shown in Table 6.

Table 6. Weight of criterion.

W_j	W_1	W_2	W_3	W_4	W_5	W_6
Value	0.120110	0.239118	0.188429	0.117355	0.133333	0.201653

Put the weight information calculated above into the VIKOR method as follows:

- Step 4: Determine the comprehensive weighted decision matrix and positive and negative ideal solutions according to Equations (15) and (16). The results are shown in Table 7.

Table 7. Positive ideal solution and negative ideal solution of alternatives.

A_i	Positive Ideal Solution	Negative Ideal Solution of Alternatives
A_1	[0.8, 0.9, 1.0, 1.0]	[0.7, 0.8, 0.8, 0.9]
A_2	[0.8, 0.9, 1.0, 1.0]	[0.2, 0.3, 0.4, 0.5]
A_3	[0.8, 0.9, 1.0, 1.0]	[0.4, 0.5, 0.5, 0.6]
A_4	[0.8, 0.9, 1.0, 1.0]	[0.4, 0.5, 0.5, 0.6]
A_5	[0.5, 0.6, 0.7, 0.8]	[0.4, 0.5, 0.5, 0.6]

- Step 5: Calculate the maximum group utility S_i, the minimum individual regret Ri, and the compromise quantity Qi according to Formulae (4)–(6). The results are shown in Tables 8 and 9.

Table 8. Maximum group utility si and minimum individual regret ri.

A_i	S_i	R_i
A_1	0.100425	0.045830
A_2	0.659151	0.054996
A_3	0.288254	0.045830
A_4	0.283595	0.054996
A_5	0.141998	0.054996

Table 9. Compromise measure Q_I.

A_i	Q_i
A_1	0
A_2	1
A_3	0.168086
A_4	0.663917
A_5	0.537203

- Step 6 Sort according to the Qi value. Since $Q_1 < Q_3 < Q_5 < Q_4 < Q_2$, according to the Q_i value, the sorting scheme $A_1 > A_3 > A_5 > A_4 > A_2$ can be obtained, so the source of the fault is A_1, which is consistent with the results in [29].
  The Q_i values, S_i values, and R_i values of the five alternative schemes are shown in Figure 4. The best scheme obtained is a1 according to the ranking of Q_i values from small to large, which is the same as the results in the literature [29], which proves that this model is feasible.

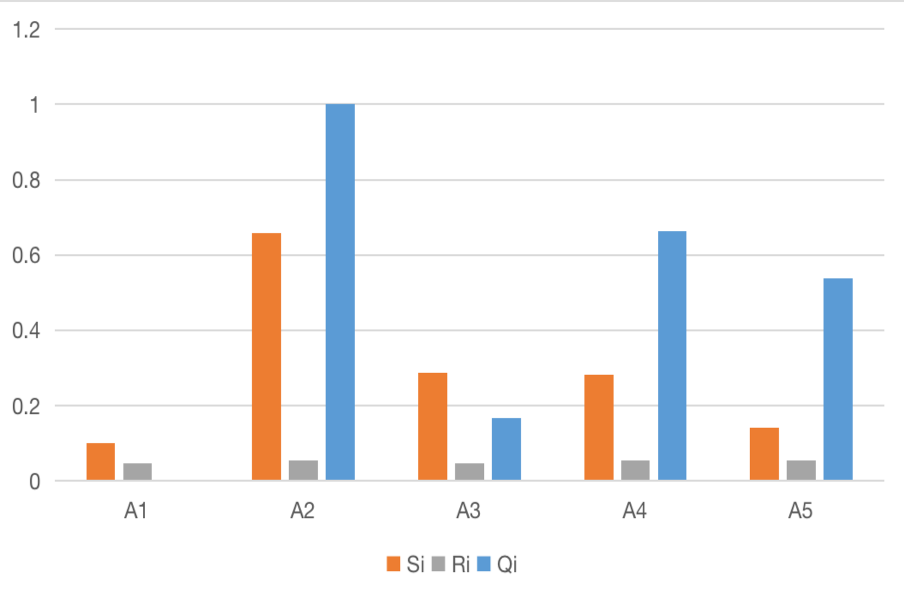

Figure 4. The value of alternatives under this model.

5. Results and Discussion

In this part, the results of different methods are analyzed and compared, and the stability of the proposed methods is discussed.

5.1. Comparative Analysis

In order to further verify the effectiveness of this method, we use the attribute weights in [29] (0.1681, 0.1828, 0.1933, 0.1513, 0.1681, and 0.1366), to obtain S_i, R_i, Q_i shown in Tables 10 and 11:

Table 10. Maximum group utility si and minimum individual regret ri.

A_i	S_i	R_i
A_1	0.107454	0.119950
A_2	0.649072	0.158154
A_3	0.259795	0.041545
A_4	0.268954	0.052718
A_5	0.115772	0.041263

Table 11. Compromise measure Q_I.

A_i	Q_i
A_1	0
A_2	1
A_3	0.154561
A_4	0.209590
A_5	0.020429

As shown in Table 11, with the same weight, the result is $A_1 < A_5 < A_3 < A_4 < A_2$. The best rescue plan is A_1 again. The order of other schemes is slightly different, mainly because of different decision-making methods and different ways of expressing decision information. The proposed model can be extended to deal with uncertainties in new neutrosophic types, such as [47,48]; We can also change the representation in the model to Pythagorean trapezoidal fuzzy aggregation operators [49] or to double exponential smoothing (DES) model combination [50],when the attribute index of the decision object is expressed as a trapezoidal fuzzy number and the weight is unknown, it is believed that the model proposed in this paper can achieve better results.

5.2. Stability Analysis

In order to test the effect of the proposed method on the ranking results, a sensitivity analysis was performed on the attitude parameters of decision-makers. First, change the size of the parameter, and then sort the three using the TFN-VIKOR method scheme, analyze the change of ranking, and verify the stability of the method in this case. Taking 0.1 as the interval and 0.1, 0.2, 0.3, 0.4, and 0.5 as the four alternatives, the test was carried out, and the stability analysis results were obtained as shown in Table 12.

Table 12. Influence of different λ values on sorting results.

λ	A_1	A_2	A_3	A_4	A_5	Ranking
0.1	0	1	0.033617	0.832783	0.807440	$A_1 < A_3 < A_5 < A_4 < A_2$
0.2	0	1	0.067234	0.865567	0.814880	$A_1 < A_3 < A_5 < A_4 < A_2$
0.3	0	1	0.100852	0.798350	0.722322	$A_1 < A_3 < A_5 < A_4 < A_2$
0.4	0	1	0.134469	0.731134	0.629762	$A_1 < A_3 < A_5 < A_4 < A_2$
0.5	0	1	0.168086	0.663917	0.537203	$A_1 < A_3 < A_5 < A_4 < A_2$

It can be seen from Table 12 and Figure 5 that the sequence of schemes $A_1, A_2, A_3A_4A_5$ did not change in the five experiments, and their relative proximity changed slightly, indicating that parameter λ had no influence on the experimental results. When a disaster occurs, if it is not dealt with in time, it will cause serious damage to all aspects. Therefore, the ranking results should be affected by subjective factors as little as possible, which further shows that the method in this paper has strong stability in decision making.

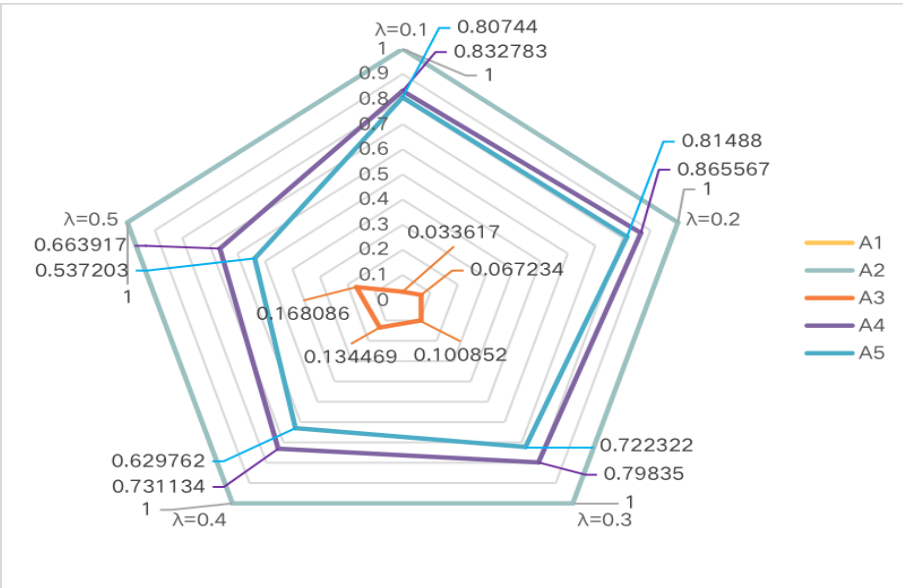

Figure 5. The effect of different λ values on sorting results

6. Conclusions

In real life, many practical decision problems have MADM characteristics. Due to the lack of knowledge or data, the decision-makers limited expertise on this issue, or the decision-making problem is an unfamiliar area for experts, the attribute weights and standard values given by decision-makers often take the form of linguistic variables. Based on the traditional VIKOR method, a new VIKOR method is proposed for multi-criteria group decision-making. The contribution of this paper is to first calculate the expert weight using the distance measurement method. Unlike traditional expert weight-calculation methods, the TFN-VIKOR method combines the TOPSIS method and geometric view and uses the distance measure between the positive ideal decision matrix, the negative ideal decision matrix, and the individual expert decision matrix in group experts to calculate the expert weight. Then, the maximum deviation method is applied to calculate the weight of attributes, and finally the VIKOR method is used to solve the problem. The proposed method is compared with the TOPSIS/ELECTRE method, and the sensitivity analysis experiment is carried out by changing the weight coefficient of the standard to verify the effectiveness and feasibility of the proposed method. The method is simple and easy to implement on a computer. It has the advantages of avoiding information distortion and information loss in the process of language information processing and can objectively and accurately obtain expert weight and attribute weight. It can be used for engineering and management problems in any other field that cannot be accurately judged due to human subjective assumptions and can and help to make the best decision. The limitation of this method lies in the fact that it cannot express the preference of decision makers due to the singleness of information expression, such as the preferred degree of an alternative scheme or the hesitation degree of their own judgment. For the limitations proposed above, different data-expression methods can be adopted, such as the intuitive Fermatean fuzzy number, which can not only express the degree of expert preference but also enable the evaluation information to obtain a larger representation range due to the characteristics of Fermatean fuzzy set. It is also possible to use a trapezoidal bipolar fuzzy number, which can have a bipolar structure using information. A bipolar fuzzy set establishes a symmetrical trade-off. Between the two judgments of human thinking, the evaluation information of decision experts will become uncertain and fuzzy. Using trapezoidal bipolar fuzzy numbers can solve such problems, and decision-makers can also use trapezoidal bipolar fuzzy numbers to determine the decision preference.

Author Contributions: Conceptualization, G.L. and X.W. All authors have read and agreed to the published version of the manuscript.

Funding: This project was supported by the Key R & D project of Shandong Province, China (2019GGX101026).

Data Availability Statement: The data presented in this study are available in [Extension of VIKOR method for multi-criteria group decision making problem with linguistic information].

Conflicts of Interest: The authors declare no conflict of interest.

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