

Supplementary Materials

Computational Analysis for Fréchet Parameters of Life from Generalized Type-II Progressive Hybrid Censored Data with Applications in Physics and Engineering

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Table S1: The APEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of δ .

(T_1, T_2) Prior \rightarrow	(n, m)	Scheme	MLE				MCMC				
			1	2	1	2	1	2	1	2	
(0.4,0.8)	(40,20)	1	0.5076	0.2045	0.3810	0.3095	0.1994	0.3161	0.3455	0.1663	0.3110
		2	0.3422	0.1799	0.3206	0.4826	0.1588	0.2970	0.3560	0.1578	0.2554
		3	0.3032	0.2071	0.3937	0.4955	0.1439	0.2387	0.3876	0.1404	0.2328
	(40,32)	1	0.5000	0.1816	0.3227	0.3418	0.1501	0.2604	0.3722	0.1482	0.2352
		2	0.4923	0.1373	0.2352	0.4275	0.1364	0.2291	0.6155	0.1337	0.2179
		3	0.4993	0.1326	0.2299	0.4741	0.1217	0.2108	0.3854	0.1025	0.1764
	(80,40)	1	0.3719	0.1436	0.2572	0.5093	0.1347	0.2109	0.5894	0.1206	0.2020
		2	0.4909	0.1317	0.2134	0.4919	0.1154	0.1879	0.4271	0.1053	0.1773
		3	0.5546	0.1040	0.1676	0.4948	0.0957	0.1528	0.4200	0.0894	0.1425
	(80,64)	1	0.4275	0.1209	0.1925	0.4059	0.1028	0.1893	0.5014	0.1020	0.1613
		2	0.4036	0.1159	0.1980	0.4973	0.0947	0.1502	0.5003	0.0814	0.1376
		3	0.4212	0.1003	0.1632	0.5003	0.0920	0.1449	0.4382	0.0770	0.1335
(0.8,1.2)	(40,20)	1	0.3554	0.1687	0.2960	0.4995	0.1651	0.2908	0.6454	0.1641	0.2807
		2	0.3608	0.1610	0.2832	0.4825	0.1564	0.2480	0.6067	0.1281	0.2147
		3	0.3777	0.1373	0.2450	0.5035	0.1183	0.2092	0.3957	0.1114	0.1857
	(40,32)	1	0.3641	0.1496	0.2596	0.3895	0.1349	0.2409	0.5043	0.1341	0.2120
		2	0.5807	0.1354	0.2377	0.3814	0.1254	0.2041	0.4953	0.1232	0.1955
		3	0.4244	0.1055	0.1781	0.5413	0.1028	0.1651	0.5275	0.0997	0.1446
	(80,40)	1	0.3656	0.1463	0.2757	0.4314	0.1278	0.1994	0.5062	0.1197	0.1870
		2	0.4064	0.1134	0.1928	0.5003	0.1089	0.1793	0.4271	0.1027	0.1734
		3	0.4728	0.0976	0.1660	0.4770	0.0865	0.1459	0.5037	0.0814	0.1282
	(80,64)	1	0.3975	0.1165	0.2065	0.4080	0.1007	0.1858	0.5029	0.0921	0.1454
		2	0.4485	0.1032	0.1767	0.4998	0.0857	0.1349	0.4513	0.0779	0.1283
		3	0.4659	0.0966	0.1559	0.5202	0.0705	0.1010	0.4679	0.0547	0.0900

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Table S2: The APEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of θ .

(T_1, T_2)	(n, m)	Scheme	MLE				MCMC				
					1		2				
(0.4,0.8)	(40,20)	1	1.9152	0.4509	0.2768	1.8784	0.4150	0.2537	1.5989	0.3735	0.1909
		2	1.6339	0.3646	0.2032	1.7685	0.3621	0.1997	1.7992	0.3148	0.1863
		3	1.5944	0.3217	0.1792	1.7555	0.3039	0.1724	1.7685	0.2907	0.1622
	(40,32)	1	1.8152	0.3725	0.2193	1.7107	0.3004	0.1626	1.7833	0.2991	0.1571
		2	1.7564	0.3384	0.1885	1.7031	0.2928	0.1552	1.7152	0.2538	0.1449
		3	1.6944	0.2866	0.1522	1.5762	0.2748	0.1450	1.7084	0.2301	0.1396
	(80,40)	1	1.6661	0.2598	0.1463	1.5345	0.2385	0.1251	1.5430	0.2171	0.1133
		2	1.7118	0.2538	0.1400	1.5596	0.2293	0.1194	1.5426	0.2158	0.1136
		3	1.6418	0.2019	0.1215	1.6013	0.1780	0.0954	1.5203	0.1577	0.0831
	(80,64)	1	1.5340	0.1991	0.1048	1.6265	0.1966	0.1032	1.5779	0.1737	0.0924
		2	1.5402	0.1937	0.0989	1.5422	0.1884	0.0986	1.5903	0.1193	0.0640
		3	1.5326	0.1829	0.0945	1.5873	0.1618	0.0866	1.5219	0.1020	0.0500
(0.8,1.2)	(40,20)	1	1.9111	0.4460	0.2742	1.6322	0.3996	0.2200	1.8173	0.3899	0.1827
		2	1.8481	0.3588	0.2021	1.5945	0.3207	0.1651	1.7195	0.3035	0.1568
		3	1.7673	0.2996	0.1782	1.7054	0.2726	0.1455	1.5586	0.2406	0.1234
	(40,32)	1	1.5844	0.3038	0.1891	1.7626	0.2792	0.1752	1.7563	0.2728	0.1711
		2	1.5966	0.3018	0.1568	1.5632	0.2607	0.1339	1.5772	0.2528	0.1322
		3	1.6633	0.2505	0.1313	1.6835	0.2290	0.1245	1.5005	0.2155	0.1108
	(80,40)	1	1.6998	0.2568	0.1443	1.7158	0.2365	0.1372	1.6557	0.1766	0.1019
		2	1.6953	0.2379	0.1331	1.6780	0.2208	0.1223	1.5782	0.1715	0.0994
		3	1.5443	0.1973	0.1004	1.6824	0.1899	0.1019	1.6320	0.1559	0.0837
	(80,64)	1	1.6640	0.1846	0.1100	1.6172	0.1818	0.0956	1.5250	0.1712	0.0897
		2	1.5032	0.1725	0.0923	1.5302	0.1693	0.0903	1.4390	0.1652	0.0888
		3	1.4867	0.1403	0.0834	1.4580	0.1380	0.0744	1.5487	0.0936	0.0506

Table S3: The APEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of $R(t)$.

(T_1, T_2)	(n, m)	Scheme	MLE				MCMC				
					1		2				
Prior →											
(0.4,0.8)	(40,20)	1	0.9106	0.0532	0.0486	0.9178	0.0438	0.0372	0.9689	0.0257	0.0235
		2	0.9188	0.0474	0.0388	0.9810	0.0315	0.0306	0.9713	0.0251	0.0231
		3	0.9273	0.0362	0.0322	0.9552	0.0267	0.0234	0.9544	0.0261	0.0221
	(40,32)	1	0.9062	0.0525	0.0452	0.9752	0.0296	0.0279	0.9468	0.0248	0.0210
		2	0.9874	0.0363	0.0356	0.9541	0.0268	0.0226	0.9457	0.0239	0.0196
		3	0.9827	0.0324	0.0300	0.9550	0.0257	0.0218	0.9541	0.0251	0.0217
	(80,40)	1	0.9218	0.0434	0.0369	0.9763	0.0267	0.0262	0.9523	0.0220	0.0185
		2	0.9822	0.0315	0.0315	0.9453	0.0251	0.0200	0.9535	0.0183	0.0156
		3	0.9752	0.0253	0.0245	0.9105	0.0245	0.0188	0.9530	0.0184	0.0158
	(80,64)	1	0.9388	0.0255	0.0202	0.9526	0.0199	0.0168	0.9481	0.0185	0.0151
		2	0.9386	0.0241	0.0193	0.9531	0.0187	0.0159	0.9469	0.0171	0.0139
		3	0.9382	0.0240	0.0193	0.9529	0.0186	0.0158	0.9477	0.0168	0.0138
(0.8,1.2)	(40,20)	1	0.9186	0.0472	0.0389	0.9525	0.0403	0.0303	0.9586	0.0298	0.0206
		2	0.9222	0.0432	0.0365	0.9524	0.0304	0.0255	0.9466	0.0249	0.0227
		3	0.9742	0.0286	0.0269	0.9712	0.0247	0.0220	0.9469	0.0232	0.0217
	(40,32)	1	0.9871	0.0358	0.0354	0.9807	0.0314	0.0251	0.9534	0.0266	0.0224
		2	0.9820	0.0319	0.0315	0.9539	0.0257	0.0217	0.9536	0.0237	0.0210
		3	0.9519	0.0260	0.0231	0.9539	0.0253	0.0213	0.9694	0.0229	0.0194
	(80,40)	1	0.9537	0.0263	0.0240	0.9107	0.0219	0.0235	0.9943	0.0201	0.0221
		2	0.9915	0.0200	0.0212	0.9528	0.0186	0.0167	0.9488	0.0179	0.0143
		3	0.9519	0.0186	0.0183	0.9524	0.0181	0.0152	0.9155	0.0142	0.0139
	(80,64)	1	0.9375	0.0209	0.0210	0.9522	0.0196	0.0166	0.9488	0.0178	0.0147
		2	0.9416	0.0195	0.0173	0.9536	0.0180	0.0156	0.9481	0.0168	0.0139
		3	0.9397	0.0177	0.0158	0.9364	0.0161	0.0141	0.9478	0.0159	0.0131

Table S4: The APEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of $h(t)$.

(T_1, T_2)	(n, m)	Scheme	MLE				MCMC				
					1		2				
(0.4,0.8)	(40,20)	1	1.3577	0.6881	0.8115	0.2936	0.4972	0.6170	0.7382	0.3654	0.3875
		2	1.3803	0.6714	0.7886	1.1611	0.4907	0.5365	0.7214	0.3174	0.3348
		3	1.1947	0.5434	0.5898	0.4098	0.3909	0.4696	0.7176	0.2942	0.3065
	(40,32)	1	1.2208	0.5834	0.6275	0.4253	0.3882	0.4616	0.7208	0.3009	0.3186
		2	1.0334	0.4895	0.4027	0.9774	0.3589	0.4260	0.7155	0.2863	0.3027
		3	0.4958	0.3613	0.3995	0.9510	0.3561	0.3663	0.7182	0.2847	0.2980
	(80,40)	1	0.5904	0.3644	0.4733	1.2379	0.3362	0.4458	0.7429	0.2428	0.2567
		2	0.4850	0.3622	0.3712	0.4433	0.3205	0.3653	0.7484	0.2083	0.2213
		3	0.6022	0.3022	0.3357	0.8662	0.2405	0.2578	0.7466	0.1967	0.2067
	(80,64)	1	1.0051	0.3341	0.3528	0.9012	0.2406	0.2541	0.7440	0.2100	0.2204
		2	0.9674	0.2855	0.3020	0.8927	0.2173	0.2301	0.7428	0.1960	0.2056
		3	0.9495	0.2666	0.2807	0.8692	0.2021	0.2135	0.7424	0.1947	0.2045
(0.8,1.2)	(40,20)	1	0.7543	0.4620	0.3419	0.2170	0.5623	0.7152	0.2952	0.4896	0.6127
		2	0.7395	0.3539	0.3654	1.1581	0.5075	0.5464	0.1635	0.4050	0.4855
		3	0.7191	0.2756	0.2911	0.4020	0.4106	0.4839	0.4472	0.3905	0.4490
	(40,32)	1	0.7249	0.2936	0.3101	1.1966	0.5552	0.5950	1.0833	0.3800	0.4469
		2	0.7252	0.2837	0.2970	1.1351	0.4270	0.4937	0.4377	0.3781	0.4266
		3	0.7053	0.2499	0.2631	0.5140	0.3386	0.3813	0.9372	0.3242	0.3310
	(80,40)	1	0.7311	0.2440	0.2563	0.7471	0.3141	0.3308	0.9407	0.3452	0.3561
		2	0.7389	0.2003	0.2093	0.9747	0.2942	0.3122	0.5970	0.2632	0.3005
		3	0.7443	0.1907	0.2006	0.9658	0.2817	0.3004	0.9075	0.2324	0.2464
	(80,64)	1	0.7474	0.2078	0.2184	0.9714	0.3013	0.3152	0.8947	0.2208	0.2328
		2	0.7447	0.1938	0.2037	0.6280	0.2803	0.3028	0.8706	0.2190	0.2311
		3	0.7314	0.1802	0.1926	0.9181	0.2621	0.2683	0.8117	0.1747	0.1893

Table S5: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD credible intervals of δ .

(T_1, T_2)	(n, m)	Scheme	ACI		HPD			
					1	2		
(0.4,0.8)	(40,20)	1	0.743	0.912	0.460	0.934	0.304	0.948
		2	0.630	0.918	0.415	0.937	0.299	0.949
		3	0.526	0.925	0.348	0.942	0.293	0.950
	(40,32)	1	0.578	0.921	0.383	0.939	0.285	0.952
		2	0.528	0.924	0.333	0.954	0.277	0.954
		3	0.517	0.929	0.312	0.957	0.230	0.964
	(80,40)	1	0.521	0.928	0.345	0.943	0.266	0.958
		2	0.448	0.936	0.316	0.961	0.221	0.965
		3	0.367	0.944	0.287	0.967	0.170	0.973
	(80,64)	1	0.407	0.941	0.238	0.964	0.167	0.981
		2	0.374	0.944	0.233	0.965	0.168	0.982
		3	0.344	0.946	0.222	0.970	0.160	0.984
(0.8,1.2)	(40,20)	1	0.639	0.919	0.460	0.934	0.302	0.948
		2	0.582	0.923	0.357	0.940	0.295	0.950
		3	0.525	0.925	0.300	0.947	0.278	0.956
	(40,32)	1	0.507	0.928	0.351	0.941	0.276	0.954
		2	0.477	0.931	0.323	0.955	0.226	0.966
		3	0.453	0.933	0.297	0.958	0.212	0.968
	(80,40)	1	0.455	0.933	0.314	0.951	0.236	0.962
		2	0.418	0.938	0.244	0.965	0.219	0.969
		3	0.369	0.945	0.227	0.969	0.169	0.975
	(80,64)	1	0.357	0.945	0.236	0.963	0.165	0.983
		2	0.339	0.948	0.208	0.972	0.163	0.983
		3	0.320	0.951	0.205	0.973	0.158	0.985

Table S6: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD credible intervals of θ .

(T_1, T_2)	(n, m)	Scheme	ACI		HPD			
					1	2		
(0.4,0.8)	(40,20)	1	1.377	0.949	0.861	0.963	0.619	0.978
		2	1.348	0.952	0.761	0.967	0.482	0.984
		3	1.102	0.959	0.730	0.970	0.470	0.985
	(40,32)	1	1.151	0.957	0.822	0.966	0.526	0.980
		2	1.107	0.960	0.741	0.971	0.373	0.987
		3	1.058	0.963	0.649	0.976	0.368	0.988
	(80,40)	1	0.934	0.962	0.686	0.974	0.495	0.984
		2	0.901	0.965	0.647	0.977	0.356	0.988
		3	0.742	0.971	0.538	0.980	0.317	0.991
	(80,64)	1	0.785	0.968	0.563	0.979	0.364	0.989
		2	0.749	0.970	0.521	0.982	0.315	0.991
		3	0.722	0.973	0.499	0.984	0.280	0.993
(0.8,1.2)	(40,20)	1	1.210	0.955	0.807	0.968	0.609	0.979
		2	1.184	0.958	0.742	0.970	0.464	0.986
		3	1.010	0.963	0.675	0.975	0.453	0.986
	(40,32)	1	0.974	0.959	0.791	0.971	0.503	0.982
		2	0.968	0.961	0.712	0.974	0.354	0.989
		3	0.891	0.966	0.547	0.978	0.322	0.991
	(80,40)	1	0.817	0.970	0.683	0.975	0.451	0.986
		2	0.812	0.971	0.556	0.979	0.352	0.991
		3	0.702	0.977	0.497	0.983	0.310	0.993
	(80,64)	1	0.669	0.978	0.510	0.984	0.359	0.990
		2	0.662	0.978	0.484	0.987	0.312	0.993
		3	0.612	0.981	0.464	0.988	0.274	0.996

Table S7: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD credible intervals of $R(t)$.

(T_1, T_2)	(n, m)	Scheme	ACI		HPD		
					1	2	
Prior \rightarrow							
(0.4,0.8)	(40,20)	1	0.115	0.881	0.106	0.897	0.101
		2	0.108	0.884	0.095	0.908	0.078
		3	0.102	0.886	0.068	0.917	0.059
	(40,32)	1	0.104	0.893	0.098	0.905	0.079
		2	0.099	0.895	0.072	0.914	0.065
		3	0.098	0.896	0.059	0.925	0.040
	(80,40)	1	0.096	0.907	0.088	0.918	0.072
		2	0.074	0.912	0.066	0.923	0.060
		3	0.073	0.913	0.057	0.928	0.039
	(80,64)	1	0.084	0.910	0.078	0.924	0.038
		2	0.072	0.913	0.063	0.928	0.032
		3	0.071	0.915	0.049	0.934	0.028
(0.8,1.2)	(40,20)	1	0.114	0.884	0.102	0.902	0.079
		2	0.108	0.887	0.080	0.914	0.062
		3	0.099	0.900	0.066	0.920	0.043
	(40,32)	1	0.103	0.896	0.094	0.912	0.076
		2	0.098	0.898	0.070	0.917	0.058
		3	0.097	0.902	0.058	0.925	0.042
	(80,40)	1	0.085	0.910	0.080	0.926	0.060
		2	0.072	0.915	0.061	0.936	0.055
		3	0.071	0.915	0.055	0.944	0.052
	(80,64)	1	0.077	0.914	0.070	0.932	0.034
		2	0.071	0.916	0.060	0.939	0.027
		3	0.070	0.917	0.044	0.950	0.013

Table S8: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD credible intervals of $h(t)$.

(T_1, T_2)	(n, m)	Scheme	ACI		HPD			
					1	2		
(0.4,0.8)	(40,20)	1	1.410	0.897	1.316	0.903	1.160	0.915
		2	1.311	0.905	1.121	0.912	1.049	0.919
		3	1.104	0.909	1.079	0.921	1.018	0.922
	(40,32)	1	1.292	0.912	1.058	0.922	0.997	0.929
		2	1.053	0.924	0.972	0.930	0.955	0.935
		3	1.040	0.927	0.845	0.934	0.716	0.944
	(80,40)	1	0.960	0.922	0.868	0.937	0.757	0.941
		2	0.957	0.926	0.778	0.940	0.753	0.942
		3	0.751	0.933	0.721	0.944	0.709	0.946
	(80,64)	1	0.859	0.929	0.750	0.942	0.716	0.945
		2	0.746	0.931	0.630	0.947	0.570	0.958
		3	0.727	0.935	0.539	0.949	0.520	0.961
(0.8,1.2)	(40,20)	1	1.340	0.905	1.288	0.914	1.124	0.919
		2	1.108	0.926	1.093	0.928	0.795	0.935
		3	1.029	0.929	0.909	0.931	0.784	0.936
	(40,32)	1	1.239	0.916	0.979	0.929	0.925	0.933
		2	1.043	0.930	0.817	0.935	0.803	0.939
		3	1.005	0.934	0.799	0.936	0.673	0.949
	(80,40)	1	0.939	0.929	0.830	0.936	0.730	0.942
		2	0.766	0.935	0.726	0.943	0.690	0.947
		3	0.739	0.937	0.659	0.948	0.552	0.954
	(80,64)	1	0.809	0.933	0.741	0.945	0.643	0.948
		2	0.744	0.938	0.600	0.952	0.500	0.961
		3	0.697	0.941	0.483	0.957	0.311	0.970