A Brief Overview of the Special Issue "Symmetry and Ultradense Matter in Compact Stars"

Mannque Rho [ID]

Institut de Physique Théorique, Université Paris-Saclay, CNRS, CEA, 91191 Gif-sur-Yvette, France; mannque.rho@ipht.fr

The Standard Model, comprising electroweak (EW) and strong (QCD) interactions, has been established and tested with great accuracy. What takes place in ultradense matter in massive stars on the verge of gravitational collapse is, however, almost completely not understood, presenting a vista of an uncharted domain. There is, of course, the issue of the theory of gravitation which is applicable in massive stars, but given the assumption that it is the Tolman–Oppenheimer–Volkoff (TOV) equation resulting from the general relativistic hydrostatic that should play the predominant role, what is at issue is, then, QCD in the matter density regime from a few times the nuclear matter density $n_0 \sim 0.16 \, \text{fm}^{-3}$ to $\sim 10 \, n_0$, at which point gravitational collapse takes place. There will surely be MDPI Special Issues devoted to the role of gravitational theories per se, so the current Special Issue will be focused entirely on the equation of state (EoS) of baryonic matter.

Now, how to access dense matter in QCD is, at present, it might be fair to say, almost totally unknown.

Although QCD is a non-Abelian gauge theory with UV completion, thanks to the asymptotic freedom, perturbation theory should be applicable at asymptotic densities; but the density involved in compact stars is far from asymptotic. The only known nonperturbative approach to access baryonic matter at non-asymptotic densities is QCD on lattice, but it is blocked by the (in)famous sign problem. Thus, there are presently no "top-down" approaches to compact star densities (denoted as n_{star}) that one can trust in the sense that nuclear matter at n_0—and near n_0—can be quantitatively accurately explained.

Now, turned around, going bottom-up in density scale from the well-understood equilibrium nuclear-matter regime $\sim n_0$ to star density $\sim n_{star}$, faces equally serious difficulties. Up to n_0 and perhaps even slightly above, there is Steven Weinberg's "Folk Theorem" for nuclear effective field theory (nEFT) anchored on chiral symmetry involving nucleons and pions, which is now established and works quite well. As stated by Weinberg, how the nEFT with nucleons and pions only—which can be aptly called "soft-pion nEFT"—works is as follows: In nuclear physics, "although nucleons are not soft they never get far from their mass shell, and for that reason can be also treated by similar methods as the soft pions. Nuclear physicists have adopted this point of view, and . . . they are happy about using this new language because it allows one to show in a fairly convincing way that what they've been doing all along (using two-body potentials only, including one-pion exchange and a hard core) is the correct first step in a consistent approximation scheme". However, as befits an EFT, it is bound to break down as one goes beyond the regime of soft pions, as required to access high densities lodged in the core of massive stars. This has much to do with what happens to the equation of state (EoS) of the baryonic matter as the relevant degrees of freedom change from "low" to "high" in density, as it must involve the change-over from hadronic degrees of freedom to quark/gluon degrees of freedom. It is thus unwarranted, as one often reads in the literature, to herald nEFT as the *first-principles* approach to nuclear astrophysics.

Given the near total absence of a reliable theoretical framework, what is indispensable at present is the input from Nature.

There is a fast-growing effort to look for guides from both terrestrial and space laboratories. One such effort is addressed in [1]. The strategy is to exploit rapidly developing deep learning techniques to extract the equation of state (EoS) directly from observations from terrestrial laboratories that are in operation or being built and ongoing multimessenger astrophysical observatories. It will provide inputs from Nature on how matter transforms from hadrons to the constituents of QCD, fermions (quarks) and bosons (gluons) in the regime where the most difficult aspects of strong interaction phenomena overlap in terms of EFTs and non-Abelian gauge theories. This will provide guides for model-building and then what could be trusted effective field theories.

The current research in the absence of guides consists of "groping" in two directions, bottom-up and top-down, loosely delineated at the density range $\sim(2\text{–}4)n_0$.

The bottom-up direction is characterized by the "density ladder" [2] starting with nuclei at low densities, typically in an energy-density functional (EDF) that belongs to Density Functional Theory (DFT) à la Hohenberg–Kohn theorem, climbing up to higher densities extracting the changes in the density dependence of the EoS. The prime examples are the first and second derivatives with respect to the density n of the symmetry energy $E_{sym}(n)$. The curvatures give how the symmetry energy behaves going up from $n = n_0$. They are measured in electron scattering experiments looking at the neutron skin thickness of nuclei. This sets up the first ladder. The next step [3] is to bridge over the regime where the Folk Theorem signals the potential breakdown of nEFT—or, equivalently, the energy-density functional (EDF) approach—by incorporating at a suitable density region roughly $\gtrsim 3n_0$, a cusp [4] or inflection [3] structure in the symmetry energy to capture a possible hadron–quark/gluon changeover. A microscopic model based on baryon–quark hybrid construction on a Fermi surface, referred to as quarkyonic, has also been studied in the literature. (Unfortunately, this particular development, which is closely connected to topological structure [4], is not represented in this Special Issue). At this stage, the constraints provided by deep-learning and other AI could come to help guide the bridge-building.

Given the close connection between the skyrmion structure of nucleons and the non-relativistic constituent quark model at large N_c limits, the skyrmion approach to dense matter is a potentially promising way of accessing nuclei from low density to high density. A highly detailed sophisticated study of skyrmion matter at a high density [5] reveals an intricate structure not seen in past many-nucleon approaches. Limited to the original Skyrme model consisting of pions only, however, it misses certain highly nontrivial effects of hidden local symmetry (HLS) and scale symmetry (HSC), playing a key role in increasing densities in general [6] and in compact stars in particular [4,7]. Not directly connected to compact-star physics is Seiberg duality, conjectured to be operative between hidden local vector mesons in the hadronic sector and gluons in the QCD sector [6]. While rigorously established in the presence of supersymmetry, certain Seiberg-type dualities seem to work magically, even though there is no supersymmetry, in hadronic physics with hidden local symmetric vector mesons [6]. For instance, the hidden/homogeneous (h)Wess–Zumino term in the HLS Lagrangian is found to encode the Chern–Simons topological fractional quantum Hall structure of the $N_f = 1$ baryon, which is absent in skyrmion physics. In unifying the FQH droplet for the $N_f = 1$ baryon to the skyrmions for $N_f > 1$ baryons, it is the hWZ term with Sakurai's "Vector Dominance" (VD) which has had no theoretical derivation since its inception in 1960s. There is an intriguing hint of the Cheshire-Cat (CC) phenomenon between the CC smile and the quark number charge [4].

There are two lessons in the above development that this Special Issue brings to our attention [8]. First, certain aspects of the quark-gluon physics of QCD could be dual via topology to hidden hadronic workings. Second, moving to high density, hidden scale symmetry could intervene to reveal phenomena that are not visible in QCD per se. The mechanism invoked [4] to simulate hadron–quark continuity taking place at a density where nEFT breaks down, phrased in terms of the skyrmion-half-skyrmion crossover, a topology change, without an explicit role of hadrons to quarks, involves a sort of what is known in

condensed matter physics as a "pseudo-gap phenomenon". The density-dependent vacuum quark condensate $\langle \bar{q}q \rangle$ vanishes globally at the crossover point—denoted $n_{1/2}$—but the pion decay constant f_π which is locked to the dilaton condensate $\langle \sigma_d \rangle \propto \langle \chi \rangle$ remains non-zero. This means that chiral symmetry is not restored. This structure captures what is incorporated microscopically in the quarkyonic model constructed in the Fermi sphere. This process involves no bona fide Ginzburg–Landau–Wilsonian-type phase transitions. Nonetheless, driven by the sliding dilaton condensate as density increases, the skyrmion–quantum–Hall unification [6] exposes [8] the presence of the $U_A(1)$ scalar meson η', which is considered to figure at high density associated with the baryonic parity doublet symmetry [9]. Parity-doublet symmetry, argued to be playing an important role in chiral symmetry restoration, could be an emergent symmetry [4], rather than intrinsic in QCD, playing a crucial role in leading to the pseudo-conformal model of compact stars with a sound speed of $v_s^2/c^2 \approx 1/3$ [7]. This variety of scenarios of the EoS of highly dense matter crucially depend on how the masses of the hadrons involved scale in dense matter toward the potential chiral symmetry restoration, which has been extensively studied for years in terms of the QCD sum rules anchored on the operator-product expansion [10]. Now, QCD sum-rules address on-shell properties of hadrons in medium and, hence, involve the issue of how the "mass parameters" of EFT Lagrangians figuring in many-body nuclear interactions can be related to the on-shell quantities. The issue of the chiral-invariant nucleon mass m_0, whether emergent or intrinsic in QCD [9], could be studied in this approach. This class of issues has been a thorny one in nuclear physics for a long time, and it still remains as such in interpreting experiments in terms of vacuum-dependent quantities, e.g., in heavy-ion collisions.

In resorting to effective field theories such as GnEFT [4,7] in attempting to go beyond the putative hadron–quark/gluon crossover, the cut-off scale is taken roughly at the vector meson mass $m_{\rho,\omega} \sim 700$ MeV. There are attempts to go beyond this scale, such as, for instance, holographic models anchored on gauge-gravity duality. Such models generically incorporate higher energy scales than those of the lightest vector meson mass with an unknown UV completion which must be inevitably different from that of QCD. A variety of such models based on gauge-gravity are addressed in [11]. The striking features of such models are that, due to the tower of vector excitations encoded in the instantons figuring as topological objects, the model can account for the Vector Dominance (VD) not only in the mesonic EM form factors but also in baryonic systems, giving further support for Sakurai's seminal idea. One can also see an emerging dyonic "salt" structure analogous to half-skyrmions, possible layers of "popcorns" and other exotic configurations of fractional baryon charges indicated in half-skyrmion structures found to play a key role in giving conformal or, rather, pseudo-conformal sound speed in the interior of compact stars. The notion of the Cheshire-Cat phenomenon, as hinted in [4], from low density to high density, including the asymptotically high density at which color–flavor locking figures in QCD in terms of "superqualitons"—1/3-charged skyrmions—make "qualitons", which are unstable at low densities, become stable at high densities. This plethora of exotic objects could render high-density physics a lot richer than the naive structure presented in perturbative QCD.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Krastep, P.G. Deep learning approach to extracting nuclear matter properties from neutron star observations. *Symmetry* **2023**, *15*, 1123. [CrossRef]
2. Salinas, M.; Piekarewicz, J. Building an equation of state density ladder. *Symmetry* **2023**, *15*, 994. [CrossRef]
3. Papaconstantinou, P.; Hyun, C.H. Energy-density modeling of strongly interacting matter: Atomic nuclei and dense stars. *Symmetry* **2023**, *15*, 683. [CrossRef]
4. Ma, Y.-L.; Yang, W.-C. Toology and emergent symmetries in dense compact star matter. *Symmetry* **2023**, *15*, 776. [CrossRef]

5. Adam, C.; Martin-Caro, A.G.; Huido, M.; Werenszynski, A. Skyrme crystals, nuclear matter and compact stars. *Symmetry* **2023**, *15*, 899. [CrossRef]

6. Karasik, A. From skyrmions to one flavored baryons and beyond. *Symmetry* **2022**, *14*, 2347. [CrossRef]

7. Rho, M. Psudo-conformal sound speed in the core of compact stars. *Symmetry* **2022**, *14*, 2154. [CrossRef]

8. Rho, M. Mapping topology of skyrmions and fractional quantum Hall droplets to nuclear EFT for ultra-dense baryonic matter. *Symmetry* **2022**, *14*, 994. [CrossRef]

9. Minamikayawa, T.; Gao, B.; Kojo, T.; Harada, M. Chiral restoration of nucleons in neutron star matter: Studies based on a parity-doublet model. *Symmetry* **2023**, *15*, 745. [CrossRef]

10. Lee, S.H. Chiral symmetry breaking and the masses of hadrons: A review. *Symmetry* **2023**, *15*, 799. [CrossRef]

11. Rojas, J.C.; Demircik, T.; Järvinen, M. Popcorn transittions and approach to conformality in homogeneous holographic nuclear matter. *Symmetry* **2023**, *15*, 331. [CrossRef]